MATLAB Programming for Numerical Computation
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Module No. #03
Lecture No. #3.4
Numerical Integration – Newton Cotes Integration Formulae

Hello and welcome to MATLAB programming for numerical computations. We are in the second half of module3. In this part we are going to cover numerical integration. In today's lecture we are going to cover range of integration formulae known as Newton-cotes integration formulae.

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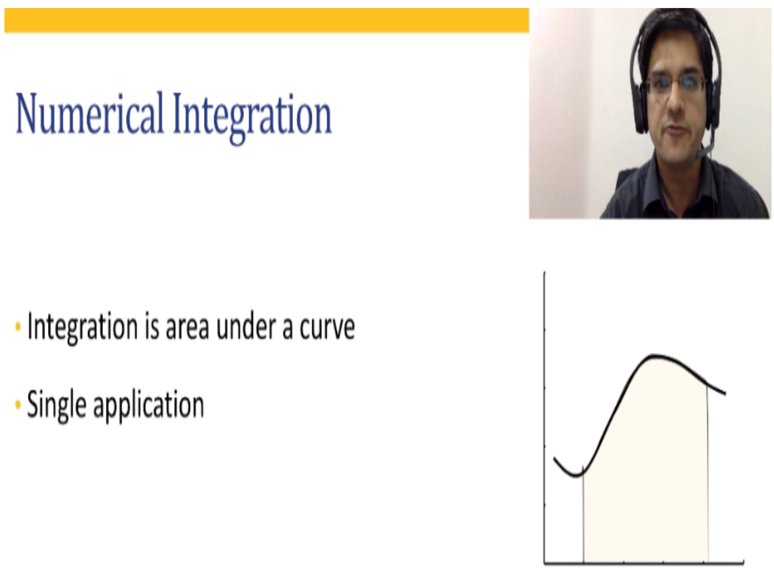

Numerical integration is primarily just finding area under a curve. For example, if a curve is f(x), then integral f(x) dx from a to b is nothing but this shaded area that you can see between the points a and b lie below this curve.

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So if we were to be use trapezoidal rule for this particular example, what trapezoidal rule amounts to is calculating the area under this trapezoid. The trapezoid is formed by joining the dots f (a) with f (b) as you can see in this cartoon over here, if we apply a single application of the trapezoidal rule, we are going to be fairly inaccurate the results are not going to match closely the actual values of integral.

So trapezoidal rule basically as we said is just joining 2 dots with a straight line and finding the area under them. Instead we can take 3 points, the point over here and the point over here and the midpoint. Join them 3 points, we can join with the second order curve and the area under that curve is going to give us what is known as Simpsons 1 /3rd rule.

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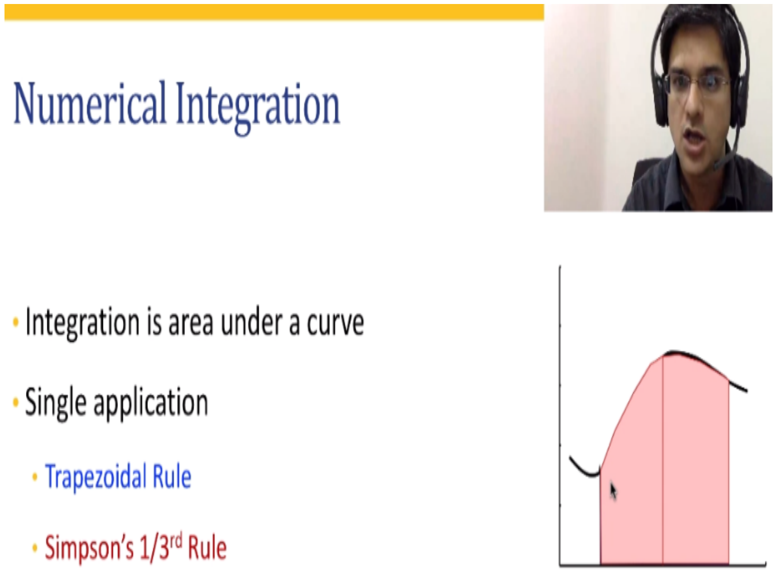

So Simpsons 1 /3rd rule is obtained by joining point a, point b and the midpoint between a and b. These 3 points adjoined by a curve. And what we obtained is known as the Simpson's 1/3rd rule. So the step size h in Simpson's 1/3rd rule single application is half of that of the trapezoidal rule. The next in Newton-cotes formula can be obtained by connecting 4 points within this region.

So if I just go back, so we had in trapezoidal rule this two points and Simpson's 1/3rd rule. One more point at the center in Simpson's 3/8th rule. What we will have is these 2 points and then two more points that are equally spaced. So if we take this point, if we take the next point at h by 3 and third point at 2h/ 3 and at fourth point at this guy okay.

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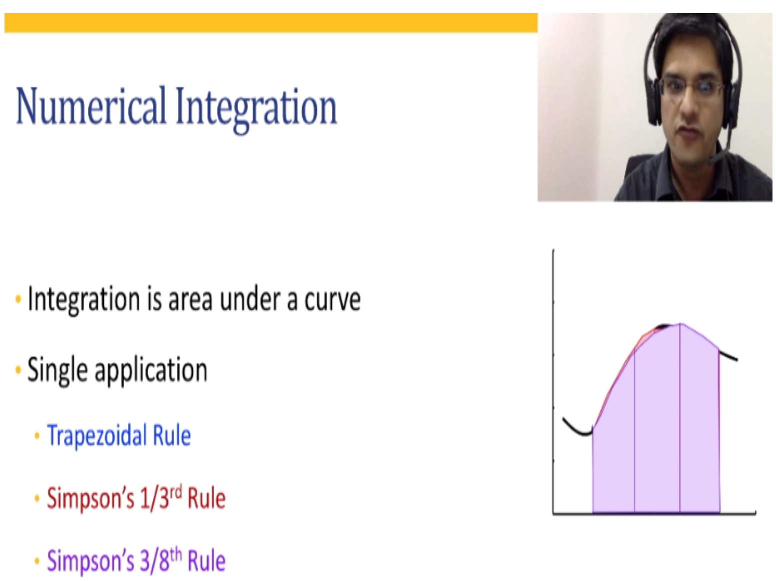

And if we were to connect all of these through a smooth third order curve, the area under that curve will be give us Simpson's 3/8th rule. In a single application of Simpson's 3/8th rule the step size h is going to be b- a / 3.

So the step size in Simpson's single application of Simpson's 3/8th rule is, one third of the step size that we used in the trapezoidal rule. We can of course have higher order formulae. So instead of 4 points if we are have 5 points and connect them with the fourth order curve, 6 points connecting them with the fifth order curve so on and so forth. We will give us higher and higher order Newton-cotes formulae.

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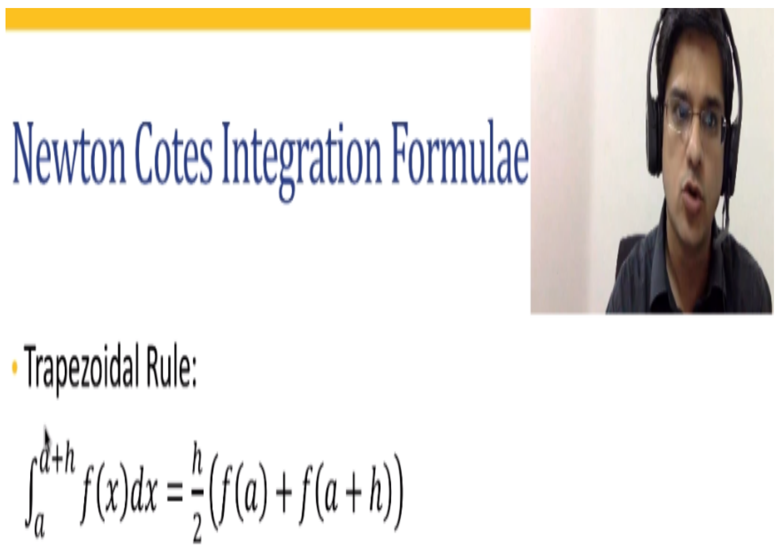

So the trapezoidal rule integral from a to (a + h) f(x) dx is going to be h/2 f (a) + f(a + h). What this means is, it is an area under this trapezoid, so it is this height multiplied by this height + this height multiplied by this base divided by 2 that is what the area under the trapezoidal is.

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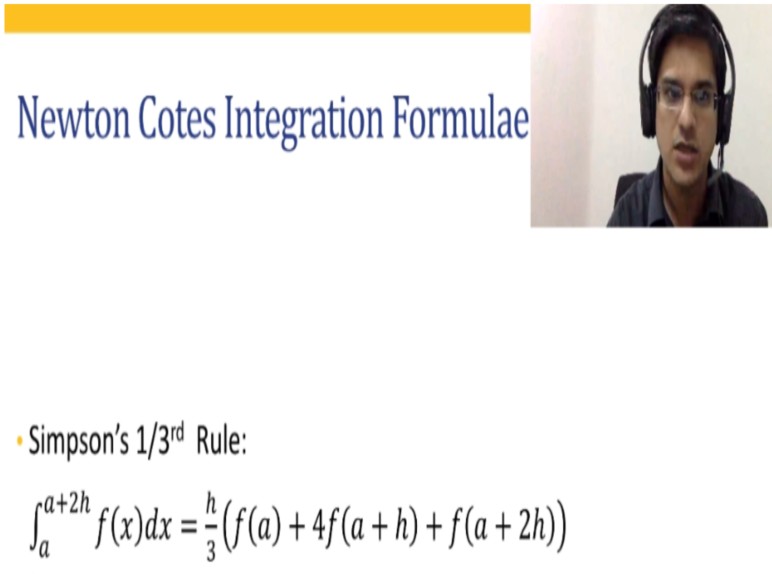

The Simpson's 1/3rd rule is given over here is, h /3 * function value f (a) + 4*f (a) + h + f (a + 2h). That is what the 1/3rd rule is going to give us, f (a) + 4 *f (a + h) + 1 * f (a + 2h) the whole thing multiplied by h /3. It will give us Simpson's 1/3rd rule and Simpson's 3/8th rule. For the sake of completeness I am just showing over here. We will not cover Simpson's 3/8th rule in this lecture.

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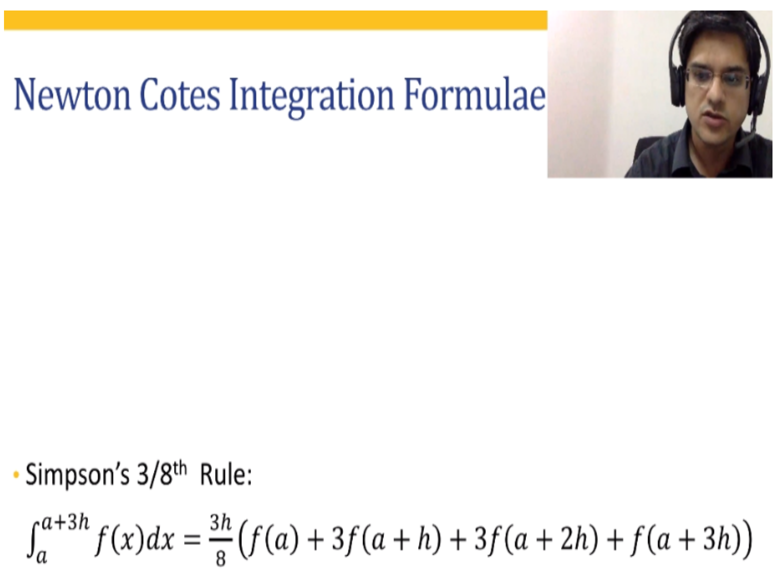

So these are the Newton-cotes integral for integration formulae.

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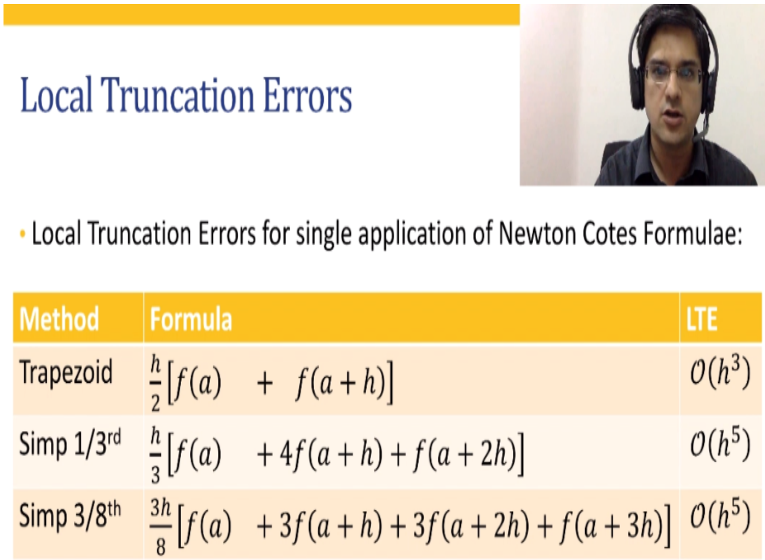

To summarize the formulae and local truncation errors the trapezoidal rule formula is this local truncation error is the h ^ 3. So the trapezoidal rule from the local truncation error, single application of trapezoidal rule is h ^ 3 accurate, single application of Simpson's 1/3rd rule is h ^ 5 accurate.

So it is significantly more accurate than the trapezoidal rule. Simpson's 3/8th rule is also h ^ 5 accurate. As a result of this usually Simpson's 1/3rd rule is more popular than Simpson's 3/8th rule and for the simplicity of application the trapezoidal rule is also very popular.

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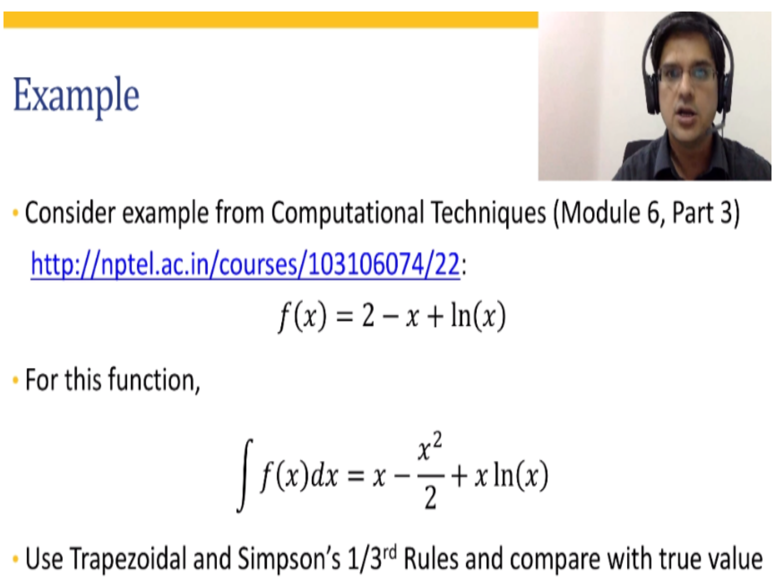

Let us take up an example and solve that in MATLAB. The theoretical aspects of this work covered in module 6 of computational techniques specifically in module 6 part 3. What we are going to focus on is, we are going to calculate integral from 1 to 2 f (x). f (x) dx using the trapezoidal rule and the Simpson's 1/3rd rule. Integral for this f(x) is given over here. So let us go to MATLAB okay and do this problem.

(Video Starts: 05:44) Edit numIntegral of Newton-cotes formula, f(x) equal to okay, so let us say a = 1, b = 2, trapezoidal rule, 1/3rd rule. trueVal is going to be equal to x- x^2 / 2 * x lnx. So b- b^2 / 2 + b * log b. If you recall, log is gives the natural logarithm of. So you can do help log. If we wanted the log to the base of 10 instead we would use the command log10 okay. so this - a- a squared by 2 + a * log a that is going to be our trueVal.

Okay, so for trapezoidal rule single application, h is going to be nothing but b-a okay. Itrap is going to be equal to, let me put underscore over here. Itrap is going to be equal to h / 2 * f(a)+ f(a + h) = h / 2 * f (a) + f (b) and we need to put inside a bracket okay. Where f (a) is nothing but 2-(x + lnx) calculated at a. So it is 2 – (a + log a) okay. And f (b) is nothing but 2-(b + log b) okay.

So let us save this err_trap = abs (trueVal- Itrap) okay. Let us uncomment both of these. Save this and let us run this and see what results we get whether we get an error or we can get results. So let us run this. It runs properly and the error is of the order of 0.04. So this error is quite substantial. (Video Ends: 09:11).

As we had seen in the cartoon over here the single application of trapezoidal rule sometimes can be very inaccurate okay. (Video Starts: 09:25) So the single application of, so as we have seen over here, the trueVal of the integral is 0.8863. Whereas, the approximate val from trapezoidal rule is 0.8466. Yeah so let us go over here. What we want to do is, something that we had done earlier in the previous lecture.

What we want to do is, instead of calculating the functional value is inline, like this we will want to give a different function to calculate f(x) okay, so let us create a new function called myFunIntegral. So function fval = myFunInt x. fval = 2-(x + lnx), 2- (x + log(x)) end and save this as myFunInt. And f (a) is nothing but myFunInt a. And this guy is nothing but myFunInt b okay. Save this and run this and see what we should get.

If everything runs fine, we should get same values as before. And that is exactly what we get the error in trapezoidal rule is, 0.04 and the integral value is 0.847. That is exactly what we have got earlier okay. In Simpson's rule h is (b- a) / 2. That is what h is going to be okay. And I_simp is going to be h / 3 * f1. So f (a) + 4 * f (a + h) + f (a) + 2 h or we can say f1 + 4 * f2 + f3. So let us just do that f1 + 4 * f2 + f3 right okay.

What is f1? f1 is nothing but function f calculated at a. What is this guy f calculated at (a + h) and this guy is f calculated at (a + 2) * h okay. What is the name of the functions that we created, the name of the function that we created is myFunInt. So let me just replace this with myFunInt again. F, I will replace with myFunInt and this f also I will replace with myFunInt okay.

That will give me I_simp and err_simp is nothing but abs (trueVal- I_simp). Display results and we should be able to get the results more easily. So the errors using the trapezoidal rule is 0.04 but the error using Simpson's 1/3rd rule is about 4.5 *10 ^- 4. So as we can see the Simpson's 1/3rd rule is significantly more accurate than the trapezoid rule okay. (Video Ends: 13:03).

So this is what we have seen the lte in Simpson's 1/3rd rule is, h ^ 5 whereas, local truncation error in trapezoid rule is h ^ 3 okay. And therefore in this particular example it turns out that Simpson's 1/3rd rule is gives better results than the trapezoid rule okay. So next what I am going to do is, and I am going to take this trapezoidal rule.

And I am going to calculate the value of trapezoid rule, the integral using the trapezoid rule for different values of h. So we will start with f (a) okay and f (a + h). We will calculate this for different values of h. That means f (b) will be calculated for different values of h.

(Video Starts: 13:49) Again we are restricting ourselves to single application. So let us go to this numIntegral, drop down, save and click on save as numIntegral2. I will call this okay. We do not want Simpson's 1/3rd rule, a is going to be this and h is going to be 1 and b is going to be (a + h) okay. And this is going to be our trueVal.

We do not need this. Now what I am going to show is, let us see we have this particular code snippet of code over here. How we can make this code bit more streamlined. You have this f (a) over here okay. We can just delete this f (a) and replace it with what existed over here. So just highlight it and drag and drop okay. Same thing over here for f (b) also.

Delete f (b) highlight this part drag and drop over here. Delete these 2 guys okay. So let us save this, let us clear all, clc and let us run this for h = 1 okay. The error err_trap that we get is 4* 10 ^- 2. I will uncomment this part okay. Now let us these h by one order of magnitude and take over h to be 0.1 okay. Our trapezoidal rule is h ^ 3 accurate.

What that means is, decreasing h by 10. We should decrease the error by 1000. So the error should be decreased by a factor of 10 ^ - 3. So let us run and see what the error we get. Now, by running this you can see that the error has dropped from 4 e -22, 7 e -5. So this is an almost 3 orders of magnitude. This is actually a 3 orders of magnitude droplet us drop this further 0. 01 and let us run and see what we get.

And we can run this. We further see a 3 order of magnitude decrease in the error. Let us drop, let's drop this even further and see what we get. Save and we will see 3 more orders of magnitude decrease the error. The error went from 8. 2e -8 to 8 .3e -11. So as we decrease the step size by a factor of 10, the error improves by a factor of 1000. The error goes down by a factor of 1000.And that is the cause trapezoidal rule is x ^ 3 accurate. (Video Ends: 16:51).

So, this particular slide itself summarizes what we have covered in the lecture today. What we have covered is trapezoidal rule and Simpson's 1/3rd rule. What we have seen is trapezoidal rule is x ^ 3 accurate. What do you may want to test as practice? Problem is verified that Simpson's 1/3rd rule is h ^ 5 accurate.

I am not going to do that in the video lecturers. Because it is simply follows the same procedure that we have done with the trapezoidal okay. And with that I come to the end of this lecture. This brings us to the end of lecture 3.4 and i will see you in the next lecture. Thanks, and bye.