Computational Fluid Dynamics

Dr Sreenivas Jayanthi
Department of Computer Science and Engineering
Indian Institute of Technology, Madras

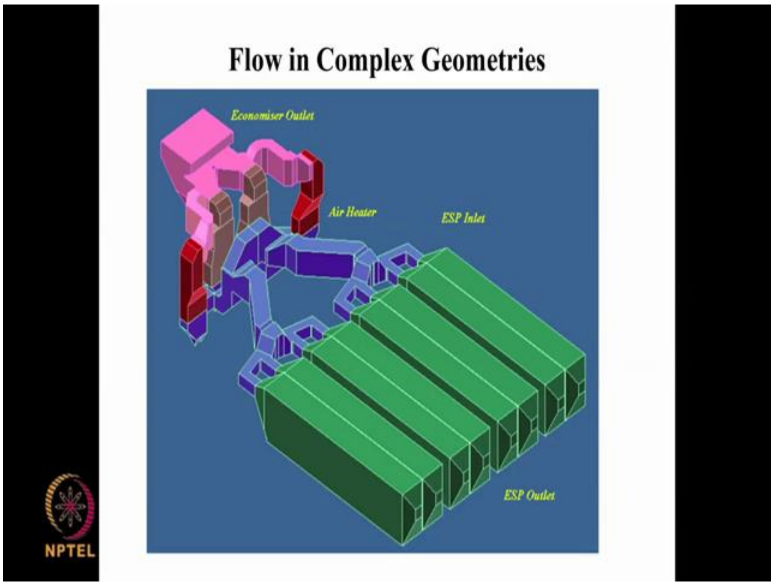
Module – 6
Lecture – 54

Issues in Practical problems, Part A:

Finite Volume Method for Irregular Geometry

I am ready to start. We have come today to the last module what we have done in the previous 5 modules is, take a look at the general CFD approach for the solution of Navier stokes equations and other related equations. In flows of very simple geometry's most practical problems are much more difficult than that. In the last module we are going to look at some issues in practical problems, and specifically we will be looking at two issues; in part a, we will be looking at how to deal with irregular geometry and in part b we will be looking at how to deal with turbulent flows. Turbulent flows are quite common and Laminar flows are much more rare and we have to have a different formulation for turbulence to account for this very strong effect of turbulence on flow and flow related phenomena like; heat transfer, and mass transfer reaction rate and how we tackle that is what we are going to discuss in the second part of this particular module, but in the first module we will be looking at how to deal with irregular geometry or complex geometry that cannot be fit in Standard Cartesian Coordinate System.

(Refer Slide Time: 01:39)

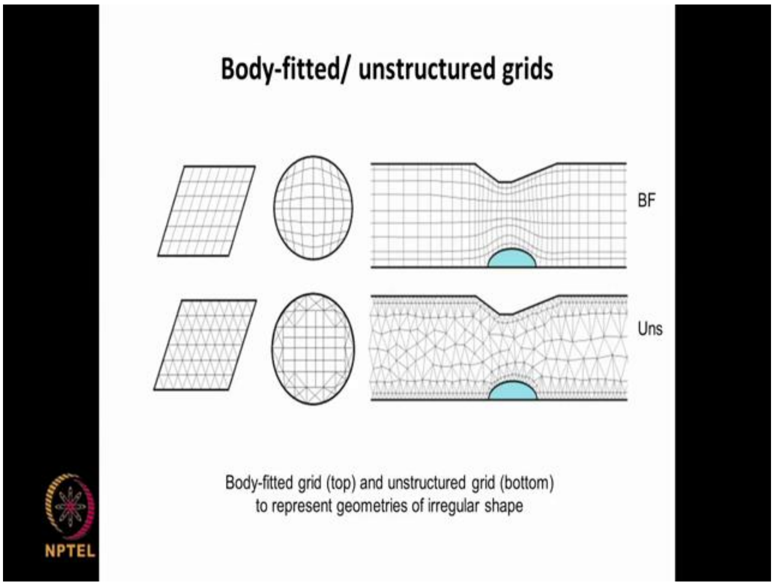


We have seen right in the beginning an example of a typical industrial geometry, and this is the case of flow coming out of a furnace and it comes and it goes through lots of heat exchangers and then finally, through some special equipment like electrostatic precipitators, which will capture the particulates in it and then it will eventually go into a stack. In a typical power plant location of these things is dictated not by fluid mechanic (Refer Time: 02:14), but by how to optimize the usage of the available space within the power plant. So, we have very irregular flow, shape flows ducting like this and all of this is typically very large. So, that this civil infrastructure to support these and then there will be other things to make sure that safety is assured for the personal working in this, and we should also have spaces for people to work in.

All those kind of tactical issues are there and those issues will actually govern the layout of the fluid flow duct and we as engineers; fluid flow engineers will have to take account of those and design our fluid flow equipment to do the required performance. To deliver the performance that is required of this. So, in such case we cannot deal with simple Cartesian coordinates to describe for example, how the flow is going through this and then how it splits into here and how it splits goes into this and how it goes and exchanges heat in this heat exchanger which is typically fairly complicated and shape and operation and then it comes back merges and then splits comes here, splits into 2, and then it goes into this further kinds of splits. So, all these things are fluid mechanics things and we have to make this happen we have to design all this ducting in such a way that this happens. So, when we are deal with that we need to know what kind of velocity profile, what kind of pressure drops and what kind of pressure drops should be induced what kind of flow resistance is to are to be created, inside this passages in such a way that the required amount of flow will be going through this track and the some of the pack will be going through this. All those issues we have to tackle and in order to do that we need to be able to get a flow feel, as the flow takes goes through this kind of thing. So, typically these kind of irregular geometries are commonly encountered in industrial practice.

We have two types of approaches for dealing with this flow through irregular geometries and these are.

(Refer Slide Time: 04:43)

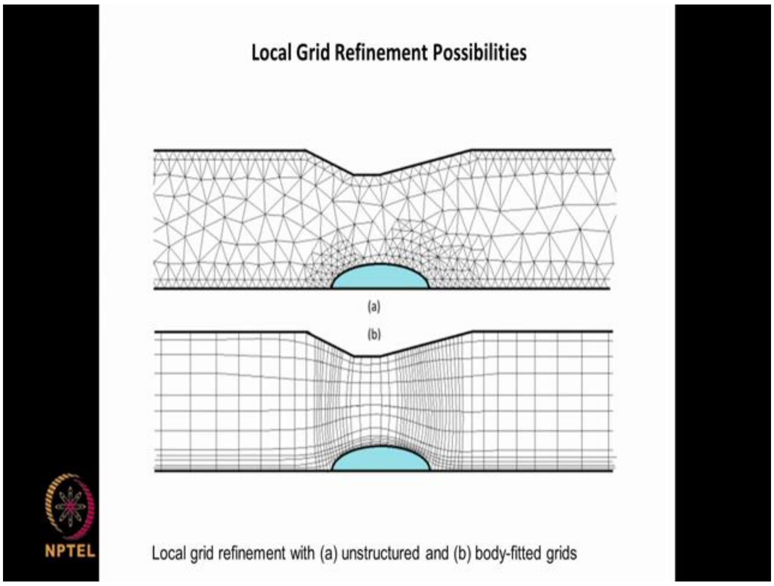


We can clearly see that in a case where for example, in the first case which is shown here we have a quadrilateral geometry which is not like rectangle it is rectangle we can put an x y coordinate system, Cartesian coordinate system, but here if you want to cut this into tiles, you cut them in this parallel way and not like that step kind of thing that may we may get with using Cartesian coordinates here. Then in a case like this we have to deal we have to abandon trying to represent the geometry in Simple Cartesian Coordinate

System and we have to have some specialized way of representing the geometrical boundaries these four walls along lines of constant coordinate plains. So, for that we use what is known as body fitted grid, where for example, the grid lines curve in order to fit the available shape this is this here, these lines are parallel horizontal, but these lines are inclined to fit with this and here we have these grid lines curving like this in order to account for this curvature here. These are curving like this and these are curving like this, And here we have a duct with a throat section and the obstruction here again the grid lines here are like in Cartesian Coordinate System, but as we coming to this here the bending the bending. So, as to accommodate the shape, and this particular boundary actually this boundary line takes the shape of the boundary here. So, if you say that this is m j line m j line is actually one of the boundaries with grid points here.

And similarly here we have this one going through like this, and then taking a bend and then comes out like this. This is known as a body fitted grid and this is a still structured grid in the sense that the grid nodes here are located along intersection of constant horizontal lines and constant vertical lines here and here is a along constant coordinate lines which are curvy linear, which are not linear, but which will be taking shape of the of curves. So, this is one particular approach to dealing with complex goemetry that may be there another approach is not to have this structured grids where the grid points are along lines constant intersection of this constant grid lines, but to have this kind of triangular elements of different shapes which, are made in such a way as to have the boundary line in one of the segments of the in this particular case a triangular cell. So, even something like this can be made into segmented linear segments each of which forms a base of one triangle; triangular element and then you can create a network like this. Here we have close to this curved surface here we have triangular elements as you come into the interior you have rectangular elements. So, all these are unstructured grid where the points where you evaluate the variables are not at the intersection of lines constant coordinate lines at points of constant intersection of these lines. So, these two are two different approaches. They have their advantages and they have their disadvantages.

(Refer Slide Time: 08:45)



So, one of the disadvantages with the body fitted grid, is when you look at the local grid refinement possibilities. For example, we have this as grid for this we have done a flow computation and we found that flow around this is of interest to you and you would like to resolve this well and when you say you want to have resolved well resolved flow we are looking at accuracy in computation and we know that accuracy depends on the grid spacing. So, we would like to have fine grid spacing small grid spacing here and as we go away from the wall and the domain of interest may be you can have large grid space. So, you are looking at possibility of local grid refinement and if you want to do that you can make smaller cells here, and there is smaller grid here smaller cells here that line here that since the grid notes in a body fitted structured grid are always at intersection of this coordinate lines. If you want to fit if you want to make small cells here so; that means, that the spacing between the coordinate lines here must be small. This has to be taken all the way out to this there is a small mistake here it has to be all the way up to this. So, even though you are getting a small grid cells here you are also getting small grid cells here at least in one direction. Similarly here you have small cells here, and here you do not have small cells. So, it can reduce the number of grid points that are used to describe this flow geometry.

Whereas, here you do not have that actually this thing is going all the way to this. So, in that sense local grid, grid refinement is done more efficiently with an unstructured grid more efficiently in the sense that we use fewer numbers of points to refine a local grid. Whereas, that kind of possibility is not there with largely it is not there with body fitted grid.

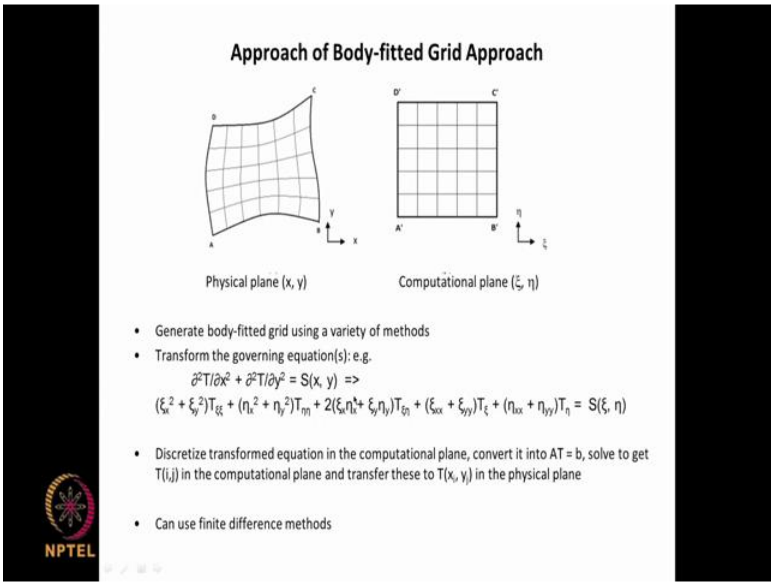
There is also an advantage for the body fitted grids in the sense that it is very much like our finite difference method, and you we have already seen the finite difference method finite volume method in the very first module when we dealing with flow in a through a rectangular duct and flow through a triangular duct. We saw that doing a problem with finite difference method was fairly easy compared to doing it with the finite volume method where we had to do all kinds of cells and then the orientations and the fluxes all that difficulty had to be done. So, there is each a programming that is possible with body fitted grid, and you can make use of the simple much simpler finite difference methods with this and the resulting equations like a psi equal to b will have diagonal structure that is expected in this and we have seen a number of efficient methods which can be used for the solution of a x equal to b type of equations with diagnostic structure all those things can be applied here, but only some of them can be applied for a unstructured grid here.

So, they these are some kind of advantages and another advantage with body fitted grid is that when you have flow going in essentially in the parallel to the wall like here, or here well away from this. Then changes are taking place only in the x direction and y direction there is only for example, velocity profile is changing. Ideally you do not need to have a very fine grid in the x direction because there is hardly any change between this point and this point.

So, you can have elongated cells in the body fitted grid here. Whereas, here it is not possible because you have a three dimensional shape here, it is not possible to have highly elongated triangles to represent this. So, when you have essentially 1-D flow or essentially 2-D flow you can make better use of the elongation capabilities in the body fitted grid rather than in the unstructured grid. If you make highly skewed triangle, then you are going to have introduced lot of numerical waves. So, that is one of the disadvantages of the unstructured grid combination. So, there are certain reasons for it, but there are so, many practical advantages grid generation is much simpler with this as we will see than with this which makes body fitted the unstructured grid as the standard default approach to be used for close in irregular geometries.

So, before we look at the finite volume method or formulation of the finite volume method for unstructured measures. We would like to just take a look at how we would solve the fluid flow equations in the case of a body fitted grid because they there are some changes that are required.

(Refer Slide Time: 14:09)



And that we will just discuss briefly, without trying to fully understand it. So, the approach of a body fitted grid this repetition of the approach here is that you have a physical plane in which you have this kind of odd shaped computation domain. It is not rectangular you have this curved side walls like this. You would like your grid here to respond to these things. So, that it curves here like this and is curving here like this and here it is curving in the opposite way. So, these are curvy liner coordinate lines which are making this into a number of rectangular tiles, not rectangular tiles. But quadrilateral tiles and the intersection of lines of this line and this line will locate this particular point.

So, we distinguish between a physical plane which is actual real geometry that we see and a computational plane. In a real in the physical plane we have a complex shape that is associated with actual geometry, this is transformed into in the computational plane into a simple rectangle here, and in this rectangle we choose to have this line which is curvy linear in here to be just vertical and this line which is curving up like this and then curving changing direction here is actually just a straight line like this.

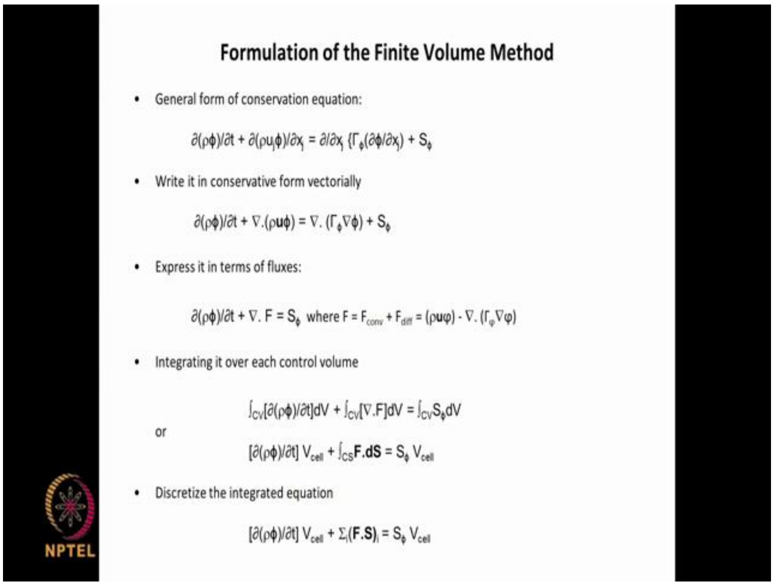
So, the idea is to transform this grid lines from these horizontal and vertical lines in a computational plane into this curvy linear lines which will describe the shape of the domain properly. So, this transformation is part of the grid generation and that is done using a number of methods. So, we do not have time definitely in this course to go into that. But we have that, kinds of grid generation. So, we generate a body fitted grid using a number of methods that are available to do. So, that we convert the problem for example, of temperature given by this 2-dimensional steady state heat conduction problem where we want to know t of x y, t x y at each of these grid points here. So, for this instead of solving this equation in the physical plane we transform everything into the computational plane, where we have uniform grid delta x and delta y and all the things well and good and you have uniform spacing and constant coordinate lines.

So, we transform this equation into computational plane and then there we do the discretization. So, if the domain in the x y plane is described by in the physical plane by x and y boundaries here. In the computational plane the corresponding x direction is indicated as psi and the y is indicated for example, as theta. So, this is the theta direction and this, the psi direction. This side a b corresponds to a plane b plane and this boundary b c corresponds to b plane c plane and c d here to c plane d plane and d a here to d plane a plane and each of this interior curve lines are like this. We would like to solve everything here we would like to write finite difference approximations here because these are very easy things to do because you have constant grid spacing whereas, here you have non uniform grid spacing and if you want to represent the gradients at the boundaries then we have difficulty here because these lines are not straight. So, in that sense we would like to work in the computational domain where, everything is very neat and tidy. So, for that we have to express now this t of x y in terms of t of psi theta. So, we have to go through the rules of transformation of derivatives and all that can be done can be done by everybody, but it takes time to do it. So, this equation which has only dou square t by dou x square plus dou square t by dou y square equal to s gets transformed into this equation here, where this subscript indicates derivative. So, this is t side square means dou square t by dou x i square and this is dou square t by dou theta square this is dou square t by dou x i dou theta and this is dou t by dou x i and this is dou t by dou u i and here psi x square is dou psi by dou x whole square dou x i by dou y whole square and similarly e cross here here dou psi by dou x and dou theta by dou x and all these things can be derived these are plain rules of transformation and but what we say is that.

So, instead of solving this simple equation in this complex geometry, we choose to solve this rather more difficult equation in this simple geometry. So, here we have dou square t by dou x i square. So, we can use something differencing for this and here you have i comma j and this i comma j in the computational plane corresponds to the similar i comma j in this. So, if you want to know the value here then you find out, out here and how do you find it to you write finite difference approximations for this derivative this derivative this derivative and convert all of this into an equation like a t equal to b that will have some diagonal structure you can use Gauss-Seidel method or whatever method you would like. And then solve this a t equal to b in this domain by discretizing this transformed equation and you get tau t i j in the computational plane and you know for every point here there is a corresponding point in physical physical plane, which you know a priori because you chose to have this body fitted grid you decide on how to do this. So, you know the variable value at t at this point this point this point to this point.

So, you get t i j not by solving the original equation you get t i j by solving the transformed equation in the computational plane using finite difference grid lines. So, this is the approach of body fitted grid way of dealing with computative geometry, this is developed in in the mid eighties and it became very instantly successful, but over a period of the next 10 years it got slowly replaced by unstructured grid approach in which we make use of the finite volume method for the discretization.

(Refer Slide Time: 21:45)



So, in the rest of the 4 lectures or so, for this module this part of the module we will be looking at how we can solve a similar kind of problem where, we have this kind of irregular geometry not using the body fitted grid approach, but using the unstructured grid approach. In this lecture we will just try to contrast this approach of the body fitted grid with the approach of the finite volume method in the unstructured grid and we will work out the full details in the next lecture. So, this is just to look at formulation of the problem in the finite volume method. We have a general form of the conservation equation like this dou by dou t of rho phi plus dou by dou x j of rho u j phi plus dou by dou x j of gamma phi the (Refer Time: 22:39) of phi dou phi by dou x j plus s phi. So, for different values of psi this can be the mass balance equation, x momentum balance, y momentum balance, p system conservation, energy conservation and so on. we have seen that.

If we want to do the finite volume method first thing is that we write this equation in vectorial form and then we put it. So, this becomes rho rho phi by dou t plus del dot rho u phi where; u is the vector here psi is the scalar and del dot sigma phi gradient of phi plus this and these 2, this is advective flux and this is the diffusive flux. So, we club this 2 in the form of a flux here and write this expression as dou by dou t of rho phi plus del dot f; where, f is a flux which is a convective plus diffusive flux convective flux is rho you phi and diffusive flux is minus del dot gamma phi gradient phi. There is small mistake in notation from phi here and phi here.

Once you have this equation you integrate it over a control volume. So, you write this as integral of control volume over control volume of dou by dou t of rho psi d v plus integral of control volume of del dot f times d v equal to integral of this s y d v. So, we assume that since we have divided the whole control volume into very small cells, within the cell the value of phi is constant. So, we have essentially discretized it and then we say that this term can be written as dou by dou t of rho psi times volume of cell, plus we make use of the Gauss divergence theorem to convert this integral over a closed control volume of the divergence of del dot psi here, into f dot d s and then we can write it like this. This part is discretized evaluated separately over each surface which covers this cell here.

So, this is the discretized form, and now the problem of solving this equation this equation or this equation becomes evaluation of this term and evaluation of the flux so; that means, that how we determine the convective flux and how we determine this and then multiplying taking the top product to the surface of each bounding surface and then you sum over all the phases and then you have source term evaluated and you multiply the volume of the cell. So, we need to know the volume of the cell and we need to know all the surfaces which envelope the cell and we need to know their direction vectors because we are taking a dot product and we need to be able to evaluate the fluxes.

If we do this then we can use this equation to develop an algebraic equation for the value of psi in that particular cell. We do that for all the cells to come up with a set of algebraic equations, and then we can use Gauss-Seidel method for example. So, in the finite volume method, we are using only the governing equation without any transformation we are solving we are doing all these things in the physical plane, there is no separate computational plane whereas, in the body fitted grid approach we are doing all the discretization and all those things in the transformed of the transformed equation in the computational plane, when we are sending back the solution at the grid points of the variable back to the physical plane. So, these are two different approaches, we will not go much more into the body fitted grid approach, but from the next lecture onwards we will be looking at this part and how we can generalize it and how we can evaluate it a general finite volume cell, and then we will also look at how to break up the domain of irregular shape into the cells. So, the grid generation essentially grid generation finite volume methods and we will put together the whole set of formulation that can be used for finite volume method for an irregular shape. We do all these things in 2-D because that is the simplest thing that we can readily understand, 1-D we can do, but it is really not it does not offer much scope for complexity and we cannot illustrate the complexity part of it. So, we will do it in 2-D. So, next lecture we will start with this and then take it further.

Thank you.