Introduction to Time-Frequency Analysis and Wavelet Transforms
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Lecture - 7.6
Wavelets Part 2/2

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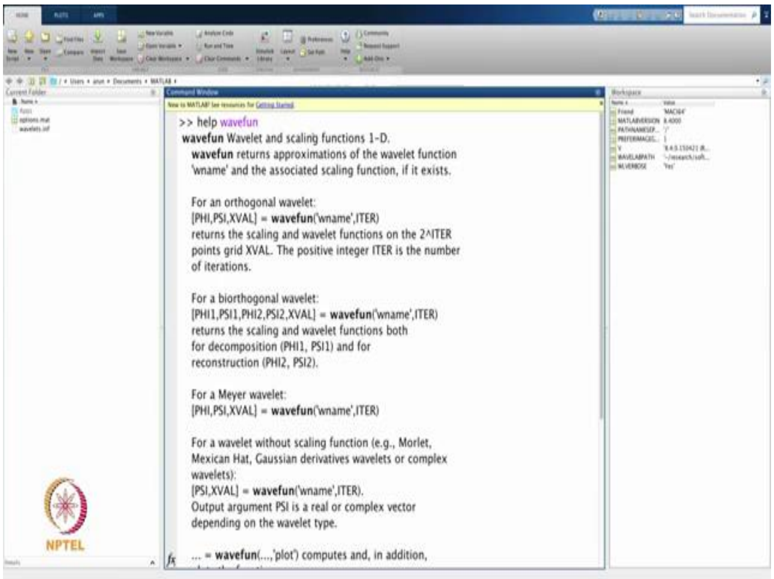

First, I shall show you how to generate wavelets. We have looked at this before as well. The most important function when you are looking at wavelets at the, that is, when you are looking at generating wavelets at the command prompt, is the wave fun. This is a routine that comes with the wavelet tool box of mat lab. As usual, I use the Release 2014b. And, it also says it returns its scaling function, but these scaling functions are for DWT, not necessarily the one that we use in CWT.

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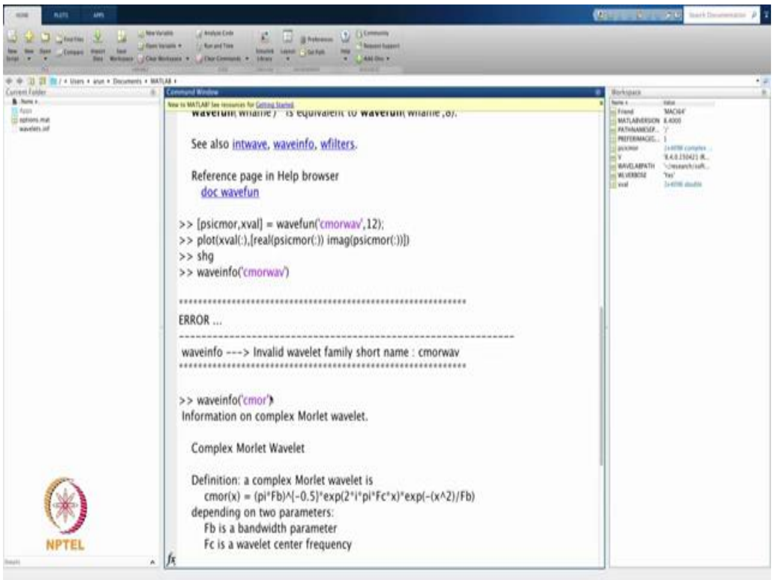

So let us, for example, pick a complex morlet wave and ask for the wavelet. And, a complex morlet wave does not have a scaling function even in the CWT sense. So, the command to generate the complex morlet wave is cmor w a b. You can also directly use is cmor wave F, which will generate the complex morlet function for you. We have done that before. So, I suggest that you also look at that option. Let us plot the complex morlet wavelet. We will plot both the real and imaginary parts. We will make sure that we are dealing with column vectors.

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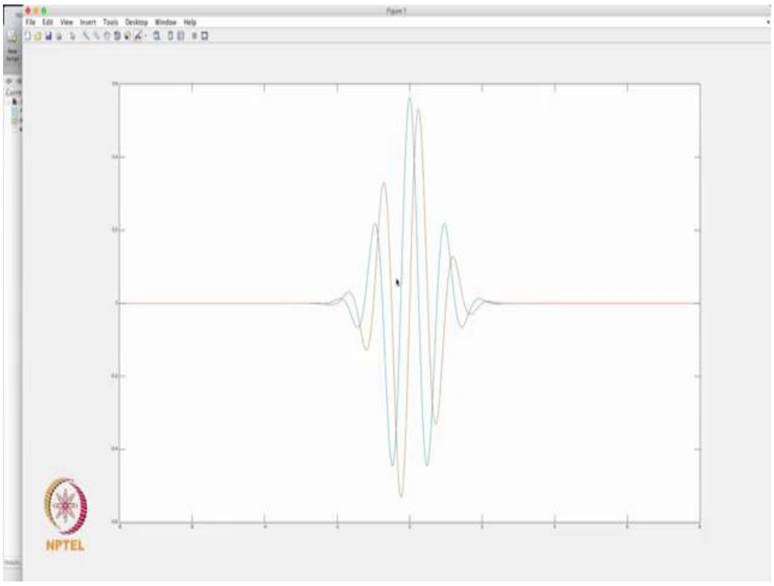

So, here we have the real and the imaginary portions. The real one is the blue line and the imaginary one is in the red line there. And, it is shifted in face with respect to the real part. Otherwise, more or less the shape looks similar. This is the familiar plot that you must have seen many times in many text books. The real part of this is essentially the real morlet wavelet. Now, it is also useful to get some information on this wavelet.

When unfortunately you have to use a different string to get the information on the complex morlet wavelet, the routine that gets you the information for you is the wave info. It is again a built-in function into the tool box.

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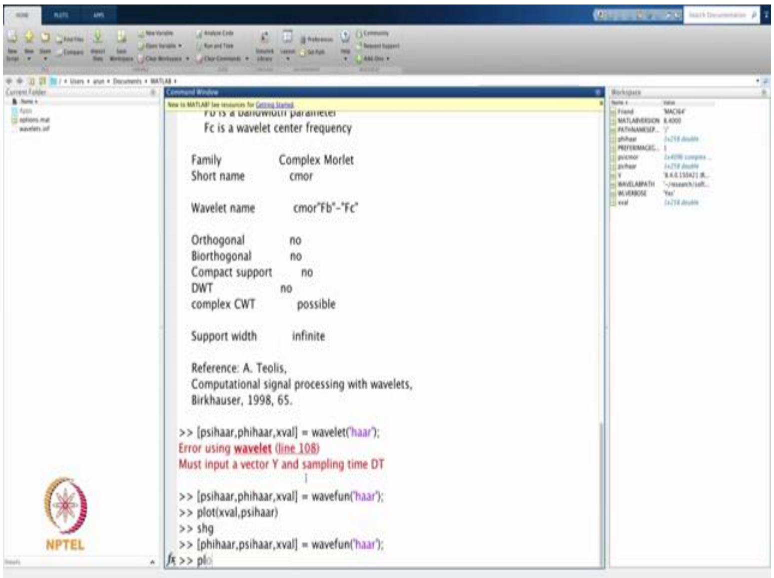

So, here we have information and complex morlet wavelet. The expression is given. We have studied this before; F b is a bandwidth parameter, F c is the wavelet center frequency. And then, the family is complex morlet wavelet, whether there exists orthogonal or biorthogonal versions of this wavelets. As I said earlier in the lecture, the terms or the properties orthogonality and biorthogonality all refer to the wavelets that are used for DWT. But, the complex morlet wave is not suitable for DWT. Therefore, it is not even applicable. It does not have compact support; that means, it does not die down in infinite time. It cannot be used for DWT. The complex CWT is possible with this wavelet. The support width is infinite because effective support width is somewhere between minus eight and eight. So, this way you can obtain information on different wavelet families.

Let us look at a haar wave for example. So, here we have the haar wavelet with us. Let us plot the haar wavelet. The first argument, unfortunately is the scaling function. So, let us rewrite here.

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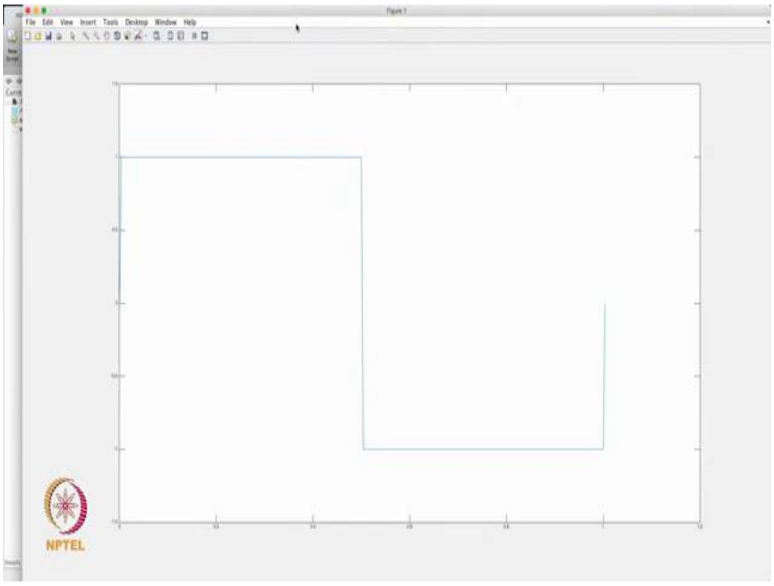

So, here is the haar wavelet for you. As you can see, it is one up to half and then minus one for the remaining of the interval and zero outside this interval; it goes with outside. Now, we showed earlier that this haar wavelet has only one vanishing moment; which means effectively it has satisfies the default zero average condition. It does not have any higher order vanishing moments. Whereas, at the Daubechies wavelet of higher order vanishing, that is higher vanishing moments, remember this haar wavelet is the Daubechies wavelet with one vanishing moment. Let us look at the Daubechies wavelet with three vanishing moments and see how it looks like.

Now when it comes to generating the Daubechies wavelet, if you recall in the lecture on scale to frequency, I had mentioned that Daubechies wavelets do not have closed form expressions. So, therefore they are generated iteratively and we shall again learn how this is done, when we look at DWT. These are generated iteratively using the corresponding filter coefficients. And, one has to specify the number of iterations to generate this wavelet. And, that is what we are going to do. Let us say we go through twelve iterations to generate the Daubechies three wavelet. We have done that. So, let us plot the Daubechies wavelet here.

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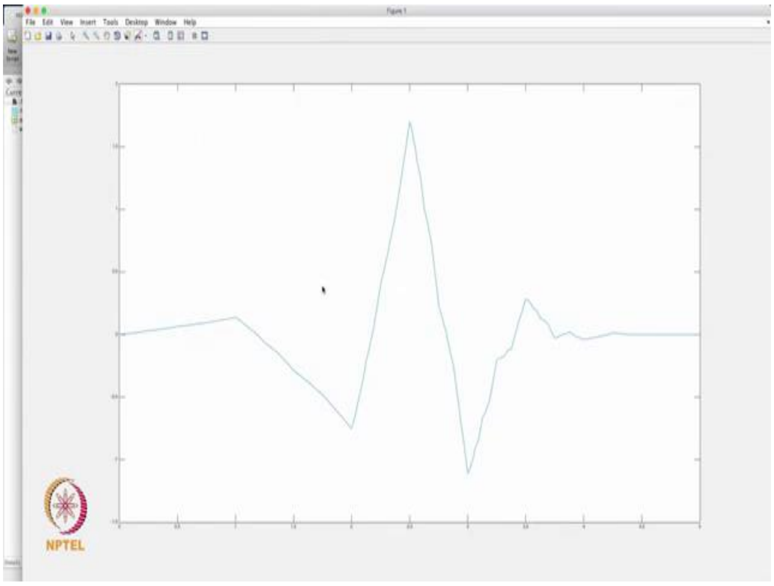

So, this is how a Daubechies wavelet with three vanishing moments looks like. It is definitely smoother than the haar wavelet. But, it is not as smooth as you would expect. So, one could generate it. For example, Daubechies wavelet with five or six vanishing moments and see how this smoothness improves.

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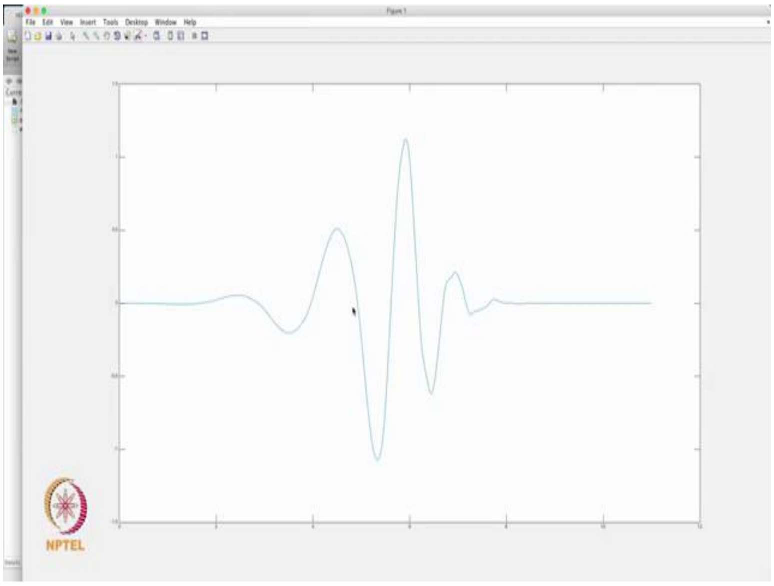

As you can see, now the Daubechies wavelet with higher vanishing moments is much more smoother. And therefore, if you want to detect the discontinuities in the signal, this Daubechies wave is not suitable for that purpose. In fact, Daubechies wavelets are not used that frequently for discontinuity detection. But, this wave can be used for detection of singularities in the derivatives of the signal. So if there is a discontinuity, let us say in the third derivative or the fourth derivative and so on, then this is much better suited. Why because very simple; when I compute wavelet transform, what I am doing is I am correlating the signal features with the wavelet.

So, where ever the wavelet at that scale and translation looks very similar to the local signal feature, the coefficient is going to shoot up. When I have a discontinuity and I am performing analysis of that signal in the vicinity of the discontinuity with this Daubechies wave, obviously the matching is not going to be very high. And therefore, it fails to pick the discontinuity.

On the other hand if I use the haar wavelet at some translation parameter and scale, the discontinuity in the signal will match the discontinuity in your wavelet. And therefore, the coefficient is going to shoot up. So, like basis or like atoms, like information. In fact, it may be nice to see how the derivative of this Daubechies wave looks like. That is, is it also smooth or is there discontinuity.

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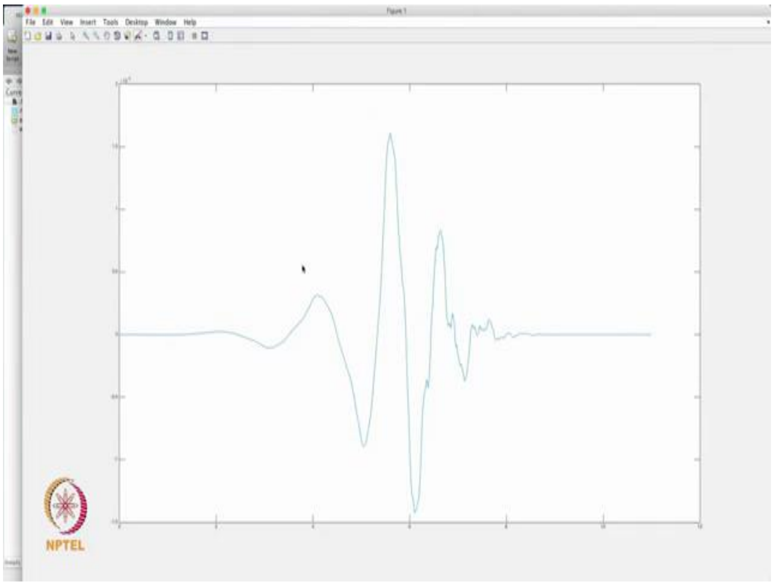

So, you can see that the derivative is also fairly smooth. This shows that the wavelet has one vanishing moment, at least. In fact, (Refer Time: 09:26) two vanishing moments because if it was discontinuous, then it could be having only one vanishing moment. In fact, we can evaluate. Let us say the third or fourth derivative of this and see at what point it becomes really discontinuous. Right. So far, for instance I can generate a Daubechies wavelet with two vanishing moments.

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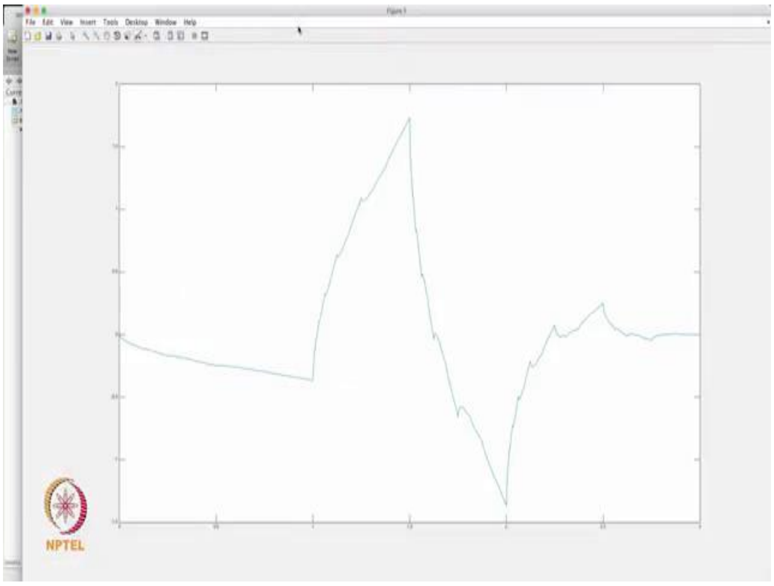

So, this is how a Daubechies wavelet with two vanishing moments looks like. It looks definitely much smoother than the regular haar wavelet. But, it is obviously not as smooth as the Daubechies wavelet with six vanishing moments. So, it is clear now that higher the vanishing moments, more as smooth is the wavelet or more regular is the wavelet. The regularities, something as I said earlier is in qualitative terms. We feel; we can have a qualitative feel of what is regularity. We know all of this. But, there are quantitative measures such as Holders exponent and Lipschitz exponent, which quantify the extent of regularity. So, let us look at the derivative of this and see if the derivative of this is discontinuous; because it is has two vanishing moments unlike the haar, which has only one vanishing moment.

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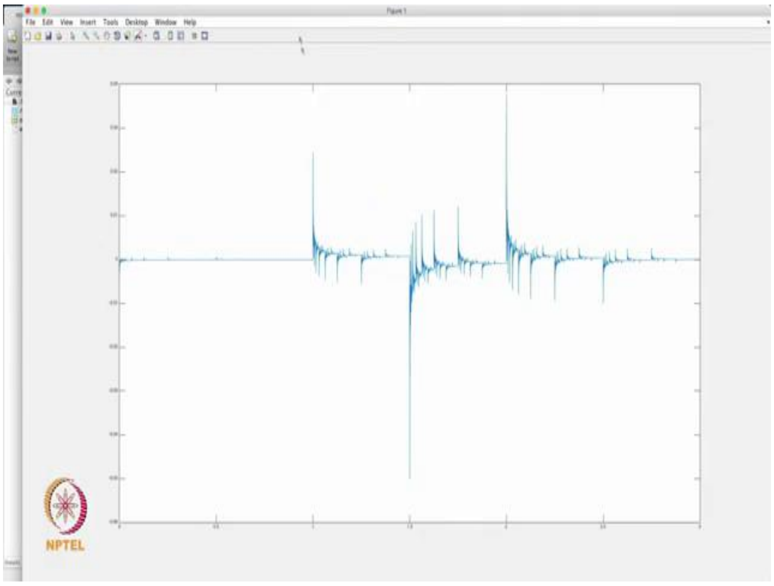

As you can see, the first derivative itself has discontinuities in it. Right. And, we should expect that because the original wave has two vanishing moments. By differentiating, we have got in red of one of the vanishing moments. And, now this derivative of d b two has only one vanishing moment.

So, this way it is clear now why this wavelet, that is, d b two is suitable for detecting discontinuities in the derivative of the signal. The derivative wavelet has a discontinuity. Therefore, it can detect the discontinuities in the first derivative of the signal. Right. So, I hope now with this, you are more comfortable with the concept of vanishing moments and have a fair understanding of how this vanishing moments is related to the smoothness of the wavelet and its suitability for detecting discontinuities in the signal and its derivatives. Once again, you can obtain information on each of the wavelets that we just looked at. Sorry

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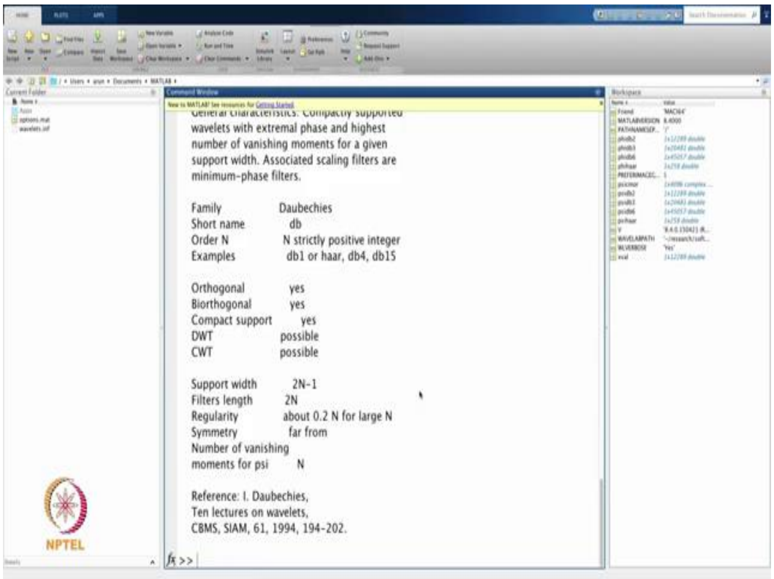

So, there is a general information on Daubechies wavelets. You can go through this. It is suitable for DWT and it is characterized by this order N, which is a number of vanishing moments; clearly says d b one is haar. And with the Daubechies wavelets, you can generate what are known as orthogonal wavelets or biorthogonal. It has compact support clearly. It dies down in finite time. And, it is suitable for both CWT and DWT, but rarely do we use Daubechies wavelet for CWT may be occasionally in singularity detection. But, that is about it. And, its support width is two N minus one. This is a point that I was mentioning earlier.

That Daubechies showed that there exists a connection between the vanishing moments and the compact support. That is, the extent over which or the width in time over which the wavelet exist. If I have a wavelet of vanishing moments n, then it has to have at least a compact support of width two N minus one. This is what Daubechies proved.

And, we look at that result more in detail in DWT, but what this tells us is as I want the wavelet with higher and higher vanishing moments, I need to tolerate wider and wider mother waves. So, d b one will have the least compact support that is N equals one. So, this support width is one; interval one. If I have d b two, then the support width is three and if I have a d b six, then the support width is eleven and so on. So, we can go back and see if that is the case for d b two.

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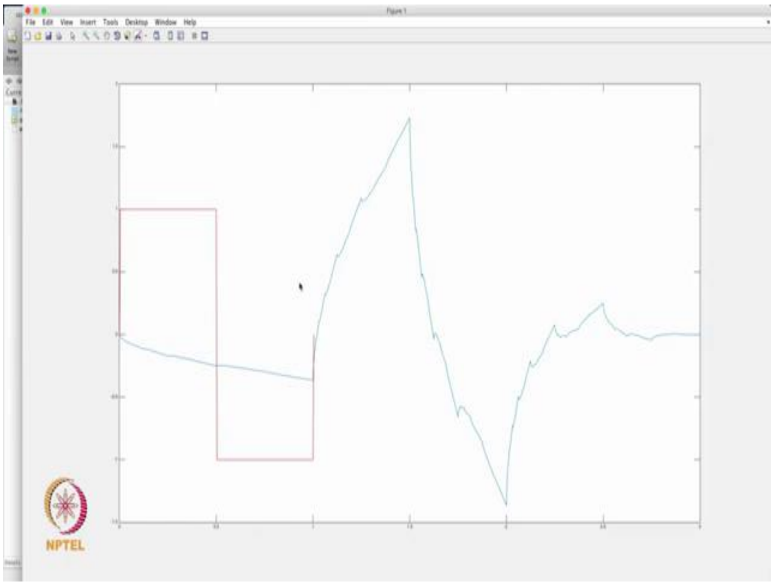

So, if you can see here the support width, that is effectively where the wavelet is non zero, is around this interval here; zero, effectively it is three. Right. So, you have one, two and three. So, exactly you have between zero and three. That is what this results says. Whereas with a haar wavelet, you can see that the haar wavelet exists for a much shorter interval. Once again, this is to show you the connection between the vanishing moments and effective width of the support of wavelet. In fact, please do not say effective here, say exactly the width of support. As the vanishing moments increases, the wavelet needs longer and longer duration to vanish. Ok.

In fact, now you can imagine the extreme case being a sine wave. For example, sine wave which is a Fourier atom exists infinitely over time. The question is how many vanishing moments this sine wave has. Can you take a guess? Well, the answer is infinity. Right. In the sense you can, actually it is very very smooth. It is highly smooth. It is continuously differentiable. Therefore, it has a number of vanishing moments; which means the sine wave is not suitable for detecting discontinuities. And, that is where you run into Gibbs effect and so on. So, all of that now make sense with the property of vanishing moments.

So, I invite you to explore the other wavelets in the family. There are not only the Daubechies family, but there are many other wavelets that I have not demonstrated. But hopefully, this demonstration has given you in a foundations to go and explore other wavelets families. There is also a GUI version of the wavelet tool box, which is brought up by typing wave m n g r, sorry, wave menu. And, this wave menu brings up this window for you; which allows you to perform many things.

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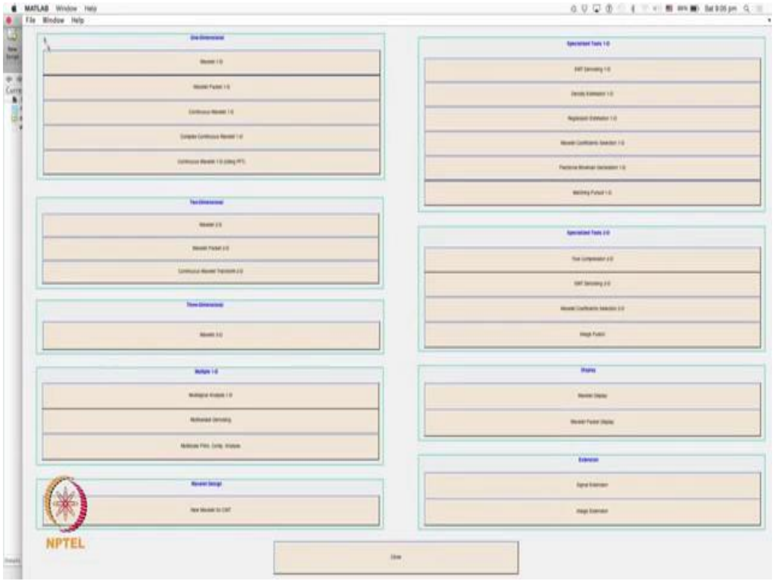

In fact, all the things that you can perform with the wavelet tool box at the command prompt, you can do a discrete wavelet transform, continuous wavelet transform; you can perform signal estimation, signal compression; you can perform multi wavelet analysis. You can also generate different wavelets families. At the bottom right, here, just above the bottom right, you have the wavelets, you can display wavelets and you could choose your wavelet families.

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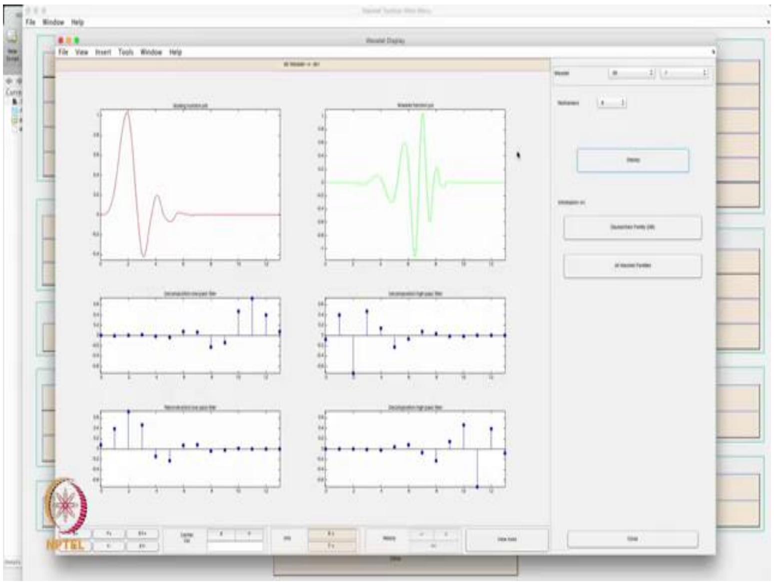

For example, if I want to plot a haar wavelet, this is how the haar wavelet looks like. On the right you have, in the green, the haar wavelet and on the left you have this scaling function. And, this way you can generate many different wavelets. For example, I can generate the Daubechies wavelets here. This tells me the number of iterations. So, here is how the Daubechies wavelet of a high vanishing moments order looks like. And, we have; I am just showing you what we done earlier as well. So, this is Daubechies wavelet of vanishing moments two. So, you could do that all of this with the click of a mouse as well. So, I invite you to also explore this GUI; because it saves quite a bit of time and allows you to spend more time on the analysis.

Of course, you can export these wavelets to the command space or command work space. And, there you can perform what our calculations you want to make such as the average being zero or not, how many moments and so on.

So, hope you really enjoyed the lecture on this different type of wavelets and how you could really display, calculate this wavelet, generate this wavelets in mat lab. And particularly, the relation of the vanishing moments property with the regularity and the extent of compact support of the wavelet.

So, see you in the next lecture, which will be the concluding lecture for the continuous wavelet transform. We shall look at; as I mentioned, three different applications of the CWT. One is in discontinuity detection, the other is the instantaneous frequency and other extraction of time varying feature and thirdly filtering using inverse CWT. See you in the next lecture.

Bye.