Computational Fluid Dynamics
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Lecture – 17
Energy Conservation Equations

Today we have the first lecture of the 4th week of this particular course, in which we are going to look at additions to the navier stokes equations for practical applications and the first of this is the Energy Equations or the Conservation of the Energy. We can derive this conservation of energy just as we have derived the conservation of linear momentum and conservation of mass.

So, if you take our control volume box having a length of delta x, delta y, delta z in the three directions then, the conservation of linear momentum and mass by applying a corresponding statement to our control volume box having length of delta x in the x direction, delta y in the y direction, delta z in the z direction.

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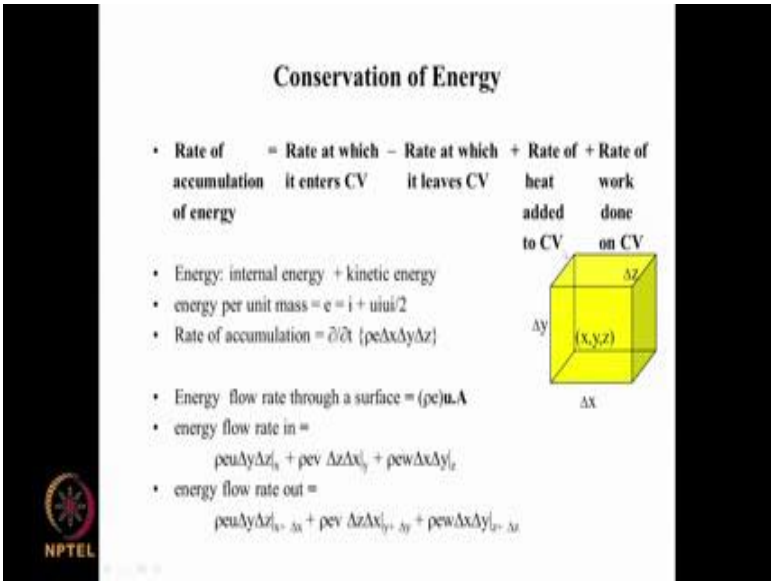

And the statement that we make about the conservation of energy is that the rate of accumulation of energy possessed by the fluid, within this control volume is equal to rate at which energy flows into the control volume minus the rate at which it leaves the control volume plus the two process by which the energy content within the fluid within the control volume can change. One is the rate of heat addition and the rate of work done on the system.

So, in the case of mass we did not have any sources by which the mass can be changed. Since we are neglecting nuclear reactions and we are neglecting the relative considerations. In the case of momentum, we said that if we have external force acting on it then that could be a potential way of changing the momentum which is processed by the fluid within the control volume. In the case of energy, we are identifying two extra factors to what is coming in and going out which can change the energy content of the fluid within the control volume. This is heat addition and work done on the system.

So, work done by external forces is acting on the system and heat added to the system from outside, both of these will lead to increase in the energy content. So, heat is added to the system then the accumulation of an energy take place that is why we have it as a plus and work is done on the system that increases its potential energy or kinetic energy or some of those things and that will lead to an accumulation of energy. So, all these process can lead to change of energy possessed by the fluid within the control volume and that is what this viable statement is here.

There what we mean by energy? We mean the internal energy and the kinetic energy. We are not considering potential energy because potential energy that we are dealing with in the absence of any other forces is that arising from gravitational energy, and the gravitational head or gravitation induce potential energy is already going to come through in the form of rate of work done by external force acting on this because we identified gravity as one of the external forces. So, that will be included in this term and so that is not included in the energy term itself.

So, you have internal energy which is usually defined as i in this it is a specific internal energy per unit mass and kinetic energy is uiui divided by; half of uiui. So, that is half of u square plus v square plus w square where u v w, are the velocity components in the Cartesian coordinate system which we are following here. And these two small i, the specific internal energy and u square by two is per unit mass. So, when you multiply by rho times volume then you get the total energy that is contained.

So, if you multiply by rho times and the sum of these things we are denoting as the small e here that is the total energy per unit mass possessed by the fluid element. So, that time by energy here we mean internal energy and kinetic energy because potential energy is coming is attributed to gravitational forces. Gravitation force is something that we identified as external force, so that will be accounted in the rate of work done term when we valuate, which will do shortly. So, we are only considering internal energy which is associated with the temperature of the fluid and kinetic energy which is associated with the velocity of the fluid.

So, kinetic energy is half u square plus v square plus w square and that can be expressed in index notation as uiui, where i is a subscript and a this i is a internal energy associated with the temperature example - i is cv times dt where or t minus t defines a, that can be one definition in terms of the specific heated constant value.

So, now we can define with this notation for the small e which is the total energy per unit mass we can write the rate of accumulation term of energy as the rate of change of the total energy possessed by the fluid within the control volume. So, this is it; the total energy per unit mass times density times the volume will give us the total energy possessed by the fluid within this control volume. The rate of change is the rate of accumulation if this is positive its accumulation, its negative it is a decrease.

Now this, whether it is a decrease or increase depends on the contribution of all the other terms that are coming here, so as we have defined earlier the energy flow rate through a face is the volumetric flow rate times density times the energy per unit mass. So, rho e times u dot a, and since we have taken the surfaces of this box has been perpendicular to the three coordinate directions, the definition of u dot a - is nothing but the each velocity component and the area of the face through which the fluid will be entering with that particular velocity. So, for example, for this particular box we have the left face and the bottom face and the back face of the through faces, through which the flow is coming in bringing around with it and energy content of small e per unit mass.

Therefore the energy flow rate into the control volume is density times energy content per unit mass times the velocity times the area. So, u this is the u dot a and this is the energy content of the fluid that is coming in per unit mass and per unit volume and that gives you the total energy of flow rate coming in through the left face. And similarly through the bottom face and through the back face, because these are defined at x - which is this one, and at y - which is this particular thing, and z - this is the back face because we fixed a origin at this particular corner.

The energy flow rate out is through the other 3 faces that is through the right face, through the top face and through the front face. So, these fluxes, these energy out flow rates can be worked out as rho e u times delta y delta z at x plus dx which is the right face at y plus dy which is the top face, but here we have to multiply v times the corresponding area and through the front face we multiply the w times the corresponding area here, to get the energy flow rates out of the flow domain. So, the difference between the two is what is going to lead to the rate of accumulation here in the absence of the heat and the work done.

But these are two processes by which the energy content of the fluid can change and we need to have expressions for these.

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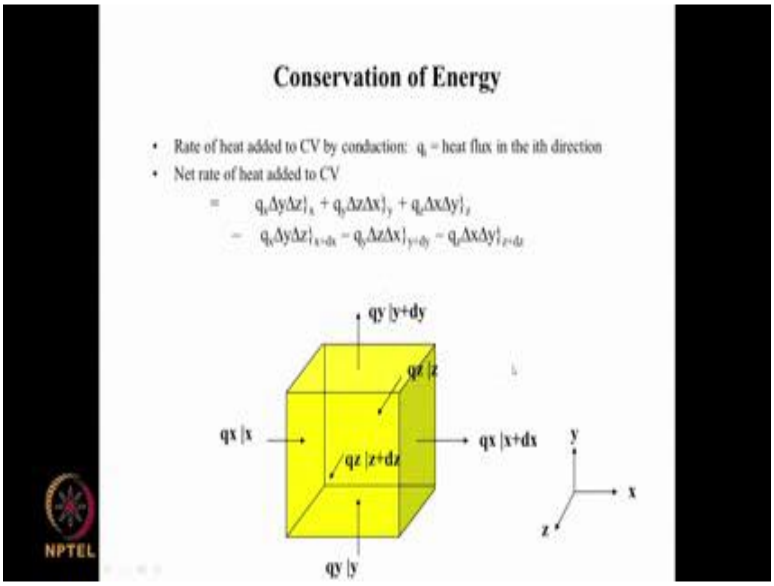

For the heat we can consider a heat flux. So, heat flux is something that is passing through the face and so in the ith direction we identify the heat flux as qi, this is the heat per unit surface area that is, watt per meter square. So, you can say that rate of heat added to the control volume by conduction. So, this is part of the fluid, it is a small part of the fluid continued, and adjacent to that you have another fluid element or control volume. If there is temperature difference between the two then they can be conduction. So, that conductive heat transfer is what is being considered here.

So, just by the fact that there are temperature differences you can have heat conduction, just as in the case of concentration variation they can be mass flux. Here you can also have heat flux associated with temperature differences and that heat flux is given therefore, the corresponding coordinate direction as a subscript here. And the rate of heat added to the control volume is what is coming in minus what is going out, and we are defining the heat flux to be positive always in the positive x direction. So, at this point qx positive means that it is coming in and at this point on the right face qx positive means it is going out.

That means, through the right face heat is going out by conduction and through the left face it is coming in through conduction. What the actual heat flux is? Whether it is in this direction or the other direction depends on the temperature variation, but as if now as per the notation that we are following that is qi is the heat flux in the positive ith direction.

So, using that we can say through the right face, heat is leaving by an amount which is the value of the heat flux times the surface area here. So, if you now consider the heat added to the system is through the x face qx at x times delta y delta z, through the bottom face it is q heat flux in the y direction on the yth face multiplied by delta x delta z, and through the back face it is q in the z direction that is what is coming in through that particular face times the face area here, and what is going out through the right face is given by qx times the corresponding face area at x plus delta x and similarly at y plus delta y z plus delta z.

So, this identifies for us what is a heat flow in minus flow out by conduction through the 6 faces of this control volume.

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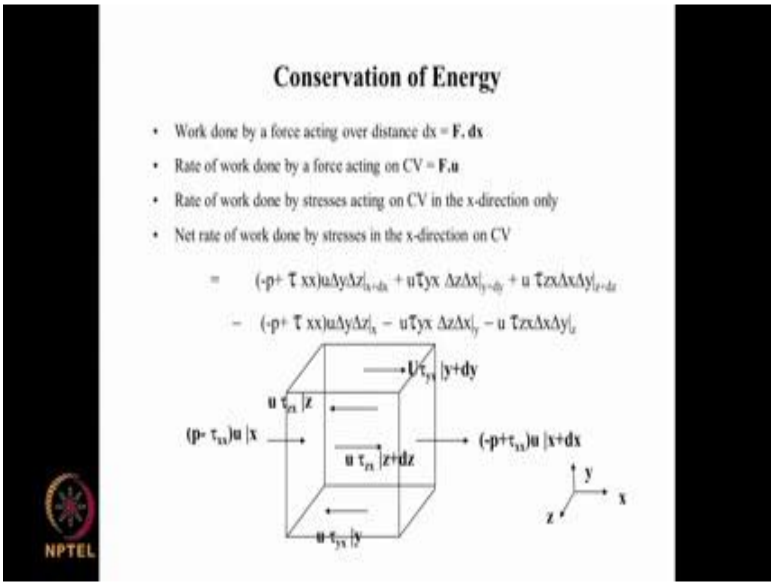

And similarly we can consider the work done by forces acting on it and we have slightly more complicated situation here, but the basics are well understood. Here we looking at work done by force acting over a distance dx is defined as f dot dx. If the force is acting in a particular direction and the displacement is also in the same direction then its fdx, otherwise in the general case its f dot dx. If you have force acting on the control volume the rate of work done by that particular force is f dot dx by dt and therefore, f dot u.

And the forces that we have considered here, the external forces that we have considered on the control volume are the gravitation force and the surface stresses, the stresses that are acting on it. So, the gravitational force component in the u direction will give us the work done by that and we also have identified pressure as one of the forces acting, stresses acting on this, so the pressure times the corresponding area times the velocity component, similarly the stresses times the velocity times the corresponding area will give us the work done.

So, in order to make it systematic here we are considering here the work done by the stresses acting in the x direction alone. So, what we have in this figure are the stresses acting in the x direction here and for example, through the right face you have tau xx and you also have minus p that is compressive force, compressive pressure at this particular point, so this x plus dx. And through the top face in the x direction there is no compressive force; compressive force would be in the negative y direction. The only stress which is acting in the x direction is tau yx at y plus dy. Similarly on the bottom face there is no pressure force there is only tau yx and its acting in the negative direction because it is in the negative face here.

Similarly through the back face you have tau zx which is acting in the negative x direction and through the front face you have tau zx at z plus delta z acting in the positive x direction. Each of this is acting in the x direction, so this has to be multiplied the u velocity component. So, that is what we have done here.

And we have to take the dot product of the force with the velocity component and we can see that on the right face we have this is in the positive x direction. So, this times u. So, u times the area delta y delta z times minus p times tau xx at x plus delta x is the total work done by tau xx and the compressive force, it is a rate of work done by the stresses acting on this and on this I think you have negative thing here because tau xx is acting in this direction, so we have a corresponding negative term here. And here tau yx is acting in this direction and here it is acting in the negative direction, so you have u times tau yx times the corresponding area delta z delta x at y and here you have with the plus sign at y plus, the same quantity y plus delta y.

Similarly on the front face tau zx is acting in the positive z direction. So, it is coming out as u times tau zx times the area delta x delta y at z plus dz. On the back face you have the same terms with the negative face, negative value evaluated at the corresponding face location z.

So, these are the 6 terms plus 2 pressure terms, each multiplied by the velocity and a corresponding area which are contributing to the rate of work done by the stresses acting in the x direction. Similarly you have stresses acting in the y direction, there are another 6 surface stresses and 2 pressures. So, that is another 8 terms which will be coming because of the stresses acting in the y direction and another 8 terms coming from stresses acting in the z direction. So, all these 24 terms will constitute the rate of work done by the external forces which we have identified as a pressure and the viscous stresses and we also have the gravitational force which is g dotted u.

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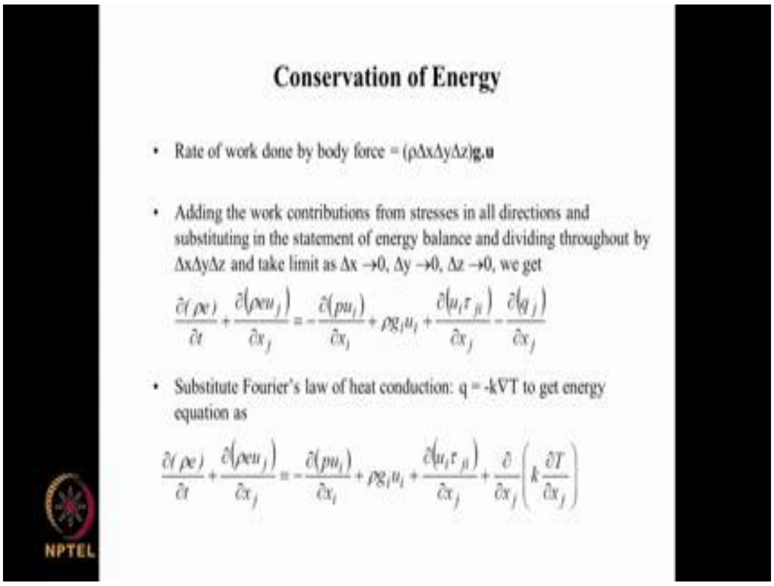

So, gravitational force is mg, mg dot u will give us the gravitational force. So, this has again 3 components and the 3 directions. So, all these things put together, so that is the flow rate terms, the accumulation term, the 6 terms associated with the heat fluxes, and the 24 terms associated with the surface stresses, and the 3 terms associated with the gravitational force rate of work. All these things will contribute to the conservation of energy and adding all these work contribution terms from stresses in all directions and you put that into the conservation statement of energy balance and as usual take divide by the delta y product of delta y delta x delta z and take limit as each of these tends individually to zero.

And if you do that and then if you work out the index notation then you have this particular equation here - which has dou by dou to t of rho e plus dou by dou xj of rho uj times e equal to minus dou by dou x i of pu i minus rho g i u i plus dou by dou xj of ui tau ji minus dou by dou xj of qj. We will write down these formulas in a tutorial later on, but this is the total energy equation and we can see that the energy equation has contribution coming from the flows because of the flow there is a net input and net out flow is what is given by this.

The pressure force is also creating some work done and gravitational force is creating rate of work done contributing to energy and this is what will be the contribution the potential energy here and then you have the viscous forces that are leading to changes and then you have heat fluxes heat transfer.

So, in the special case where there is no heat transfer, temperature is constant you do not have this term and in the special case where viscous stress are zero then you do not have this term, but in the general case we have all these terms contributing to this and at this point we need to specify what this heat fluxes are. Since we are considering heat conduction we can substitute the empirical form of the heat conduction equation which is the Fourier's law of heat conduction which is q equal to minus k gradient of t.

So, this qx means that delta gradient of x in the x direction y direction z direction like that. So, qx will be equal to minus k dou t by dou x, qi will be equal to minus k dou t by dou y, once you substitute all that, you have the final form of expression - the conservation of energy equation like this dou by dou t of rho e plus dou by dou xj of rho u j e equal to minus dou pu i by dou x i plus rho g i u i plus dou by dou xj of ui tau j plus dou by dou xj of k dou t by dou xj.

So, this is a general form of a the expression and when you look at this what are the terms that are new here, rho is already there in a navier stokes equation velocity is already there in a navier stokes equation this e here consists of a contribution from velocity which is already there. And the internal energy for which we need to have the specific heated constant volume. Pressure is already there in the previous expression and velocity is already there, g is a known constant and u is, this term is known here and in this term we have the stresses, but stresses are already known from the momentum equation so there is nothing new in this and here we have thermal conductivity and temperature as the new variables.

So, you have in this, 2 new variables that is the internal energy which is coming here and the temperature variation. Usually internal energy is related to the temperatures through the specific heat at constant volume, so that is a known material property. So, if you know the energy equation brings in a new variable temperature which is changing within the domain, and it also brings in 2 new material properties - the thermal conductivity and the specific heat constant volume, all the other terms in this are known quantities. So, this is a statement of energy equation the general case, there are other forms in terms of enthalpy, in terms of a mechanical energy, all those things are there.

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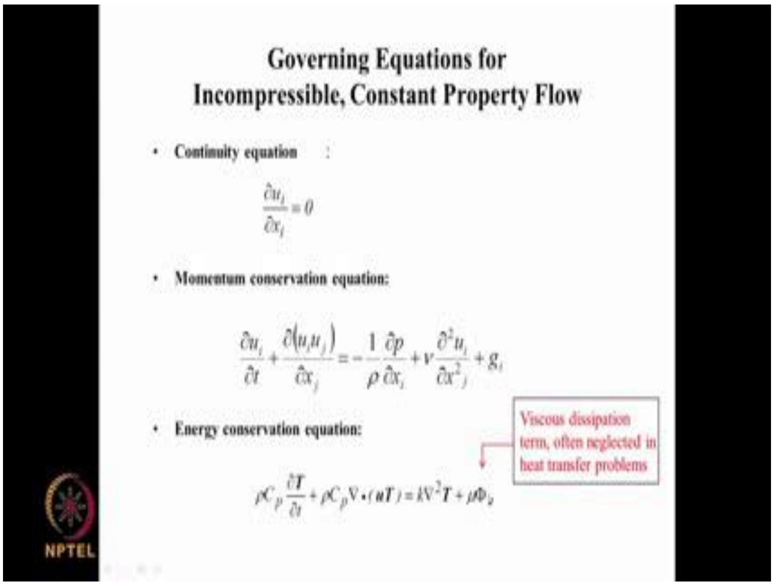

But let us consider a simplified form for our analysis in this particular course. So, constant property including specific heat and thermal conductivity, incompressible flow, constant viscosity for all those things we have a simplified equation for continuity dou ui by dou x equal to 0. And we have the simplified form or momentum equation which is a relatively simple compared to what we would have in the more general case and energy equation also reduces to a simple form, it can be put in this form in terms of specific heat at constant pressure not at constant volume. So, this is like the enthalpy here.

So, this is rho cp times dou t by dou t. So, that is temporal variation of temperature plus rho cp times dell dot ut. So, this is advection term equal to k dell square t plus mu times the viscous dissipation term which is this term divided by mu. So, this tau ji is expressed term of mu times dou ui by dou u xj, all that term is expressed in this particular form here. And when you are dealing with real heat transfer problems, where the heat fluxes is are strong and then you have a temperature variations associated with the flow and all those things are very strong, the contribution of viscous dissipation of energy is usually very small and its neglected. So, in such a case you can leave out this term completely and you have much simpler equation here.

So, in any practical cases you do not need to worry about a this particular thing as long as we are interested in velocity induced temperature variations and heat fluxes associated with conduction and all that, you do not worry about this.

But if you have highly viscous fluid like glycerol, petroleum, crude oil and all that, then maybe we will have to consider the viscous dissipation of energy arising out of the work done by the viscous forces. Otherwise this can be neglected; and we have simplified equations which describe the flow of an incompressible constant property flow like this. For compressible flow you have additional things which we will see later on.

So, this is an end of this lecture. In the next lecture we will consider the implications of these in terms of practical situations that we come up with.