Introduction to Time-Frequency Analysis and Wavelet Transforms
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Lecture - 7.4
Scalogram and MATLAB Demonstration
Part 2/2

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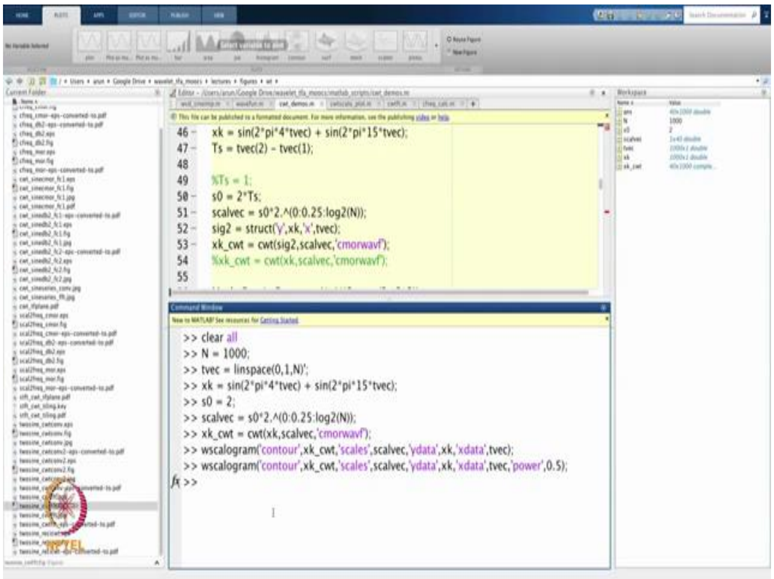

So, what we shall do is we shall first generate a signal that consists of two sine waves an example that we have seen earlier, let's do that. So, let us do that x k. So, if generated a signal that consists of two sins running in parallel of frequencies four and fifteen hertz respectively. The first step in the computation of scalogram is the computation of the cwt itself. For that we can we need to supply the set of scales over which we want to compute the cwt. So, let us set the initial scale as 2, let us assume that the sampling interval is one, you can always rescaled scales with the appropriate sampling interval. Just to make things simple we can say s naught is 2, and then or scale vector we can probably computed up to log 2 of N by 8 or maybe when run up 2 N, that's not matter.

And let us choose this our as our set of scales, so over which we want to compute the cwt, now compute the cwt, we are using convolution algorithm here, we are not really ((Refer Time: 02:40)) for time. So, let us use the convolution algorithm, and in fact the scalogram routine, that is the w scalogram routine in mat lab wavelets toolbox assume that we have computed the cwt by using convolution algorithm. But nothing preventive from using the cwt based on fourier transform algorithm and passing that to scalogram. So, this scale the w scalogram is not going to breakdown. So, now I have these coefficients computed from this cwt, and all I have to do is now plot the scalogram. What I shall do is, I shall first issue the comment and then go over quickly the help. So, what I am doing here is, I am using of course the w scalogram routine which is bundle with the mat labs wavelet toolbox.

And in the first argument as specified if I want contour plot or an image plot, this w scalogram is a capable of returning the computed scalogram values, right now I am not taking that I am not asking for the scalogram as the output, I just want to plot of the scalogram. And I want to a contour plot; the first argument, the second argument is the set of coefficients that you are computed from cwt, this could come either from you cwt or the fourier transform the one based algorithm. The only difference is if you are computed using the convolution algorithm, then directly you can pass the output of that here as you have second argument, but if you are computed using the fourier transform based algorithm, then remember there it written a structure, therefore you have to access that specific field and pass it along ((Refer Time: 04:28)), that is only difference. Then the remaining arguments are optional. So, for example, I supply the vector of scales at which I have computed, and I do this by specifying this scales property . And if I do not do that when it assume that it has you are computed from certain set of scales and so on better to always pass the scales at which you have computed.

And then I also plot the I also pass on the signal to the scalogram. Now this is again optional, if I pass this signal then it plot this signal on the top and the scalogram at the bottom, it makes a life easier when it comes to comparing the features pointed out by the scalogram, and the features present in the signal. And here I can pass on also the time vector at which have computed and for this I can specify by specifying the x data property and that’s our time vector, and final power that I have is point five. What is this power? Well, it is essentially got to do with in normalization, remember these amounts to the normalization of the scalogram amounts to scaling the coefficients, cwt coefficients by 1 over root s. So, this power is essentially the one over s raise to some power and that power is 0.5, if you want a normalize scalogram. If you do not want a normalize scalogram, then do not pass this argument then by default the wavelet coefficients are not re normalized. So, let us look at the un normalized scalogram, and see what plot is obtained. So, this is the plot that we have.

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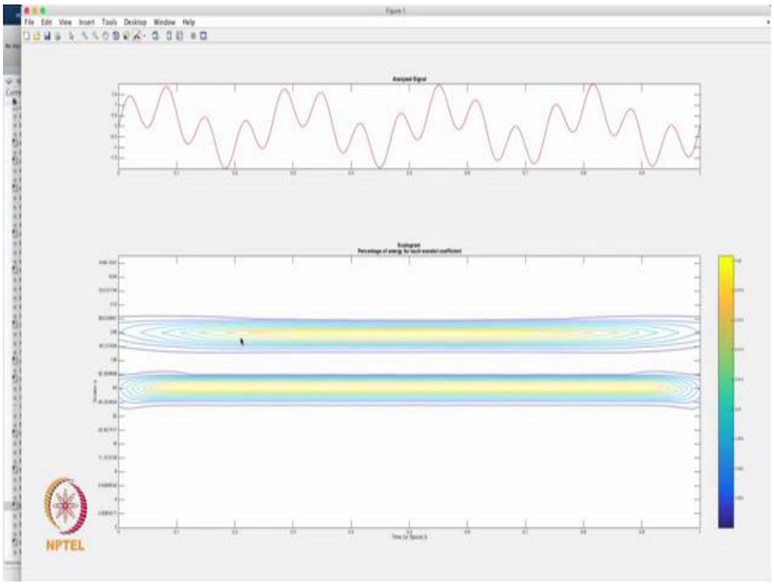

So, look at what we have here - the signal on the top, and the scalogram here. So, what has happened here is of course you should expect the this scalogram to be intense around two scales. By these two scales when you convert them into frequencies, you will be able to find the appropriate frequencies, you can use the conversion that we have done from scale to frequency. But what is what comes out from this un normalized scalogram is somewhat misleading, it is says that this signal at the higher scale, I have ask for a contour plot, higher scale would correspondent to a lower frequency that seems to have more energy than the one at the lower scale. And this is in line with what we discussed earlier, the wavelets at higher scales take in more of the signal at any tau, and therefore have a larger value for the coefficient compare to the wave, coefficient obtain by wavelets at smaller scales.

Now, what we shall do is, we shall normalize this scalogram, and see whether it makes any difference. So, now things have fallen some somewhat in order that I have now this scalogram showing me that the energy of these signals that which is essentially sinusoidal components at both frequency’s around both scales are of comparable or comparable the energies are comparable. So, this makes now lot more sense considering the wave, we have generated the signal.

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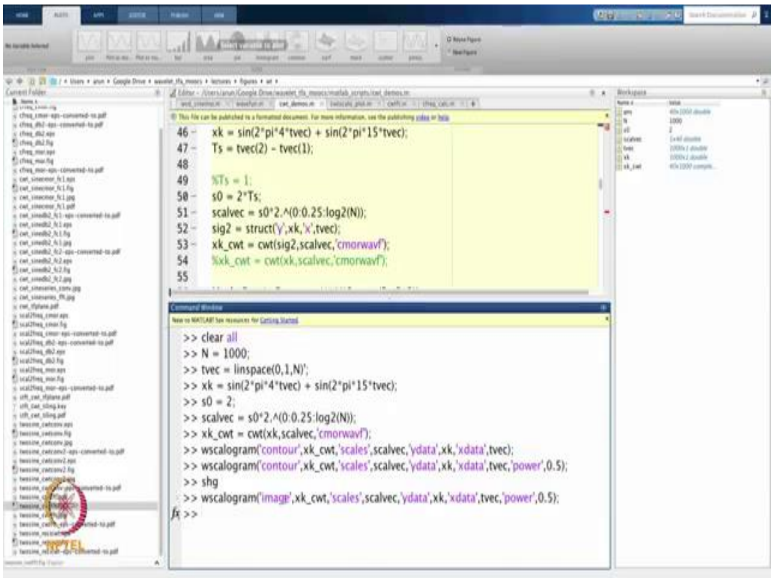

And you can also obtain an image plot which probably looks more colorful, the contour plots do not looks so colorful necessarily.

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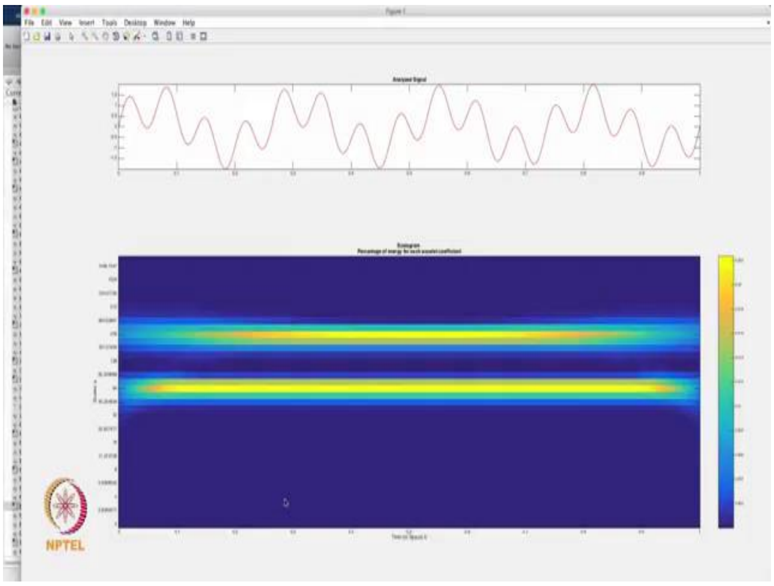

So, the image plots make it look more drastic, but notice that whenever you plot and image plot the scales may be reverse, because images are usually produced in such a way that the y axis is reversed. So, keep that in mind, so hopefully now you understand the difference between normalize scalogram, and un normalize scalogram. Why you should be working with normalized scalograms.

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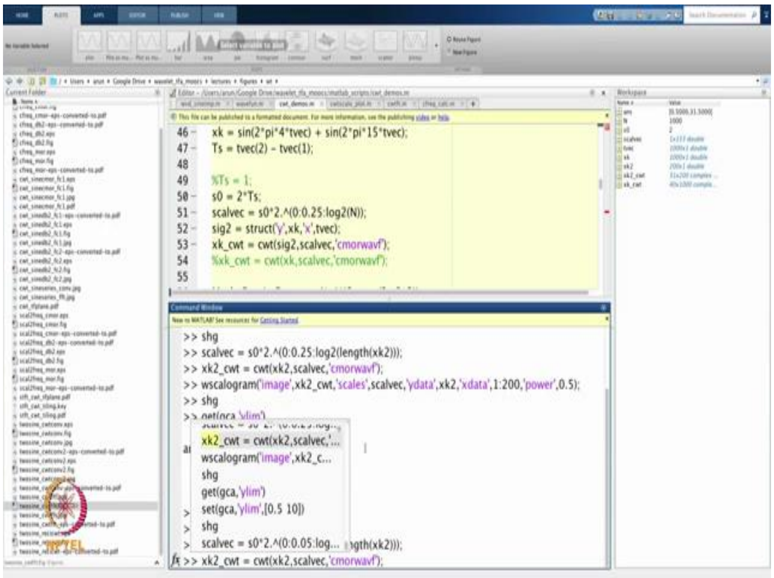

Let us also look at the scalogram of an impulse, sorry... Before I do that let me take you to the documentation on the scalogram. So, here is our documentation I am not sure, this is clear to you, but what it is says is essentially the default syntax, what is the default syntax, and what it returns? It return essentially this scalogram computed at different scale values and tau values, and the optional argument as well. And then what is this? What is a default value for those optional arguments. And that is that is about it essentially the power is something that sorry, the power is something that as what to do with how you scale the coefficients with the normalization that we have chosen with we normalize a coefficients with 1 over s power half, and that half is a power that it is taking about that is about it.

So, let us now generate an impulse of first let us have a vector of length 200, and let us arbitrarily pick a value at which we want an impulse. Let us say the value of the impulse is five at 80, we could choose a pulse as well, that is what essentially. So, what we have here is a pulse, sorry an impulse located at eightieth instant.

Now we want to know how the scalogram of this impulse looks like. So, I have x k to underscore cwt coming out of the continuous wavelet transform routine, we can use the same scales that is the advantage of working with sample data, that is assuming sampling interval to be one, but of course a length of n is different. So, let us recompute length of x k 2 s naught is the same in both cases, that is a starting scale is a same. So, let us compute the, sorry continuous wavelet transform x k 2, sorry x plus cwt is cwt x k 2 and scale we shall particularly using complex morlet wavelet, that is what we have now, and we go back and here change are time vector as 1 to 200, and change the signal to x k 2, and change the wavelet transform to see what is happening? Well, we have computed at more scales and necessary, we can go back and actually change the scales up to which we are computing or alternatively change the limits for the y axis here on the image plot. We can also do a contour plot, if you like, but let us change the limits here. First you can determine what the limits are? So, these are the limits, we can change these limits to 0.5, let us say to 10, yeah.

So now, we are able to see the scalogram, of course we are not computed at a very fine set of scales, and therefore you see this square boxes, we could recompute this at very fine scales to get at to get a very fine display. So, to do that you can go back and change this spacing here for this scalogram, and we can ask a 0.05 spacing here. Of course in terms of powers of 2, and then recompute or continuous wavelet transform and replot, sorry or scalogram.

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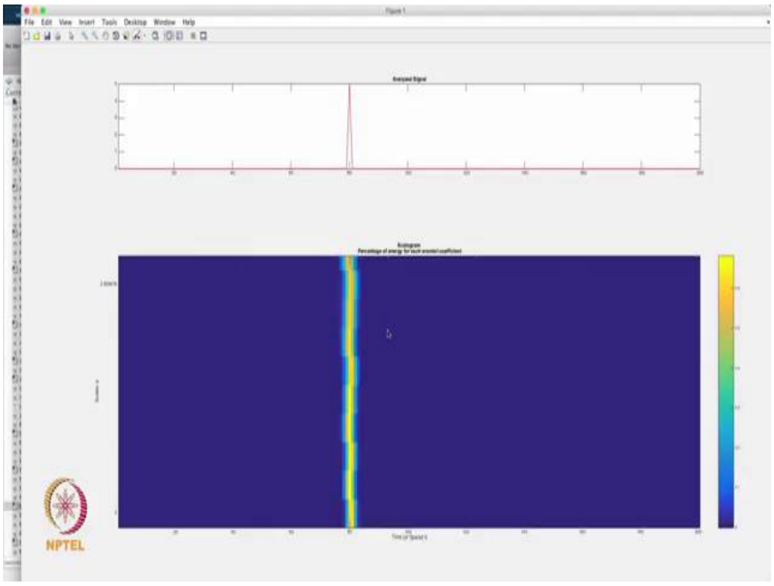

And we can actually further. So, this is the situation that we have here, we could yeah... So, in fact you can play around with the y axis limits, but an important point to observe here is as you are moving the scales here, let me even change I ((Refer Time: 14:41)) for able to see the font size again, I will change the font size for you. So, as we are moving up the scales. In fact we can move up further. What is happening is that the energy is spreading and that should be expected, because the wider wavelets that is the wavelets at higher scales result in a larger smearing in time. Therefore, the ability of the wavelet to deduct the up to localize the energy in time deteriorates as you go up the scales, and therefore the scalogram kind of the diverges as you in terms or spreads in time as you move up the scales, and becomes narrower as you move down this scales.

So, this is something to keep in mind, because and this is of particular interest, because what happens is when we compute cwt, we are computing cwt of a finite length signal. And we have talked about this earlier also in different context, particularly in computation of cwt, we have edge effects whenever we compute cwt. So, at the borders what happens is you are essentially running into discontinuities. If you are using a convolution algorithm, then it assumes something about the signal beyond outside the observation interval, and if you are using an fft based algorithm it assumes that the signal repeats itself and so on. In fact to the convolution algorithm you can change the assumptions that you want on how the signals behaves outside the observation interval. But normally what happens is eventually when you are computing the cwt of a finite length signal, the wavelet really is not seeing the signal is unable to see the signal outside the interval, if you extended then it will able to see it, but whatever may be the extension there are going to be edge effects, and how this edge effects d k depend on the extension.

If you do not perform any extension then that is in the sense you assume that the signal to be zero outside the interval, then you have discontinuities occurring at the edges. In other words, as if the wavelet seeing an impulse located at the beginning, and the at the end of the signal. What is the impact of this discontinuity on the scalogram? Well, the impact is straight forward. In fact this is captured in what is known as a cone of influence. The impact is very can we intuitively understood when I use the wavelet of high scale that is wide wavelet, then when I am situated either at the beginning and the end or the end up huge portion of the wavelet is outside the observation interval. In other words, when I if I want to if I want to really analyze the signal properly I have to make sure that the entire wavelet is located within the observation interval. For wide wavelets, it takes a long time for the wavelet to really that is the entire width of the wavelet to be within the observation interval. For narrow wavelets, there width is narrow, therefore it does not take too long for that too long meaning as you are traversing along the signal, it does not take too long for the wavelet to come within the observation interval. Let me show that you on the board.

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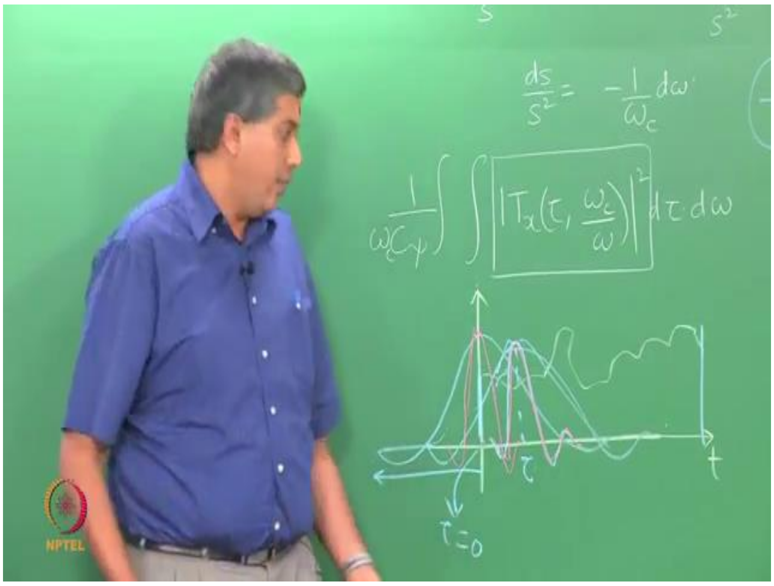

So, look at this here, this the situation when I am using a wide wavelet then essentially what is happening is let us say this wide wavelets was initially located at this point, then this would how the wavelet look like initially, that is when I am looking a tau equal zero. So, what is happening is, there is a huge portion of the wavelet, large portion of the wavelet outside the observation interval. And it this produces artifacts in your cwt. Of course, depending on how you extent the signal outside the interval, but if I assume that the signal is zero, then you are introducing the discontinuity at this point and at the end points.

So, the artifacts that are produced by this wide wavelet at this point is more or less the same as a wavelet of an impulse located at time zero. What you have seen earlier in example is an impulse located at eightieth instant, and we notice that the energy is spread in time quite heavily. So, what happen is here the effect of the discontinuity is felt for a long time intuitively how long well the time it takes for the wavelet to be completely within the interval of the signal. Now compare this situation of a wide of a narrow wavelet. So, let us color it differently here, the narrow wavelet is red in color. Imagine this narrow wavelet being placed here, that is at the beginning of the signal. So, that the center of the wavelet is exactly at the origin.

Once again of course you have portion of the wavelet outside the signals observation interval, but the time for this wavelet to be drawn within that is to within the signals observation interval is not going to be too long. For small values of tau, the wavelet is going to be completely within the signals interval. It is the another other way of saying that is, if the energy spread of an impulse located at origin obtain from high frequency wavelet is not going to be spread too much in time as we have seen in previous plot. Therefore, the time for this effect of discontinuity to be felt at finer scales, that is lower scales is going to be much smaller.

Now these times that at taken by each of these wavelets at different scales for the effect of discontinuities to vanish are called e folding times, and this e folding times can be calculated for different wavelets, it is given in the ((Refer Time: 21:34)), combo, and so on. Rather than worrying about exactly the e folding times, hopefully I have understood the concept of this influence time of influence of this discontinuities at the edges, and normally what happens is when you look at it in the scalogram this appear as the that is there is a cone of influence that can be drawn. And only the scalogram values within that cone can be considered for any statistical analysis, the values outside the cone of influence are cannot be subjected to any particular statistical analysis, because there are quite of few artifacts introduced by the discontinuities at the beginning and end.

In mat lab, you can calculate the cone of influence using a routine called con of Inf, but using that and along with the scalogram plot is not so straight forward instead you could use open source wavelet tool box by ((Refer Time: 22:44)) the that tool box supplies essentially routine called wt which computes a cwt.

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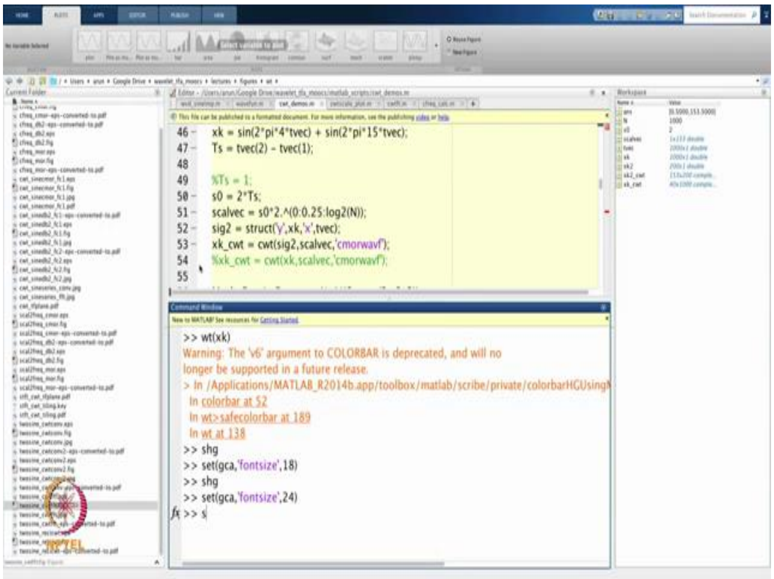

And using that we can obtain the cone of the influence as well as the scalogram. So, let me just show you how to do that with that routine wt. So, you have to install the wavelet tool box which is freely available from ((Refer Time: 23:15)) website or of course you can use this c o n o f I n f routine that is available with the wavelet tool box, but I fine using the wt routine from the package by ((Refer Time: 23:28)) much easier. Therefore, I will just show you it is called the wtc tool box wavelet coherence tool box, and bundle with that is this wt commen, and simply invoking wt for example, x k on the previous signal that we have that we had produces scalogram as well as the cone of influence.

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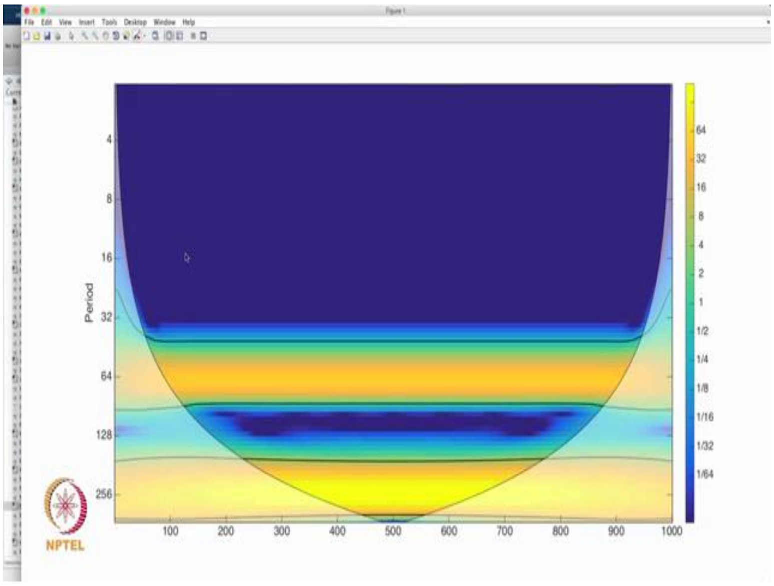

So, look at what we have here, what is plotted here on the y axis, let me enhance the y axis, font size is for you hopefully now it is lot more visible or maybe I can this change it to. So, what we have here is period which is nothing but equivalent of scale, and this scalogram has been computed using the complex morlet wavelet, this is a normalize scalogram, and what you see here is the cone of influence. Without the cone of influence, it is hard to find out which parts of the scalogram can be subjected to scientific analysis or statistical analysis. Now with this cone of influence what this what I know is at low scales that is at low periods, the scalogram values calculated at these times that is starting nearly from zero to the end of the signal are reliable. There are no artifacts there or the artifacts introduced by the discontinuities at the edges vanish in no time. Whereas, when you come to high scales, that is high periods the time taken for the edge effects is longer, and therefore I have to wait for quite sometimes before the wavelet is completely drawn into the observation interval. And hence the scalogram values available are reliable scalogram values or only over a narrower period of time.

So, always one should use this cone of influence when you want to make meaningful interpretations out of a scalogram. Of course, in this case this scalogram outside the cone of influence also corresponds to what we see in the signal features, but then the the outs the scalogram values outside the cone of influence are influenced by the discontinuities at the edges. So, this is what I wanted to show you in this lecture, and of course we have looked at a definition of sclogram, and now we have learnt how to do this in mat lab.

And what we shall do in the next lecture is we shall talk of an interesting cousin of wavelet which is wavelet function, which is called scaling function. Where we will introduce essentially the low filter equivalent of the wavelet function, the wavelet access a band pass filter, but it is nice to have both the low frequency filter and the band pass filter. So that we can traverse the entire frequency range, and see you in the next lecture.