Computational Fluid Dynamics
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Lecture – 57
Triangulation: The Advancing Front Method

We have seen the Nodalization in the last lecture and in this we look at one method for triangulation. We mentioned that triangulation is not a trivial thing; this method of advancement method advancing front method has been developed in date seventies.

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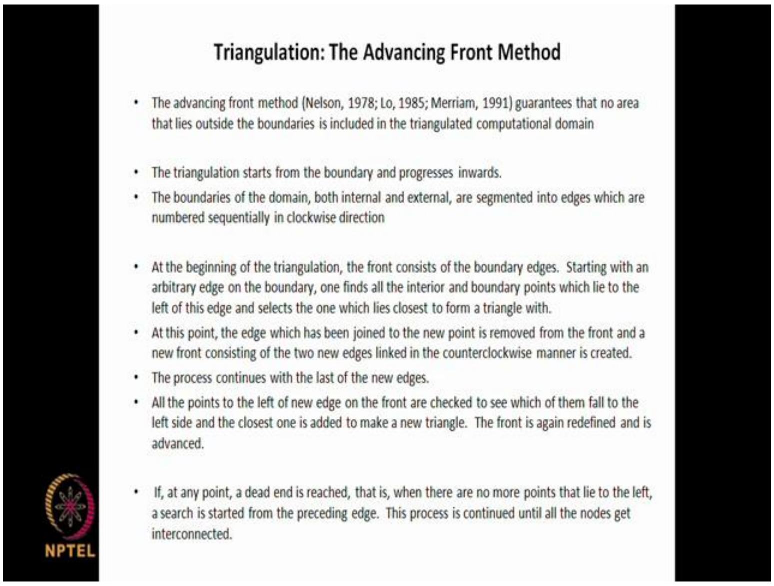

It has one attractive feature that it guarantees that, no new area that lies outside the boundaries is included in the triangular duct computation domain. So, this has a guaranty one of the conditions that we have said was that there should not be any new area that lies outside the computational domain to be included and this algorithm guarantees that. The triangulation starts from the boundary and progresses inwards so that we do not have this problem of including external boundary and the boundaries of the domain both internal, external are segmented into edges which are numbered sequentially in clockwise direction.

So, you start with the boundary with one boundary point and then you move along the segments; boundary segments which you have already identified along the nodes and put them in a sequential order while going through in a clockwise direction, for all internal points, internal domains we go in the anticlockwise direction. At the beginning of the triangulation, the front consists of boundary edges starting with an arbitrary edge on the boundary, one finds all the interior and boundary points which lie to the left of this edge and selects the one which lies closest to form a triangle with.

At this point the edge which has been joined to the new point is removed from the front and a new front consisting of the two new edges, linked in the counter clockwise manner is created. The process continues with the last of the new edges, all the points to the left of new edge on the front are checked to see which of them fall on the left side and the closest one is again taken to make a new triangle. So, the front is again redefined and then we continue like this.

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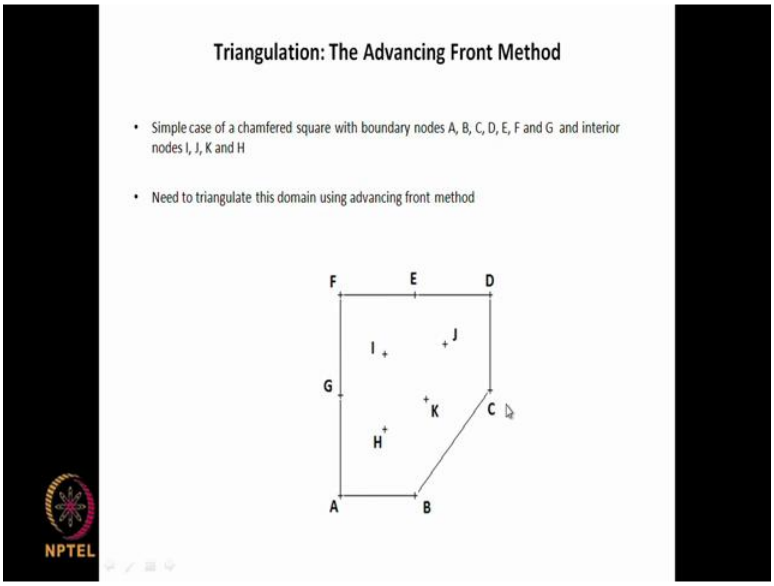

So, let us see what it is; by taking a very simple example, you have here a chamfered square and it has been the boundaries of this have been segmented with these linear segments here a, b, c, d, e, f, g and then you have four internal nodes h, i, j, k. It is just to illustrate the process rather than to generate a grid which is very good for this, so if you have this, then how does the advancing front method work to triangulate this particular domain which consists of this chamfered edge. So, we do not want any joining of k with b, c in a way that it goes outside so there is a possibility; for example, b, h, c and b, h, k to be triangulated. So, we would like to see whether how that kind of thing is avoided in this.

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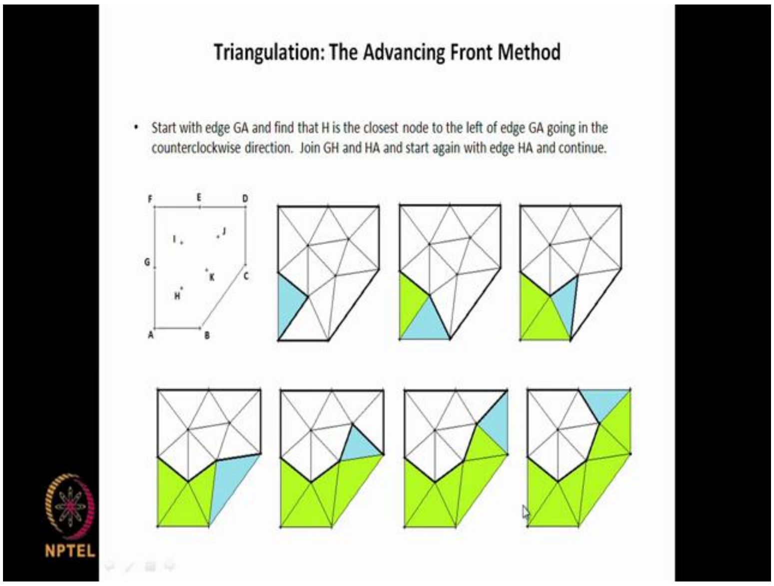

So, we start with edge g a, so we started with a b, b c, c d, e f, g h, f g, g a like this and so we start with this edge and then you find all the boundary and interior nodes which lie to the left of this. So, when you are looking at edge g h; this is to the left of this, this is to the left of this. We are going in counter clockwise direction, so we are going in this direction so this edge, this edge; in fact, all these edges here lie to the left of this.

Now, you say which of this nodes lies closest to g a; in such a way that we can make a triangle. So, how do we do that for example, we take a rope or a thread and then we say what is the length of the thread which joins, which goes up to h and then gets back to a, what is the thing if we where to wrap it round this edge g to i and i to a, g to j and j to a like that.

So, the one which has the least length is the one which is the closest; for example, we can say what is the distance here and what is the distance here, what is the total length; distance plus this distance total length. So, given two points we can find the distance and from that we can find out what would be the total length, if we were to make a connection like this and the smallest of that will be the point with which we should make a triangle.

So, using that kind of criterion we can identify h to be the one which is closest and we join this and then we have a triangle that is formed. So, at any point we have the front which is advancing towards those unconnected, as it unconnected points. So, when we start out we have a front which consists of a b, b c, c d, d e, e f, f g, g a that is a front which is propagating inwards all the time. So, once we have identified this h here, then our front now is ab, bc all this and from here, here and then comes back to this.

Now you start at this point here and you see. What are the nodes which are to the left of this line? So, this is one and if you prolong it like this, you can see that this is also to the left of this, this is also to the left of it and all these others are to the right side of this; of this edge as we go from h towards a. So, out of these three points; which one is the one which is closest to this edge again you can see that this is the one which makes the smallest triangle. So, smallest in this sense of this combined length here and therefore, we join this and then we create this new triangle.

So, the new triangle that is created at each stage is shown in blue colour here and the already create triangles are shown in green colour here and the advancing front is always shown in dark black line here. So, now we are at this point so this is a last edge that we were using. So, again here we see which are remaining nodes, boundary nodes which are to the left of this and we see that this one, this one, this, this, this, this, this, this one, this, this are all available and we find that this is the one which is closest.

So, this time we join it like this and we have a new front which is going now like this, one new triangle created and two old triangles and now we start with this and then we find that this is the one which is closest. So, we create this edge, we join this thing here and then we have a new triangle, three old triangles and now here what are the points which are to the left of this, this one, this one, this, these points and this point.

Out of this, this is the one which is closest, so we create this triangle here and this is our advancing front like this. So, now we are here and here we look at all the nodes and we find that this is the one which is closest to this, we create this triangle and then we have this and then at this point; we find that this is a new triangle, this is the one which is closest to this and then you create like this. So, we have a shrinking, advancing front here and increasing number of triangulated triangles and the latest triangle which is created.

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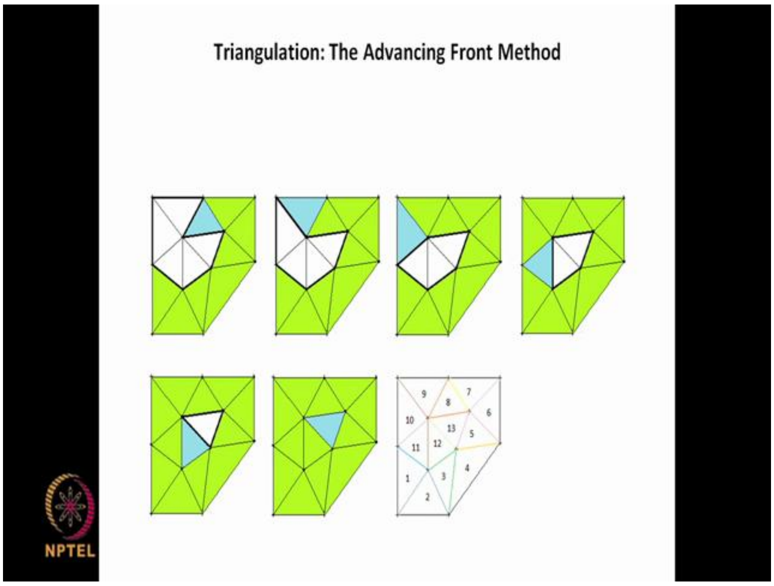

Now, we were here and this is advancing front and we are starting from here; the latest edge to be created and we find that this is the one which is closest to this, so we join this to make the new triangle. Now at this point, the front consist of coming to here like this, like this, like this, like this, like this, like this, and like this. So, this is the latest edge and we join this now to make this and then you have created this edge here and we join this here to create this and then we join this one; to create this and then this and then this.

So, we have created triangles making use of all the nodes because we start with one edge here and then we complete all the nodes here and this is the order in which we have created those triangles. So, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 this is a very robust method that will be a that is used especially for the desirable feature that you are always looking at points which are inward so; that means, that points which lie outside are not linked to form triangles, and that is one very good feature of this. It has some one distinct disadvantage that this method in this form does not consider.

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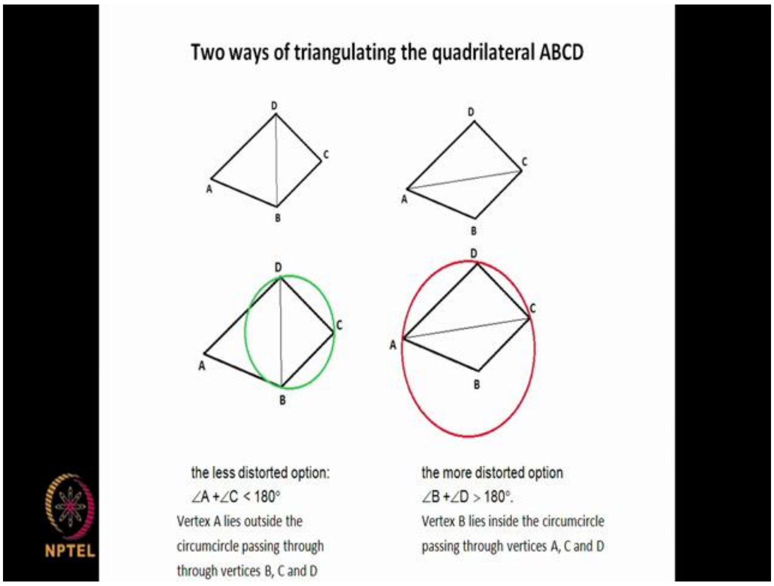

That it if you take a a quadrilateral a, b, c, d and you want to make triangles out of this, you can think of a two different ways of triangle triangulating it.

For example; you can join b d and make two triangles, or you can join a c and make two triangles. So, the same 4 points this can be divide into two triangles like this or like this. Which of these is more desirable? The one which has less distortion, the one which has all the 3 sides more closely linked to each other, the three sides which are roughly of the same sizes or with the least aspect ratio. Aspect ratio we can say is the maximum side length divided by the minimum side length.

So, if you define it like that, distorted triangle will have larger accept ratio compare to the one which has which is less distorted. So, then we can see that now there are two possibilities; one like this and another like this. So, which of them is more desirable is that for example, if you take if you take this 3 points and you draw a circle which passes through all this points, and if the fourth point is lying outside then, this is good because this is producing 2 triangles here and this is almost more equilateral than this, you have a long length here and a short length here.

This one has these particular combinations results; in a high aspect ratio triangle rather than this one. In this, even these three triangle these 3 sides have much an accept ratio which lies close to one here, for this triangle and for this triangle where as if you make it like this then you have this length which is much longer than this length.

So, you have higher aspect ratio. So, this is not desirable and this is desirable, one methodical way of finding this is that, in this desirable way you have angle a plus angle c is less than 180 degrees, and if we join it like this angle b and d, which are the other vertices which are not joined together, now here these are greater than 180 degrees and one other criterion is that if you make a circumcircle, passing through these 3 points then, the forth vertex which is not part of this joined diagonal here lies outside, and if that is a case then you are not good distribution of triangle. Whereas, in this particular case you draw the circum scribing circle like this and then you find that this is inside. So, this means that this is not a proper way of joining the triangles.

So, a given quadrilateral can be converted into 2 triangles, by joining either the diagonal b, d or the diagonal a, c. Which of them is lies gives us triangles with smaller aspect ratio more of more uniform size is a question that needs to be asked, and it is prefer to have a triangles with smaller accept ratio. So, this particular check does not come into picture n making this in the advancing front method in the triangulation.

So, this is a disadvantage, i think any correction of the triangulation is to be done after you have triangulated. So, after you have triangulated you can look at each triangle and see what is the aspect ratio, and see take a adjacent triangles, and see if there is a better way of doing this triangulation by doing this circumcircle option, and see whether some correction can be made after triangulation.

But there is a different method of triangulation, which is known as the Delaunay triangulation or the Boyer algorithm which incorporates this checking for the good triangulation of a quadrilateral concept in the way it generates the triangles. In the next lecture we look at what that method is and how it works, and will look at a general description and we will come back to this and then see how it works in this particular case.

Thank you.