Computational Fluid Dynamics
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Lecture – 37
Compressible flow to in-compressible flow

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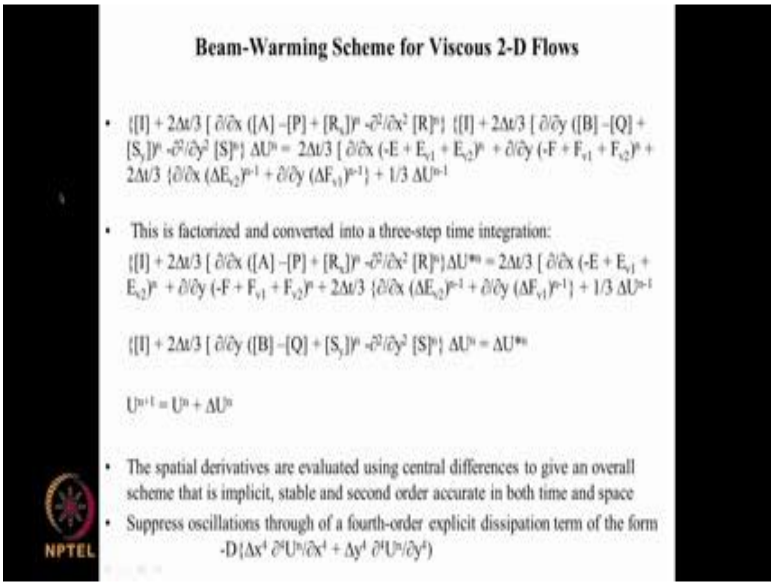

And once we do that then we have tri-diagonal matrix method applied twice in order to get to the final solution. Some more details about spatial derivatives are evaluated using central differences. To give an overall scheme that is implicit we can see the implicitness coming from here, this delta un-involving second-derivative and first-derivative, and that is implicit stable and second order accurate in both time and space.

Something that can be applied for viscous 2-D flows, so, these schemes like these things, can give us to oscillations and we have seen that already for the scalar transport equation, involving both advection term and diffusion term. We have seen that, when we use central differences we can get oscillations, but central differences guarantee a second order accuracy. So, if you will not have second order accuracy and if you do not want have some oscillations then we need to suppress those oscillations and those oscillations one of the ways they have to suggested earlier on these are 1970s mid 1970s and earlier

1980s.

So, you would introduce in explicit dissipation term fourth-order method term like this which can suppress those undue oscillations which are which arise because we have making use of central schemes for the space derivative which are second order accurate. So, we have here a method which looks fairly complicated, and that is typically true of the kind of methods that we need to solve that we need to put together when we want to solve Navier stokes equations.

So, it is no longer a simple case of substituting the derivatives using finite difference methods. We have to take care of several factors and the factors that we are looking at that we are taking care in this method, and also in the case of the Mac Cormack method are that we have not one equation, but we have four equations or five equations or three equations like that coupled equations. These are coupled equations and non-linear equations and when we are considering viscous effects they involve cause derivatives they may involve normal derivatives.

So, each of these things has been taken into account in coming of with an overall method, which is implicit and second order accurate in its conception and therefore, one can expect it to be stable unconditionally stable unlike the case of Mac Cormack method that explicit Mac Cormack method, but we can see that we are not solving it in. So, clean a way in the process of accounting for non-linearity and coupling.

We have introduced coefficients a p r which are and s b q s these are being evaluated at nth times step at the previous time step values. So, these are not the latest values it is not fully, fully implicit part of it is explicit and part of it is implicit. And what this means is that if you take very large times steps because, you are scheme is stable for any value of delta t then, in the process of taking long time step you are freezing the contribution of the non-linearity and coupling to the previous time step value.

That can give us two problems. So, when in a case where essentially what you are saying is while you are solving the u equation you are expecting the contribution to these things coming from the v a and w a and those kind of things are frozen at the previous values, previous time step values that is possible only for small values of delta t not very large values of delta t. So, when you take two larger time steps then you have problems that can arise because of this lagging of the coupling and non-linearity terms and. So, that can mean that can limit your delta t values that you can take.

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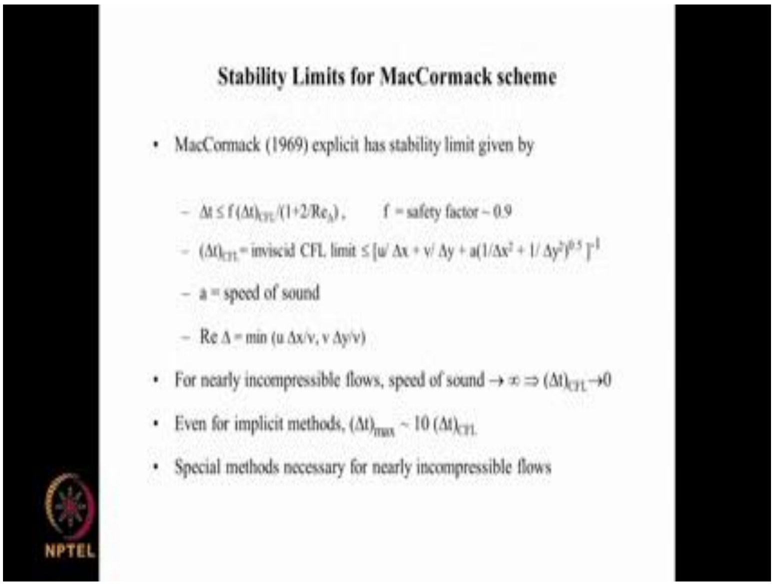

When we are looking at compressible flows we have seen examines two popular, well put together methods and what we can see from this is that these method requires some special putting together in order to make them solve Navier stokes equations and the governing equations in different forms, and we have to take special considerations to bring in to take account of non-linearity coupling and the cause derivatives and normal derivatives and we have seen that these schemes are limited in that time step.

In the case of the Mac Cormack scheme that delta t as we have seen is a function of the speed of the sound, and the speed of the sound becomes very large for nearly in-compressible flows and its infinity for totally in-compressible flows, where density does not change and when a becomes infinity the delta t CFL becomes as many and so; that means, that you cannot progress in time you can progress with in time with infinites with time steps and that is realistically not possible.

If you go for the implicit methods you can go for larger times steps, but again you do not want to have such a large time steps that so, the delta t is too large because as you are changing u v and w are also changing and you are not taking account of that. So, there are densities also changing and enthalpies are changing. So, all those things are simultaneously changing, and for implicit methods also one usually recommends that delta t which is not more than ten times the delta t that you dues with the corresponding explicit method.

So; that means, that the methods that we have looked at for compressible flows, cannot be straight away used for in-compressible flows. So, this gives us a problem have been successful in extending the compressible flow methods like some of which we have seen two in-compressible flows, but there is also been a separate movement towards developing special methods for in-compressible flows. As part of this course in the next half of this module 4, we will be looking at methods that have been specially derived for in-compressible flows.

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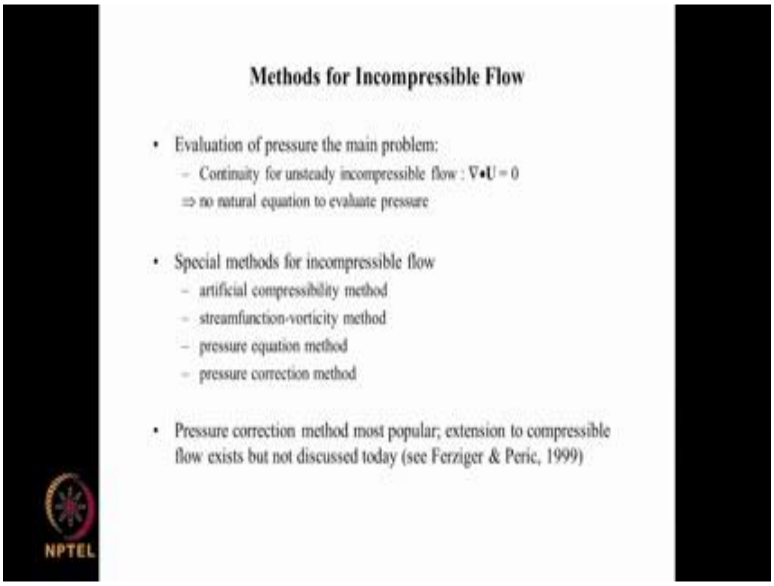

And here we have we lose the advantage, that is present in compressible flows of a natural linkage between the continuity equation and the momentum equation. Because in the continuity equation you have dou rho by dou t plus dou by dou x of rho u plus dou by dou y of rho v equal to 0. You have that equation and you can use that equation to get density and once you have density, and the internal energy from the energy equation then you can use an equation of state to get pressure and temperature. So, that kind of linkage between the continuity and the momentum equations is not present in the case of in-compressible flows because there is no density term that is coming in the continuity equation.

So, continuity equation is also in two-dimensional cases it is just dou u by dou x plus dou v by dou y equal to 0. So, there is no pressure there is no density and this creates a special problem because of this lock of loss of linkage and you can say that there are 3 or 4 different approaches for dealing with this and these are the ones that we are going to discuss in the second part of this module. So, we will just look at what these methods are; So, first one is about artificial compressibility. So, if compressibility does not exists, if there is no relation between density and pressure.

If there was a relation then we could have used we could have evaluated the density from the continuity equation and gotten the pressure from the equation of state which would be linking pressure in density, through the compressibility. So, if there is no compressibility the, we could artificially introduce compressibility and then solve creates a linkage between the continuity equation and the momentum equations. So, something like this we can be created and the resulting method would work, but it is a problem of the equations the equations have been tampered with by introducing an artificial compressibility. So, what we will see is that this method will work and is correct it is solving the correct equations in the limiting case of steady flows.

So, that is one approach that has been taken. If you want to do steady in-compressible flow calculations they then you can introduce an artificial linkage between the continuity and the momentum equation, by introducing a fictitious equation of state and fictitious compressibility and then you can get a solution or steady state and under steady conditions the equations are correct the equations related to solving are correct. But the transient part is not really truly reflective of that.

So, there is limitation limited applicability, but that is one approach by which you can solve for steady in-compressible flows. It is an extension of the methods that you have already developed for compressible flows, but there is a different way of looking at it different way solve it. Since pressure is the nuisance variable in the sense you have pressure is there in the x momentum of equation which will give us the u velocity, it is there in the y momentum equation which will give us the v velocity, but it is not there in the continuity equation which should give us density and from which the pressure in the case of compressible flows, and in the case of in-compressible flows we do not have that luxury and pressure does not appear in that. So, is it possible to eliminate pressure completely from these 3 equations?

We can do that by introducing two new variables stream function vorticity and the 3 equations that is the continuity the x momentum and the y momentum equations can be rephrased into two coupled equations involving stream function vorticity, in which there is no pressure and the variables are stream function vorticity and you can solve these two equations for these two variables and deduce from that the velocity and after having gotten the velocity you could then separately solve for pressure.

So, this is a method which can get rid of that vexing question of pressure by completely eliminating pressure from the Navier stokes equations. But unfortunately this method is is applicable only for 2-D flows only in 2-D flows you have the concept of stream function. If you extended it to three d then you get into quite large number of coupled equations even more than what you have delt with and that makes it very complicated. So, if you are looking at either steady or unsteady 2-D flows only, then in whether its cartage and coordinate system or cylindrical or polar you could come up with the stream function vorticity method to solve the corresponding equations without worrying about the pressure.

So, this is a different approach, different from what we have seen in the first case which is the artificial compressibility. There is another possibility that is since pressure evaluation is something that is difficult for us, if you can rewrite; if you can derive any equation for pressure, from the continuity and momentum equations and if we then what it would mean is that if you know the velocity you can calculate pressure, and if you know pressure you can calculate velocities.

So, you would first solve for velocities and then from that you would get pressure and then you put the pressure back and then get velocities. So, you can have either an explicit pressure equation method or an implicit pressure equation method. You can alternatively solve the momentum equations with assumed pressure and then solve for velocities and then use the velocities to get an improved value of pressure and then you can put it back and then forward. So, you are solving a pressure equation, you are solving a reformulated set of Navier stokes equations, still using the same primitive variables u v and w and p and that is known as a pressure equation, the variants like max scheme and all that and.

So, that is one possibility which can be used for transient 3-D flows unlike artificial compressibility method which can be used only for steady 3-D flows, unlike stream function vorticity method which can be used for transient flows, but only 2-D flows pressure equation method can be used for both. Pressure correction is a in a way its a variant of the pressure equation method where you are trying to say that i have a guest pressure field and it is; obviously, not solving the satisfying the continuity equation. So, i would like to correct the pressure by a small amount and then in such a way that the velocity also gets corrected and the corrected velocity will satisfy the continuity equation.

So, that idea is formulated in the form of an algorithm, where you will be solving not for pressure directly, but for pressure correction. So, these are the 4 methods that will examine for specifically for in-compressible flows and of these the pressure correction method is the most popular especially for internal flows. It is been used in a reverse way a method which has been developed for in-compressible flows has been extended to compressible flows.

But it is something we do not discuss as part of this. So, you have truly compressible flows methods that can be extended in theory to in-compressible flows, and methods that have come from in-compressible flows, extended to compressible flows. So, in a way there is some sort of merging of the of the streams, but people still use hard core compressible flow, people use specialized methods which have been developed for compressible flows.

Hard core in-compressible flows like automobile engineers or may be chemical engineers and these kinds of people. Who consider not just the flow, but lots of other things like combustion, chemical reactions and all that kind in such a case the pressure correction method is found to be more robust and it enables you to model allied phenomena in an elaborate way. So, that is where it is in that that is probably the reason why pressure correction methods have become very popular and often used for in-compressible flows and slightly compressible flows in many process and districts.

So, in the second week of this module 4, we will be looking at the basic ideas in these four methods i would like to end this module this part of the module by emphasizing that, we are not looking as part of this course developing methods which are developing course which can do all this kind of calculations. We are just trying to understand the principles and understand the concepts which have been developed to deal with the real Navier stokes equations, and we can see a lot of difference between the way that we are solving the Navier stokes equations and the simple concepts of stability consistency and convergence those kind of concepts that we introduced at in module 3.

So, we can still see some traces of those in the methods that we actually use for Navier stokes equations. But the expediency of practicality of having to deal with viscosity turbulence and all those kind of things, which involves more and more coupling and more and more non-linearity and all that. So, we can no longer afford that luxury of consistency and total guaranteed stability, we have to compromise on that and come up with methods which can give rise to a solution which can gives us a solution and its always therefore, necessary for us to validate our CFD simulations of real flows, and that is something we would like to keep in mind.

Thank you.