MATLAB Programming for Numerical Computation
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Module No. #08
Lecture No. #8.3
Practical aspects of ODEs - Method of Lines for Transient PDEs

Hello and welcome to MATLAB programming for numerical computations. We are in module 8 and this module, we are covering practical aspects of solving ordinary differential equations, initial value problems. Specifically, we have been looking at odd odes with multiple variables. In both the lectures, we had higher order odes initial value problems, which were converted into a set of first order odes.

In today's lecture, what we are going to cover is, how we can convert partial differential equations, which are partial differential equation in time and space into ordinary differential equations in time, using a method called method of lines. So, this was an example which we had covered in lecture 4.5. In lecture 4.5, we had covered this in context of tri diagonal matrix algorithm.

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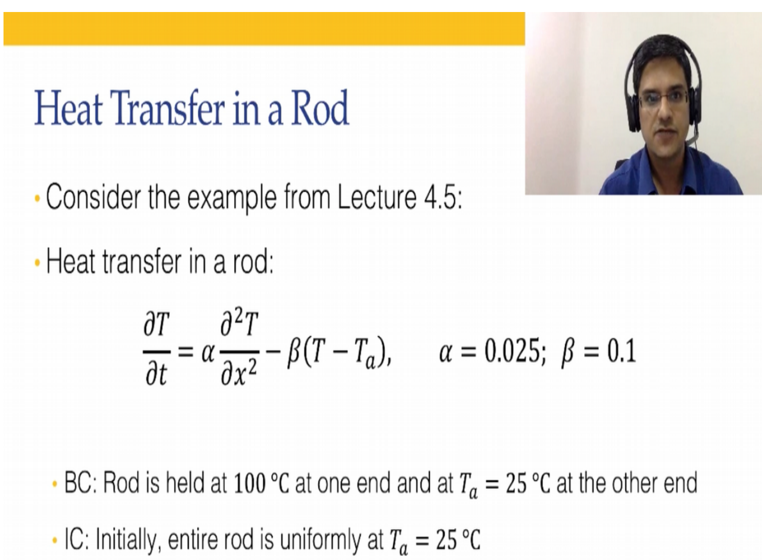

In that case, we had considered a steady state solution, in which dT /dt was equal to 0. However, in case of transients, the model is going to look like this. So, consider, that we have a rod which is 1 meter long. Let us say, that the rod initially is at the room temperature, which is 25 degree Celsius. At time 0, we start heating one end of the rod and maintain that end at 100 degree Celsius.

The other end, is maintained at room temperature of 25 degree Celsius. In addition to this, the rod is also heat, losing heat to the surroundings. The heat loss is based on this particular term, the heat conduction is based on this term, and overall the transients, how the temperature goes from 25 degrees Celsius uniformly in the rod to the final temperature profile with 100 degrees at one end and 25 degrees at another end, is defined by this partial differential equation.

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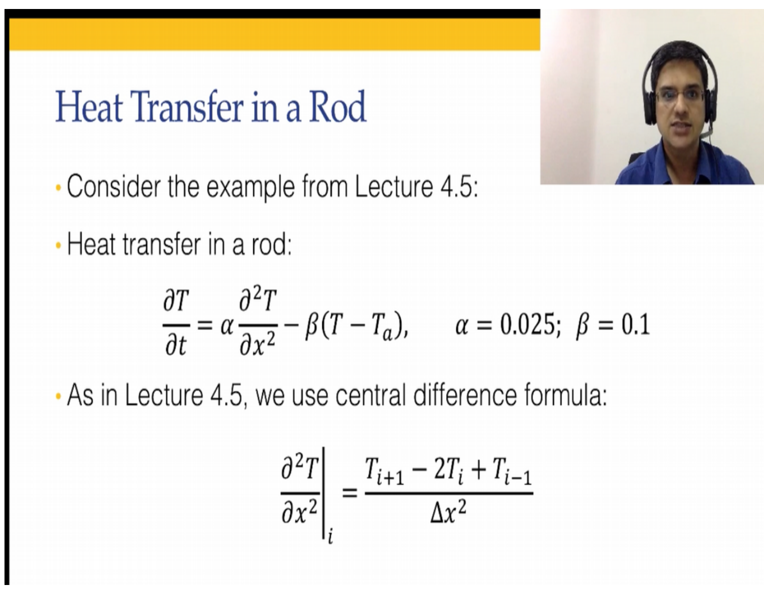

Now, how to solve this partial differential equation? One way to solve this is, what is known as method of lines, as we had done in lecture 4 .5. In method of lines, what we are going to do is, use central difference formula for differentiating in space. We leave the differential part in time as it is. So, let us say, we are going to take that rod, which is 1 meter long and divide it into 10 intervals. We are going to have temperature, going from that initial point of 100 to the final point of 25 in some way.

The central difference formula will result in at any point i d square t/ dx square is written in this form. If the 1 meter rod is divided into 10 zones, out delta x is going to be equal to 1 /10. If we have 10 zones or 10 intervals, we will have 11 nodal points. The first one is t= 100, the last one is going to be t= 25 and for the all middle points from t2 to t10 we are going to have the equation, this particular equation satisfied, once we substitute this guy over here.

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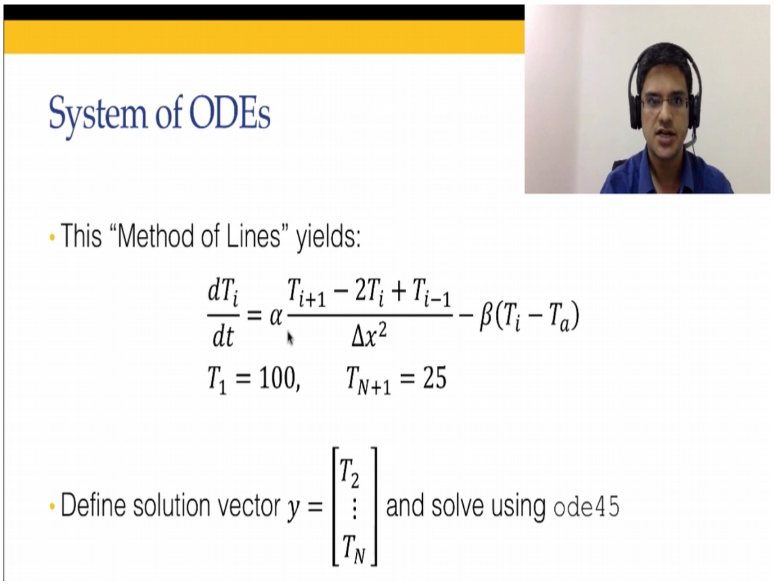

When we do that the method of lines is going to yield us dT/dt equal to this equation for all the center points and t1= 100 and t (n+ 1) = 25 at the 2 end points. Now what is changing with time is, ti for (t2: tn). So, what we are going to do is, define the solution vector y as t2, t3, t4 and so on up to tn. We do not need to solve for t1 or tn +1 because those values are already known to us. So let us go on to MATLAB and try to do this problem okay.

(Video Starts: 04:22) Edit, let us call this rod conduction, rodcon or rodConduc and function okay. As we have always been doing this, fval equal to file name rodConduc (t, y) okay. Getting temperatures, t1 is going to be equal to 100. We know that okay. We also need to find out, what n value is. So n is going to be nothing but length(y+ 1). Why is that so, because y goes from 2 to n. Okay, so the length (y) is n- 1 and therefore n is going to be equal to length (y+ 1). t2 to n is going to be equal to y and tn+ 1 is going to be equal to 25.

That ambient temperature, parameters, ambient temperature ta is 25 and tn+ 1. We will replace this as ta. So, now we have our temperature vector that is completely defined okay. Let us define dT /dt okay. dTdt is going to be equal to let us say zeroes (n+ 1, 1) okay. So, dT/ dt we are going to again define for all of the ts. However there is no way to define dT1/dt and dT (n+ 1)/dt. So we are going to initially make all of this but eventually we are going to discard dt1 and dT (n+1) okay. So, we will do that in a bit okay.

We also need to define various other parameters. The other parameters, that we need to define are alpha= 0.025 and beta= 0.1. We also need to get the step size h or delta x okay. If n is the number of intervals, the step size is length / n, 1/ n. Why because the length of the rod is 1 meter okay. So, now we have to do the final part and that is to define dT / dt and hence to define fval okay.

From i= 2: n, so, that is what we will write, so for i= 2: n. dTdt (i) is going to be equal to alpha multiplied by this guy, the central difference formula. So alpha / h^2 * T (i+ 1) - 2 * T (i) + T (i-1) okay. So, that is the first part, that is alpha / h ^2* T (i + 1) - 2*T (i) + T (i- 1) - beta multiplied by something. So, we will write that part also -beta *T (i) - Ta okay. We also need to define, alpha and beta which we have done already. We also need to define h that also has been done already okay. So, that should complete our definition of dT/ dt.

Finally, we want to extract fval from dT / dt and to do that, we will say fval is going to be equal to dTdt (2: n) okay. And that is our fval and thinks we have done over here. Let us save this okay. In order to run this, runRodProblem. We will create a script, to run transient heat conduction in a rod okay. Let us say, the number of intervals was 10 okay. Our initial values are going to be T (1) = 100, T (2 : n) = 25 and T (n+ 1)= 25.

We also need to ensure that is the column vector. So let us write this as T (1, 1), T (2: N, 1) and T (N+1, 1) okay. Let us call tSpan as (0:20) okay. And (tSol, TSol) solutions is going to be ode45 @ (t, y) and rod conduct (t, y). I think we should just put a space over here. We have tSpan and our T0. Let me just write this also, as T0, to ensure that, we know that this means our initial condition. Plotting results, so, what we want to do is, p let us plot tSol and ySol there are going to be nine lines over here, but that is actually fine no problem.

We will just do that. While our T0 goes from (1; n+ 1), our y0 actually should go from T (2: n) that was what our definition was. And this is the change that we need to make okay. And plotting our result is going to be this okay. So, let us run this and we get an undefined variable that is because the variable name that we have used over here. We have used the T0. So this also should be changed to T0. So, let us save this and run this okay.

So, the point that is closest, to the hot end has reaches a temperature of around 85 degree Celsius, after 20 minutes, the second point reaches a temperature of about 75 degree Celsius, some 62 degree Celsius so on so forth. So, this is the point closer to the hot end and this is the point closer to the cold end. So, this is how the temperature varies from the initial value of 25 degree Celsius to the final steady state value okay. (Video Ends: 12:46)

So, what we have done over here, is to use method of lines to convert a partial differential equation with transients, into an ordinary differential equation in time. And we have used ode45 to solve the resulting ordinary differential equations. (Video Starts: 13:05) Finally, what I am going to show over here is the power of ode45. So, let us say, instead of 10 intervals, if we were to divide the overall domain in 100 intervals, let us just plot everything.

Let us say, we want to plot the middle point okay. So, let us just do that or midpoint. Let me just say n/2. Let me just run this okay. It does take a little bit of time in order to solve, but finally it has solved the problem and you will see, this is how the temperature varies at the middle of the rod. If we want to check how the temperature varies at closer to the hot end of the rod. And we also want to see how it varies closer to the cold end of the rod okay.

So, we are going to plot these multiple lines and let us see okay. So this is how the temperature is varying closer to the hot end of the rod and this is how the temperature varies closer to the cold end of the rod. (Video Ends: 14:47)

So, with that we finish this example, of using method of lines to solve of partial differential equation. And indeed we come to the end of lecture 8.3. What we have done in this lecture, is taken a partial differential equation and converted it into a series of ordinary differential equations and solves the resulting ordinary differential equations using ode45. We also showed that, the ode45 is a fairly versatile method and it is has, the ability to solve a really large set of odes. With that, I come to end of lecture 8.3 and I will see you in the next lecture. Thanks and bye.