Computational Fluid Dynamics

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Module - 6
Lecture - 59
Issues in Practical Problems, Turbulence Flows

We have come to the last week of this 12 week course, we have done 11 weeks of CFD and in the process we have learnt how to solve numerically the exact equations which govern the motion of a Newtonian fluid. We have seen that while we are making approximations at several stages during the evaluation we have a quiet confidence that we can solve these equations faithfully and expect to get solution which would match with the exact solution pretty closely. We also have seen in the last two weeks lectures how we can deal with irregular geometry using finite volume methods. So, all this gives us a quiet confidence in terms of the capability of CFD.

This is very well and good that, when it comes to practical flow problems we have number of other phenomena which erode this confidence in many cases we would not be able to solve the exact equations because the equations cannot be not enough knowledge is known not enough is known about this phenomena or not enough is known to come up with mathematically accurate models for these phenomena. We may not be able to solve those mathematical models which are best for the phenomena in the context of a CFD solution framework.

We note that in CFD we are able to solve the scalar transport equation very well, but if we have some integral partial differential equations which are for example, govern the radiative transfer in a participative medium then that kind of mathematical model is not readily amenable to CFD type of solution. There may be some other mathematical representations like Markov methods, probabilistic methods, which again may not be readily solved easily in the context of a CFD type of approach. So, there are definitely mathematical models which do not lend themselves to CFD type of solution and if we have phenomena which occur of that type then tackling them in a CFD context is going to be difficult and very often we have to make close approximations.

One such difficulty with regard to practical problems where we have to make a significantly simplifying assumption is the case of turbulent flow. Turbulent flow is known to everybody, I think you all have heard about what turbulence is and you would have seen many movies talking about air turbulence and the way that passengers fell in an aircraft as the aircraft hits a pocket of turbulence.

So, that kind of familiarity is very much there is our conscience. We also have the basic idea of turbulent chaotic life is one such again a connotation which is pretty accurately descriptive of the turbulent flows that very often happen in many industrial natural conditions. So, we can say from a fluid mechanics point of view turbulence is a something that is quite common.

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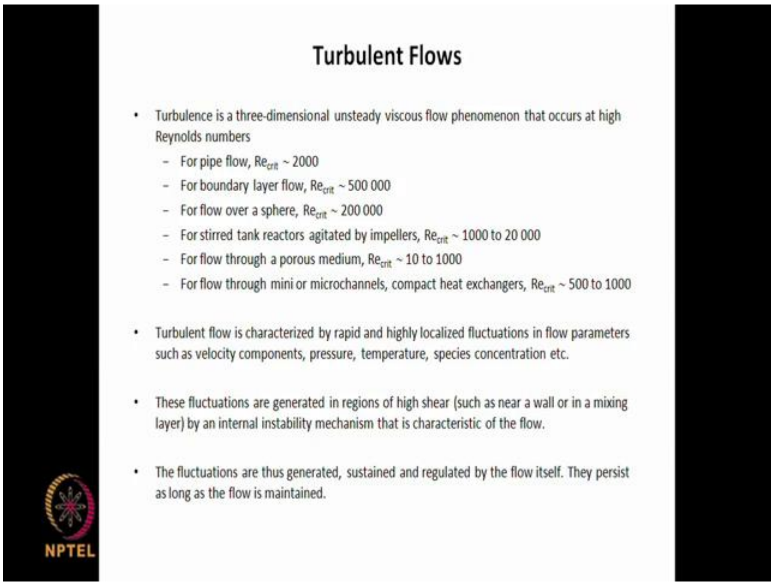

And we see that every day as water flows through the pipelines and as it comes through the faucets into buckets or whatever, as we take shower we have turbulent flow of water falling on us. So, it is quite common and at the same time it is also quite complex for us to treat mathematically we are going to look at that particular aspect in this last week’s module starting with some idea of what turbulent flow is and why we have to take so much care about it at this stage in the CFD solution.

So, turbulence can be taken to be a three dimensional unsteady viscous flow phenomenon that occurs at high Reynolds numbers. We all know about Reynolds numbers and we all know about viscous flow which is the default type of flow that would happen in a whenever we have fluid, except in the case of inviscid flows which are not possible in real life conditions we have viscous flow and when the velocity of the fluid is high then there is a possibility of the flow becoming turbulent. This transition from laminar to turbulent can be quite marked; abrupt in certain cases it may in other cases, it may be quite slow transition gradual transition from a state of being laminar flow to a state of being fully blown turbulent flow.

We know from our basic fluid mechanics essence that if we define the Reynolds number as a characteristic velocity terms characteristic length divided by the kinematic viscosity of the fluid then for pipe flow our Reynolds number, critical Reynolds number at which the transition takes place is that out of 2000. Maybe slightly typically more than slightly more than 2000 and definitely less than 3000-4000, one can expect by a few thousand of Reynolds numbers the flow to have become fully the flow to have transitioned into 100 percent turbulent flow not intermittently turbulent flow.

For pipe flow we definite Reynolds numbers as the average velocity times the inner diameter of the pipe divided by the kinematic viscosity of the fluid if we consider the other simple case of boundary layer flow over a flat plate then you define Reynolds numbers based on the free stream velocity that is the approach velocity of the fluid onto the plate and the length is defined as the distance from where the plate begins, where the flow of the plate begins. So, as the fluid is flowing over this plate the Reynolds numbers continuously increases because the distance from the beginning of the plate increases. At some Reynolds number typically of the order of half a million to 1 million there is a transition to turbulent flow. You have the characteristic features of turbulent flow appear and they become firmly established by about a million Reynolds numbers of about a million.

If you take the another simple case of flow over a sphere again it is a well characterized flow, in the sense that you have a sphere in an infinite fluid medium, sphere has a characteristic length scale which is the diameter. Again we are looking at the upward velocity or the free stream velocity of the fluids as the characteristic in velocity. So, u infinity times d by nu is the Reynolds number in this particular case and here the transition to turbulence happens at a Reynolds number of 200,000.

So, that is pretty high although we do not expect it and people who expect this transition to be more like 50 or 100 or a few hundred. But around the Reynolds number of around 50 to 100 what we have transitioned is from a steady flow into an unsteady flow leading to vortex shedding is the transition that takes place around Reynolds number 50 to 100. So, that vortex shedding type of flow exhibits some characteristic of turbulence, but it is not really turbulence flow one would have to wait for or increase the velocity to a very high value such that Reynolds number goes up to about 100,000, 200,00 for turbulent flow over the sphere to set in.

If you take other kinds of practical equipment like stirred tank reactors where you have a vessel cylindrical vessel and you have an impeller which is rotating at a particular speed. Then we define the Reynolds number based on the impeller tip speed and the diameter of the impeller divided by the viscosity of the fluid. In cases like this it is not like a clear geometrically clean flow, in the sense impeller can be having a quite complicated shape and it is not a 1-D type of thing, it is a 3-D three dimensional (Refer Time: 10:32) coming and the flow does not have the same sustained character as in the case of for example, pipe flow.

So, very close to the impeller you have lot turbulence and far away from this you have little turbulence, but there is still the confining wall of the vessel that is pretty close. So, it is not like flow over a flat plate where the physical dimensions of the fluid domain are infinite and similarly flow over a sphere. Again you have an isolated sphere in an infinite infinity long and wide medium. So, for all these cases it is difficult to precisely say this is Reynolds number beyond which you have transitioned to turbulence. Typically anywhere between 1000 and 20,000 can be a transitional resume and beyond 20,000 also one would expect the flow to have become fully turbulent.

If you look at flow through porous medium, one would not imagine porous medium and the high pressure drops and all those things we need to turbulent flow, but if you have a

Reynolds number of less than 10 one would say that it is an laminar flow and Reynolds number greater than 1000 is probably what is required for the flow to have become turbulent. So, the Reynolds number in this particular case includes the porosity as a variable in the Reynolds number definition.

On the other hand as we go to very tiny channels, micro channels or compact heat exchanges with very small length scales for example, the gap between plates of a compact heat exchanger can be out of 1 or 2 millimeters. In such cases even though these are like flow through ducts and pipes you have a transition to turbulence flow which happens at much lower Reynolds numbers of the order of 500 to 100. Here one should also keep in mind that in many of the cases here when we talk about transition to turbulence flow, especially in the cases of pipe flow or in the channel flow type of situations we are looking at the Reynolds number at which the linearity variation, linear variation between the pressure gradient and characteristic velocity that stops being linear is considered as Reynolds number at which turbulent flow transition in to turbulence flow begins.

So, in that sense it is not based on the usual characteristics that are exhibited by turbulent flow as we will see. So, when you say that the Reynolds number at which the velocity pressure gradient relation becomes stop becomes linear is the Reynolds number at which the turbulence flow is supposed to have happened is a not such a clear definition of what turbulent flow is. Turbulent flow in the more rigorous sense is characterized by rapid and highly localized turbulent localized fluctuations in flow parameters such as velocity components, pressure temperature, species concentration all the usual flow parameters exhibit rapid and highly localized turbulent fluctuations.

There is a characteristic signature of turbulent fluctuation that is present, when that characteristic signature is there then we say, yes this is a turbulent flow; if that is not there the fact that there is velocity fluctuation or the drag fluctuation does not make it turbulent flow. The case in point is the flow over a sphere. At a Reynolds number of 100 for flow over a sphere you begin to have vortex shedding which induces certain variation, time depend variation of velocities at different points within the flow close to the body. But still the flow is not turbulent it becomes turbulent and it exhibits those turbulence like signature fluctuations only at a much higher Reynolds number of the order of 200,000.

So, that is a very important distinction between an unsteady flow and turbulent flow turbulent flow is characterized by rapid and highly localized fluctuations. These fluctuations are generated in regions of high shear where you have strong velocity gradients are usually the locations where these are created, such as near a wall or in a mixing layer where again you have two fluids of different velocities are brought together. In these regions of high shear there is an internal inherent instability mechanism that arises from these high velocity gradients. That leads to the said characteristics of turbulent flow which is rapid fluctuations, and highly localized fluctuations and these fluctuations are therefore, generated by the flow itself and these are sustained by the flow and although these are instabilities they never grow beyond a certain limit and in that sense they are regulated by the flow itself.

So, that instability does not lead to breakdown as the flow, just like when we say we have an unstable bridge then we expect those kind of images where the bridge starts oscillating very widely and then at some point breaks down. So, that kind of breakdown does not happen in this. Just that it exhibits very rapid fluctuations and these fluctuations will persist as long as the flow is maintained that is if you have a flow through a pipe going at a Reynolds number of say 100,000 then, it you are able to measure the velocity fluctuations at a particular point then those fluctuations never really die down. They persist as long as Reynolds number is maintained at that 100,000. These are not externally generated; these are generated by the flow inside the pipe.

So, you do not need an external impulse for these to be generated. The only thing that needs to be given, so as to maintain this fluctuations is the driving force that needs to be maintained in this particular case the pressure gradient imposed from the flow is to be maintained. So, that the flow is sustained at the Reynolds number 100,000.

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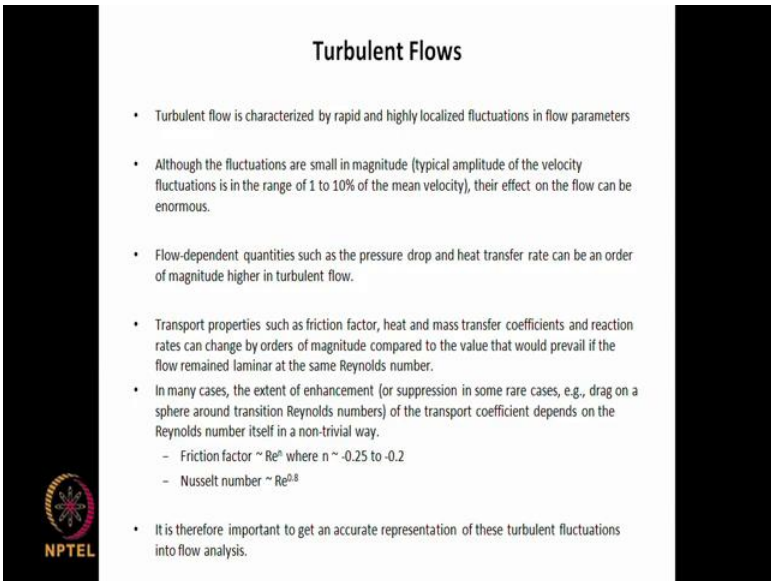

So, there are some special characteristics of turbulent flow which are definitely very important for us when we are looking at fluid flow in the related phenomena. One such thing is that the fluctuations that we are talking about of associated turbulence are pretty small - for example, typical amplitude of velocity fluctuations is in the range of 1 to 10 percent of the mean velocity. So, if you have a mean velocity of one meter per second typically the under turbulent flow conditions at a particular location it may vary between 0.9 and 1.1. Still very small compared to the overall velocity of one meter per second. That despite this being small you have very important strong effects that are associated with turbulent flow.

If the features that that we look for - for example, features like what is a pressure gradient what is a friction factor, what is our heat transfer coefficient, what is mass transfer coefficient, what is a reaction rate, these are the matters of interest to an engineer. Now these things are strongly affected by this low amplitude fluctuations, these low amplitude very high frequency fluctuations. So, flow dependent quantities such as the pressure drop, heat transfer rate can vary by orders of magnitude. So, we will see that in the next lecture, but in this lecture we will just talk about it and then we will become more (Refer Time: 20:01) in the next lecture.

Transport properties such as friction factor heat and mass transfer coefficients and reaction rates can change by orders of magnitude compared to the value that would prevail if the flow remains laminar at the same Reynolds number. So, if you look at the pressure gradient or the friction factor in flow through a pipe at a Reynolds number 100,000. If you look at what it would have been if the flow is laminar then the ratio of the two would be very high out of 10 or 100 like that, depending on what kind of flow and at what Reynolds number you are talking about.

In many cases the enhancement or suppression in some rare cases for example, the drag on a sphere around transition number is something that decreases as you go from laminar flow to turbulent flow that is a rare case and we know why that is so. But typically otherwise we have an enhancement of the transport rates, transport coefficients, and this enhancement depends on the Reynolds number in a non trivial way. It is not like a multiplicative factor of 10 or 50 or 100 that multiplicative factor value itself depends on the Reynolds number, it depends in a non trivial way. For example, friction factor in laminar flow is proportion to 1 by Re, that means, it is proportion to Re to the power n where n is minus 1 in turbulent flow that proportional the power factor n is point minus 0.25 minus 0.2 it is much less, that means, that the friction factor is going to be much higher in turbulent flow.

Similarly, Nusselt number which represents the heat transfer coefficient in non dimensional way through a pipe, in laminar flow it is a constant value maybe 3.65 or 4.33 for a two cases of constant wall temperature and constant heat flux, it is only of the order of 3 to 4. But in turbulent flow it is proportional to Reynolds number to the power 0.8 that means that if you have a Reynolds number of for the order of 10 to the power 5. Then the nusselt number can be of the order of 20, so that gives an enhancement of the factor of 50. So, the nusselt number the heat transfer coefficient is 50 times more than what you would have estimated if the flow were laminar or if you neglected the effect of the fact of turbulent flow.

What does that mean? That means, in a typical heat transfer problem you would like to estimate what is the heat transfer surface area, the surface are if you have a flow through a pipe then it translates itself into the length of the pipe that is required. So, if you make a

50 fold increase in heat transfer coefficient then the length of the pipe will be reduced by factor 50. It may not quite happen that way because in a typical heat exchanger the heat transfer coefficient on the inside and outside and through the pipe wall all these things come into picture. But what it means is that your heat transfer equipment the heat exchanger can be too large for the (Refer Time: 23:53) that it is supposed to be.

Similarly, if you under estimate the friction factor assuming it to be laminar flow then the pump that you are required to maintain flow though a particular pipe at a particular Reynolds number can be small by an order of magnitude. You probably would have required a 10 horsepower pump and you brought only one horsepower pump because that is what you have estimated. So, it can be like that. So, you can grossly over design or under design your heat transfer or fluid flow equipment if you neglect the fact of turbulent flow. So, this is including the effect of turbulent flow is very important in our design calculations.

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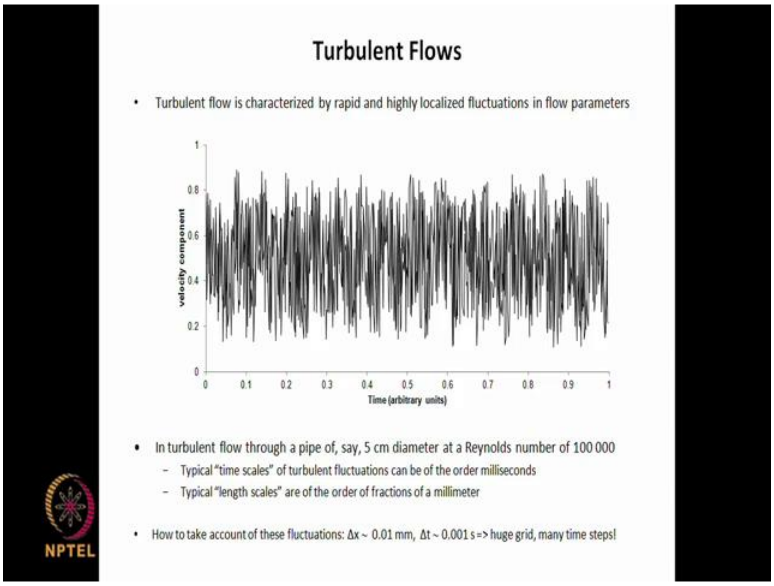

Now, there is a problem with that. The problem is shown in this figure, it is a typical variation. We mentioned that turbulent flow is characterized by rapid and highly localized fluctuations in flow parameters and the squiggly variation that we are seeing here where you have this, it is a schematic you have velocity components which has an average value of 0.5, but it is not 0.5 it is changing wildly like this very rapidly. On the time scale on the horizontal axis you have time and although it is on the average steady it is changing very rapidly and something like this is the characteristic feature. If we were to neglect these fluctuations and then just take the average value to be 0.5 and keep it that way in our calculations then they going to closely under predict or over predict or heat transfer coefficients and pressure drops.

Now, if you want to take account these fluctuations you have a different problem. For example, in a case of flow through a pipe of diameter of say 5 centimeters and at a Reynolds number of 100,000 which not a typical. You have a velocity of 2 meters per second of water. So, water flowing through a pipe of 2 meters per second in a pipe of 5 center diameter will have a Reynolds number 100,000.

For such kind of flow the time axis here which is given as 0 to 1 here is such that the velocity fluctuations, we talk in terms of times scales such fluctuations. So, that is how rapidly it is changing if you say like this as (Refer Time: 26:54) variation. What is the time period of oscillation? So, that can be of the order of a tens of milliseconds; and the variations are not only with respect to time they are also with respect to space so that means, that velocity at this point and the velocity at this point are going to be not the same, they are not going to be related in the same way. For example, if you have a velocity gradient like this then this may be 2 meter per second this may be 2.1 meter per second, but in turbulent flow this will not remains 2 meters, this 1 to will not remains 2.1 meters both are fluctuating rapidly like this.

So, even if you bring it down to very close points then again they can they can be fluctuating. So, the length scale over which you can see significant differences in velocities because of fluctuation are the length scales typical length scales and these can be fraction for millimeters. So, if you want to resolve this variations over fractions of millimeters and tens of milliseconds you need to have a delta x of the order of say 0.01 millimeter and delta t of the other of 0.001 second.

That means, that if you want to represent your 5 centimeter diameter with the delta x of 0.01 you can imagine you will have 50 millimeters 5000 grid points only in the radial directions or 2500 grid points in the radial direction and that makes it a very huge grid. you will have millions and millions of grid points and you will have to run time dependent calculations over 1000s and 1000s of time steps. So, that will be humongous expensive to do computational. So, this is the difficulty with the turbulent flows and how we resolve this and how we overcome it through some modeling is what we are going to discuss in the next 3-4 lectures.

Thank you.