Introduction to Time-Frequency Analysis and Wavelet Transforms

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Lecture - 7.7
Application of CWT Part 1/2

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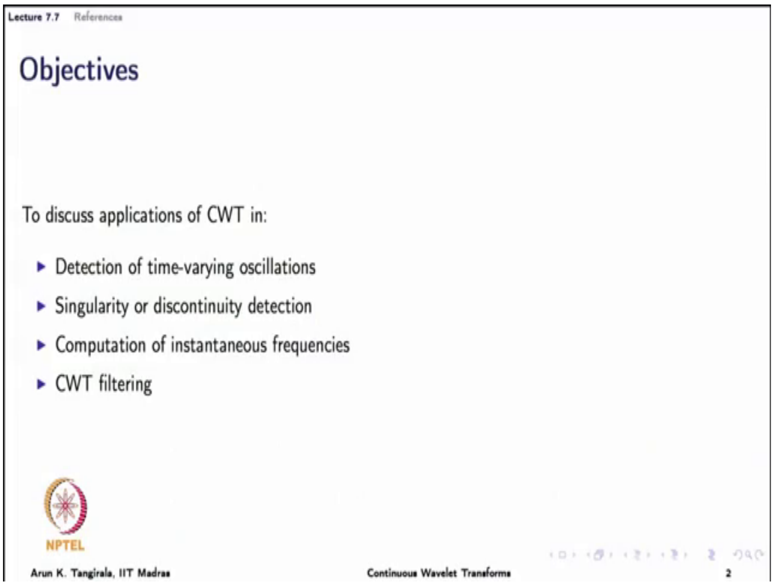

Hello friends. Welcome to lecture 7.7 on Applications of Continuous Wavelet Transform. This is the last lecture in the unit on Continuous Wavelet Transform, where we shall discuss applications of CWT in four different problems. One is on the detection of time varying oscillations which is quite important in many domains, including process, engineering, atmospheric sciences and so on. The second one is on the detection of singularities again which has wide spread applications. The third one is on the computation of instantaneous frequencies like the way we did with spectrogram and finally, we shall talk about CWT filtering. Normally filtering is performed using discrete wavelet transform, but is also possible to filter using continuous wavelet transform, and I will show you how to do that in Mat lab as well.

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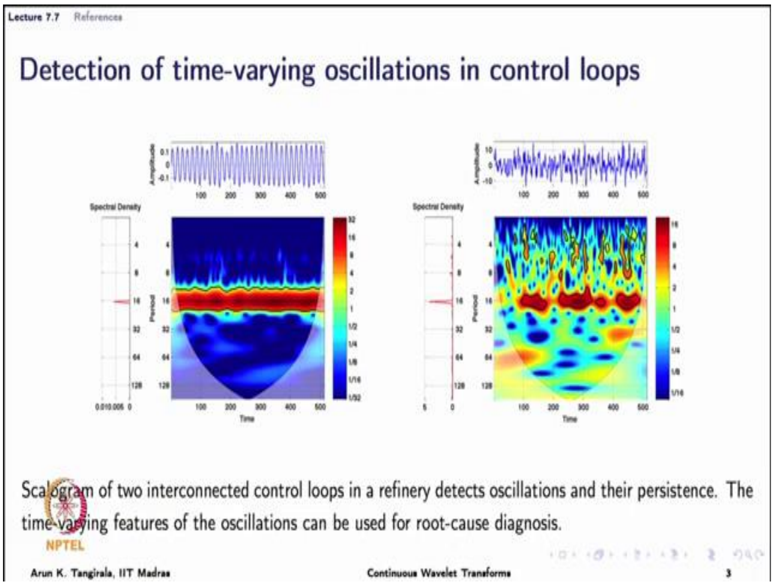

So, let us look at the first application which is on the detection of time varying oscillation in control loops. As I said the problem of detecting time varying oscillation is quite prevalent in many domains, not just in process engineering, but also in atmospheric sciences, meteorological data sciences and so on, where one is interested in knowing how the periodicities of certain atmospheric phenomena vary with time. In this example here or in this application I should say what we are looking at is large refinery, where number of control loops or in operation, and normally one is interested in a health of these loops pretty much like we are interested in health of a biological process, such as ourselves. One of the symptoms of poor control loop performance is oscillatory behavior or oscillatory characteristics in control loops.

You do not have to be an expert in control to know that the main purpose of designing a controller and having a control loop is to make sure that process variables or whatever variable that we are controlling is supposed to be maintained more or less very close to the set point without any oscillation. There will be always some fluctuation in the process variable, but those fluctuations should be ideally random in nature. When you have oscillatory process variables despite having control, then obviously there is some serious cause for concern pretty much like when the temperature of human body which is supposed to be regulated very well by the human body, when it starts oscillating, then there is a cause for concern with respect to the health of the body as well as that particular temperature control loop.

So, here we have selected two loops from a refinery process. One loop is upstream of the other loop. In other words, both these loops are interconnected and we do not know which is the cause or which is the effect when it comes to oscillations in these loops. There may be many causes we do not know, but the first thing is to detect oscillation. Of course, detection of oscillation in process variable can be easily done by using spectral analysis like the way you see here. In both these plots on the top you have as usual the time series data and on the left panels, you have the Fourier spectrum. The presence of clear peak is an indication, is a very strong indication of the oscillatory nature of these variables. Of course, one can also visually see none of the series that the variable has an oscillatory nature to it.

Now, what is important for us is to know whether this oscillation persisted throughout the observation period or did it come and go, did it just appear and vanish. And here is where the scalogram plots which you have now been generated using the wavelet coherence toolbox by Grinsted A. Moore. You could generate these plots using the wavelet toolbox as well the nice thing about these course is that you get the ((04:36)) of influence also drawn for you. So, at the scalogram plots for each of this series reveals the time varying nature of the oscillation. For loop one, it is clear that these oscillation persists throughout the period of observation whereas, for loop two, the oscillations really do not persist throughout. In fact, they are intermittent. In fact, if you look at the time durations corresponding to the frequencies where you have oscillations, in fact we draw here with the y axis as period and not a frequency, but you can always convert period to frequencies.

So, if you look at this zone which has the oscillatory nature to it, the line corresponding to the period that we have here we see that the initial portions for the loop two, there are no oscillations. Then, you see oscillatory behavior which persists for quite some time. So, what does this mean to us? Does it really give us more information than just a Fourier plot? Yes, of course it tells me that the oscillation here did not persist throughout and that information is useful to me in root cause diagnosis. So, for example, if only these two loops were there and they were connected, then which one really started off first or was the controller really trying to fight out the disturbance. If both these had a common source, did the source enter the loop two in a delayed fashion and so on. So, there are number of questions that can be answered by looking at these scalogram plots.

Of course, you could use the spectrogram also, but we have already studied the advantages of scalogram over spectrogram. I do not have to really choose any window, window size and so on, all that. I do need to do is choose a mother wave and here we have chosen a morlet complex, morlet wave for the analysis. One can also look at the phase of these plots to get more information. So, this is to give you a feel of where the scalograms are applied, and how they could be applied in the technique time varying oscillation. There are number of papers that you can read up in the oceanography or the atmospheric data sciences literature, where wavelets have been widely applied.

Please go and do a quick search in the literature and you will be able to find many such applications. As far as this application is concerned here, we have managed to figure out that in loop two, oscillations really do not persist throughout and that we could use information in loop cause diagnoses. We will not really go further into these because we have a number of other interesting applications coming up.

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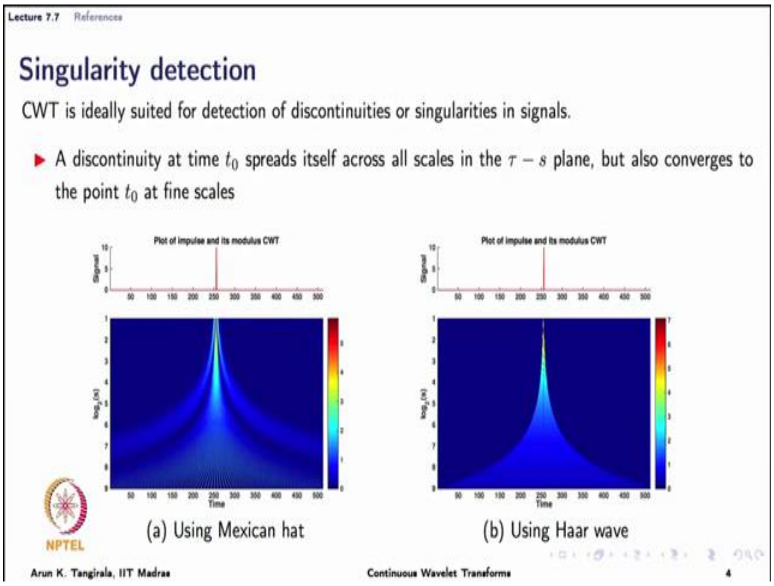

The first or the second in line is singularity detection. As mentioned, this is a very wide spread application in many fields, not just in engineering that everywhere what is essentially singularity essentially is a discontinuity and we already know through our wavelet analysis that a discontinuity at time t naught in a signal spread itself across all scales in a time scale plane, but importantly if you look at the scalogram plots that I have shown here with two different wavelets, the scalogram or in fact, I do not show this scalogram here. Apologies. I show here the modulus of the continuous wavelet transform, but you would see similar features in the scalogram as well. So, the magnitude of the CWT converges to the same point at finer scales. What we mean by finer scales is lower scales. Same point meaning the point at where the discontinuity is observed. So, that is the key feature that is exploited in similarity detection and this is also the cause for the cone of influence that we talk about because whenever we perform CWT at the borders, we have discontinuities and the effect of discontinuity at the borders persist for a longer period of time at courses scales, and for much shorter period of times at finer scales.

In this problem, we are interested now in knowing what are those features in the modules plot that converge to finer scales? If I fine a feature or in the sense, if I fine the modules of the CWT converging like the wave is, see here from the courses scale to finer scale, then that clear indication of the present of a singularity. Of course, this is a qualitative statement. Now, we need to quantify this statement further.

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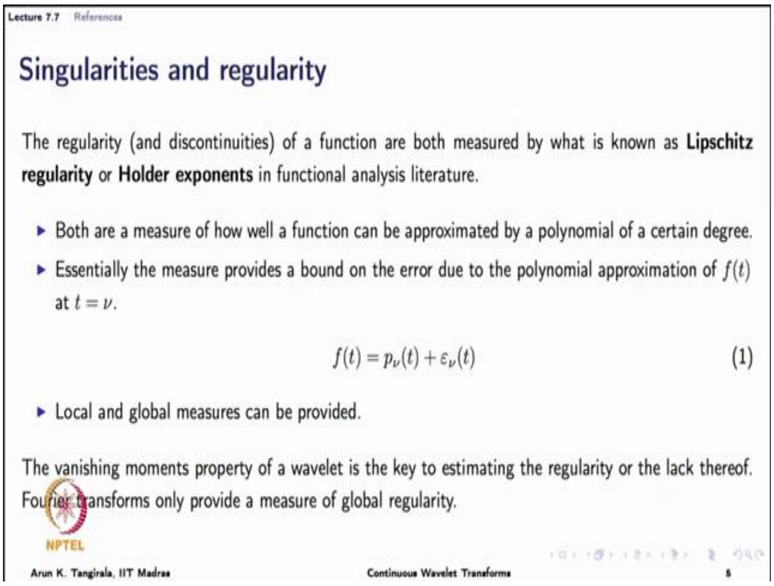

For that purpose we first quantify what we mean by regularity or singularity? This regularity and singularity have complementary characteristics of each other. If a signal is regular or a function is regular, then they are no discontinuity. It is essentially smooth. Singularity is the complete breakdown of regularity at particular point, and the problem of quantifying regularity was studied long ago. In the mathematics literature we have two measures which have basically used a same definition. These measures are known as Lipschitz regularity or holder’s exponent. As I said both use a same definition. What is the basic idea? Well, the basic idea is to see how well a function can be approximated by polynomial of a certain degree locally say when we talk about regularity of a function or a signal, we can talk about the local behavior of the function or global behavior of the function.

The first thing to do would be to define the local regularity and then, extend to an interval. Before I really give you the mathematical definition of Lipschitz regularity or the holder’s exponent, intuitively what these measures do is they provide a bound on the error that you ((10:50)) by approximating f of t locally with a polynomial of certain degree. So, in other words, you are breaking up the function into two parts, a polynomial at nu. So, nu is the point of interest to us. We would like to know if there is a singularity at that point. We approximate the function with the polynomial and epsilon is the error (( )) due to the approximation. Of course, if f of t is exactly the polynomial, then the error is 0.

What these measures give is essentially a bound on this error expressed in a certain fashion, and both as I said local and global measures can be provided turns out that the varnishing moments property of a wavelet is a key to measuring the Lipschitz regularity and the holder’s exponential. The basic idea is to perform a wavelet analysis and get an estimate of this holder’s exponent, where which we denote by alpha and I will define in the next slide the vanishing moments of the property. If you recall the ability of the wavelet to locally approximate the function with a polynomial of (Refer Time: 12:00) certain degree, so whatever is left out will be reflected in the CWT that is the basic idea.

If I have a wavelet with p vanishing moments, recall it can approximate polynomial exactly approximate of polynomial of degree t minus 1. So, what if you ((12:21)) function has more than this polynomial, then that will appear in your epsilon and you can use that to quantify the regularity or the holder’s exponent. That is the basic idea and of course, Fourier transform will also be able to give you, but unfortunately it gives you measure of global regularity. That is how the function behaves over all in time, not how the local regularity which is what we are interested in. So, before we learn of course how to use CWT for quantifying this regularity, we need now the definition of the Llipschitz regularity of the (( )) exponential itself as I mentioned.

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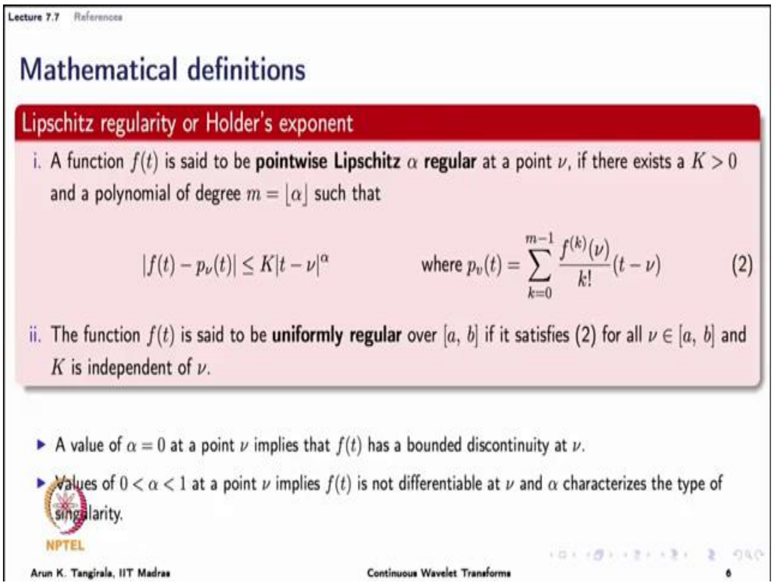

The basic idea is to approximate f of t with the polynomial and then, the measure is going to give you a bound on the error. So, p v of t is the polynomial approximation of f of t at a point nu, where I am interested in as a (( )) the regularity of f of t. What is this polynomial? This polynomial is nothing, but the tailor series expansion of f of t around nu up to the decide order. What order m? The order here is denoted by m. Sorry, what we are doing is essentially replacing f of p with the tailor series approximation up to order m and then, collect consolidating on the remaining terms into error terms.

What is Lipschitz regularity is doing is, it is basically giving a bound on that error and the exponent of this bound is this Lipschitz regularity. So, it says if the error, if you can show that the error in your approximation can be the bound on a error in the approximation can be expressed as k times modulus t minus v rise to alpha, and there exists a k greater than 0 when alpha becomes the Lipschitz regularity or holder’s exponent. So, you can see these as a generalization of the error that you get from tailor series approximation, and any function is said to be uniformly regular over some interval if it satisfies this expression. That is at every point in the interval you should be able to show that there exists an k and then, alpha such that the error is bounded in these fashion and these scale should be the same at all points in these interval. Then, we say that f of t is uniformly regular or uniformly lipschitz and so on over the interval.

This case should be independent of that of where you are looking at that there in the interval. Now, what we would like to know is how to detect singularities. What has the Lipschitz regularity got do its singularity? Well, a value of alpha equal to 0 at any point new implies that f of t has a singularity. That is the key. So, if I fine from a CWT that alpha is 0, then I conclude that there is a singularity. We can still do it visually, but here we are able to do it quantitatively and that is the most important thing. Any value of alpha between 0 and 1 means that f of t is not differentiable at that point, and the value of alpha characterized as a type of singularity. We do not go too much into this aspect at this moment, but let us look at things with an example.

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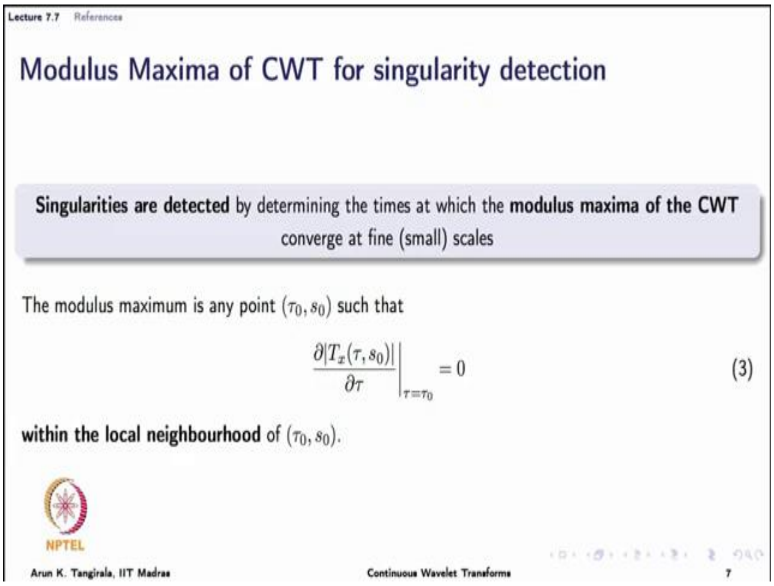

Before we do that, the main result is as follows. What we have learnt just now is how to quantify singularities or regularity. Now, we are turning to what feature of CWT will help in estimate that Lipschitz regularity or hold as exponent alpha. That is the thing. So, two things, the definition of regularity are quantification of regularity which is given by the holder's exponent alpha, and now we are searching for features in CWT which will allow may to estimate that alpha is the point. Now, it turns out that you can prove we will avoid all the theoretical proofs that singularities are detected by determining the time at which the modulus maxima of the CWT converge at fine scales.

Now, we have looked at this in the case of the impulse. We have looked at how the modulus converges. We have not looked at modulus maxima. What is this modulus maxima? Well, the modulus maxima is, the line is the maximum of the modulus locally at a certain scale at the certain time. In fact, here we are interested in certain time. So, I pick a time t and I see locally what the maximum is. Then, that we call as modulus maxima. So, I can now construct modulus maxima in the entire time scale plane, and all I have to do is follow the line that goes from the course scales to the fine scales. If there is a line that goes from course to fine and at whatever point this line converges that at that point, there is a discontinuity. When I show you the example, it will be lot clearer to you. So, that is the first feature that we follow the lines of modulus maxima.

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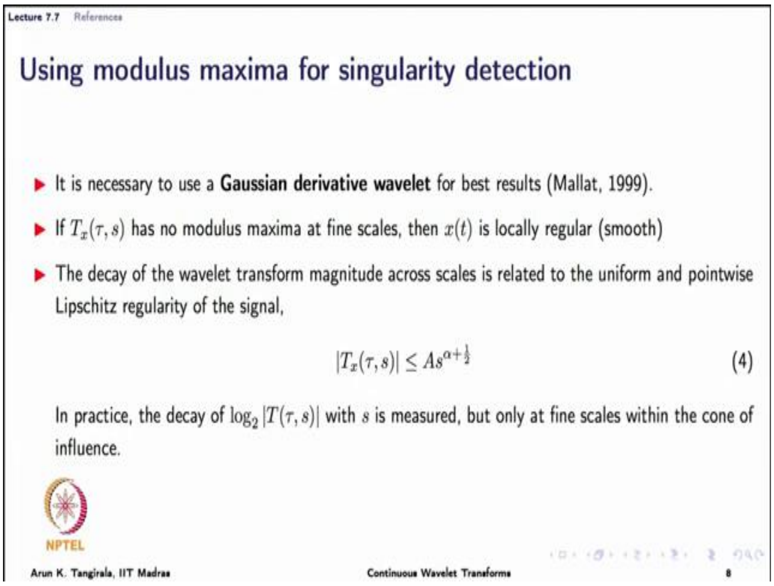

The second one is that it is necessary to use a Gaussian derivative wavelet for best results that is if you do not want any ambiguity. What kind of ambiguity can arise? Well, what can happen is even if there is no singularity, you may still end of getting a line that is the modulus maxima line that will converge from course to fine scale as I will show you in the example which I have borrowed from Mallat’s loop. If you have to resolve that ambiguity is best to use Mexican hat or Gaussian derivative type wavelets. You could use other wavelets, but then you may run into some ambiguities and consequently now as a (( )) of what we estimated if the wavelet transform has no modulus maxima at these scales and x of t is locally wavelet.

We are assuming that we are going to use a real wavelet here. We are not using a complex wavelet. Therefore, whether you taught of t x are not the modulus, then magnitude should be fine. Essentially if t has no modulus maxima at fine scale, then x of t is locally regular. You could have modulus maxima at course scale also. Secondly, we would like to get an estimate of alpha. Suppose there is no singularity exact discontinuity, but breakdown of the differentiability of f which is also important in signal analysis and more. So, in image analysis where you are looking at edge detection images, you would like to know what kind of edges you have. Discontinuities you have run into whether it is a singularity or a breakdown of the differentiability of f. In other words, you would like to estimate alpha because alpha characteristic type of singularity again will keep away all the theoretical results. You can see that the modulus of the wavelet transform or the magnitude of the wavelet transform is bounded above by this factor a time (( )) a constant a times s to the alpha plus half l alpha is the same holder's exponent of the Lipschitz regularity what we just learnt. So, what I do is, I plot algorithm of the magnitude of CWT versus log 2 s and look at the slope, and from the slope I (( )) alpha. That is the basic idea.

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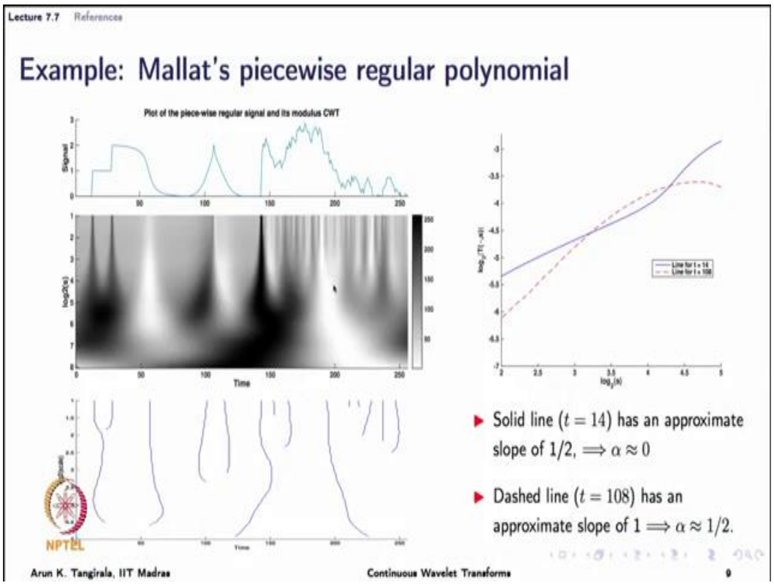

Let us look at an example which I have borrowed from Mallat's book. In fact, this example has been reproduced the help of wavelet which has a very nice feature. There is a directory in wavelet which contains all the code for reproducing entire set of figures in Mallat's book and I have borrowed the code corresponding to this chapter. This figure appears in chapter 6 of Mallat's book. I am not talking of the latest edition, but the previous edition and here Mallat's uses a very nice signal. It is called a piecewise regular polynomial you can say. So, here you have discontinuities of different types. So, here in the beginning I have discontinuity here and then, as smoother one because it is this kind of a jump is different from the jump that you see here, more importantly (( )) kind of jump that you see at t equals 1 naught 8 is different from the kind of discontinuity that you see at equals 14.

So, let us look at the magnitude of the CWT plot which has been computed using the Gaussian derivative that is Mexican hat wavelet. If you see here the magnitude, the magnitude of the CWT converges from course to finer scales at several points, especially where they are discontinuities. So, look at t equals 14 that is a time 14 and then, the next one here and then, you have of course convergence to fine scales, but if you look at a color bar, the intensity of this is very weak. Now, this is the problem that you can run into if you only follow the feature which goes from course to fine scales that is we said singularities are detected by following the lines that converge from course to fine scale.

If you only use that feature, then you may run into some ambiguities. It is better to always look at the holder's exponent also to get a field of what kind of discontinuity, whether it is singularity or some other kind of discontinuity and so on. So, you can see wherever there is a jump in the signal, you see this feature of CWT converging from course to fine scale. So, rather than just visually looking at it, it is nice to draw the modulus maxima line that you see at the bottom again generated by the code given wavelet, and you can see the difference in the way these lines converge. For example, if I look at the discontinuity at time t equals 14, there is a line that start from course scales that is high scales and then, converges to fine scales whereas, if I look at the next discontinuity, the line really does not follow. That is it does not start of in the course scales. You only see some kind of a maximum line, the modulus maxima line that starts off from relatively finer scale to even finer scales. This does not mean there is no discontinuity, but this discontinuity here at the second point here is not as sharp as the one that you see here.

In fact, when you look at the (( )) that you have here at this point at t equals 1 naught 8 that discontinuity is also different. As we just said we would like to really quantify rather than keep qualitatively classifying these discontinuities as is done in Mallat's book, I have picked two modulus maxima lines here corresponding to time t equals 14 and t equals 1 naught 8 and plotted the logarithm of the CWT versus the logarithm of the scale. Why did you do that? This is based on the result given 4. Remember when we plot this slope of this will get me an estimate of alpha. So, if I look at the blue line here corresponding to t equals 14, the slope of it is roughly half if you look at the slope. Therefore, that gets me alpha equals 0 whereas, for the red dash line which corresponds to the modulus maxima line at t equals 1 naught 8, the slope is approximately 1 which means alpha is half. Both are indicating the discontinuities, but there are different types. This is a perfect singularity here for the function at t equals 14 whereas, this is more of a loss of differentiability for a f. Of course, there is some kind of discontinuity here, but you see visually itself you can see there is a difference between the types of discontinuities here, and this is how you quantify singularities or regularities using CWT. There are number of other examples that are given not in Mallat’s book, but in a book that.

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Let me just take you to the reference in this book here by Misiti and others. You should look up at the concluding chapters where they talk of number of applications of wavelets, and one of those applications is the singularity detection where they show you how you could really use the holder’s exponent, compute the holder’s exponent from CWT and classify the type of singularity. Let me take you back now.