Introduction to Time-Frequency Analysis and Wavelet Transforms
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Lecture -7.4
Scalogram and MATLAB Demonstration
Part 1/2

Hello friends, welcome to lecture 7.4 on the unit on continuous wavelet transforms. In this lecture we are going to learn what is a scalogram this is a equivalent of spectrogram and periodogram with respect to short time fourier transform and fourier transform respectively. And also look at how to compute scalogram using the wavelet tool box in MATLAB. In the previous lectures we have learnt what is the definition of CWT, how to convert scale to frequency, and how to compute the CWT itself. Therefore, this is a natural sequel.

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As I have just said we will first go through definition of scalogram, and then look at how to compute this in MATLAB.

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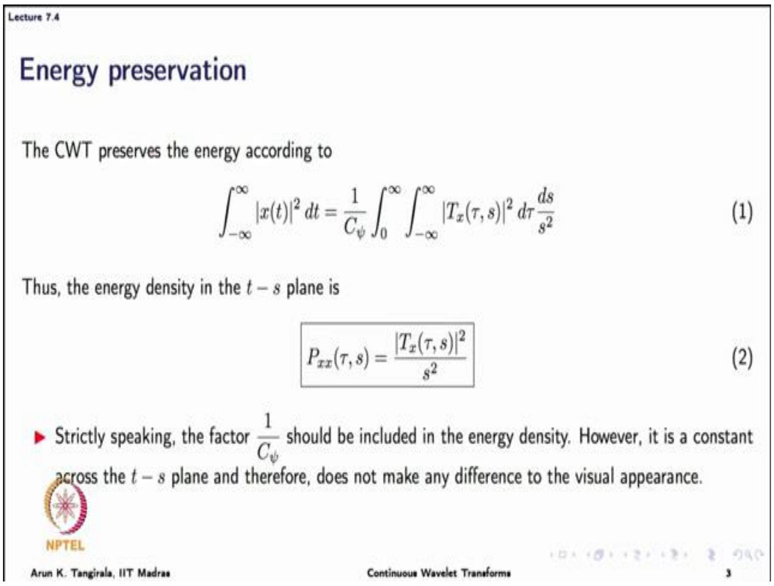

The scalogram, as I said, is equivalent to that of spectrogram and periodogram, both of which are derived from the energy conservation or preservation relations in the respective transforms. When it comes to CWT, the energy is preserved according to equation 1, where we have the C psi as a admissibility constant, T x as usually see continuous wavelet transform.

Now, if we look at the equation, on the left hand side you have the energy of the signal based on the signals representation in time domain. And then, on the right hand side we have this double integral. Notice that, the scales run from 0 to infinity, and the translation parameter tau runs from minus infinity to infinity. Therefore, the energy density in the time scale plane is the, mod T of tau comma S square by S square, because the integral or the double integral of this quantity here gives us the energy; of course, barring a factor 1 over C psi.

So, strictly speaking, I should have a, 1 over C psi, tagging along with, P of tau comma S, but we ignore that because visually the, 1 over C psi, is going to be a constant across the time scale plane, and therefore, does not make any difference. However, if you want to recover the energy correctly or if you want to compare this with the signals energy or that obtained from spectrogram and so on, you have to bring in, 1 over C psi. So, keep this point in mind.

Very often we are interested in the time frequency plane, as we had argued when we wanted to convert scale to frequency and so on. So, in signal analysis, the time frequency plane is of interest.

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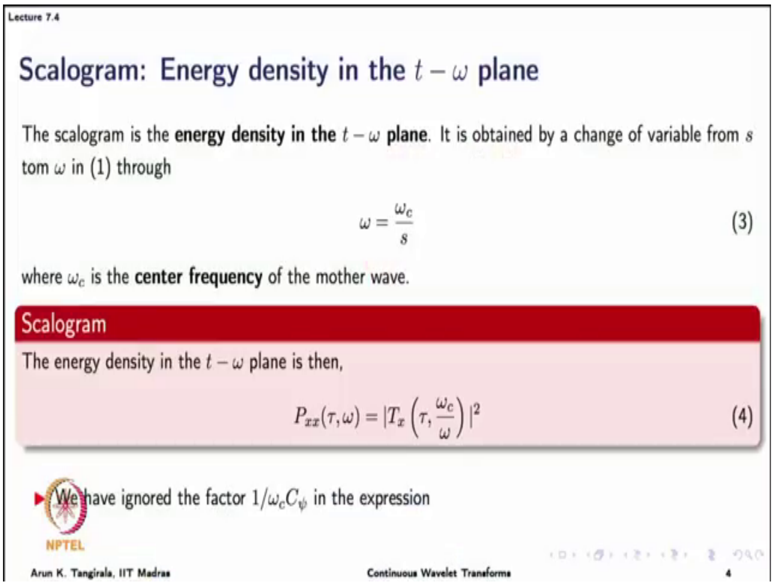

And, therefore we convert the scalogram which is now the scalogram; the earlier one is also called scalogram, but strictly speaking if you look at the literature, scalogram refers to the energy density obtained from CWT in the time frequency plane. So, you have to convert the expression given in equation 2 to a quantity which is in terms of, tau and omega.

And, how do we do that? Now we go back to our scale to frequency conversion expression. We know approximately this pseudo frequency relation, omega equals omega c by s, where omega c is the center of the pass band frequency of the wavelet. And, as I had mentioned in the unit on scale to frequency, you could replace this omega c with another reference omega which could be a peak frequency and so on, but it is usually the center frequency.

Now, when you bring in this relation between, omega and s, and take it back to equation 1, so, we did not directly substitute in equation 2 per say. We first rewrite equation 1 in terms of tau and frequency. So, what is the expression that results when we make the substitution? One has to first derive the relation between d s and d omega, as I shall show to you on the board.

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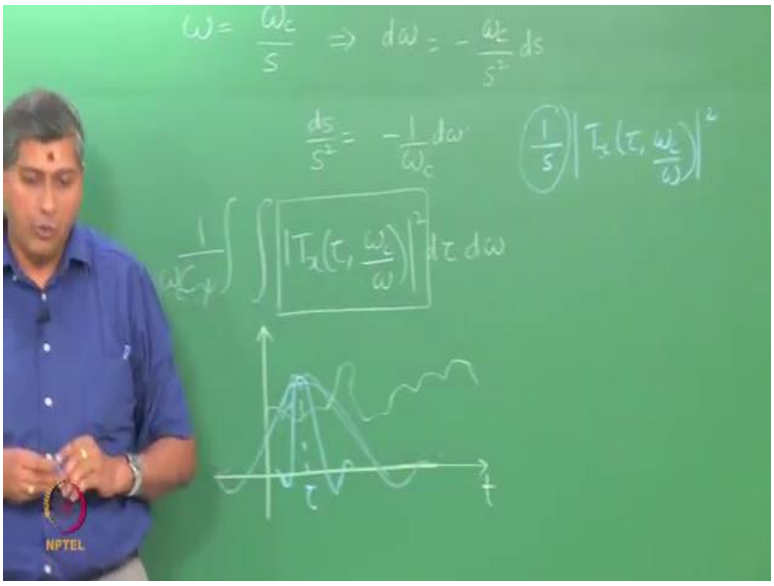

We have this relation, omega equals omega c by s, therefore, d omega is, minus omega c over s square times d s. And, as a result, I have, d s over s square as minus 1 over omega c d omega, right. And, now we take this relation and plug it into equation 1, as a result of which I have the double integral now as, T x tau; s being replaced by omega c over omega, so, I am going to evaluate the continuous wavelet transform at omega c over omega; times d tau; of course, I have also 1 over c psi, and then omega c coming in; the minus sign is taken care in the integral itself; therefore, I have, d tau d omega, right.

And, that is about it, because you have a minus infinity to infinity, you should be able to really not worry about the negative sign per say. So, this now is my scalogram in the time frequency plane because the area under this gives me the total energy. Of course, strictly speaking, I should also include this, 1 over omega c c psi. That is exactly what we have in expression, equation 4. And now, notice that this scalogram is a function of tau and omega.

And again, to reiterate we have ignored, 1 over omega c c psi, in the expression because it does not make any difference to the visual appearance. However, if you want to recover the total energy you will have to put it back, and then, into equation 4, and then integrate that in the time frequency plane. In fact, only if you put back, 1 over omega c c psi, into equation 4, you can make it comparable to the results that you obtained, for example, from spectrogram or winger ville distribution and so on, because they give you the correct energy density expressions in time frequency plane.

Now, normally what happens is, the raw scalogram, as you have defined in equation 4, does not give you the correct comparison between energies at different scales. And, why does this happen? That is because at each scale the wavelet has a different width. So, when I am calculating the energy density at a particular tau, and, at a particular scale I am calculating the energy at a particular tau, the amount of energy that is the value, let us say, of the energy density at a particular tau, depends on the width of the wavelet because the value is obtained by correlating the signal with the wavelet at that scale, right.

What is T of tau comma s, or tau comma omega c over omega? It is essentially the correlation between the signal and the wavelet. So, let me just illustrate that for you with a sketch. So, what I have is that, let us say, I have some signal of this nature, right; and here I have time t; and I am going to bring in, a wavelet into my analysis.

If I choose, if I am looking at wavelet at high scales, that mean I am looking at wide wavelets, then the CWT computed in such a wavelet is essentially the correlation, let us say, I am standing here this is the current tau that I am looking at; this is where the wavelet is centered at, then the wavelet is going to look like this because I am looking at a wide wavelet.

So, what happens is, the T is essentially the correlation between the signal and this wavelet. And, because this wavelet is wide it is going to cover a huge portion of the signal, and therefore, the number is going to be higher, than compared with the situation where I have a narrower wavelet. That is, I am looking at a wavelet now, at low scales. In such situations, let us say, I am looking at a very fine scale wavelet or a high frequency wavelet relatively, then what happens is, by design of this wavelet, this is going to be much narrower than the wavelet at the higher scale.

Consequently, this wavelet here located at the same center tau, is not, is going to see a smaller portion of the signals. So, when the energy calculations are done, it is unfair in some sense to compare the energy computed with the wavelet at this scale and the energy computed with the wavelet at this scale. So, you want to be fair and therefore, normally what is done is we choose a factor of 1 over s to rescale or renormalize the scalogram, so that the scalogram computed at higher scales are scaled down to account for the fact that this high scale wavelet is able to see a larger portion of the signal.

And, likewise, the scalogram computed at lower scales are kind of scaled up; they are broad on on part with the one that we are computing at high scales. Of course, this is not some regress factor here, you could choose some other factor as well, but we have chosen 1 over s; you could actually choose some other factor, but essentially that factor should bring the, should account for this phenomenon that happens at the high scale wavelet see a larger portion of the signal and the smaller scale wavelet see a smaller portion of the signal.

As long as your normalization factories able to account for it, you are ok. Generally what is used in literature is 1 over s. This amounts to saying that I am scaling the coefficients with 1 over root s, that is what it amounts. I will show you how to do things in MATLAB, how to plot the normalized scalogram and the unnormalized scalogram this way.

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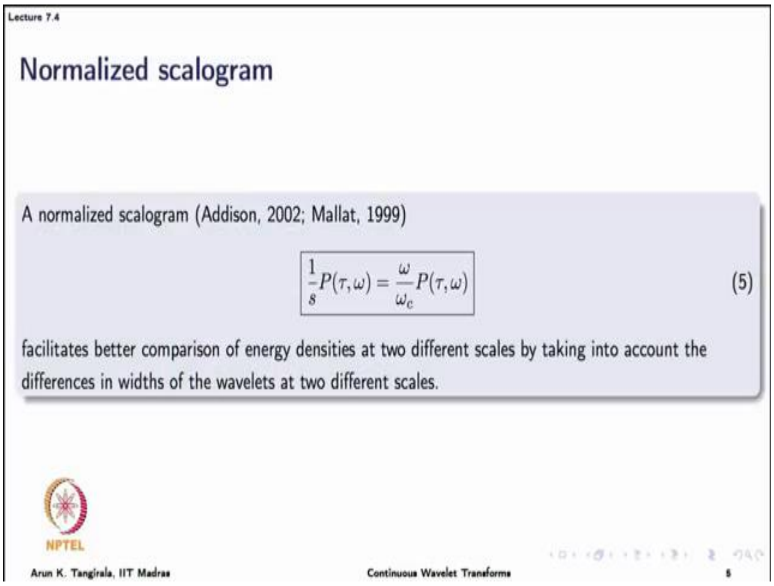

So, as we have just discussed, normally one uses a normalized scalogram with the normalization factor being 1 over s. And, 1 over s, is essentially omega over omega c, once again by virtue of the scale frequency relation.

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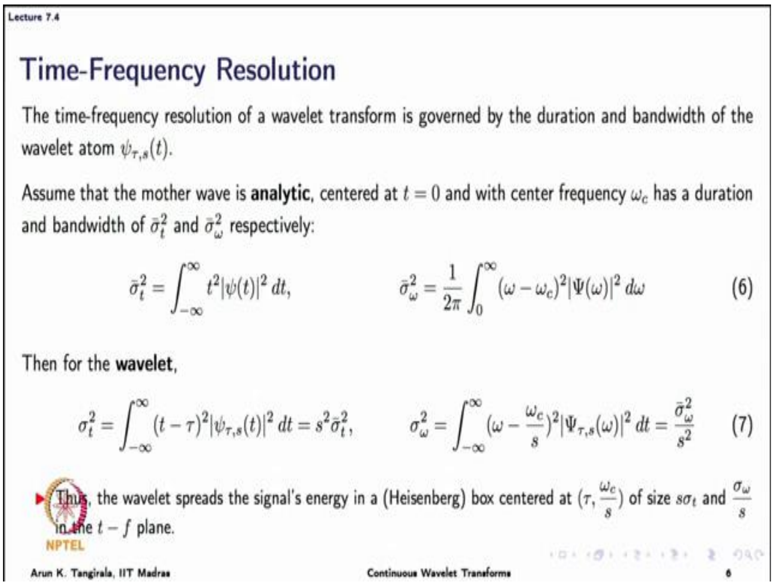

Now, before we really go and learn how to compute this scalogram and plot them in MATLAB, let us look at what this scalogram essentially is doing. It is essentially spreading or capturing the energy of the signal, not at a point in the time frequency plane, but it is capturing in a box. Normally this boxes are called Heisenberg boxes because the box itself comes into picture because of the uncertainty principle or the duration bandwidth principle. And, because again of the parallel between the duration bandwidth and the Heisenberg's uncertainty principle, this boxes are called Heisenberg boxes.

What are the widths of this boxes in time, along time and frequency axis? Well, to determine that let us start with a wavelet. And, normally because we use analytic wavelets we have restricted this discussion to analytic wavelets. You can, of course, extend this discussion to real wavelets as well. So, let us assume that we have an analytic mother wave which is centered a t equals 0. It is a fair assumption. And, let us assume that it has a center frequency omega c.

And, further, let us denote the duration and bandwidth of this analytic mother wave by sigma square t bar and sigma square omega bar. Now, what you see in equation 6, essentially that is the expression for sigma square t bar and sigma square omega bar; or, they directly follow from the definitions of duration and bandwidth that we have learnt in unit 4.

Notice that, we have assumed the wavelet to be located at 0; that is, in the sense, center of wavelet is 0 in time; therefore, we only have a t square here. Now, when you consider the mother wavelet, that is what we would like to know, because what happens is, the smearing of the signals energy is determined by the smearing, the energies spreads of the wavelet in time and frequency which is, which are essentially given by the duration and bandwidth of the wavelet.

Because the wavelet is a scale and translated wave, and then there is 1 over root s factor, you will have to recompute the duration of the wavelet, but the expression for the duration comes from the definition itself. So, I have sigma square t as integral minus infinity to infinity, of t minus tau square. Remember, that the wavelet is centered at tau, whereas the mother wave is assumed to be centered at 0. Therefore, the center time or the meantime is tau.

And, I have mod psi tau comma s t square; all I have to do to get to the answer, that is as s square time sigma square t bar, I have to substitute for the expression for psi of tau comma s which is 1 over root s psi of t minus tau by s, and do a change of variables; a very simple 1 or 2 step derivation to arrive with the answer. So, I leave that to you. It is a very simple exercise. So, all you have to do is replace this with 1 over root s, psi of t minus tau by s, and do a change of variable and we should be able to get this expression. We have qualitatively talked about this when we were talking of the scaling properties of fourier transforms in unit 3.

And then, we have sigma square omega; again, falling out of the definition of the bandwidth. And, I have here omega minus omega c over s. Remember, when I scale the mother wave by factor s, the center frequency shifted to omega c over s; therefore, I have to use that here in the definition psi, the big psi of tau comma s of omega, is the fourier transform of the wavelet now.

And, once again what you do is, you first derive the relation between the big psi tau of, tau comma s of omega, and the big psi of omega which is the fourier transform mother wave, using the scaling property of the fourier transforms. We have done this in deriving the f of t algorithm for computing CWT. So, you should be able to get that relation there as well.

Just do that, substitute that here, and then again do a change of variable. You should be able to show that essentially your sigma square omega is sigma square bar omega over s square. That means, now what is happening is as expected when I am looking at high scale wavelets that is wide wavelets, they have larger duration than the mother wave, but then narrower bandwidth compared to the mother wave. Likewise, for high frequency wavelets, that is wavelets at small scales, they have much smaller duration than the mother wave, but a wider or a larger bandwidth than that of the mother wave.

In other words, what the wavelet is doing to the signal energy is, it is taking the signal energy and spreading it, like you have seen on the board earlier. It is analyzing a large portion of the signal if you are using a wide wavelet, or it is analyzing a smaller portion of the signal if you are using a high frequency wavelet. So, it is spreading the energy nevertheless, it is not analyzing signal exactly at a frequency or a time point; it is spread over time and energy.

And, this signal energy is therefore smeared in this box, Heisenberg box of widths given by s time sigma t bar, and it is a 1 over s times sigma omega bar, along the time and frequency axis respectively. So, therefore, you should not be able to see the ideal time frequency energy density but a smeared energy density in the time frequency plane, that is anyway expected by virtue of the duration bandwidth principle. So, what we shall do now, is learn how to compute the scalogram in MATLAB.