# Computational Fluid Dynamics
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## Lecture – 18
## Practical cases of fluid flow with heat transfer in CFD point of view

In the last lecture, we derived the energy equation. And the set of equations which govern incompressible constant property flow with heat transfer is given here in this slide.

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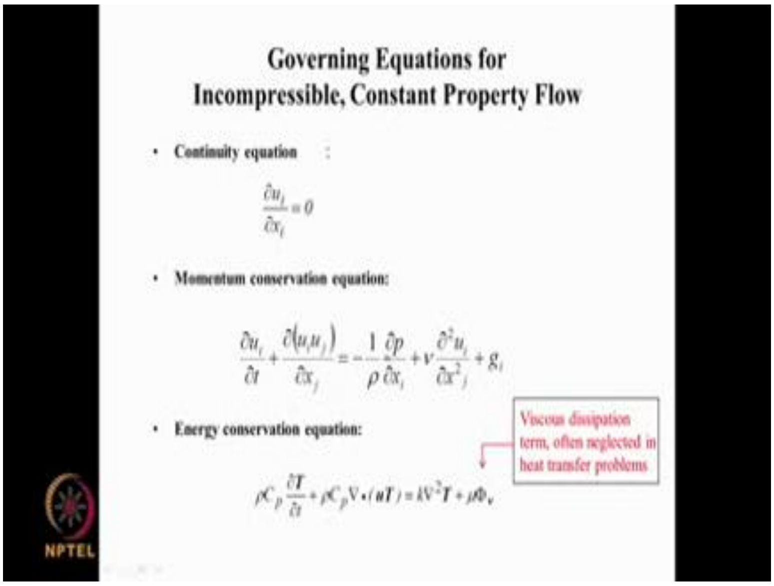

Governing Equations for Incompressible, Constant Property Flow

- Continuity equation :

$$\frac{\partial u_i}{\partial x_i} = 0$$

- Momentum conservation equation:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x^2_j} + g_i$$

- Energy conservation equation:

Viscous dissipation term, often neglected in heat transfer problems

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \nabla \cdot (uT) = k\nabla^2 T + \mu\Phi_v$$

We have the continuity equation, the three momentum conservation equations for a Newtonian fluid with constant properties, so that the viscosity - the kinematic viscosity here is constant; and the energy conservation equation reduces to a much simpler form and expressed in the form of a rho C p T that is specific heat, constant pressure. And therefore, from in order to get to this particular form, we have rewritten the earlier equation which we had written in terms of internal energy that has been rewritten in involving a enthalpy. If we do that process, you will get an equation like this.

And as already mentioned, this is viscous dissipation term and when it is negligible unless you are dealing with large viscosity fluids large viscosity liquids. For example, if you are studying the mixing of a highly viscous fluid using an impeller, at that point, we would have to consider this because this would generate a sufficient amount of local dissipation and local increase in temperature which may have a consequent large change in the viscosity which would then be coming into the momentum equation through this term and that could change the velocity field.

So, in such cases, you would have to consider this, but if you are looking at light fluids, light viscosity fluids liquids and gases, then you could neglect this particular term and you could take this as essentially the thermal energy conservation equation. And solution of this will give us temperature as a function of x, y, z; solution of these will give us u, v, w and p as a function of x, y, z. And from these, we can get the friction factors or pressure drops, the heat transfer coefficients, the amount of heat transferred, the wall temperatures, all this information we can get from this.

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Heat Transfer Applications

- Convective heat transfer
- Conductive heat transfer
- Radiative heat transfer (not in the syllabus for exams)

So, what we will do now is that using these equations, we will see heat transfer applications, and how we can relate these equations to our conventional non CFD conception of heat transfer. So, we are looking at heat transfer applications involving the three modes of heat transfer - convective heat transfer, conductive heat transfer and radiative heat transfer. This radiative heat transfer especially from a point of view CFD is quite complicated, so it is not relay in the syllabus as well as the exams is concerned, but I do want to spend a few minutes discussing this, so that you know that there is much more to it then what we have seen so far in this.

So, that it is not something that is included in what we have doing. So, to that extent, this more an awareness it is not going to be there from the point view of exams. So, the idea is that having formulated a problem in terms of having a derive the governing equations, how can we establish that what we have done is capable of dealing with convective heat transfer and conductive heat transfer and radiative heat transfer, so that is a question that we would like to examine now.

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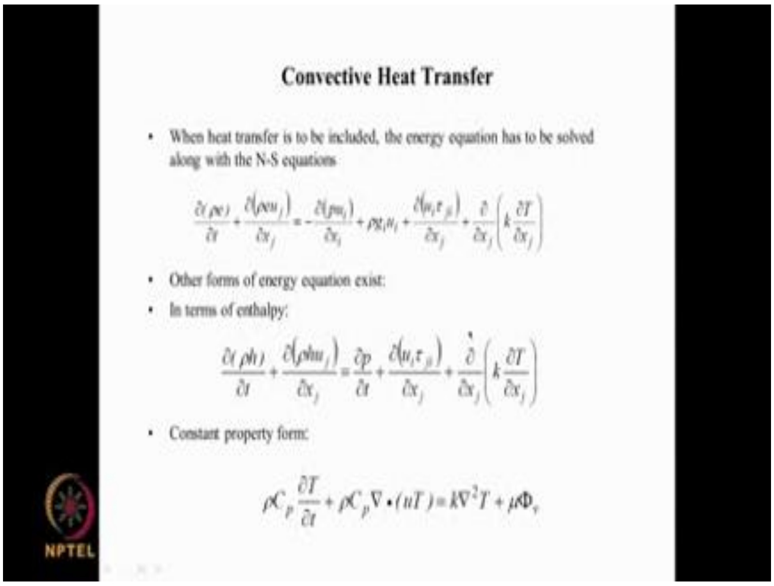

## Convective Heat Transfer

- When heat transfer is to be included, the energy equation has to be solved along with the N-S equations

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial(\rho e u_j)}{\partial x_j}=-\frac{\partial(p u_i)}{\partial x_i}+\rho g_i u_i+\frac{\partial(u_i \tau_{ji})}{\partial x_j}+\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right)$$

- Other forms of energy equation exist:
- In terms of enthalpy:

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial(\rho h u_j)}{\partial x_j}=\frac{\partial p}{\partial t}+\frac{\partial(u_i \tau_{ji})}{\partial x_j}+\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right)$$

- Constant property form:

$$\rho C_p\frac{\partial T}{\partial t}+\rho C_p\nabla\cdot(uT)=k\nabla^2 T+\mu\Phi_v$$

So, when we look at convective heat transfer. So, when heat transfer is to be included, the energy equation has to be solved along with the Navier stokes equations. Because it is only from the energy equation, we get temperature and only when we have temperature we can talk about heat transfer. So, but we also see that the temperature equation has a velocity component that is coming here and there is also viscous dissipation.

So, even if we neglect this we still have a velocity component that is coming here; and in that sense the velocity field also needs to be resolved and that means, that we have to solve the energy equation together with the momentum equation and the continuity equation. So, here we have one form of energy equation written in terms of the total energy which is internal energy plus kinetic energy. This small e is a specific internal energy plus specific kinetic energy, so that is i plus half u i u i is what this internal energy is.

But we can also write in terms of other energy equations. For example, in terms of enthalpy, this may be useful if you are doing for example, turbo machinery application and in such cases the enthalpy difference will give you the amount of work that is done by the system on the turbine blades, which is then use for extraction of power. So, in such cases, for example, you could be using enthalpy. And through manipulation of this and other equation of states and those kinds of things, you could rewrite this in the form of enthalpy like this.

And the form is very similar, you have dou by dou t of dou e here and you have dou by dou t of rho h the enthalpy. And you have dou by dou x j of rho e u j this is what we call as a advection term, so this is associated with flow. The fact that flow is coming in and going out of the control volume brings with it anything that the fluid process including the x-momentum, y-momentum, z-momentum, internal energy, enthalpy, entropy all those thing are concentration, all those things are brought in. And the net thing arising out of the coming in and going out of the flow through the control volume. And you have dou p by dou t plus the rate of work done by stresses - viscous stresses and heat conduction the net heat conduction through the surfaces.

And in the specific case of constant properties, this reduces to this particular form, and you can neglect this. So, the solution of energy equation with appropriate boundary conditions takes account of convective heat transfer for the fluid. So, the important thing is that we do not have to do anything special to simulate convective heat transfer, it is already included in the governing equation for the energy, because of this particular term here, the advection term here is only place where velocity is coming, and this will take care of the convective heat transfer.

And solution of this energy equation along with the governing other governing equations is sufficient for us to distinguish between convective heat transfer and how much of convective its Reynolds's number dependents, and all those kind of a things will be to get. By direct solution of these equations without having to worry about Reynolds number effect and all that. Reynolds's number is included in the equations; we do not have to bring it out especially in the form of correlation.

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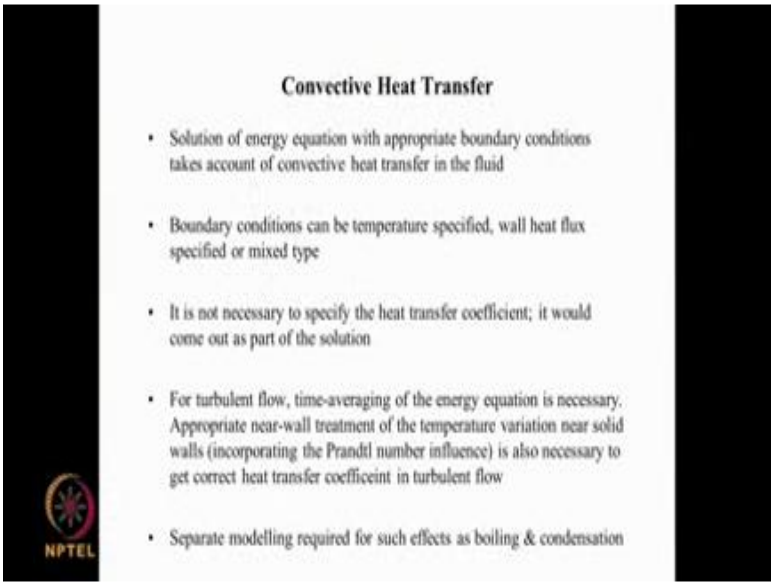

Convective Heat Transfer

- Solution of energy equation with appropriate boundary conditions takes account of convective heat transfer in the fluid
- Boundary conditions can be temperature specified, wall heat flux specified or mixed type
- It is not necessary to specify the heat transfer coefficient; it would come out as part of the solution
- For turbulent flow, time-averaging of the energy equation is necessary. Appropriate near-wall treatment of the temperature variation near solid walls (incorporating the Prandtl number influence) is also necessary to get correct heat transfer coefficeint in turbulent flow
- Separate modelling required for such effects as boiling & condensation

And what we need to do is that we (Refer Time: 08: 26) equations and we identify the domain, and we specify the boundary conditions so that is all we need to do, we do not have to do anything special for convective heat transfer. So, the boundary conditions what do we want here we want boundary conditions can be temperature specified, wall heat flux specified or both can be specified.

For example, in the case of convective boundary conditions, which will see later on, we specify the ambient temperature and a heat transfer coefficient for example, by natural circulation from the wall and that will be something like a mixed type of boundary condition. Whereas temperature boundary condition is you specify the wall temperature at the boundary or you specify the heat flux, you specify the gradient of the wall of that temperature at the wall like this.

And it is important that it is not necessary to specify the heat transfer coefficient, it would come out as part of the solution. Now the heat transfer coefficient that we talking about is the heat transfer coefficient at the wall inside the flow domain, if the heat transfer evacuation external to the flow domain is by a process, which is determined by the external conditions. For example, you have a fuel cell stack and you are cooling it by having a fan blowing over it, then what is the boundary condition on the walls of the fuel cell that depends on what is the heat transfer coefficient that you are creating outside the computational domain, outside the wall, outside the fuel cell which is you are interest.

And what is the role of the fan, for example, if you put in high speed, then there is more heat transfer; and if the fan is in low speed, then there is less heat transfer; if there is no fan, then it is natural circulation. So, the heat transfer external to the flow domain is something if it is relevant you have to specify. But heat transfer internal to the flow domain is something that will come out of the as part of the solution, so that that is a very important aspect. We do not has to specify the inside heat transfer coefficient, all we need to do is to write down the energy equation in one of the three forms in as a internal energy or enthalpy or in the constant property form like this we can neglect this.

And each of this will give you solution of this will give you h as a function of x, y, z; and this will give as a i as a function of x, y, z. And if we know the C v or C p, then from the internal energy or from the enthalpy you can find the temperature. And solution of this with specified values of C p and k will directly give you T as a function of x, y, z. And from that you can compute the heat fluxes through the walls; and from that if you still want you can get some idea of the heat transfer coefficient, but if you are directly getting heat flux then heat transfer coefficient is not necessary to be computed in the CFD solution, it is a post processing thing.

If you want find out what is the heat transfer coefficient, then you can do that. It is same as when we do the calculation of flow field, we do not specify any friction factor. For example, in the first example of flow through the rectangular duct, and the second example of flow through the triangle duct, we did not specify any friction factor and we dint specify that friction factor is given by 16 by Reynolds’s number where or 64 by Reynolds’s number nothing was given.

We took the governing equation, we apply the condition of the no slip and that was all; and from that for the specified pressure gradient, we got the flow rate. The normal non CFD rule will have been that you have the specified pressure gradient, you bring in a friction factor correlation in terms of the average velocity, you plug it into the formula; and from that you get the average velocity. In order to get the average velocity, you need to have friction factor specified, but if you are doing C F D if you are solving the original equations accurately at every point within the flow domain, then it is not necessary to have any friction factor.

If you still want friction factor, you can deduce it from impose pressure gradient and the calculated average velocity. In the same way here, if we specify the wall boundary conditions, and if we specify the flow rate and those kinds of things, then you will be able to get a temperature distribution and the corresponding heat flux through the walls. And from the heat flux through the walls and wall temperature and the ambient temperature, you would be able to get heat transfer coefficient if you wanted. So, heat transfer coefficient need not be specified here, the convective heat transfer coefficient, it is already taken care of and it is no where explicitly calculated unless you want to have it. So, heat transfer coefficient is not a concept that (Refer Time: 14:17) in with CFD.

But we are talking about heat transfer coefficient inside the flow domain. If heat transfer coefficient outside the flow domain is necessary as part of the specification of the boundary condition that is still needs to be done. We need to specify the heat transfer coefficient because outside the flow domain, we do not know what the temperature variation is, we are not computing that. If you were to include part of outside domain also in the computation domain then you do not have to even specify that, but that is a different story. So, let us not go too much into that at this stage.

So, for turbulent flow, we have slightly more difficulty, we will see this in the last module of this particular course. And what I would like to mention is that what we are talking about is the single phase convective heat transfer. When you have special phenomena like boiling and condensation, which you know change of phases, so you have two different phases or may be several different phases, multi compounding mixtures, in such cases we have to do some special treatment, and those special effects are not included in this.

So, single phase convective heat transfer does not require anything to be specified here except the appropriate boundary conditions at the walls. The equations already have inside them, the information necessary to consider that convective effect inside the flow domain.

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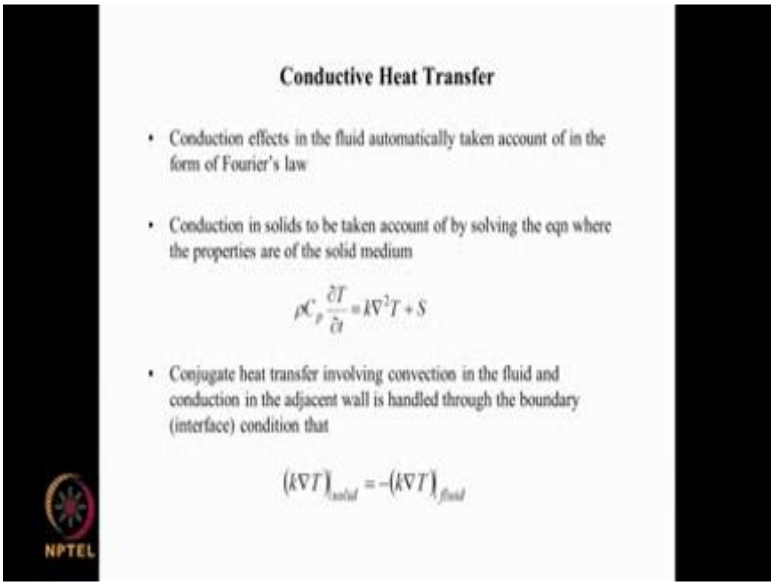

## Conductive Heat Transfer

- Conduction effects in the fluid automatically taken account of in the form of Fourier's law
- Conduction in solids to be taken account of by solving the eqn where the properties are of the solid medium

$$\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T + S$$

- Conjugate heat transfer involving convection in the fluid and conduction in the adjacent wall is handled through the boundary (interface) condition that

$$(k\nabla T)_{solid} = -(k\nabla T)_{fluid}$$

Now, what about conductive heat transfer? Conduction effects in the fluid are automatically taken account in the form of Fourier law, because when we wrote down the heat flux through the walls through the surfaces of this box like control volume, what is the heat flux that heat flux is a for a control volume a (Refer Time: 16:32) control volume, which is embedded in the entire fluid continuum.

So, there we are talking about heat flux coming from the neighbouring control volume in to this because of temperature differences, and so that is conduction heat transfer within the medium. So, the conduction heat transfer within the medium is already included is our basic formulation in the form of heat fluxes here.

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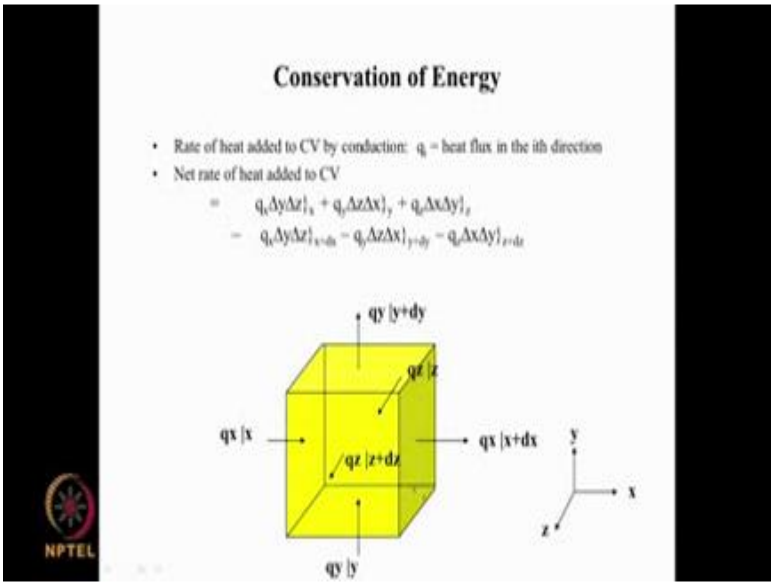

Conservation of Energy

- Rate of heat added to CV by conduction: $q_i$ = heat flux in the ith direction
- Net rate of heat added to CV

$$= q_x \Delta y \Delta z\}_x + q_y \Delta z \Delta x\}_y + q_z \Delta x \Delta y\}_z$$
$$- q_x \Delta y \Delta z\}_{x+dx} - q_y \Delta z \Delta x\}_{y+dy} - q_z \Delta x \Delta y\}_{z+dz}$$

So, this is a (Refer Time: 17:02) control volume, it has no walls it is not these are bounding surfaces and so this is part of the fluid domain with fluid walls and you have next cell which is here. And between these two there is a wall there is a wall which is made up for the fluid itself; it is not like a physical wall, which is separating the two.

So, depending on the temperature here and temperature here; if the temperature here is more than the temperature here then heat would come in through this way and q x would be negative. If this temperature is greater than this, if this temperature is less than this, then heat would be flowing out, in which case q x at this phase would be positive and that is already included.

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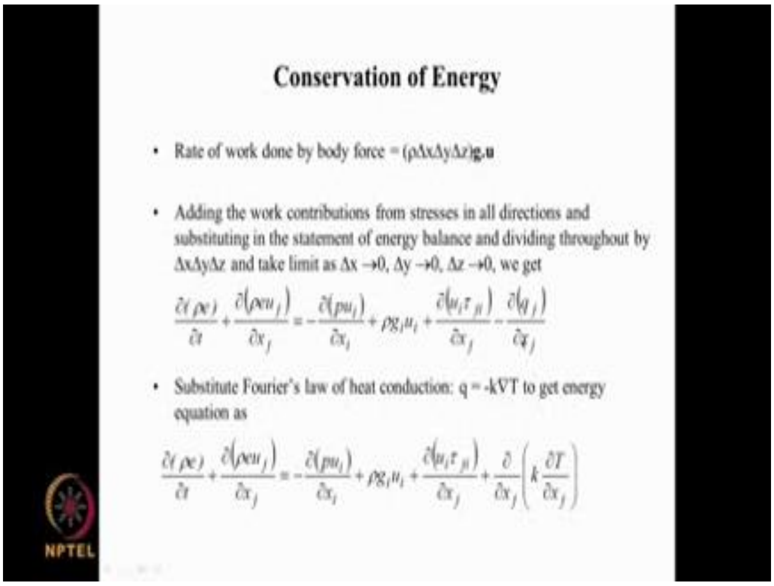

## Conservation of Energy

- Rate of work done by body force = (ρΔxΔyΔz)**g.u**
- Adding the work contributions from stresses in all directions and substituting in the statement of energy balance and dividing throughout by ΔxΔyΔz and take limit as Δx →0, Δy →0, Δz →0, we get

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial(\rho e u_j)}{\partial x_j}=-\frac{\partial(p u_i)}{\partial x_i}+\rho g_i u_i+\frac{\partial(u_i \tau_{ji})}{\partial x_j}-\frac{\partial(q_j)}{\partial x_j}$$

- Substitute Fourier's law of heat conduction: q = -k∇T to get energy equation as

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial(\rho e u_j)}{\partial x_j}=-\frac{\partial(p u_i)}{\partial x_i}+\rho g_i u_i+\frac{\partial(u_i \tau_{ji})}{\partial x_j}+\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right)$$

Because in going from here to here, we have made use of the Fourier’s law of heat conduction that this q here is minus k dou t by dou x, so that is included in this. So, both conduction heat transfer and convective heat transfer inside the fluid domain are included in the equation that we are solving. And both of them require only the appropriate boundary condition to be specified, so that is what we are saying here.

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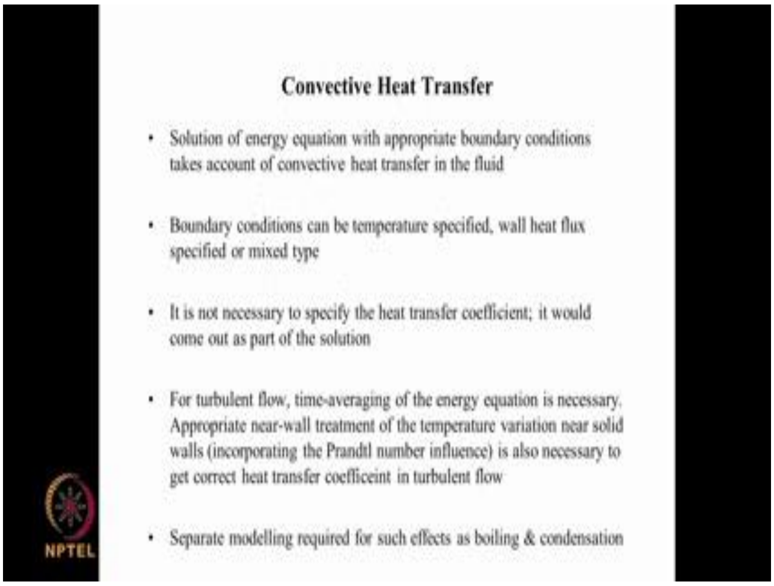

## Convective Heat Transfer

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Conduction in solids should be taken account; conduction effects in the fluid are automatically taken account in the form of Fourier’s law which is applied to evaluate the conductive heat transfer within the fluid continuum. Now if you are talking about the solid for example, we have a fluid here and then the solid in between and then the fluid here, then they can be heat transfer from this hot fluid into the cold fluid through this intervening wall, and so our computational domain will be partly this and partly this separated by the wall. But heat is transferring from the bulk of this fluid to the tube wall and from the tube wall through the other side of the tube wall into this. So, there you have what is known as a conjugate heat transfer problem.

So, in such cases the conduction through the walls needs to be considered by an equation like this. It is essentially the same as the equation for the fluids except that this term is 0, and this term is 0. And s here refers to an arbitrary source term there may be some heat generation terms which may be present, for example, due to a chemical reaction or due to some friction or due to some nuclear reaction that may be going on. So, in the absence of this then t this will give us the equation for transient heat conduction.

So, conjugate heat transfer involving convection the fluid and conduction the adjacent wall is handled through the boundary condition at the interface, such that minus k gradient of temperature within the solid, which represents the heat flux from the solid side of the wall and that must be equal to k d minus the gradient on the fluid side of a wall.

And so these two fluxes must be the same at the interface which is separating the solid liquid interfaces solid fluid interface. So, this becomes a boundary condition for the solids and this becomes the boundary conditions for the liquids. We do not know exactly what this wall temperature is, but it can it eventually be found out when we solve the entire problem involving convective, conductive heat transfer within the fluid and conductive heat transfer within the solid. So, we will come back to these in a tutorial problem.

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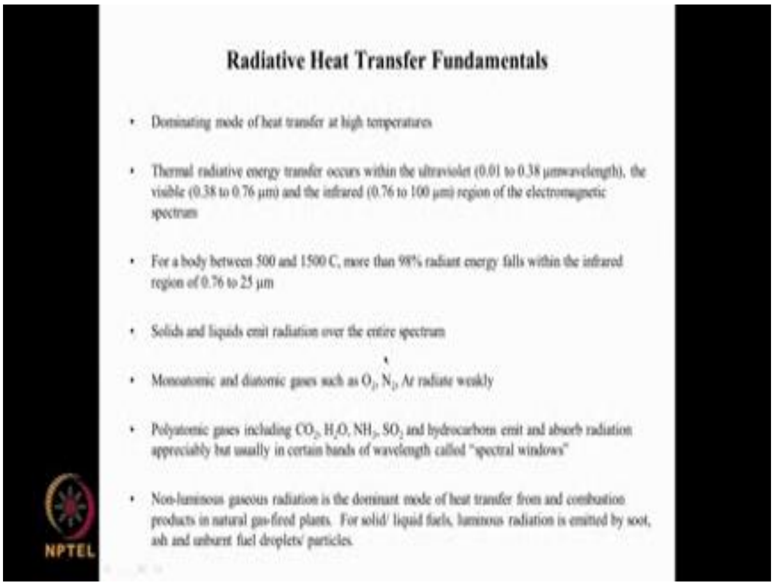

## Radiative Heat Transfer Fundamentals

- Dominating mode of heat transfer at high temperatures
- Thermal radiative energy transfer occurs within the ultraviolet (0.01 to 0.38 μmwavelength), the visible (0.38 to 0.76 μm) and the infrared (0.76 to 100 μm) region of the electromagnetic spectrum
- For a body between 500 and 1500 C, more than 98% radiant energy falls within the infrared region of 0.76 to 25 μm
- Solids and liquids emit radiation over the entire spectrum
- Monoatomic and diatomic gases such as $O_2$, $N_2$, Ar radiate weakly
- Polyatomic gases including $CO_2$, $H_2O$, $NH_3$, $SO_2$ and hydrocarbons emit and absorb radiation appreciably but usually in certain bands of wavelength called "spectral windows"
- Non-luminous gaseous radiation is the dominant mode of heat transfer from and combustion products in natural gas-fired plants. For solid/ liquid fuels, luminous radiation is emitted by soot, ash and unburnt fuel droplets/ particles.

Now, of the two modes of the heat transfer convective heat transfer and conductive heat transfer nothing more needs to be done from a CFD point of view. We need to identify the domain, we need to identify the equation the extra energy equation, and bring out the essentially the specific heat and thermal conductivity of the liquid of the fluid as a special properties that need to be specified. And we also need to specify thermal boundary condition at the surfaces which are bounding this flow domain computational domain.

When you come to radiative heat transfer, we have an entirely different based; usually radiative heat transfer is not taught that much in our under graduate curriculum. So, I would like to just put in some simple facts related to radiative heat transfer for us to appreciate what is the important about it. So, this is the dominating mode of heat transfer when the temperature is high. For example, in the furnace, where the temperature can be 1000 degrees, 500 degrees, 1500, 2000 degrees centigrade. And at such high temperatures, radiative heat transfer is the dominant mode of heat transfer and unless this is done properly, we would not get a proper temperature distribution. So, in such cases radiative heat transfer become an important issue to be tackled.

Now, thermal radiative energy transfer occurs within the ultra violet the visible and the infrared region of the electromagnetic spectrum. So, we are looking at 0.01 micron to 100 micron region, and I would like to bring out the very thin spectrum which is 0.38 to 0.76 micron within which the visible light comes and then the infrared region goes from 0.76 to 100 micron.

So, it is a very wide spectrum for the infrared region and we have fortunate to have eyes which are sensitive to this very thin band through which sunlight most of the sunlight energy is received. So, anyway that is beside the point. For a body between 500 and 1500 degree centigrade more than 98 percent of radiant energy falls within the infrared region of 0.76 to 25 minus micron. So that means, that within this range 98 percent of the total radiant energy that is being emitted by body at temperature which are up to 1500 degree centigrade or less, then most of the energy is going to in the infrared region and that spectrum needs to be resolved.

Solids and liquids emit radiation over the entire spectrum, mono atomic and diatomic gases such as O 2, N 2 and argon, radiate weakly, they do not participant significantly. Polyatomic gases including carbon dioxide, steam, ammonia, sulphur dioxide, and hydro carbons emit and absorb radiation appreciably, but usually in certain bands of wave length called spectral windows.

So, why are these gases important? Because when we are dealing with heat transfer applications like furnaces, then and if we are looking at hydrocarbon fuels then the carbon gets oxide to carbon dioxide, the hydrogen in the hydro carbon fuel becomes steam. And under certain conditions, we can have ammonia produced, and sulphur dioxide may be produced, if sulphur is present in the fuel. So, these are also special gases which participate in the radiative process, whereas oxygen nitrogen and argon these are essentially transparent, they do not participate, they let the radiation go through without interacting with the protons that are passing through

But and these are also special kind here, because only in certain bands of wave length do these gases absorb and radiate and reradiate and scatter and interact with the radiation. And whether this is the role of these things in the role of radiatory transfer significant a significant depends on the temperature. And whether there is significant amount of radiation in which these bands are the spectral windows of these gases are important. So, non-luminous gases are the dominant mode of heat transfer, and combustion products from combustion production in natural gas-fired plants. For solid liquid fuels, luminous radiation is emitted by soot ash and unburnt fuel droplets and particles.

Again when we are looking at furnaces, we can have gases fuels, and we can have solid fuels and liquid fuels. So, the solid fuels and liquid fuels are typically heavy hydro carbons, so that means that in the process of combustion they go through lots of chain reactions and produce intermediate compounds, and they may also have more of inert materials inert solid materials which comes out as in the form of hash.

And the structure the fuel may be sufficiently complex that all the fuel is not released nor all of it is burnt, so you have unburnt fuel droplet or particles. And these small sized particulate substances are can have a significant role in radiative heat transfer, and they can be like bright sparks, they can produce luminous radiation, whereas gases are essentially non-luminous radiation.

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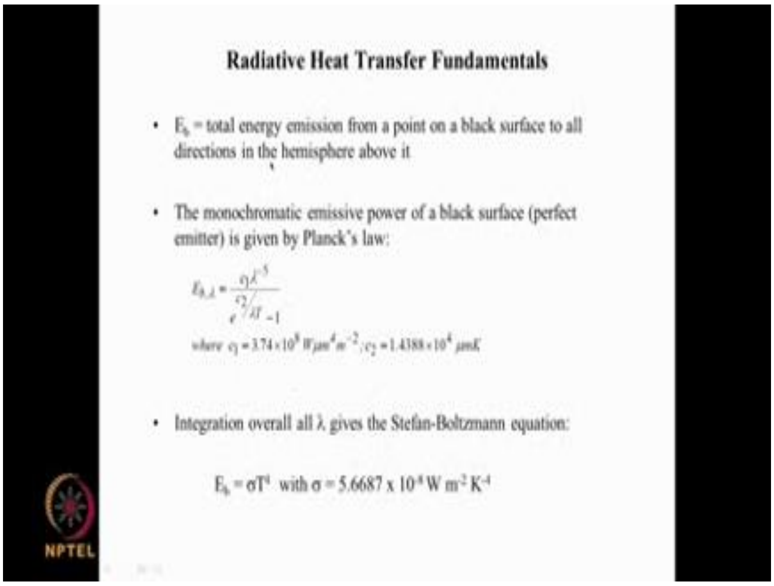

Radiative Heat Transfer Fundamentals

- $E_b$ = total energy emission from a point on a black surface to all directions in the hemisphere above it
- The monochromatic emissive power of a black surface (perfect emitter) is given by Planck's law:

$$E_{b,\lambda} = \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T} - 1}$$

where $c_1 = 3.74 \times 10^8\ W\mu m^4 m^{-2}$; $c_2 = 1.4388 \times 10^4\ \mu mK$

- Integration overall all λ gives the Stefan-Boltzmann equation:

$E_b = \sigma T^4$ with $\sigma = 5.6687 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$

Every hot body produces emits radiation according to the Planck’s law, where the energy see we have E b is the total energy emission from point on a black surface to all directions within the hemisphere about it. And so monochromatic, so that is the given wavelength emissive power of a black surface or that of a perfect emitter, these are all radiation heat transfer language (Refer Time: 28:48) is given by the Planck’s law where you have two constants with which huge numbers. But you have lambdas which are in microns, so the lambda is very small here. And so integration over all wavelengths gives the Stefan-Boltzmann equation. It is the total radiation radiant energy emitted by a body at a certain temperature T is given a various as T to the power 4, so the higher the temperature the more will be the radiant energy.

It is true of even conduction convective heat transfer, but there the amount of heat flux is proportional to the temperature difference. And here it is two the power of 4, T to the power of 4, so that means, that this can be this can increase rapidly. And fortunately we have the Stefan-Boltzmann constant which 10 to the power minus 4, so this becomes significant only when temperature is pretty high so at high temperatures because of the dominant mode of heat transfer.

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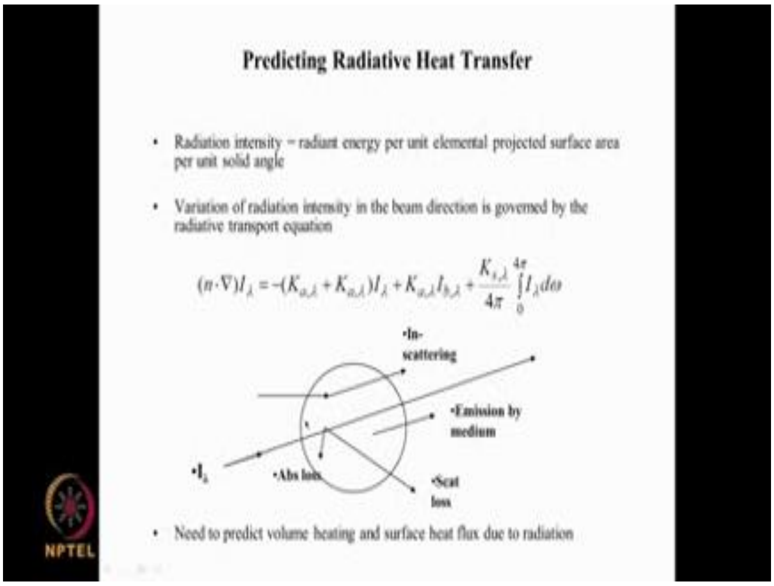

Predicting Radiative Heat Transfer

- Radiation intensity = radiant energy per unit elemental projected surface area per unit solid angle
- Variation of radiation intensity in the beam direction is govened by the radiative transport equation

$$(n\cdot\nabla)I_\lambda = -(K_{a,\lambda}+K_{s,\lambda})I_\lambda + K_{a,\lambda}I_{b,\lambda} + \frac{K_{s,\lambda}}{4\pi}\int_0^{4\pi} I_\lambda d\omega$$

- Need to predict volume heating and surface heat flux due to radiation

So, if you want to predict radiative heat transfer considers a beam of photons which is passing through medium here. And this medium can be a solid medium, it can be gases medium, or it can be a liquid, or it may be gases medium with lots of particles ash particles, soot particles, drops or different sizes and all of these can interact with the photons. And they can either absorb them, so that that particular photon does not come out, and the energy associated with that photon is taken away by the absorbing medium, this medium so that it becomes hotter. Or it can scatter because this photon is going in straight line here it can be deflected out in a different direction, so that this is not accounted for in this particular beam.

Or the medium can scatter other beams in the direction of the same beam here, so that you can have in scattering or out scattering; while in scattering increases the intensity of this radiation as it is going, out scattering will decrease the radiation. And the medium itself because of its temperature and because of the Planck's law and all that it will be emitting radiation in a certain wavelength of a certain length as per the Planck's law. So, all these things together can be written in the form of an integral of differential equation.

So you have differential part is coming from this gradient here and then the integral part is coming from the integration of the scattering kernel. And then you have absorption here. In scattering, out scattering, and then emission from the medium, and absorption all these thing will come out in the form of an integral of differential equation. So, this equation is the very different from the equation that we have considered and we have to solve this in order to get a proper treatment of radiative heat transfer, so that there are number of methods that have been developed and these are very different from the usual CFD type of solution methods. Under specialised cases, these can become like that C F D kind of things,, but that is very rare.

So, you have very special methods that are used to solve this equation. And if you want to do the radiation heat transfer properly unlike in the case of conductive and convective heat transfer, we have to solve an extra equation which is extremely difficult to solve in the conventional CFD framework. So, you have to do this and only then you can get proper treatment of heat transfer in high temperature flow situations as in the case of furnaces and boilers and those kinds of things we have to consider this.

So, in the next class, we will look at mass transfer. What we have done about conductive heat transfer, convective heat transfer, all this is sufficient for us to deal with conventional applications involving essentially low temperature, ambient temperature, heat transfer applications.

We will look at how we can deal with mass transfer and what more is required for the mass transfer, because that is also an important aspect when we dealing with industrial applications. Then we will see what kind of problem formulation is needed to deal with mass transfer. We have seen that when we want to deal with heat transfer, we have to bring in the energy equation, and we have to bring in the temperature from that. Similarly, what do we from the mass transfer that is what we will do in the next class.