Computational Fluid Dynamics
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Lesson - 41
Pressure Correction Method

In the last lecture, we have looked at how to discretize the momentum equation on a staggered grid.

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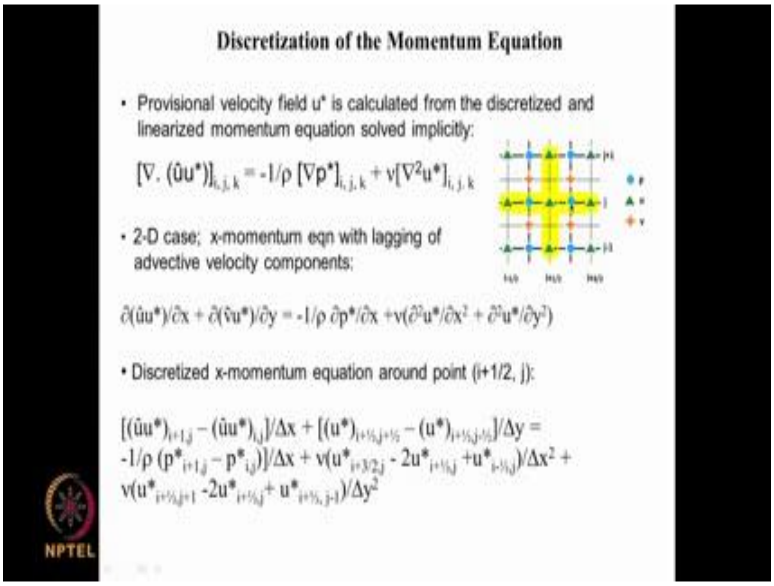

Here, we have staggered grid where pressure is evaluated at the circles at intersections of lines of constant i and constant j. Here it is i, j and here it is i minus 1 j, i j plus 1 and so on. Velocity is evaluated the u velocity component is evaluated at half grid distance displaced in the x-direction which is this green triangles, and that is evaluated at therefore, i plus half and j, and i plus 3 by 2 j, and i minus half j. And vertical velocity component to be is evaluated at the star positions, which are displaced with respect to the position where p is evaluated in the vertical direction by half grid spacing. So, these are evaluated at i j plus half, i j minus half, i minus 1 j minus half, i minus 1 j plus half like this.

On this grid, when we are discretizing the x-momentum equation, this is the momentum equation and we are taking the x-component and that x-velocity at any location i, j, k is denoted here by u star it is a provisional velocity. It is provisional because we are we can solve this only a pressure is known. So, we assume some guest pressure p star, and you substitute in this and by discretizing this you can get and solving it, you can get u star at i, j, k. Now each of this terms is being evaluated i comma j comma k; and this is evaluated in the in the following special way.

For the 2-D case, we can write this equation as dou by dou x of u hat u star plus dou by dou y of v tilde u star equal to minus 1 by rho dou p star by dou x plus nu dou square u star by dou x square plus dou square u star by dou y square. In this equation, this u star is the variable that we want to evaluate, it is equal to the phi in the scalar transport equation for the variable phi and coefficients are this u hat, v, v tilde and nu these are all either properties of the fluid or the terms the non-linear and coupled terms, which are coming from other equations.

So, when we are solving this equation we expect that we know this coefficient u hat we do not know this, this is being evaluated here. So, we make use of the previous situation value and we therefore, denote this as u tilde. And similarly, here in this equation v is not known, because v can be known only when we are solving the momentum equations. So, we make use of the previous situation value to get this. And we solve this equation only for u star as per this approach here. And we also make use of upward differencing upwind differencing for the advection terms so that is this one and this one, and we make use of the central differencing scheme for this. And this also made central differencing.

So, with these things, we can write this as u tilde u star i plus 1 j minus u tilde u star i i minus i j divide by delta x, and similarly at the edges of this boundaries. So, we express it like this and we then evaluate what this u tilde at i plus 1 j is and i plus 1 j is this particular point here. And at this point, if you want to find u tilde here we take this to be the average of the two neighboring values of u that is available; and similarly this term is coming from dou by dou y, so this is evaluated at i plus half j plus half and j minus half. So, in this case, we need to have v tilde at j plus half and j minus half that is obtained as the average of these two.

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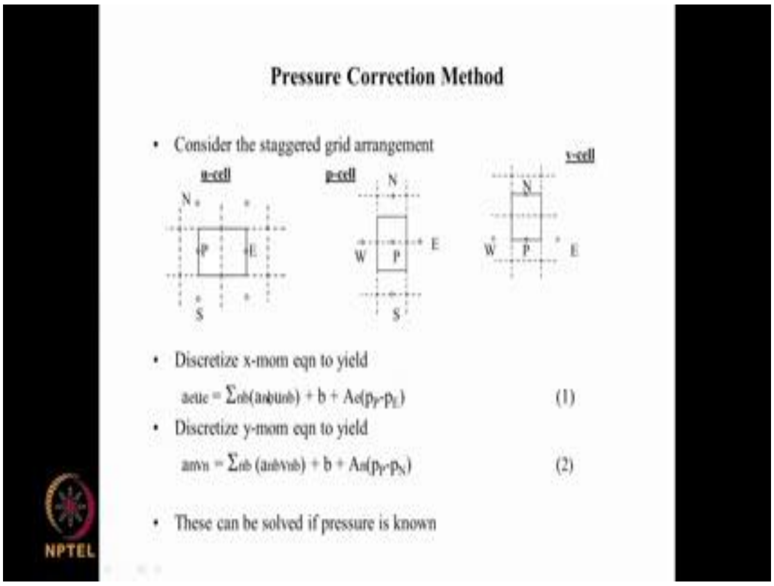

So, using these kinds of approximations, we can come up with a discretized equation. What we going to look at in this lecture is to make use of this approach, and see how we can solve the all the three equations together that is the continuity, the x-momentum and y-momentum equation for a two-dimensional problem.

So, in this pressure correction method, we are using a staggered grid. So, we have three different grids; one for the x-momentum equation which is for the u cell; one for the y-momentum equation, so this is for which will give us the v component, and one for the continuity equation, and we make use of the continuity equation to get the pressure. So, for each case, we are introducing slightly different notation here, so that we can understand very easily this particular integration, and the notation is like this, where you have capital P, and capital E, capital N, capital S these things are where pressure is evaluated, and at the corresponding.

So, if you take this particular cell here for the pressure, so pressure is evaluated at capital P, the immediate point at which pressure is evaluated on the east side is denoted as capital E; the immediate location where the same quantity P is evaluated as the capital N, capital W and capital S. And if we put this as a control volume which is lying half a between the two points on the east side, west side, north side like this then we can associate with the faces of this control volume the midpoints. So, this midpoint is evaluated on the east face this is evaluated on the north face, west face, and south face.

And if you notice our staggered grid system with respect to capital P point where pressure is evaluated the u velocity component is evaluated at the east face and the west face. And the v velocity component is evaluated on the north face and the south face, because east face and west face are displaced in the horizontal direction by half a grid point to the left to the right and half grid point to the left. Similarly, north face and south face are displaced in the vertical direction by half a grid distance in the positive y direction negative y direction. So, this constitutes, if this is i comma j this is i plus half j and this is i minus half j. Similarly, north face point is i j plus half and this is i j minus half. So, this is our control volume.

When we are looking at solving the x-momentum equation for u, we make a control volume which is centered around the east face of the p cell and, so it as the east face as the two mid neighbors as capital P and capital E and that is what we have here. So, over this control volume, this spans half a grid direction distance in each direction that is east direction, west direction, north direction, south direction.

We can integrate as we have done here and then we can come up with a discretized x-momentum equation, which is now discretized here. And the variable is the velocity u component at the small e that is the east face of the pressure cell here. So, this is the variable which is being solved and this is being returned in the form of an algebraic equation which constitutes the discretized algebraic equation discretized as per this kind of formula here.

Now, the this discretization is of, it involves two separate kind of discretization, where we have this these terms here which are coming from the advection term and the diffusion term. So, if you look at this you have an advection term involves dou u by dou x and dou u dou u by dou y, so that brings in the neighboring points. And similarly the dou square u by dou x square dou square u by dou y square terms also bring in the neighboring points and neighboring velocity points. So, these are what are coming here and these are denoted as n b meaning neighboring points.

So, the velocity component at the neighboring points together the coefficients some of which are like that kinematic viscosity and some of them are the v and u components from the previous iteration values. So, these are the coefficients. So, this implies over this implies summation over the four neighboring points. So, his implies a west face u west plus a north face u u north face like that. So, those are the immediate neighboring velocity components that are coming here.

So, since we are writing the x-momentum equation for this grid, so u e s here and then four neighbors will be u further at this point and u at this point and u at this point and u at this point. Those are the four neighboring u velocity components the taken here. Just like when we integrate this equation we would be getting the pressure at capital E, capital west, capital north, and capital south. Here it is this point the velocity at this point the velocity at this point and the velocity at this point. So, those are the four things that are coming here.

And the pressure gradient term, this one is evaluated at point at this point in an pressure gradients dou p by dou x. So, at this point, dou p by dou x can be given as dou p e minus p p divided by delta x, and divided by delta x all those things are substitute in this coefficients here. So, when we discretized a x-momentum equation over its cell, what we have is the the value of the velocity component at the center of this cell being given in terms of the value of the velocities of the same component at the neighboring cells plus may be other terms which may be constants which may be coming, and the values of the pressure at the immediate two neighbors capital P and capital E.

Similarly, when we discretize the y-momentum equation, y-momentum equation is evaluated at a grid point at at a grid cell which is centered around half or distance displaced in the vertical direction with respect to p. So, this is for this one this is centered around the north face of the p cell. So, here again we can go through the discretization and express the velocity at this point which we are calling it as v n small n indicating i is a north face velocity with respect to the capital P here.

This is expressed as in terms of the neighboring velocity components that is this point the velocity vertical-v at this point v at this point and v at this point and v at this point plus the pressure gradient in the y direction at this point which is given as p north minus p capital P divided by delta y, and divided by delta y is comp computed in this.

So, the discretization of the x-momentum equation gives an algebraic equation for x velocity component at the center of that particular cell in terms of the neighboring x velocities and pressure difference between capital P and capital e that is the two immediate neighbors in the x-direction. Discretization in the y-momentum equation gives the y velocity component in terms of the neighboring y velocity components and in terms of pressure at two neighboring points displaced in the y-direction here.

So, these equations can be solved, if pressure is known if the pressure field is known that is if we know the pressure at capital P and capital E for this location, so for the next point capital P and capital E for that. So, if we know the entire pressure at every grid cell for this capital P, capital E, capital N, capital like this, so that is p at i j for all the points then we can solve this equation for u e that is velocity at every i, j at every i plus half j. And again if pressure field is known at every point at every grid cell i j, then we can solve this this set of a b equal to b equation for velocity at every i j plus half. But question is how you know the pressure.

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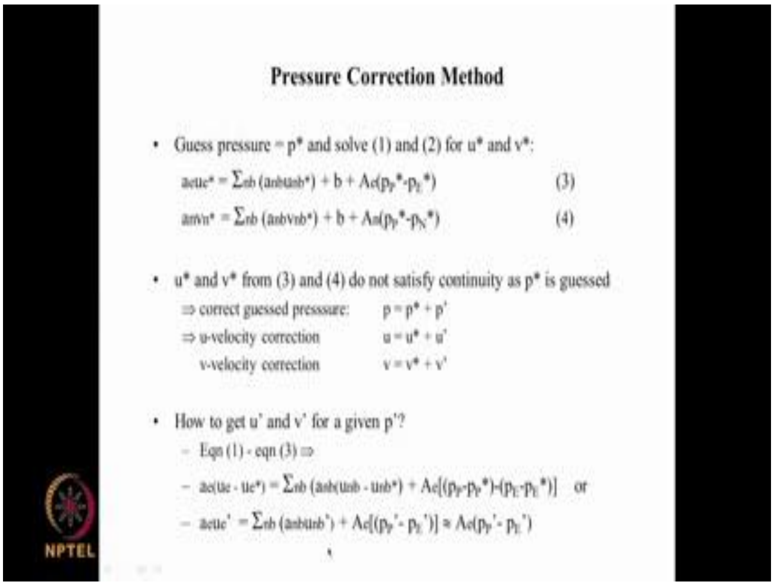

If you start with some guest pressure, and if you solve the x momentum the discretized x-momentum and y-momentum equation we get velocities, which are consistent with the guest pressure field. Since it is a guest pressure field is not wholly correct, we denote these provisional velocities as u star and v star. So, we start with an assume pressure field and then we get we solve this equation and right hand side this is known b is known and this gives you an implicit equation for the velocity component u. So, this becomes an equation like a u star equal to b, so that will be solved for u star; and similarly this becomes another a 1 v star equal to b 1 and that can be solved for v 1.

So, once we assume pressure at every grid point i, j then we can get velocity at every grid point i plus half j, and every grid point i j plus half for the two velocity components here. But these velocity components are not necessarily correct and they do not satisfy the other equation the third equation which has not been used which is the continuity equation.

So, what we therefore, suggest is that let us try to correct the pressure the assume pressure by adding a component p prime which is yet to be determined. So, it is a pressure correction in such a way that the new velocity field we know that if you change pressure the velocity changes if you change the pressure field both u and v change. So, there is also when we make a pressure correction, there is also velocity correction u prime and a velocity correction v prime which is different at every grid point and it is given by the new pressure field substituted here.

So, a pressure correction p prime involves a velocity correction u prime and v prime, we can make a a rough estimate for what will be the velocity correction for a given pressure correction. For example, if we subtract this from this equation. So, you get a e times u e minus u e star equal to sigma n b a n b times u n b star minus u n b minus u n b star like that and again p p minus p p star p and p e minus p star e.

So, those terms will come here. So, you subtraction you as a e times u e minus u e star plus the sum of all these things again where each of them is expressed as u n b minus u n b star like this. So, now, we know that u e minus u e star is u prime from the definition. So, we can say that subtraction of three from one will give us a e u e prime equal to sum over the neighboring points of a n b u n d prime plus a e times these two are pressure correction at p and pressure correction at e.

So, at this stage, we do not know what the pressure correction is, but if there is a pressure correction which is not necessarily the same at every grid point which is varying at every grid point pressure itself is varying at every grid point. So, if there is a pressure correction given at every grid point, then there is then they add up in such a way that the velocity correction is given by this whole equation. And if you neglect this component then you can say that a e u e prime is roughly equal to a e times p p prime minus p e prime, and what this gives us is that neglecting this, if you make a pressure correction then you can find out what is a velocity correction here.

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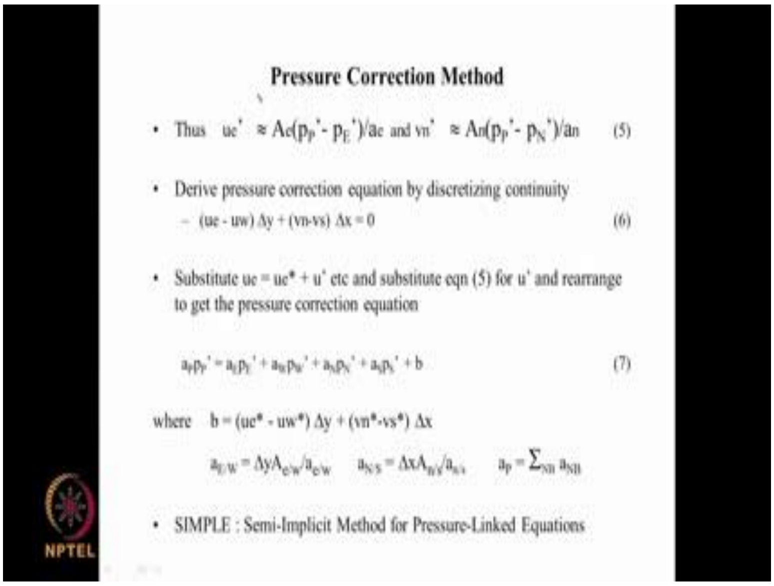

And similarly we can also make an estimate for v n prime the velocity correction in the vertical component velocity is a n times p and p prime minus p n prime. So that means, that at this stage, we are saying that let us make a pressure correction and let us say that this pressure correction introduces a velocity correction u prime which is given by this and velocity correction in v prime which is given by this, but what pressure correction to make. So, for this, we make use of the continuity equation and continuity equation is dou u by dou x plus dou u by dou y equal to 0, so that can be written as so dou u by dou x is u east minus u west divide by delta x and v n minus v s divide by delta y.

So, multiply both by delta x delta y and then you have this. So, this is the discretized form of the continuity equation over this cell here. So, we are saying that continuity equation dou u by dou x plus dou v by dou y is being evaluated over this cell. So, dou u by dou x is this velocity minus this velocity this velocity is u e this velocity is u w both e and e r, e n w are small letters indicating the face of this particular cell. And similar dou v by dou y is given by velocity vertical velocity at this point which is v small n minus v small s here so that is what we have written here. So, this is the discretize continuity equation.

And here we substitute u e to be equal to u e star plus u prime and u w is u w star plus u w prime and v n equal to v n star plus v n prime and v s equal to vs star plus vs prime. And what is a advantage, at this stage, we have already calculated all the velocities u u star at all this points because we are solve this. So, the star quantities in this are already known. So, this gives us an expression like u u prime e minus u prime w divide by delta y times delta y plus v n prime minus v s prime time delta x equal to zero.

So, now we already know what this u prime is roughly in terms of p prime and pressure corrections. So, we substitute u e prime equal to a e times p prime p minus p prime capital e by small a e now all this coefficients are known from the geometry which will see in the next lecture we can evaluate those things and. So, we can substitute all with corresponding pressure differences pressure correction differences. And similarly all the v s can be substituted in terms of the pressure differences pressure corrections in differences involving the gradients in the north and south directions here.

So, once you substitute the equivalent expressions in in this equation all the velocity corrections will go and this equation becomes an equation involving pressure corrections only and the pressure corrections are at capital P and capital N capital E and all that. So, we can rewrite this continuity equation as an equation for pressure correction, where the pressure correction at capital P is given in terms of pressure corrections at capital E capital N, capital W capital S and that is of this particular form. Where somehow the things the star quantities are coming in the right hand side of this, and we already know these things because we have solved the corresponding discretize momentum equations.

So, if you look at this equation here in this equation b is known, and the coefficients a and w, a and a w a capital E these can be determined as part of the discretization. So, this becomes an algebraic equation for pressure correction at capital P in terms of the weighted sums or the pressure correction at the corresponding things. So, when you write this for all the grid points i, j then you have an equation which is like A p prime equal to b, so that is the matrix equation in which p prime at the grid cells i comma j is the are the unknowns.

So, this enables us to solve for the pressure correction. So, once you get pressure correction then we can put that here and get a new pressure and you can also estimate the pressure correction are the velocity correction induced by the pressure correction here and we can get velocity corrections. So, we get new u and new v. So, using these new values of p u and v we go back to the discretized x-momentum equation and then we come here and we substitute the new values and get a new estimate for velocities and then we keep on doing this.

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So, this method is known as the simple method. It is known as the semi implicit method for pressure linked equations. Obviously, these are linked through this pressure correction here and through the pressure guest pressure field in the x and y-momentum equation. And it is semi implicit, because in this equation here the velocity is being evaluated implicitly. So, here it is implicit, but the pressure part is not being evaluated implicitly at this stage.

And if you come to the pressure correction equation this is the true pressure correction equation, but in this we are neglecting the component the contribution from these and then we are just directly putting this one here. So, this implicitness of the evaluation of the velocity correction is not being considered therefore, this is only semi implicit method. So, this is a semi implicit method for pressure linked equations. We do not need to worry too much about the nomenclature, but we need to understand how we go about doing this.

So, we start with some guest pressure field and velocity field why do we need the velocity field because when you are solving the x-momentum equation, we also need to know the previous value of u and v. And when you are starting out there is no previous value. So, you make some guess values for u star, v star and p star at all i, j, k, i plus half j, j and i j plus half. So, wherever a particular variable is being evaluated, there we make guess for these things.

So, now, we have p star, and u star and v star at the correspondent locations where there to be determined. And using p star and using this discretize form of the continuity equation of the x-momentum equation, we solve for u velocity component; and using the same p star we also solve the discretized y-momentum equation to get v, v star.

So, in the first step, to begin with we make guess velocity and pressure fields, and then we solve the discretized x-momentum equation for u star, solve the discretized y-momentum equation for v star. And once you know u star and v star then we can discretize the continuity equation and the continuity equation is reformulated as a pressure correction equation. And once you know u star and v star all the terms that are there in the pressure correction equation are now known. So, the pressure correction equation can be rewritten as an algebraic equation a p prime equal to b and that is solve for pressure correction at every point i, j. And from these pressure corrections we make the approximate velocity corrections u prime and v prime.

So, by the term we come to step five here we have we know what is the initial pressures and velocities and the corrections that we need to make. So, at this stage, we have new values of pressure and velocity and u velocity v velocity and if there are other things like temperature turbulent kinetic energy and all that those can be calculated. So, now you have updated pressure and velocity components, and these updates are use to now go back to step two, and solve again the x-momentum equation and then y-momentum equation and pressure correction equation. We do this iteratively.

Why we need to do this iteratively is because in the discretized equation we are making use of the all pressure field and. So, we get this and again here we get this. And when we are solving the pressure correction equation, we are making use of the new values of u star v star like this, but these new star new values are obtained based on the assume pressure field. So, even these are not fully correct. So, that is why we have to solve the pressure correction equation and the discretized x-momentum equation and the discretized y-momentum equation iteratively, so that each time we are making updates and updates. And eventually and hopefully it will converge and converge solution is such that the pressure corrections and velocity corrections are zero.

So, this is the simple algorithm for solving the continuity x-momentum, y momentum and even z-momentum equations in a specific way. Such that if the problem is steady we solve the steady form of the Navier-stokes equations; if it is unsteady we can extend this also to the unsteady form of the Navier-stokes equations and we can use this. So, this has proved to be very popular, since it is introduced by in 1968 by Patankar and Spalding and number of coworkers. Numbers of variants and improvements have been made to this and you have SIMPLEC, SIMPLE R, SIMPLEST, PISO, and all these methods are variations or inspirations of this method.

Although this method is been described here for staggered grid, this can also be extended to collocated grids and it can also be extended to compressible flows and all that and its used in many CFD computations, for the solution of Navier-Stokes equations for compressible and incompressible flows primarily for incompressible flows.

And because we are doing it in so many iteration kind of things it is possible to bring in more complicated physics like turbulence and reaction and all those things they increase the number of equations. So, when you have so many numbers of coupled equations we have to make incremental variations. And this method allows those kind of incremental variations to be brought in properly and that is also one of the reasons why this as become very popular for incompressible flow calculations, where usually a lot more physics of the problem is tackled than incompressible flows.

So, simple method is probably in terms of its variants and a latest variant is one that is invariably used for many computations of incompressible or nearly compressible flows. So, we can see that to have a number of methods for the solution of incompressible flows special methods.

In the next tutorial class, we will make a flow chart for the simple method both for steady flows and unsteady flows, and see how we can conceptualize an overall scheme of solution which keeps in it the concepts that we have discussed in module three about solution of the discretized equations. So, those concepts are still there, but in very hidden form and we have to do much more before we can solve the coupled equations. So, this is an illustration of that.