MATLAB Programming for Numerical Computation
Dr. Niket Kaisare
Department of Chemical Engineering
Indian Institute of Technology, Madras

Module No. #06
Lecture No. #6.4
Regression and Interpolation – Interpolation options in MATLAB

We are in module 6 where we are covering regression and interpolation. In lecture 6.1 we gave an introduction to regression and interpolation techniques. In lecture 6.2 and 6.3 we have covered linear and nonlinear regression techniques in MATLAB. In this lecture we are going to cover techniques for doing interpolation. Specifically we are going to talk of 2 interpolation techniques in MATLAB called spline and pchip interpolation okay.

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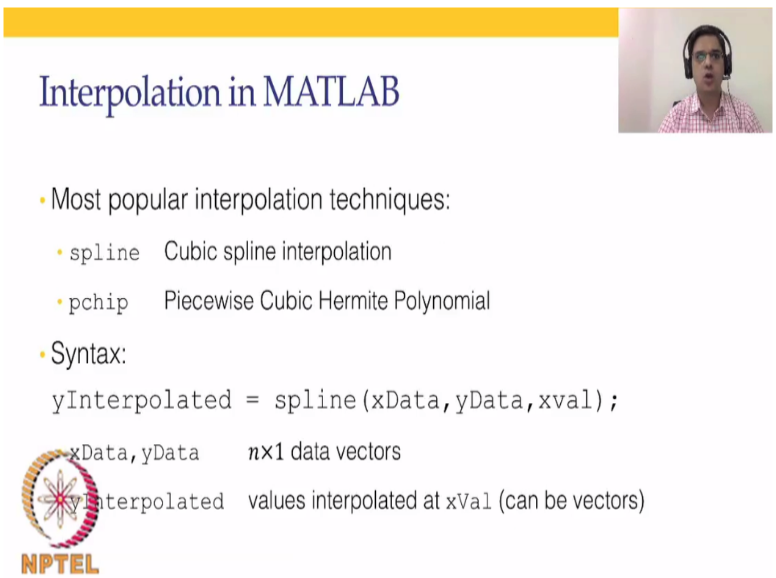

So what are these 2 interpolation techniques? Spline is known as a cubic spline interpolation. So that's the command that will use to obtain cubic spline interpolation. That's nothing but fitting the data with a cubic polynomial which satisfies certain properties so that we get an overall smooth curve that fits all the data that we have. The other option in MATLAB is known as Piecewise Cubic Hermite polynomial.

We will take 2 examples to show spline and pchip interpolation and we will use this example to demonstrate when or when is it advisable to use pchip interpolation. And in all other conditions we are going to use spline interpolation okay. The syntax for both spline and pchip are very similar to each other. The syntax for spline I am giving over here. All you need to do to run pchip is just to replace spline with pchip and you will be able to use pchip instead of spline.

The syntax is simple yInterpolated that means the output of the function spline is the interpolated dependant data that is given by spline (xData, yData). xData is that independent data, yData is the dependent data okay. And so this is the data that we have in with us. Now we want to interpolate between some x values between these original data points okay. And that is given by xVal. We will come to that example in in a little bit okay. So spline and pchip are 2 different interpolation techniques that we are going to cover in today's lecture.

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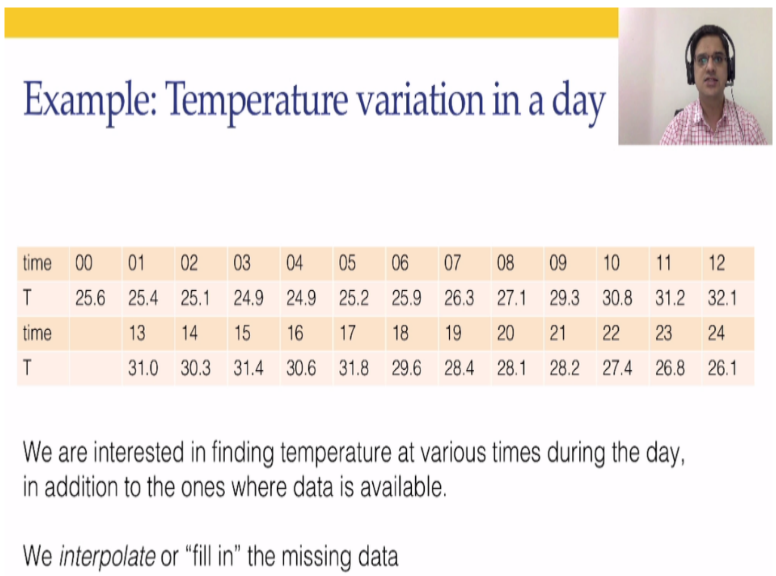

Let us consider the example of temperature variations in a day; this data was taken for temperature in Chennai on 30th of October. This starts with midnight of 29th of October and ends at midnight of 30th of October and we have temperature data in degree Celsius at various different times within the day on 30th of October.

So we have temperature data at 1 AM, 2 AM, 3 AM and so on. Let us say we wanted the data at 2:30 AM or 6:30 AM or let us say 10:15 AM and so on okay. In that case we do not have this data directly available from the website. So what we are going to do is, we are going to use an interpolation function in order to calculate the data at those intermediate points.

(Video Starts: 03:35) Let us go to MATLAB and see how that works okay. So I have already created a file called interDiurnalT to have to have the data for us ready okay. So let us go and check what the data looks like. So at time 0, the temperatures 25.6, at time 1 its 25.4 okay and at time 23 is 26.8 and time 24 is 26.1. So, 26.8 and 26.1 okay. Interpolating temperature values.

So let us say we want the hI, let us say that contains the hours. Excuse me; the hours, at which we want the values to be interpolated. So 2:30 AM is going to be 2.5, 6:30 AM is going to be 6.5, 10.15 AM is going to be 10.25 let us say we also wanted the data at 5 pm okay. This is just to check how our interpolation works at 5 pm we expect our data to be exactly the same as what we have over here.

So the last value should be equal to 31.8. So we will check whether that's what we get okay. And interpolated values TI are going to be nothing but spline let us go to the syntax spline x data yData our xData that is the independent data is in the vector h, the dependant data is in vector t. Keep in mind that the size of vector h and t are both 25. So they are row vectors of length 25 okay.

That is important, it is important that h and t are of the same size and hI are the interpolated at rr, the hour values at which we want our interpolation to be done and TI is going to be the temperature values at those points so let us run this okay. And let us go and check what the values of hI and TI are okay.

So at 2.5 hours the temperature is approximately 25 degree Celsius, at 2.5 temperatures is approximately 25, at 6:30 hours temperature is 26.13, at this point the temperature is 26, 13 at 10, 10 hours 15 minutes the temperature is 30.9 and at 17 hours that is at 5 PM temperature is exactly the same as data which is 31.8 okay.

The temperature at this point is exactly the same as what the data shows over here okay. So these are the temperatures calculated at the intermediate data points using the spline interpolation okay. Let us go on to another example and this example is of a car moving on a busy Chennai street okay. (Video Ends: 07:02)

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Example: Vehicle speed in front of Govt. Hospital

time (s)	0	10	20	30	40	50	60	70	80	90
speed	45	32	0	0	7	12	20	15	29	55

This is the car that is moving in front of the government hospital in Chennai. And there is a signal right in front of the government hospital. So the car takes one and half minutes to go from this end of the road to the other end of the road. The road that we are currently monitoring it is. When it enters this particular road the speed of the car is 45 kilometers per hour. It comes to a halt at the signal under speed becomes zero.

It is stays as 0 for something between 10 to 20 seconds okay. After which the car starts moving again and the car accelerates to go up to 55 kilometers per hour okay. So this is the data. (Video Starts: 07:52) Let us go to MATLAB and let us bring up the file which I have already created with this data. So this is the time and this is the speed okay. Interpolation using spline okay. ,TI, let us say our let us say we want our speeds at 5 seconds, 15 seconds, 25 seconds, 35 seconds so on and so forth.

So let us say 5:10:90. I could have also made at 85. That is not a problem, so the time values are going to be 5, 10, 15, 20 sorry, the time values are going to be 5, 15, 25, 35 up to 85 okay. The speed interpolated using spline is the command is same as before spline independent data, dependent data and the vector at which we want the interpolated values okay. So let us run this okay.

So, when we run, we have we now have the interpolated data in vectors tI and sI. So tI, sI, let us type this out okay. So at time 5 the speed is 45.37. So the speed has kind of slightly increase from 45 to 45.37. In this interpolated data at 15, the speed has dropped to 13.9, at 15 seconds of speed has dropped to 13.9. Now there is something interesting that happens at 25. The interpolated value of speed shows -3. 2 okay.

What that means is that between these 2 points although we know for a fact that the car is stopped at that signal. So between these 2 points the speed has to be 0. The interpolated value shows that the speed is nonzero. The reason for this is because splines tries to fit a smooth curve to all these data points. It tries to fit a smooth curve and when it tries to do that, it gets the value between 20 and 30 to be negative okay.

Let us go and change this code a little bit. So between 5 and 90, we want the interpolated values at all data points okay. And we also want to plot them. So let us say we will plot our original data t, s. We will plot them as blue circles. Let us say hold on okay and we will plot. Let us move this up here and we will plot our interpolated values tI, sI as black line okay and let us see what is spline does.

So in fact tI 5 to 90 let us go right from 0 to 90 in steps of 1. So the interpolated value SI is going to be speeds at 0 seconds, 1 second, 2 seconds, and 3 seconds and so on up to 90 seconds okay. So let us run this and see what we get okay. So this is the blue lines are the original data and black lines are the interpolated data.

As you can see that the spline interpolates is the data in form of a smooth curve and because this particular speed has dropped rapidly to 0 and stayed at 0 for a while. When we try to fit a smooth cubic spline to this particular data, we get negative values between 20 to 30 seconds okay. Now we know for a fact that the speed will now is not going to be negative because the car is not expected to go in the reverse direction.

So that kind of constraint is actually there in that data. So let us say now we want that particular physical situation to be handled more gracefully under such situations. We will use the other interpolation formula which is the pchip interpolation formula okay. (Video Ends: 12:34) So let us see how we will use pchip okay. What we said was the syntax is exactly the same we just replace spline with pchip okay. And that is exactly what we are going to do in MATLAB.

(Video Starts: 12:45) So this is what we had written earlier. So let us just copy paste this interpolation using pchip and sI_pc equal to instead of spline, pchip okay. And let us just do this for tI from 5 to 90 in steps of 10 okay. For now we do not need plot. So let us comment this out okay. And let us clear all, close all and clear the screen oops! Yeah, let us run this okay. Yeah we did not need the figure plotting okay.

So let us see what tI sI where, this is what tI sI where and let us type tI and sI_pc and see what they were okay. So let us focus on the point at which we got negative values of the speed okay. And that was 25. At time 25 the value of speed you are using the spline interpolation was -3.23. Whereas, using pchip is zero.

So pchip maintains this kind of monotonicity in the data. What that means, we will come to that in a minute but this particular expectation that the speed is not going to go below 0 when the vehicle is at halt is maintained by the pchip interpolation. Whereas that is not maintained by spline interpolation. At other data points, we have spline and pchip behaving in more or less similar manner.

There is some difference between the interpolated values but that difference is not very significant. The most important difference comes in at this particular data point where we want pchip or we want an interpolation to match the requirement that when the vehicle comes to a halt its speed is at 0 okay.

So let us plot this. Let us change this back using control z to get the data, to get the interpolated data at all points between 0 and 90. And let us plot this also and plot sI_pc. And let us plotted using red- line okay. So this is the new command that I have okay. I am plotting tI versus sI_pc as red- line and that I will use to compare with this spline okay. Okay let us clear everything okay. So this is what we have and let us run this okay.

So the blue circles are the original data points, the black line that you see over here is the spline interpolation that we have done earlier. And a red dash line is the pchip interpolation, the piecewise cubic hermite interpolation. Now you see the difference between spline and pchip okay. There is no difference for example or there is very little difference between these 2 points or between all of these points but there is significant difference over here and over here okay.

And the difference arises between pchip and spline because of the following reason. What pchip does is it maintains monotonicity. What that means is between these 2 data points when we try to interpolate the y value, is always going to lie between these two y values okay. As you can see spline the value at this intermediate point is greater than the value at this point. Pchip does not allow that to happen all the values in the intermediate point strictly lie are below this value and above this value okay.

That is the other reason why when this p is 0 and 0 over here. We get this flat line using the pchip interpolation because pchip ensures that the monotonicity is maintained. So let us go to power point and finish, what, what we started off with. (Video Ends: 17:38) So what we did in todays lecture is we considered 2 popular interpolation techniques spline interpolation and pchip interpolation and we demonstrated that using 2 examples.

The first one was temperature variation near during a day. In this particular example we were happy with what the spline interpolation gave us. In this particular example we had a constraint that the speed is not going to fall below 0 and pchip was therefore found to be more suitable because pchip maintains the monotonicity in the data. So that is primarily what we have covered in this particular lecture. This is the last lecture of module 6. Just to recap very briefly, what we did in module 6?

In lecture 6.1, we went to introduction of regression and interpolation, in lecture 6.2 and 6.3 we covered several examples of regression techniques, linear as well as nonlinear regression techniques using MATLAB. In today’s lecture we have covered 2 specific interpolation techniques those were a spline interpolation and pchip interpolation. So with that we come to the end of module 6. I will see you next week in the next week in module 7. We are going to cover ordinary differential equations. So see you in the next week. Thank you.