

**MATLAB Programming For Numerical Computations**  
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**Module No. #06**  
**Lecture No. #6.4a**  
**Tutorial on Regression**

Hello since there have been several questions on regression linear or non linear regression on the forum I thought I would make a quick tutorial today. So this tutorial will take up 1 example and see what we mean by non linear regression and what we mean by linear regression okay.

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## Consider the Following

- To fit a model of the form:

$$r = \frac{a}{b+t} \text{ to the following data}$$

- $t = 10 \quad 20 \quad 30 \quad 40 \quad 50$
- $r = 0.29 \quad 0.22 \quad 0.18 \quad 0.15 \quad 0.13$

So, the example that we take is let us say the model is  $r = a / b + t$  okay.  $a$  and  $b$  are the things that are unknown and the data is collected through experiments for the values of  $r$  for different values of  $t$ . So these are the values of  $t$  and these are the corresponding values of  $r$ .

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## Using Nonlinear Regression

- To fit a model of the form:

$$r = \frac{a}{b+t} \text{ to the following data}$$

$$t = 10 \quad 20 \quad 30 \quad 40 \quad 50$$

$$r = 0.29 \quad 0.22 \quad 0.18 \quad 0.15 \quad 0.13$$

- The above model is nonlinear in a and b → `lsqnonlin`

Now as it will it might be obvious by looking at this okay. Right at the outside, this particular functional non linear in a and b. Because it is non linear in a and b, we need to use non linear regression method such as `lsqnonlin`. (Video Starts: 01:05) So let us go to MATLAB and solve this using `lsqnonlin`. fun okay now what does this function.

What does this function supposed to return? So for that lets do help `lsqnonlin` okay. What we want to do in non linear or linear least squares is find out the values of a and b such that the error between this data and this model is minimized okay. If we see the help of `lsqnonlin` let us see where it is, what it says is `lsqnonlin` attempts to minimize the square of fun.

So what should fun return? fun should return the errors so that a and b that minimized the sum of square errors are formed. So function `fErr = lsqFun` and what should be the argument. We want to find out the phi's we want to find out a and b for which this value is minimized. So let us write down the data. The data is `t=10, 20, 30, 40 and 50` okay.

That the first thing I have said multiple times in this course is when we have a vector we want those vectors to be column vectors. So multiple numbers of rows and single column. So, therefore as return t and r we are going to transpose. Next is, get the parameters from the phi. So get parameters okay. a is `phi1` and b is `phi2`. Compute the error. In order to compute the error we first need to compute r model and r model is given by this equation.

So rModel is given by  $a / b + t$  okay.  $a / b + t$ . Now  $t$  is a vector and therefore we need to do this as  $./ b + t$ . And our error which is fErr is nothing but the difference between  $r$  and rModel okay. And we save this and we are done with the function for lsqFun okay. Next what we want to do is say make out regress solve regressionSolving okay.

This is going to be our script. So for solving using lsqnonlin okay. So what do we need in order to solve using lsqnonlin we want we need the data and we want the initial values. So let us say the initial values were 10 and 10. So that is our initial values and our final solution is going to be equal to lsqnonlin okay, function and phi0. So the function is going to be @ phi, (@p lsqFun (p), phi (0)) okay.

And when we do that that is all we need to do we don't need to do anything else. We run this and we are get we got these values of phi. So, if we write down the phi, phi is 9.4 and 22.4. Now we can also solve this using linear regression but not in the form that it is written.

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### Can this be Converted to a linear regression form?

- To fit a model of the form:

$$r = \frac{a}{b+t} \text{ to the following data}$$

- $t = 10 \quad 20 \quad 30 \quad 40 \quad 50$
- $r = 0.29 \quad 0.22 \quad 0.18 \quad 0.15 \quad 0.13$
- Let us invert the above model:

$$\frac{1}{r} = \frac{b}{a} + \frac{1}{a}t$$

- This is linear in parameters  $b/a$  and  $1/a \rightarrow$  Linear least squares

In order to use linear regression it has to be converted into a form which is linear in the parameters. To do that we can invert this model and we will get if we represent  $y=1 / r$  then we will have  $y = a_0 + a_1 * t$  okay or rather  $a_1 * x$ . So linear least squares so our  $y$  is  $1 / r$  and our  $x$  is  $t$ . So our  $x$  is just  $t$  okay.

What did we do in linear regression is we created a matrix  $x$  okay. For in the case of  $y = (a_0 * 1) + (a_1 * x)$ . We created the matrix  $x$  was ones  $(n, 1)$  okay that represent this. And  $x$  that was our  $x$  matrix okay. So we are going to make an  $x$  matrix in a similar way. Our  $x$  matrix is going to be similar ones  $((n, 1), x)$ . We need to also specify what  $n$  is going to be. And  $n$  is going to be length  $(t)$  okay.  $\phi$  for linear is going to be equal to  $(x^T x)^{-1} * x^T y$ .

That is going to be our  $\phi$ . Once we get that  $\phi$  we can calculate the values of  $a$  and  $b$  okay.  $a$  is nothing but  $1/\phi^2$ .  $a$  is 1 and what is our  $b$ ?  $b$  is going to be  $\phi - a$ . So let us save this and let us run this okay. There is an error on line number 10, column 22 okay we have forgotten to put this multiplication sign. So  $(x^T x)^{-1} * x^T y$ . So let us save this now let us run this undefined variable  $r$  okay.

That is because we have not taken this data so let us copy this data and paste it over here. A linear least squares and for solving using `lsqnonlin`. So now hopefully this should work. Linear transformation and obtain original parameters. Let us save this and let us run this okay. And we will see our  $\phi$  is 9.4 and 22.4. Our  $a$  is again 9.4 and our  $b$  is 22.6. (Video Ends: 08:39)

So what we have done is solve the same problem using non linear regression as well as using linear regression so hopefully this was helpful for you thanks.