Introduction to Time-Frequency Analysis and Wavelet Transforms
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Lecture -1.2
Brief Tour of Wavelet Transforms

Now, one of the key applications of Wavelet Transforms is in detecting discontinuities.

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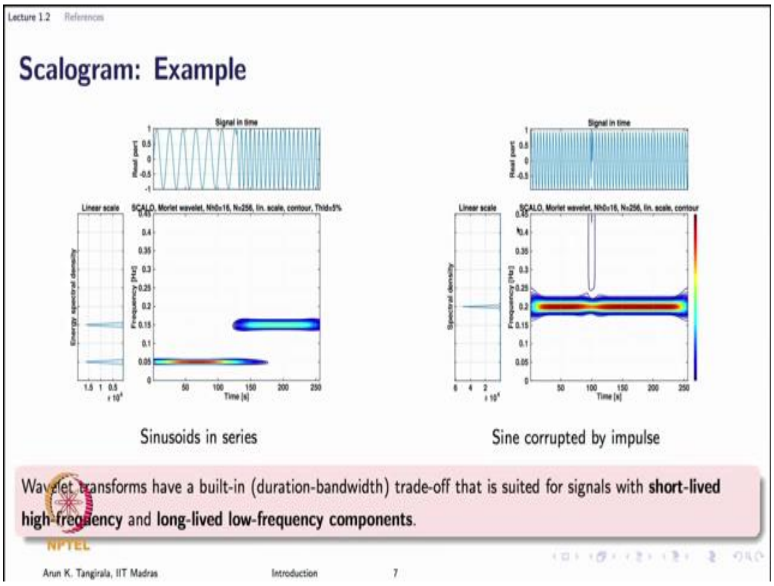

And if you look at this example that we have here on the right it has done that job for you. We have a sinusoid corrupted with an impulse disturbance, without any user intervention, it is able to detect the discontinuity as well as sign wave. Whereas, in the spectrogram all though we did not discusses in detail, one has to adjust this width effectively to capture both the impulse and sine wave you have do it by trial and error. Whereas, with wavelets they are it is very nice, it is very ideally suited for these class of signals.

Because, the impulse is short lived and it is nicely captured by the very high frequency wavelets. And since impulse any way has a broad spectrum, you do not have to worry about the frequency localization. And a sine wave at least in this signal exists for a long period of time and it is f much lower frequency, then any of the frequencies present in impulse at least the high frequencies.

Therefore, it is nicely captured in time very well. But, the energy has been smeared in the frequency direction or dimension. Always compare with what you would get with the spectrum. Because, the Fourier spectrum will give you global features, it is important not to discard that information, when your performing time frequency analysis. One of the first steps in time frequency analysis is to perform Fourier analysis get an idea of global features and then move on to the time frequency analysis. Obviously, when it comes to capturing the frequency contain, nothing can beat the Fourier analysis for you.

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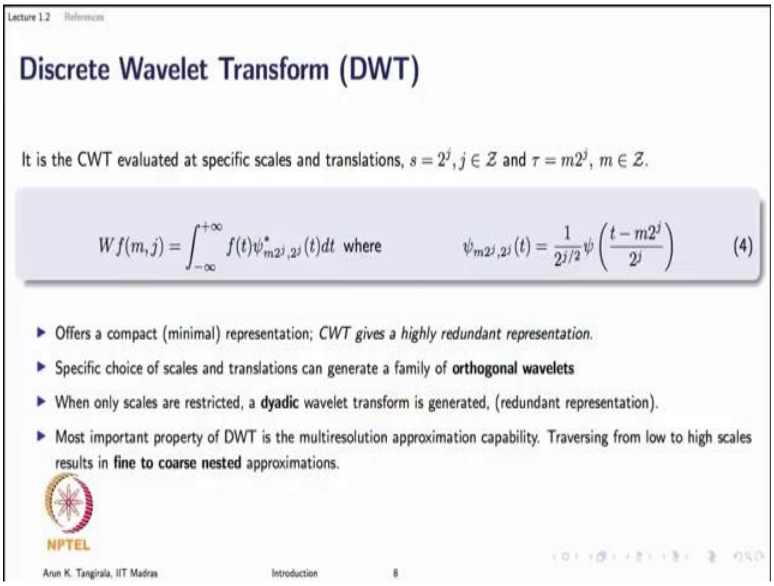

So, will close this discussion with quick overview of what is known as a discrete wavelet transform. Until, now we have talk about continuous wavelet transform. In fact, we had when we gave the definition of the wavelet transform that is the continuous wavelet transform. What is continues about it? Do not please attribute that to the continuous nature of the signal.

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The continuous prefix there has got to do with the continuous with these parameters here tau and s. When, these are continues value, then we call them as continuous wavelet transform, pretty much like with other transforms. What we are talking about hear is discrete wavelet transform. Again here, discrete does not refer to the signal,, but refers to how you choose a tau and s. When, I say discrete wavelet transform it means, that both tau and s are now going to take one specific values on a grid that I will choose unlike in a continues wavelet transform where tau and s are continuous value.

So, that is the prime difference the continuous and discrete I have got nothing to do with the signal, in both cases signal is continuous. So, why do we require discrete wavelet transform? The simple answer to that is, if I look at the continuous wavelet transform I am going to evaluate the transform at every possible value of tau and s. Of course, computational that is not possible. But what that means, is I am going to choose a very fine grid for tau and s to get the picture.

Now, when I do that what happens islet us say I choose scale equals 1 that is I am looking at the mother wave itself. And I am going to analysis the signal with this mother wave. Then, I translate with the in very fine steps in time, then what happens is there are going to be huge overlaps between the signal segments that I am going to analyze. And that will generate lot of redundant information. Remember we have going from a one dimensional signal representation to a two dimensional representation. And likewise scales as well, when I look at too very finally, spaced scales. And that is a analysis a two finally, space scales again I am going to generate a lot of redundancy. And this is not good or useful desirable in many applications. And the classic application being signal compression, where I would like to represent the signal in as few numbers as possible.

So, if you think a think of such an application which is of course,, very widely encountered then, I would like to choose tau and s in such way, that is the translation and scaling in such a way are ultimately when I am done with analysis of the signal. This coefficients that I generate by virtual transform are going to be minimal. That means, those are the minimum number of coefficients that I will require to represent the signal in the time frequency plane. And it turns out, that if you choose this particular grid for the scaling parameter in the tau translation parameter. Then, you have what is known as a compact representation in the time frequency plane, on the time scale plane. And the particularly when you choose powers of two, we call is as a dyadic discrete wavelet transform. But, normally we leave out the dyadic, it is understood by default discrete wavelet transform always works with dyadic scales.

And you should see here, that the translation parameter is been chosen proportional to the scale. What this mean is, if I choose a particular wavelet of this width, I should shift I should march ahead proportional to the width. If I choose a wider wavelet, then I should march ahead my steps of marching should be longer, larger. Whereas, when I choose a narrow wavelet, this steps that I take in time for translation should be smaller.

Because, then I would have covered only a small portion of the signal at each value of the tower. And that is the reason we have the proportionality for tau. That time to gather the grids for grids spacing for tau and s makes this a very compact representation. And will talk about the mathematics of it, how is a guaranteed choice of s and tau will generate a compact are what is a known as a orthogonal family of basis function for analysing signal.

Otherwise, the original transform is always CWT. DWT is not a new transform, it is very popular because of the applications, where it is applied. Signal compression, signal estimation, image analysis and. So, on DWT is very popular. But, please do not think that DWT is the best technique always, it has its own draw backs and CWT is very good for example, in feature extraction. Because, it has is redundant information and you can get very nice features. Whereas, we DWT may not able to get the features very well. And there are other draw backs such as invariance to features. When a feature shift in signal in time, DWT cannot detect that shift. So, that is something call shift in variant wavelet transform and. So, on. So, DWT is the answer for many application, but not for every application. And CWT also has it own place.

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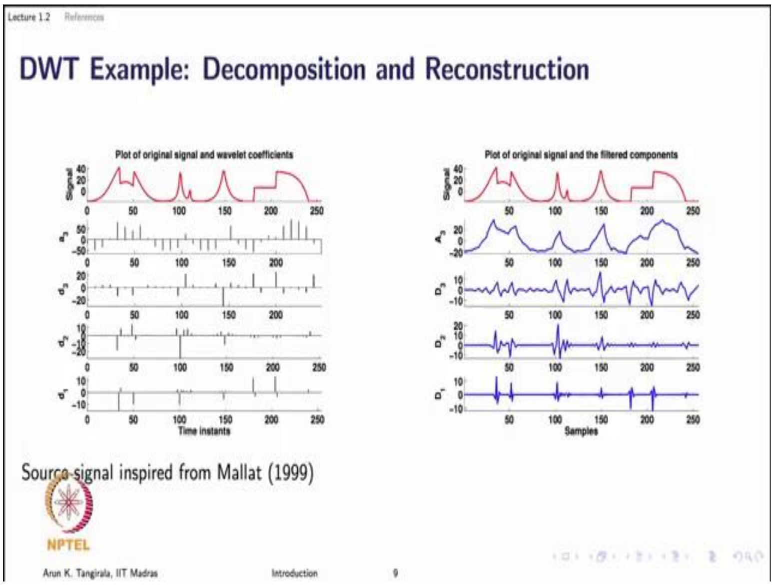

So, just to give a feel of how DWT looks like, I have an example for you take an from Mallat's book. That is signal itself if take an from Mallat's book, that the figures have been completely regenerated. So, look at the left figure that we have here on the top I have what is known as piecewise polynomial signal, it has features such as discontinuities very sharp, smooth, changes and so, on.

What is typically done in DWT is you would actually break this signal into different components at different scales exactly what the way you do in CWT. But, the terminology is what makes it a bit obscure. What you see on the bottom four panels here are essentially the coefficients. That is after your perform the transform at different scales. The second panel here consist of what is known as approximation coefficient, which is the transform taken of the signal at the courses scale. What we mean by courses? The scale at which the low frequency wavelet exists. And then you have at the bottom, the bottom most the coefficients computed at the finest scale. That is the narrowest wavelet, that you are using in your analysis.

Now, what is the use of these coefficients that I have computed. Notice that, the width in time for this coefficients is different at each scale, that is got to do again with the choice of tau. We said we will march ahead in time depending on the scale at which I am computing. Now, what is this coefficients give me? They give me the energy contained in the signal at this respective scales. Now, predominantly what you see is a low frequency trend in the signal with occasionally some discontinuities.

Whenever there are discontinuities, you will see them reflected in the finer or the high frequency scales. That means, values of s less than 1 and the remaining, the most part of the signal lives in the low frequency domain or high scale. And typically we say trends are approximations. And that is why we these are call approximation coefficients. Now, what is you see on the right is again the signal on the top. But, now I have plotted continuous versions of the signals components what are these exactly. Again with the signal on the top.

The second one that you see, which is denoted here as a 3. I have how did I generate this? Well, this simple way to generate this signal here is by zeroing out all the coefficients d 1, d 2, d 3 on the level. So, I assuming that none of the detail should be present in the signal. There where there by some mistake and I am going to eliminate them, then I am going to construct the approximation of the signal. So, that is the coursest approximation of the signal that I am constructing at this scale of my analysis. I can even construct courses are approximations.

Then, what I have in the remaining three on the right hand side here. The upper case, notice the difference between the notation use uppercase for the reconstructed components and lower case for the coefficients. So, for the d 3 also it is a same story. What I do is in constructing the big d 3? I will retain only the coefficients in small d 3 and then ignore or zero out the coefficients at the remaining scales, the idea is to reconstruct only that part of the signal, that is the idea.

So, what we are doing is, we are reconstructing the signal at individual scales. And if you add up all the blue curves that I have they are you should get the red one. That is a perfect reconstruction property, that the discrete wavelet, the wavelets are use for DWT should satisfy generally speaking.

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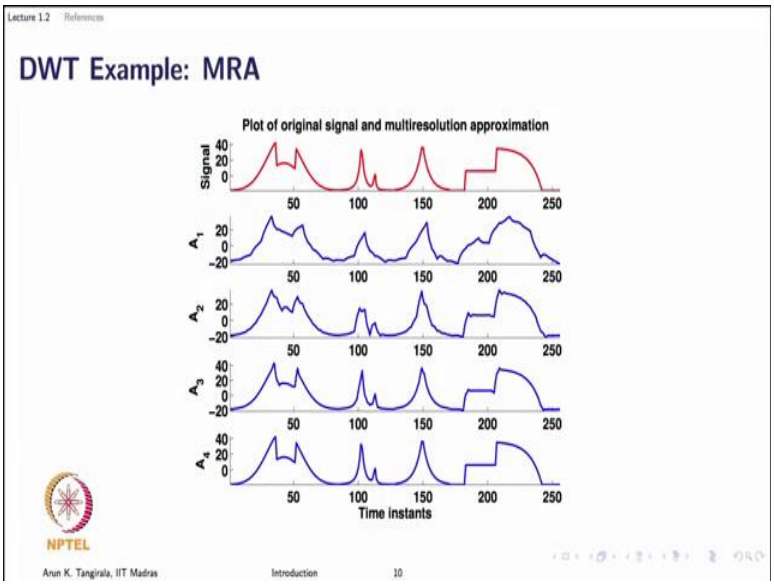

Now, this is idea that is used in multi resolution approximation. That is very common and very effectively used in computer based on image analysis where a construct approximations of images and different scales. In fact, if you were to see in many modern browsers, you would see if you are viewing an image, the image will be displayed to you at different approximations, starting from the courses to the finest. That is a same idea here.

I start again I have the signal on the top, I have the coursest approximation that I constructed earlier. Now, what I am going to do is? I am going to add this detail big d 3 to approximation and get a better approximation of the signal, and then even the better approximation, and finally, the signal itself. Because, we said when we add up all the broken up components I should get, recover the original signal itself. So, this constitute what constitute what is known as a multi resolution approximation which is very popular in wavelet theory, we will talk about this in details later on.

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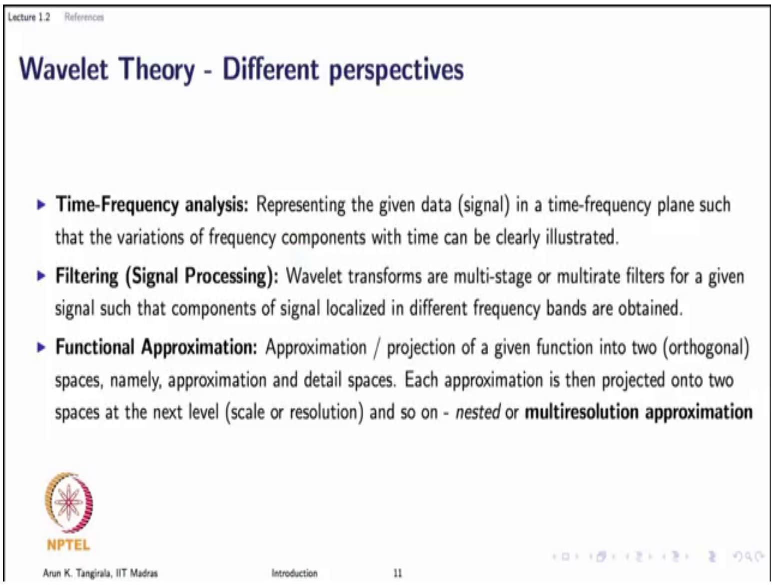

Now, to conclude let me tell you that whenever you read literature on wavelet theory, you will find several text book written by different peoples Mathematician, Physicist, Engineers and Econometric people and so, on. Each one takes a different perspectives simply because, wavelet have such enormous and diverse set of applications. For mathematicians wavelets are approximating function, for engineers wavelet are filters and for physicists is the time frequency analysis tools.

So, depending on the book that you read it may be heavy notation or not and heavy ingrate symbols or not and. So, on. As engineers, what is important is to know how the wavelets are applied. Rather than really getting bogged down with all the theorems and. So, on. Therefore, you should try and read material that are presented to you by engineers who are able to give you good perspective, particularly the filtering and time frequency analysis perspective.

What we shall do in this course is, we will start off with the time frequency perspective and then gradually move on to the filtering perspective. Of course, occasionally will talk about the approximating part, there is no escape, whichever perspective you take, you will inevitably be forced, compelled to talk about the other perspectives. But, predominantly will take the time frequency and the filtering perspective of the wavelet theory because, those are the most wildly encountered applications in engineering. And you should always remember that all three are equivalent, it is just that the perspective and the utility is different.

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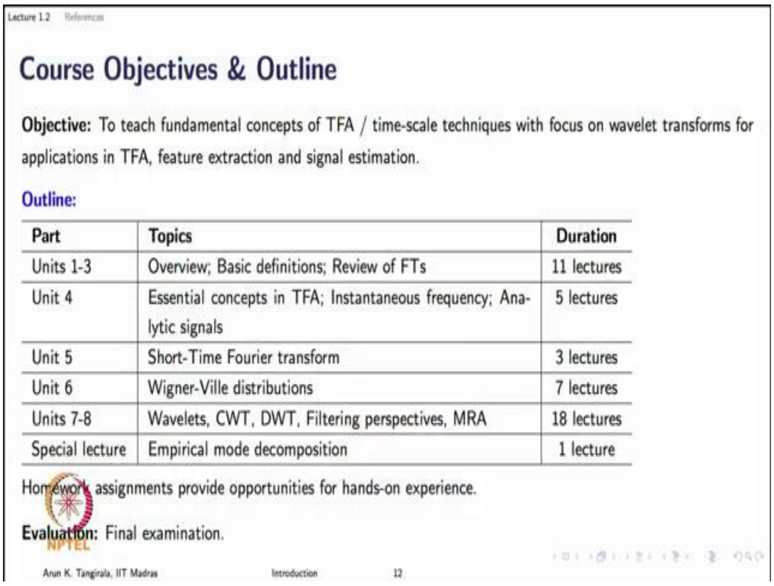

So, with those words with me close this lecture by of course,, giving you the course outline as well. And the course outline is fairly straight forward in the sense it is already given on the course website for you. The course is divided into several units or you can call them as modules, and each unit as a set of lectures, and I have given the distribution for you. The final unit talks about the empirical mode decomposition, that is more of a special topic, we will not go as much as in detail as we do for the other topic such as, short time Fourier transforms, Wigner-Ville and wavelet transforms.

The empirical mode decomposition is the most recent one, if you look at the historical developments. As I mention it takes an instantaneous frequency perspective, like the Wigner-Ville whereas, the spectrogram and the scalogram take the regular frequency perspective. Of course, there is this time scale perspective that wavelets have take us one. So, let us go through this course together and enjoy the journey.

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Here are a few reference text for you, that we will follow. Of course, predominantly the material may be borrowed from this. And as I mention in this previous lecture, there are going to be notational differences. Please do not get confused between notional differences. What is important is to be consistent notation that I use this course. If you find any errors in the consistent you of notations within the course materials is please feel free to point them out and to bring them to our attention.

The software that will use is primarily mat lab. And in this mat lab platform, there have been several open source tool boxes that have been developed. Really wonderful tool boxes and will use them extensively in this course, wave lab, the time frequency tool box will be use predominantly. And to a certain extent the wavelet coherence tool box which is also an open source by Grinstead and Moore and others.

And the commercial wavelet tool box, that is supplied by Mathworks to a lesser extent. So, please be comfortable with mat lab make sure that you are familiar with the basic syntax on basic working of this software.

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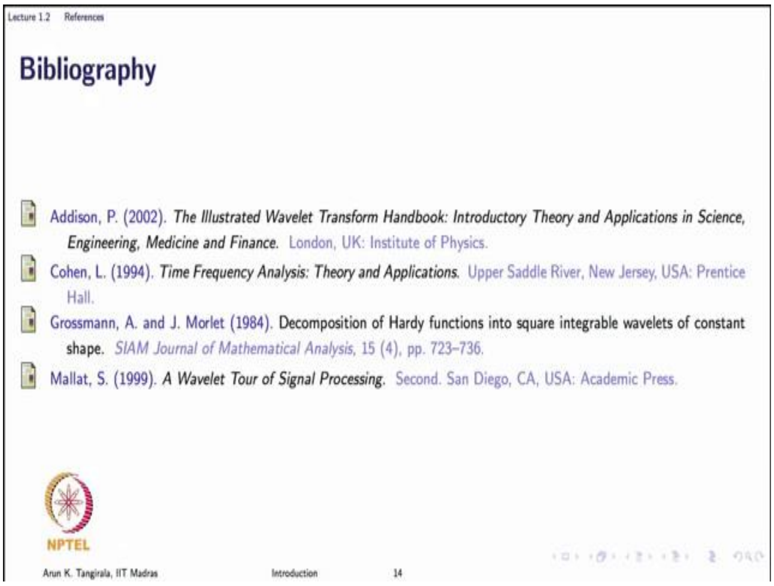

These are list of references for the lecture that we just when through. Of course, some of these references will run through the entire course. And make sure that you read up the corresponding chapters to get more details on whatever a lecture you go through. So, will meet in the next lecture.

Thank you.