MATLAB Programming for Numerical Computation
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Module No. #04
Lecture No. #4.2
Linear Equations – Gauss Elimination

Hello and welcome to MATLAB programming for numerical computations. We are in week number 4. In this week we are covering linear equations. In lecture 4.2, we are going to cover one of the most popular method of solving linear equations which is known as Gauss Elimination. In today's lecture we are going to use what is known as the naïve Gauss Elimination technique.

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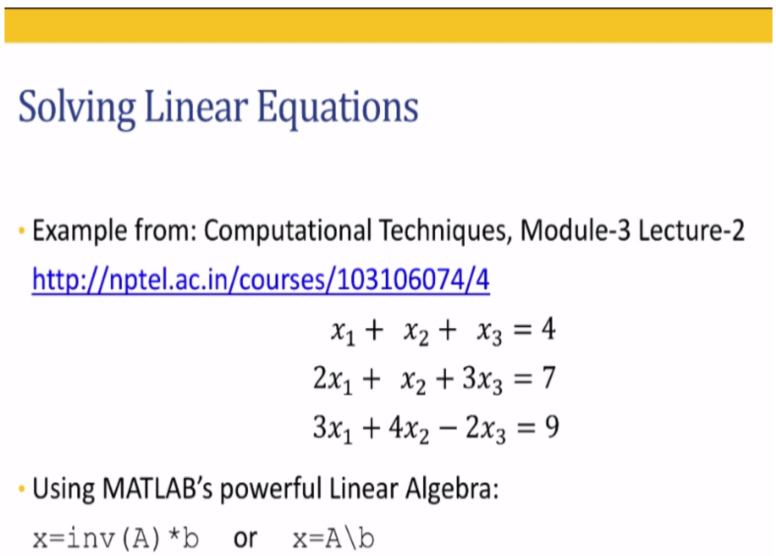

Okay let us take an example. In order to show to how we are going to solve this problem this example is from the computational techniques have video course module 3 part 2 the link for which is given over here okay. So these are 3 equations and 3 unknowns. The 3 unknowns are x1, x2 and x3 together they form a vector which is column vector containing 3 rows okay.

We will write this equation as Ax = b where, a matrix is going to be nothing but 1, 1, 1; 2, 1, 3; 3, 4, -2 and our b vector is going to be 4, 7, 9, in the previous lecture we have seen that we can use MATLAB's powerful linear algebra sweet in order to find the solution x. Either using inv(a)*b or the backslash command, what we are going to do today, is to use the Gauss Elimination technique in order to solve this linear equation okay.

(Video Starts: 01:36) And Let us say edit gaussElim and this is the one that we are going to create in order to solve the problem using Gauss elimination. Let us go on and look at the problem, our a is 1 1 1; 213. Let us write it down a as 1 1 1; 2 1 3 and 3 4 -2. Our b matrix is nothing but Let us go back and see 4, 7, 9 b is 4; 7; 9 okay. Let us save this and run this we can get a and b.

And our x is going to be nothing but a\b and that is our x which is 1 2 1. So that is the solution to our problem using MATLAB's backslash command. (Video Ends: 02:40). Okay what we want to do is, to solve this problem using Gauss elimination. So the first step in Gauss Elimination is to create augmented matrix.

(Video Starts: 02:48) And Let us go ahead and do that. Let us make comments first. Solve Ax = b, naïve Gauss Elimination okay. And augmented matrix Ab = A, b and we will end this with a semicolon. (Video Ends: 03:19) Okay Let us go back okay. Now at each step A(i,i) is the pivot element. So first we will have A(1,1) as the pivot element then will have A(2,2) as the pivot element and finally A(3,3) as the pivot element.

(Video Starts: 03:31) We do not have to do anything with the A(3,3) mainly because A(3,3) row number 3 is the last row so. Let us actually do this A(1,1) as pivot element. Yes we will have a set of computations with that. We will have set of computations with A(2,2) as pivot element and with A(3,3) as pivot element. Let us say Gauss Elimination, notice that when I have used 2 percentage signs, this has formed a section in this particular code okay. And with this enter over here okay. Now we are ready to do the next steps. (Video Ends: 04:38)

So what is the step with A(i,i) as the pivot element? So Let us say A (1, 1) as the pivot element we want to create 0's in the pivot column which means in column number 1. So column 1 over here is basically 2, 1, 3. So what we are going to do is, we are going to subtract 2 times row 1 from row number 2.

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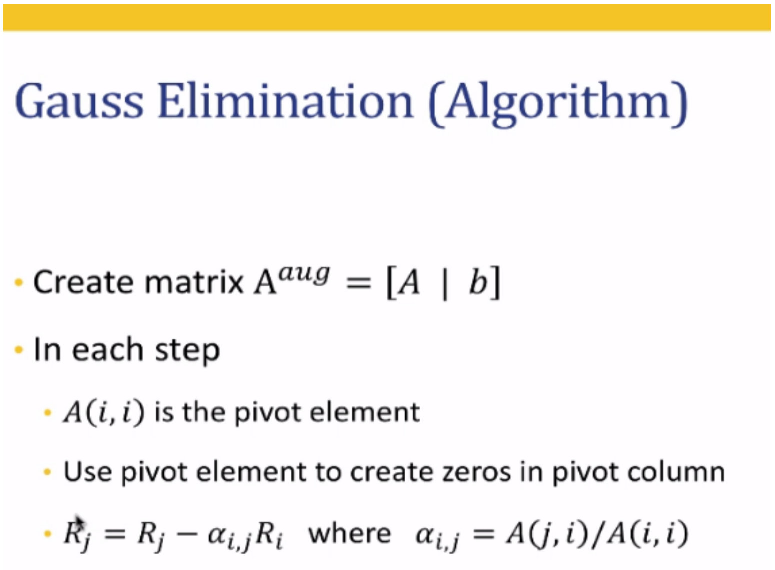

So we need to use row operations. If row 2, the new row 2 is going to be row 2, the old row 2- alpha times row 1 where, alpha is A(2,1) divided by A(1,1), A(2,1) which is 2 divided A(1,1) which is 1. So that is going to give alpha as equal to 2 and row 2-2* row 1 is going to be our first step.

(Video Starts: 05:28) So let us do that in MATLAB saying alpha = A (2,1) that is the second row first column divided by A (1,1) okay. (Video Ends: 05:44). Let us go back and see that is exactly, what we had $\alpha_{i,j}$ is going to be equal to A (j,i) divided by A (i,i). So A (2,1) divided by A (1,1) and A (3,1) divided by A (1,1) okay.

(Video Starts: 05:59) So that is going to be our alpha. Our row 2 = row 2- alpha times row 1 okay. This is not a correct way, so how do we get the row, how are we going to get the row? And we are going to be get the row of the Ab matrix and the way we are going to do that, let us actually copy paste this, and yeah so we have this Ab matrix.

So, how are you going to get row number 2? Row number 2 of Ab is nothing but row (2, :). Entire row number 2 will be extracted by this command. This is what we get. So this is going to be row 2. What is row 1? Row 1 is Ab (1, :) okay. So, what is alpha * row1? Alpha multiplied by row 1 is going to be nothing but 2 * Ab (1, :) and when we do that that is exactly, what we get so this is 2 * row 1.

What we want to do is row 2 -2 * row 1 and assigns it to row 2. So Ab (2, :) is going to be equal to Ab (2, :) which means row 2 = row 2- α * row 1. And row 1 was Ab (1, :) write this is row 1 which is exactly I have written over here. Alpha value is 2, this is exactly what I have written over here, row2 is Ab (2, :) which is exactly what I have written over here and that expression is been assigned to Ab (2, :).

So let me cut this, go back over here and paste this. So this is going to be nothing but row 2- alpha time row 1 okay. That is cool with us. Next what we want to do is, we want to repeat that with row 3. As we have seen over here. (Video Ends: 08:11)

We use the pivot element to create 0's the entire pivot column, entire column 1 we are going to use row 1 in order to create 0 over here as well as 0 over here okay. So we need now to repeat that for row number 2. (Video Starts: 08:25) And the way we are going to do that is nothing but alpha now is going to be a 3, 2 / a, sorry, 3, 1 / a 1, 1.

Note that only the row number is changing. Row number has changed from 2 to 3 and we change from 2 to 3 over here and - α * row number, what is it row3= row3 - α * row 1. So that is all the computation required with A (1, 1) as the pivot element and we have done over here.

So that is the first step. Now we need to repeat that for column number 2. (Video Ends: 09:09) So what now after at the end of this? We are going to get is, we are going to get 1 1 1. This guy is going to be 0, this guy is going to be modified number and so on and so forth. This is again going to be 0 and this we are going to have some numbers.

(Video Starts: 09:25) Let us do this. Let us save this, just run it and see what value of Ab we are getting and if we are getting any errors okay, cool. So we do not have any errors. Let us clear the screen and type Ab okay. So what we have done at the end of the first step is with the A(1,1) as the pivot element we have obtained 0 in that pivot column in column number 1, in row 2, 3 and so on. We have obtained 0's over here okay. So that is the first step.

Now the second step is going to be, to convert this as 0 using A (2, 2) as the pivot element, we are going to use the row operations with A (2, 2) as the pivot element, And Let us go back to MATLAB. Let us do that. In this case now alpha is going to be something divided by A (2, 2) and what is that something going to be.

That is going to be row number 3 right. What we want to do over here is, we want to use row 2 in order to get this guy equal to 0. Alpha is going to be this guy divided by this guy. So this is A (3, 2) divided by A (2,2). So this is a 3 sorry, 3, 2 okay. Again keeping up with the equations that we had seen a minute before. That is the value of alpha over here and our value of Ab (3, :) is going to be nothing but Ab (3, :) - α * Ab.

And this time it is (2, :). Why because our equation with A (2, 2) as the pivot element was r3- alpha time r2 okay. So r3 is this guy equal to r3 - α * r2 and that is what we get over here. So Let us save this and run this and see what we get with our matrix Ab okay. Oops! That is something wrong and Let us go back and check what exactly did we do wrong okay.

Alpha is going to be equal to a, I am sorry, this is a problem I should be using Ab over here and not a okay. So shift everything I need to change right. And alpha over here also has to be Ab because we are working with the augmented matrix okay. Because this is in the lower triangular form, we do not really need to do anything with the pivot element A (3,3).

So oops! Yeah let me move over here and delete this part with pivot element. And this is the end of our Gauss Elimination. We have done with the Gauss Elimination over here. Why did we think we just 2 steps because we had 3 /3 matrix okay. Next step is going to be back substitution okay. (Video Ends: 12:44)

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Back-Substitution

- $x_3 = \frac{b(3)}{A(3,3)}$
- $x_2 = \frac{b(2)-A(2,3)x_3}{A(2,2)}$
- $x_1 = \frac{b(1)-A(1,3)x_3-A(1,2)x_2}{A(1,1)}$

Now yes with respect to back substitution what we do is, we have our last equations as if, (Video Starts: 12:53) we go back to MATLAB and check this, our last equation is - 4 x 3 = - 4. So x3 is just going to be this guy divided by this guy okay. So our x3 is going to be equal to 1 okay. Then we go on to the second last equation, second last equation is - 1x2+ 1 * x3 = -1.

We take this on to the right hand side and divide by A (2,2) and that is going to be our solution for x2 and then we use x2 and x3 in order to find x1. That is the procedure we are going to use. So Let us do that. So we are going to do x. So x, initialize our solution okay. So our x3 is going to be nothing but Ab end, sorry, Ab (3, end) which is the last end basically means the last column and third row so Ab (3, end) divided by Ab (3, 3).

That is the value of x3 right. Again what we will get with respect x3 is, Ab(3,3) is the denominator and Ab (3,end) is the numerator Ab(3,end) is the numerator and Ab(3,3) is the denominator that is our x3. Our x2 is going to be nothing but Ab (2, end) right. Ab row 2 and last column so Ab (2, end) – Ab (2, 3) * x3 okay.

That is going to be the numerator, so we will put the numerator in the brackets okay, divided by Ab (2, 2) that is going to be our x2 and our x1 is going to be like just copy this and we will have to be change from 2 to 1 everywhere and everything else remains the same. So (1, end) - Ab (2,

3) * x3 will change to Ab (1, 3) * x3 and we will do this. We are going to add Ab (1, 2) * x2, (1, 2) * x2 okay.

So let us see what we have done. What we have done over here is the, first guy x1 is going to be Ab (1, end) - Ab (1,3) multiplied by x3 - Ab (1,2) * x2 and that is really what we have done over here in our MATLAB okay. And that will give us the solution. Once I changed the denominator also okay. Keep in mind that I have an inner bracket over here and then an outer bracket over here okay.

And save this. We run this and we check whether the solutions are met or not our value of x is 1, 2, 1 which were the same as a backslash. So this our Gauss Elimination is working to the way we desired it to do okay. (Video Ends: 16:33)
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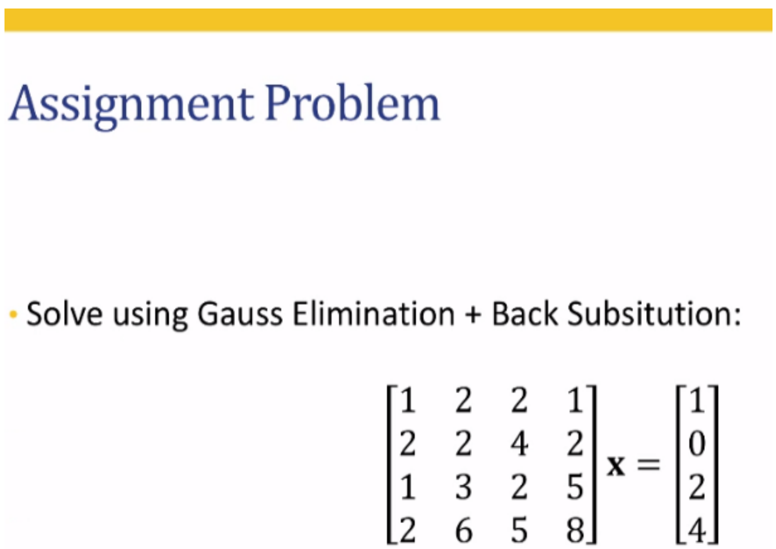

Now let us spend some time in order to make this better. What we want to mean by this is that MATLAB has a very powerful linear algebra set up tools and we do not really need to write this in the way. We have written it right now. (Video Starts: 16:53) What we need to do is, we need to put it in a loop okay. First of all let us look at this equation. Let we copy this numerator that was in the bracket and paste it over here.

Let me just put some spaces so that it will become a little bit easy for us to check okay. So we have Ab (1, end) - bunch of things okay. So Let us keep Ab (1, end) on a scroll. We do not have to worry about that part. Let us look at the thing that was there in the brackets. That was Ab (1,3) * x3+ Ab (1,2)*x2. So if i have to write Ab 1, 2 to 3.

What I am to get is, the 2 guys so Ab (1, 2) and Ab (1, 3) okay. If I write x 2 : 3 and I am going to get now vector which is the column vector and if we multiplied these 2, what I am going to get is, Ab (1, 2) * x2+ Ab (1, 3) * x3 which is exactly what we had over here okay. So let me just multiply this guy with this guy okay.

And that is going to be Ab (1, 2) to 3 * x 2 to 3 and we want to check whether the result is same as this guy that is the whether the result is equal to 3 or not. So let we press enter and we actually find. Yes that is absolutely what we get. So, let me go over here and I am going to change the thing in the bracket to what we had written in the command prompt.

The command prompt what we had written is, this copy it and paste it over here okay. And that is going to be our value of x1 okay. Now if that is value of x1 this is, if you see 1+ 2 to 3 okay. So we will just write it as sorry 1+1 okay, to n and will go up and we will write n = 3. We will have 3 equations. So, I will be write it over here will actually take this and put it right at the top okay. So 1+1 to n and 1+ 1 to n okay divided by Ab (1, 1).

Likewise, what we are going to do is Ab (2, 2+ 1) to n * Ab (2+ 1 to n) okay. Why we are writing in this way because now we have this x going from 3 to 3 and that is good enough for us okay. Let us put this n loop for i = 2 in step of -1 to 1 end. End write end this is going to be i. I have replacing 2 with i over here okay.

Everywhere where there is 2, I am replacing it with i okay. Same thing I am going to do over here but everywhere there is 1, I am going to replace that i okay. Now we compare these 2. What we will see is, that these 2 expressions are exactly the same that is how we write that I, the results in end loop okay. As a matter of that if we were to do this for x3.

What we are going to get is Ab (3, end) - Ab (3, 3) + 1, 4 to 3 and 4 to 3 is null. So 4 to 3 is going to be nothing so is going to be Ab-nothing divided by Ab (3, 3). So I will delete this and I will change this to 3 and gaussElim okay. And we will see our x is exactly the same as we got over here. (Video Ends: 21:29) With that we come to the end of lecture 4.2 and I will see you in the next lecture thanks and bye. `