Introduction to Time-Frequency Analysis and Wavelet Transforms
Prof. Arun K. Tangirala
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 7.2
Scale to Frequency

Hello friends, welcome to lecture seven point two in the unit on continuous wavelet transforms. In this lecture we are going to talk in greater detail on this topic of converting scale to frequency. Of course, we have looked at this concept briefly, particularly in lecture 7.1 where we introduced CWT and in deriving the synthesis equation for the CWT. We started off with the synthesis equation for the Fourier transform and then, slowly introduced the scale factor from where we realized, that scale and frequency are inversely related and so, on.

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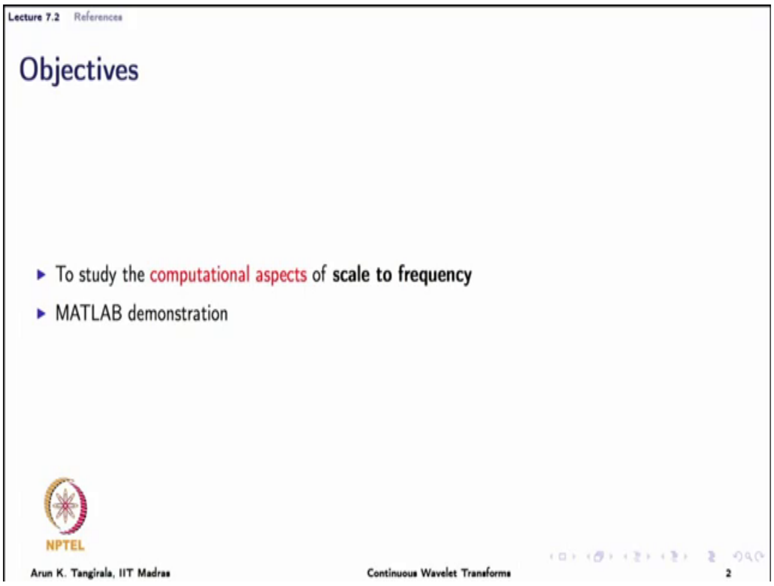

Today, what we are going to do is, we are going to dwell on this topic in greater detail and look at how to compute the conversion factor, that takes me from the scale domain to frequency domain. I will also show you a few MATLAB demonstrations.

Now, before we really plunge into the lecture, an important point to keep in mind is this entire business of converting scale to frequency, is not a well posed problem primarily because wavelet is a short lift wave, wavelet or mother wave, whereas, the sinusoidal signal, which is characterized in terms of frequency, is an infinitely long signal. And by virtue of the duration bandwidth principle we know, that the bandwidth of a short lift signal is going to be of finite value, whereas, the bandwidth of a sine wave is going to be

0. That means, essentially I will see a single peak in the spectrum of a sine wave or the spectral density plot, whereas, I would see some kind of a density function of finite width when it comes to the spectral density of a wave.

Therefore, what we are trying to do here is, when we go from scale to frequency we are asking, I have a broadband or finite bandwidth wave and I am trying to find a corresponding sine wave, which has 0 bandwidth. In other words, I am trying to replace the entire band of frequencies that a single scale corresponds to with the single frequency and we have studied this concept earlier, which is the center frequency. But the center frequency is only a representative of the band of frequencies, that the scale represents,, but it is not entirely the frequencies itself, that has to be kept in mind. In other words, converting scale to frequency amounts to finding a suitable representative frequency for a band of frequencies always; scale corresponds to band of frequencies.

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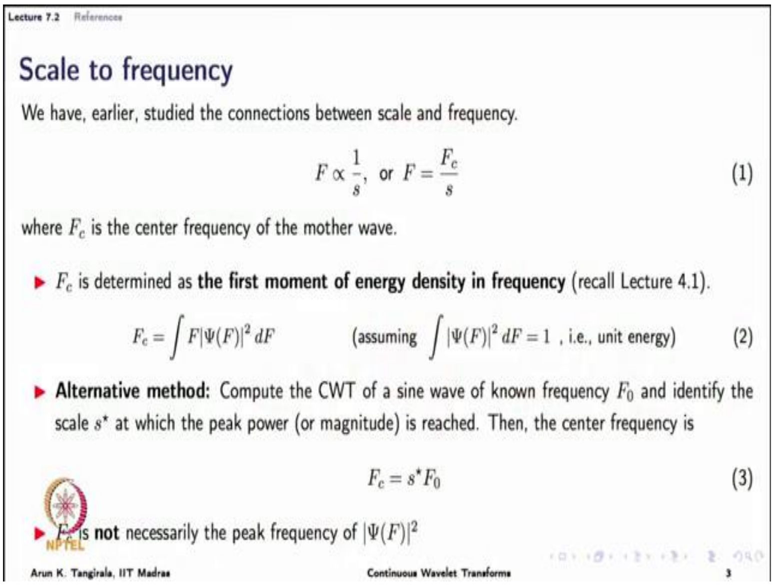

Now, with these words let us recall also from our previous discussions that we had, particularly in 7.1, that scale and frequency are inversely related and the proportionality constant, that relates to scale and the frequency is the center frequency.

Now, just now we said, that a scale in reality corresponds to band of frequencies and I am trying replace with it, that with the single number, which is a F here. Therefore, many a times F is also known as psuedo frequency. It is not really the true frequency. But why are we doing this? Because in signal analysis I am far more comfortable with frequencies and scales.

But on the other hand, in modeling, that is, multiscale modeling and image analysis I am a lot more comfortable with scales and therefore, this conversion business does not arise in those situations. Of course, in multiscale modeling there are situations where I would still be interested in the scale to frequency conversion,, but not necessarily in image analysis. So, in signal analysis this topic is of particular interest and since this course is primarily on signal analysis, this entire conversion warrents a greater discussion in greater detail.

So, coming back to the point, I, in order to convert scale to frequency I need to find this proportionality constant F c, which is the center frequency. Now, from our lecture 4.1, that is the unit 4, we can recall the definition of center frequency. It is a first movement of the energy, spectral density of any function here or function is the mother wave itself and its expression. Of course, you may recall, assume, that the function for which I am computing the center frequency has been normalized to have unit energy. Here, what; that means, is, in order to use this expression given in 2, equation 2, I will have to ensure, that the mother wave has been normalized unit energy, which is generally the convention that is followed in wavelet analysis.

An alternative method of determining the center frequency is to take a sine wave or a cosine of known frequency and evaluate the continuous wavelet transform with the wavelet for which you want to find the center frequency and then determine the scale at which the CWT reaches a peak or the power wavelet power reaches a peak, both are equivalent. Then, the center frequency is calculated as given in equation 3. It is s star times F naught; F naught is a frequency of the sine wave that you have taken, s star is the scale at which the CWT or the wavelet power has reached a maximum. That is an alternative method, which makes a lot of sense. In fact, this method can be even followed, pursued theoretically, and I will give you a reference to a paper where a theoretical determination of F c using this method is carried out.

In passing over, I want to point out that the center frequency is not the peak at which the wavelet spectrum reaches a peak. It is not the frequency at which the wavelet energy, energy spectrum reaches a peak. Very often it is confused, these two concepts are confused, that is, the one that we have just explained where I am searching for this scale at which the CWT of a sine wave reaches a peak versus where the energy spectrum density of the wavelet reaches a peak. However, in some special cases, the peak frequency of the energy, spectral density and the central frequency coincident, that is, when the energy spectral density of the mother wave is symmetric, as we will see in the examples to come.

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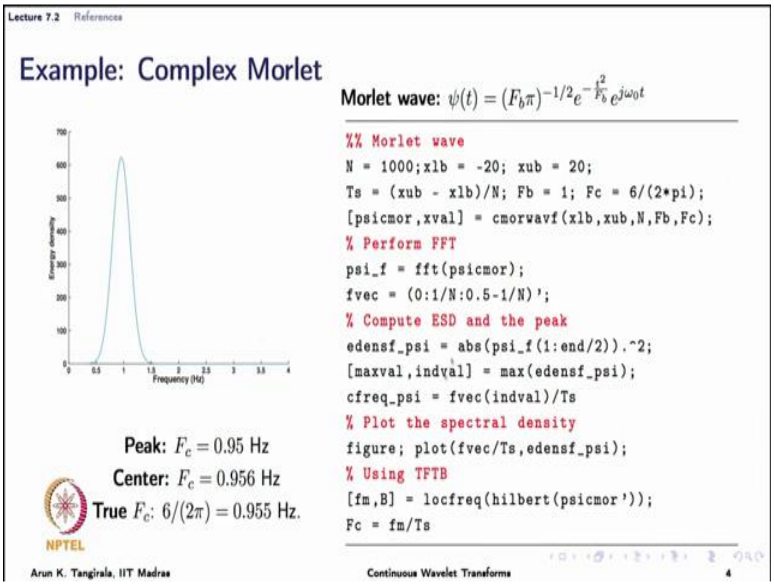

So, let us begin with the complex morelet wave, which is widely used in continuous wavelet transform. We have until this point not discussed the different types of wavelet families and when they are used and which ones are more suitable for CWT and DWT and. So, on. But at this point you can also take it from me the complex morlet wave is used widely in continuous wavelet transform. Therefore, it calls for attention here.

The expression for the complex marlet wave is given at the top. You have two parameters.

Here, F b and F c. If you look at the expression, sorry, if you look at the expression for the mother wave, it is nothing, but a Gaussian modulated complex sine wave of frequency omega naught, right, and F b here refers to the bandwidth. Earlier I said, that there is a second parameter call F c, that is nothing, but a center frequency and that is nothing, but your omega naught by 2 pi. F b refers to the bandwidth of the mother wave. What I am showing you here is for the choice of F b equals 1 and F c equals 6 by 2 pi, which means omega naught is 6.

The energy spectral density obtained by implementing the MATLAB code that I show on the right. We will go through the MATLAB code,, but let us quickly discuss the energy spectral density and the center frequency calculation now. When I choose the, when I look at this energy spectral density and I pick the center frequency visually, more or less it corresponds to the peak frequency because of the symmetricity. So, the energy density spectral density looks more or less symmetric. Therefore, I may say, that the center frequency is a peak frequency itself, which is nearly true for this wave, morlet wave. The peak frequency turns out to be 0.95 hertz and the center frequency is 0.956, whereas, a true frequency, which is 6 by 2 pi is 0.955. So, clearly the center frequency, which has been obtained by the definition that we had mentioned earlier is closer to the true center frequency.

How do I obtain the center frequency? I have once again used the time frequency toolbox. So, let us go through the MATLAB code. The first three lines here generate the morlet wave for me or the complex morlet wave where I have used the cmorwavf routine in the wavelet toolbox, that comes with MATLAB's 2014 B release and I compute this morlet wave over a grid of 1000 points in the interval minus 20 to 20 in time and we have set the values of F b and F c to this here as to the same values, that I have discussed earlier. The cmorwavf takes in the interval over which you want to compute and the values of F b and F c and the grid size, that is, a number of grid points and returns the morlet wave as well as the time points at which it has computed the morlet wave. Then, I take the Fourier transform using the fft algorithm. Locate the peak in order to calculate the peak frequency.

So, I compute the energy spectral density, locate the peak. And remember, when I am computing the Fourier transform, I am treating this as a discrete, discretized or discrete time morlet wave. Therefore, to correctly compute the peak frequency, although I call the cfreq_psi, it is actually the peak freq_psi. I will, I convert the discrete time frequency or the frequency of the discretized signal to the continuous time frequency, that is, the frequency of the continuous time wave through the sampling frequency and I have information on the sampling frequency at the top here.

And to compute the true center frequency, we use the locfreq routine from the time frequency toolbox where I take the Hilbert transform of the complex morlet wave. And here I should caution you, that strictly speaking, you should be vectorizing the size psicmor not taking the transpose. So, that is a small typographical error here. The transpose of a complex wave will actually construct at least the syntax in MATLAB. This syntax will produce a complex conjugate,, but the attempt here is to obtain a column vector of the morlet wave because a cmorwavf returns a row vector. So, the correct code here should be psicmor, bracket open, colon, bracket close and not the apostrophe.

So, I take the Hilbert transform because I know, that the analytic associates have center frequencies that are well-defined, not the signal itself. And once again, the locfreq gives me the center frequency for the discrete time morlet wave and I convert, I recover the continuous time frequency center frequency. So, this is the typical procedure that you would follow for any wavelet you generate, the wavelet or the wave, mother wave compute the Fourier transform.

If you want to locate the peak frequency or you can skip the Fourier transform part and directly use locfreq to obtain the center frequency, as we had discussed earlier, the more appropriate value or the conversion factor is a center frequency not the peak frequency. In this example, they coincide or they are very close to each other because of the symmetricity of the spectral density.

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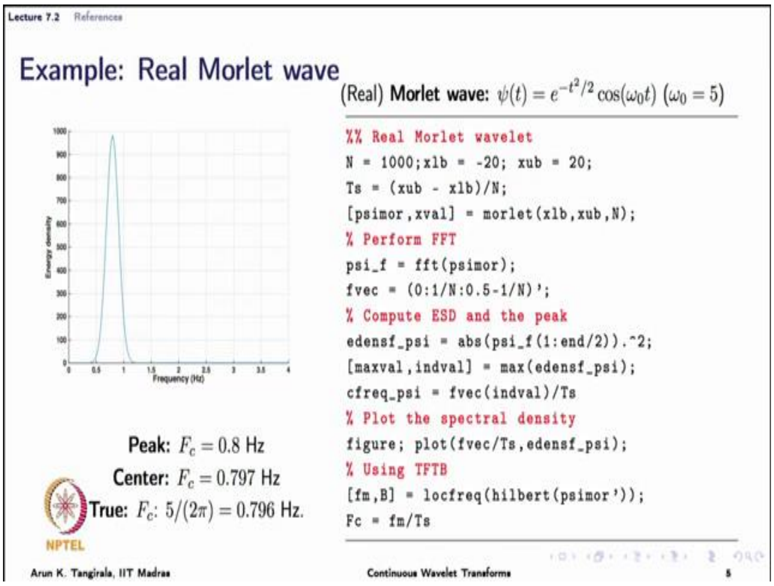

Now, moving on, let us look at a real morlet wave. This is also widely used in CWT. The story again is the same. I have given you the expression for the real morlet wave. Strictly speaking, the real morlet wave is nothing, but the real part of the complex morlet wave. But the expression, that I have given here and the expression, in fact, is same except that the parameters, that I have used are different from the one that I have used in the complex morlet wave. Otherwise symbolically, you can straightaway see, that I have plucked the real part of the complex morlet wave and that is why, it is call the real morlet wave.

Once again, on the left we have the energy spectral density and I compute the center frequency using the locfreq routine and I also locate the peak frequency. Now, here as, as before the center frequency is, calculation is much closer to the true one than the peak frequency,, but the difference is really negligible. Why the answer is same? The energy spectral density looks symmetric and therefore, the peak and the center frequency coincide or nearly coincide.

I do not have to really go through the MATLAB code all over,, but just want to point a few differences. One, that the morlet wave is computed or generated by this morlet routine in the time, in the wavelet toolbox, which asks the user to specify the interval and the grid points over which it is computing the morlet wave. In other words, the user really does not have a say on the parameters here,, but you can, of course,, code your own morlet wave, that is not a really big deal.

So, the morlet routine returns the values of the morlet wave and the time points at which it has been computed. So, psimor is not really a continuous time morlet wave, it is a discrete time morlet wave. Therefore, whether you are computing the center frequency or the peak frequency, you have to convert the frequency from the discrete time to the continuous time and that is what I do in both cases.

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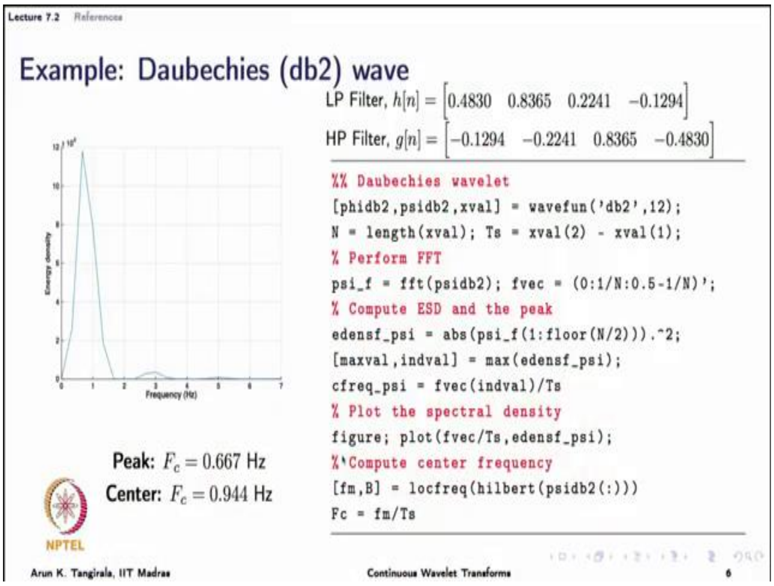

Now, let us move on to another example, which is very interesting in a few respects. First, this is, in fact, this is the daubechies wave, which is very widely used, not in CWT,, but in discrete wavelet transform. Nevertheless, I want to show you how to compute the center frequency of this wave for a few reasons.

The first point here is, that unlike the morlet wave, the daubechies wave does not have a close form expression, that is, there is no analytical expression. The daubechies wavelet or the wave, in fact, is computed with the help of filter coefficients. We know, that wavelet acts as a filter, specifically as a band pass filter. And what I have given you at the, given to you at the top or the filter coefficients corresponding to what is known as this daubechies 2 wave. The daubechies waves family is a large family and is characterized by the length or the half length of the filter. In this case, the filter is 4 coefficients long and therefore, is it is called the db2 wave. But this two also has another meaning. It represents the, it, it is in fact, the number of vanishing moments of this wave. We have not defined what is meant by vanishing moments until this point in the course,, but we shall learn later what is meant by vanishing moments. It, it essentially characterize the ability of the wave to detect singularities or discontinuities in a signal and its derivatives.

So, coming back to the point here. These are the filter coefficients of the db2 wave, that is, a high pass filter and the low pass filter are also the filter coefficients for the wave,, but it is the high pass filter, that allows me to generate the wave or daubechies wave, mother wave and the low pass filter coefficients help me generate what is known as this corresponding scaling function. Remember, the wavelet acts as a band pass filter. So, there is frequency that is left out in the low frequency region and the scaling function fills that void. We shall formally learn what is a scaling, scaling function later on.

So, third example in the discussion is what is known as the daubechies wave? This daubechies wave is widely used in discrete wavelet transforms more than in continuous wavelet transform,, but we look at this example for a few reasons unlike the morlet wave. There is no close form expression for daubechies waves. You see, on the title I have db2. The daubechies waves come in different sizes and shapes and the parameter 2 or the number 2 has a certain meaning there and I will talk about it shortly. So, you could have db2, db3 up to db10 and. So, on.

As I said just now, the daubechies wave does not have a close form expression, instead we generate the wave using what the, what are known as filter coefficients. Remember, that the wavelet acts like a band pass filter. So, the filter coefficients given in g, that is, a high pass filter allow me to generate the daubechies wave, and the low pass filter coefficients you can say is a cousin of this high pass filter, which allows me to generate what is known as the scaling function.

We have not discussed what a scaling function is. It is essentially the brother of the wavelet or the wave, which is responsible for filtering the low frequency components of a signal. The wavelet takes care of the band pass. There is a high pass, higher frequencies of components of the signal. For this db2, the filter coefficients are fixed and have been given here. We are particularly interested in the wavelet, not the scaling function. And in order to generate the daubechies wave, because I do not have a close form expression, we shall learn later, that it can be only generated in an iterative fashion, that is, I iterate the computation of the wave across different scales and. So, on,, but we will not go into those details right now. We shall only learn how to do this in MATLAB.

The wavefun routine in wavelet toolbox allows me to compute the db2 wave. This 2 here stands for the half the filter length or the number of vanishing moments. The filter length here is 4, half the filter length is 2. The other interpretation here or the other message here is, this daubechies wave has, what is known as, 2 vanishing moments and the vanishing moments has got to do with the wavelets ability to detect discontinuities in the signal and its derivatives. Again, we shall learn formally the concept of vanishing moments and, and the, in the properties of wavelets as in a different lecture. For now, let us strict to this particular wave called db2.

As I said, I need to compute this iteratively,. So, I specify in the wavefun the number of iterations over, over which I want to compute. Larger the number of iterations, more accurate is a computation of the wave. So, I specify here db2, the string comma 12 and the wavefun returns the values of the daubechies wave of this particular filter and it returns, as we have just discussed, the scaling function, the wavelet function and the time points over which it has computed the db2 wave.

Of interest was, is this psidb2 and once again, I either compute the center frequency or the peak frequency knowing very well, of course,, at the center frequency is more appropriate for comparison purposes, I compute, both the procedure is same, compute the Fourier transform, locate the peak if you want to compute the peak frequency of the wavelet or use locfreq from the time frequency toolbox to compute the center frequency. Again, you have to convert from discrete frequency, discrete time frequency to the continuous time one. So, I, here I have the center frequency reported for you, it is 0.944 and the peak frequency as 0.667.

There is considerable difference here unlike the morlet complex and real morlet. Of course, you must have guessed by now the reason for this. The reason is, the energy spectral density of this wave is not symmetric and that leads to a big difference between these two. The question is, which is more appropriate because in this case, I do not know what is a true frequency. Of course, by theoretical definition and the discussions that we had at the beginning of this lecture, we know the center frequency is a more appropriate conversion factor for scale to frequency.

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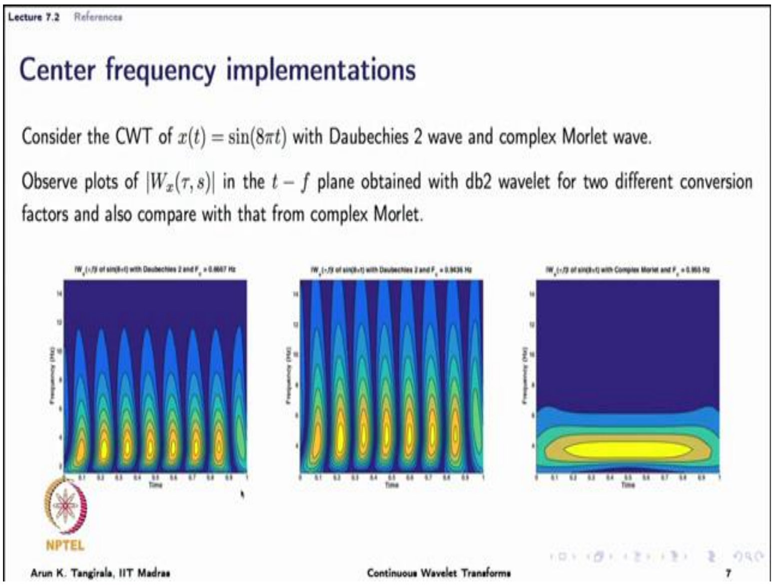

But let us look at an example where we follow the second method, that is, I take a sine wave of known frequency, compute its CWT with daubechies 2 wave and a complex morlet wave. And what I do here is I use the two different conversion factors when it comes to daubechies 2 wave because there is considerable difference between the peak frequency and the center frequency, whereas, for a complex morlet wave the difference is negligible.

So, the three plots that you see here, among the three plots, the first two correspond to the magnitude of the CWT obtained with the daubechies wave,, but with the two different conversion factors. One, with the peak frequency and other with the center frequency. It is fairly obvious, that the center frequency calculation, that is, 0.944 is much better conversion factor than the peak frequency. This should dispel any confusion that you may have with this conversion factor, F c being the peak of the energy spectrum of the wavelet because a true frequency in this case is 4 hertz and the peak that is a scale, the frequency at which the CWT reaches a peak or the magnitude of CWT reaches a peak is much closer to 4 when I use the center frequency as a conversion factor viz. A viz. Using peak frequency as a conversion factor.

Now, compare this with what I have in morlet wave besides the fact, that the CWT looks highly localized and, and. So, on. The main point here is, that the conversion factor that I have used for the complex morlet wave, whether it is center or peak, more or less gives me the true frequency. Remember, you will never get the true frequency accurately, that is why, we called is a pseudo frequency,, but this serves the purpose that we are looking at. Of course, now you can look at the differences between the magnitude of CWT with the daubechies wave and the morlet wave. You can also now understand why morlet waves are very popularly used in CWT and not daubechies waves, right. But you should keep in mind that these two plots, that is, the first two plots that we are seeing are with daubechies 2.

Now, what I welcome you to do is as an exercise. Repeat whatever we have done here including the center frequency calculation with db10 for example, that is, increasing the number of vanishing moments from 2 to 10, that does it make a big difference in the, in, in the CWT in the center frequency calculation and. So, on. I leave that exercise to you and you can come back to us if you have any questions.

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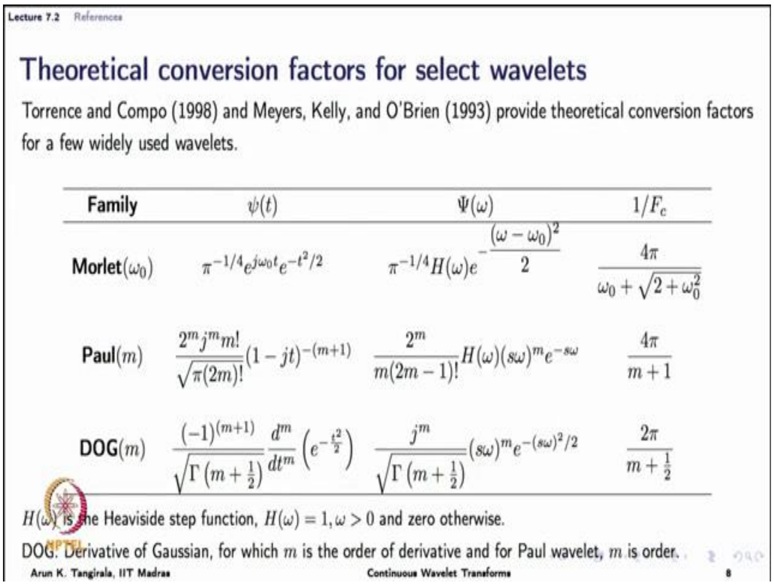

Now, it is possible to theoretically compute the center frequency as I mentioned early on in this lecture for a few wavelets and this was reported by Meyers et al in 93 and then again by Torrence and Compo in 98 in there very nice papers on, on practical use of wavelet analysis in the respective domains, particularly in metrological data analysis.

So, I am reporting these results as was reported by Torrence and Compo for three families of wavelets known as a Morlet, the Paul and the derivative of Gaussian. Each of these waves are characterized by some parameter. The Morlet wave is characterized by the center frequency omega naught or F c, the cyclic frequency. The Paul wave is characterized by what is known as an order and the derivative of Gaussian here is, essentially, the order of the derivative of the Gaussian that you are going to use. The third column gives you the expression for the Fourier transforms h of omega is the ((Refer Time: 27:24)) step function, the standard step function, which is 0 for negative frequencies and unity for non-negative frequencies.

You can quickly notice, that for the first two wavelets, because they are complex and symmetric, the Fourier transform is real value, whereas, with the derivative of Gaussian it is a real valued wave and therefore, the Fourier transform is complex. The last column is a one that is of interest to us. It gives inverse of F c. So, you can use that to convert what is known as scale to Fourier period or you can use an inverse of this to compute the, convert, to converts scale to frequency.

So, as an example, suppose I want to compute the conversion factor for the Morlet wave with the center frequency being omega naught equal 6. So, that would correspond to F c being 6 by 2 pi, then you can plug in omega naught equal 6 here and you will see, that 1 over F c turns out to be roughly 1.03 and an inverse of that will give you the conversion factor, that we had obtained here for the complex Morlet. There may be some differences between the theoretical calculation and the one that we have here because this is a numerical calculation based on the generation of the wave,, but they, they will turn out to be pretty close.

And the other point, that I want to make here is, typically we talk of converting scale to frequencies for wavelets that are. So, called analytic or regular, that is, for wavelets whose power spectrum is 0 at negative frequencies. Recall, the definition of analytic signals. We rarely talk about converting scale to frequency for wavelets that are not analytic, that is, whose frequencies or whose spectra are not 0 for negative frequencies and. So, on.

Nevertheless, we still went through such an example, for example, daubechies 2 wave is one such example. It is not an analytic signal,, but you can construct an analytic version of this. It is not that there is no analytic version of this. So, you can perform CWT with an analytic version of this. The general procedure or practice in CWT is that you use complex wavelets or analytic wavelets. So, that you can extract both the amplitude and phase of the oscillations in the signal.

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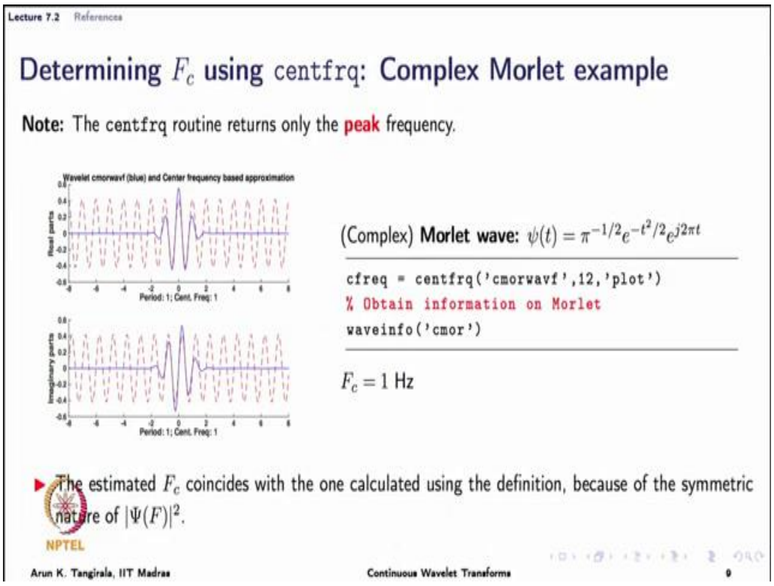

I want to conclude this lecture with the demonstration of this centfrq routine, that is, that comes with the wavelet toolbox in MATLAB and I am using the release 2014 B of the, of MATLAB and the wavelet tool box. Upfront I want to caution you that the centfrq unfortunately returns only the peak frequency, which we have been saying is not the correct way of determining the center frequency unless the energy spectral density is symmetric.

So, to demonstrate how the centfrq works, have taken the morlet wave once again for you. The expression given here for the complex morlet wave and the one that I have given earlier only differ by a factor,, but otherwise the basic expression looks same. So, the syntax for centfrq is that you specify the wave for which you want to compute the center frequency. The second argument here depends on the wave. If there is a close form expression, the second argument essentially specifies the number of grid points over which you want to compute the wave.

So, in this case, in fact, it is a power of the number of, of what you supply here, power of 2. In other words, what I am asking centfrq to do here is compute the center frequency of the complex morlet wave by first generating 2 power 12 points on an interval, that is by default set in the centfrq, that is something that you do not have a say on it, actually computes the complex morlet wave over interval of minus 8 to 8,, but over a grid size of 2 power 12 points, that is, 4096 points. And the last argument asks this routine to plot the sine wave or you can say, it is a, it is approximately that sine wave of the particular center frequency that it has computed.

So, in this case when I use the routine, the default values that it uses for the complex morlet wave is that F c equals 1. Earlier we had worked out for F c equals 6 by 2 pi, which is not exactly 1. Here, the default value is 1 and a center frequency calculation comes out alright to be 1 hertz. So, the estimated center frequency coincides with the one calculated using the definition, that is, I am not showing you what you would get with the definition.

But if you set the F, parameter F c equals 1 and F b equals 1 in the previous example, that I showed you and run the result, that is, a complex morlet wave through the locfreq, you will get the center frequency as 1. So, they both coincide, that is, a peak frequency calculated by the centfrq and the center frequency calculated by locfreq will coincide because energy density is symmetric in frequency.

And when it comes to the real morlet wave, again the story is the same. You specify the string morl to refer to the real morlet wave. The 12 here as a second argument is asking this routine to compute the morlet wave over 2 power 12 points on an interval minus 8 to 8. And you can, of course,, obtain whether it is cmorwavf or morl, you can obtain more information on this waves by typing in waveinfo,, but we will not talk about it right now. What is of interest to us is the estimate of the center frequency that this returns. It is 0.8125 hertz and this is different from what we had computed. If you recall, for the real morlet wave the true frequency is, the true center frequency is 0.796. You can say, more or less 0.8 hertz, which is correctly detected by the locfreq, is correctly calculated, whereas, here the centfrq turn calculates it to be 0.8125.

Earlier, when we calculated the peak frequency, we had 0.8 hertz. The centfrq also calculates a peak frequency,, but why does it return 0.8125, the answer is because it computes over a shorter interval, that is, it computes a morlet wave over this interval minus 8 to 8, whereas, here we computed it over minus 20 to 20. Does it make a difference? It does seem to be making a difference here, although effectively.

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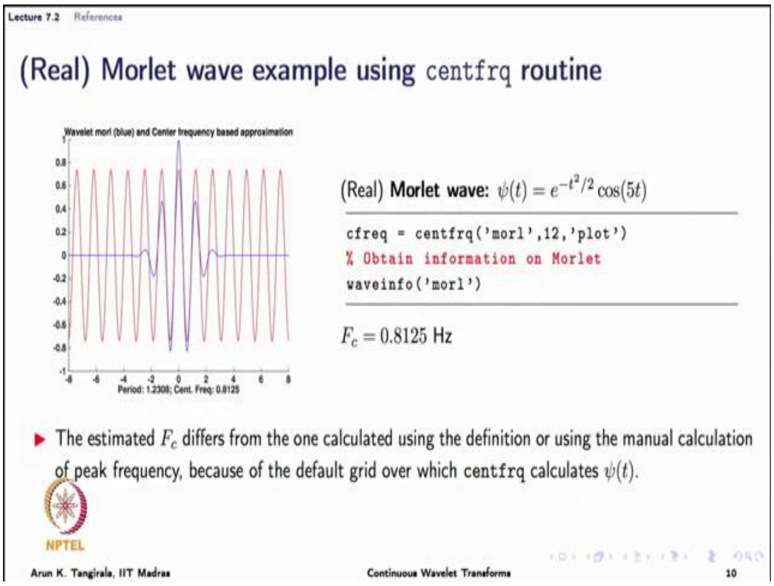

The theoretical support for the real morlet wave is some wave between minus 8 to 8, that is, it pretty much goes to 0,, but the number of points that you are using does seem to make a difference and you can check it out for yourself by changing the defaults in the centfrq.

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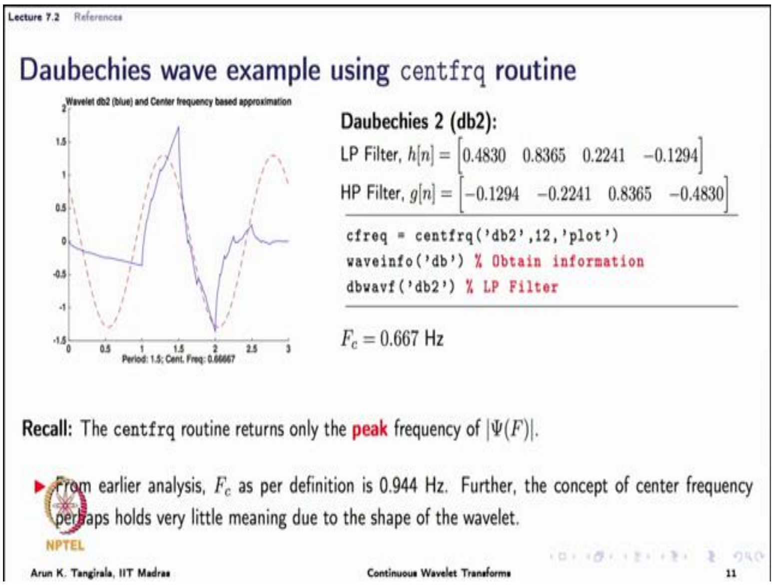

Finally, let, let us look at the db2 for which we had calculated the peak and center frequency and we found, that there is a considerable difference and the center frequencies was the more appropriate choice there and always. So, here the centfrq once again calculates db2. The second argument refers to the number of iterations over which it computes the daubechies 2 wave and the center frequency is nothing, but the peak frequency.

So, the ideal name for this routine should have been peak frq rather than centfrq because it really computes a peak frequency and not the center frequency. But again, recall from earlier analysis, that is, with the example that we showed on the wavelet transform of the sine wave of known frequency, the center frequency calculated according to the definition is a more appropriate conversion factor rather than peak frequency. So, you have to be careful when you use this routine.

There is an associated routine called scale to frqscal number 2 frq that uses this routine to convert a set of scales to frequencies for a given wave. Now, that also suffers from the same drawback as this routine. It, it, in fact, it is returning the frequencies converting scales to frequencies based on the peak frequency rather than the center frequency. It is not. So, much of an issue when you are working with complex morlets or real morlets or wavelets that have symmetric energy spectral density or that look like this complex morlet waves and. So, on. But you will have issues when you are working with waves like wavelets, like daubechies 2, daubechies 3 and. So, on, which are lower in the ladder of vanishing moments, right.

So, I will leave all the other wavelets for your own exploration,, but basically what we are trying to, what we have tried to tell you in this lecture is converting scale to frequency is a hazy business because you are trying to convert a band of frequencies to a single number and that is not a well posed problem,, but you can still do it because we do that routinely even in random variables and. So, on or even in our center frequency definitions in unit 4. We have introduced a center frequency with that very purpose. We want to get a feel of this band and the center frequency serves as a single number.

Of course, there is also this bandwidth Therefore, in some sense it is justified in converting from scale to frequencies. But you should remember, that we are working with not the true frequencies,, but the pseudo frequencies and the calculations become more and more meaningful when the energy spectral density is symmetric and when you are working with analytic wavelets.

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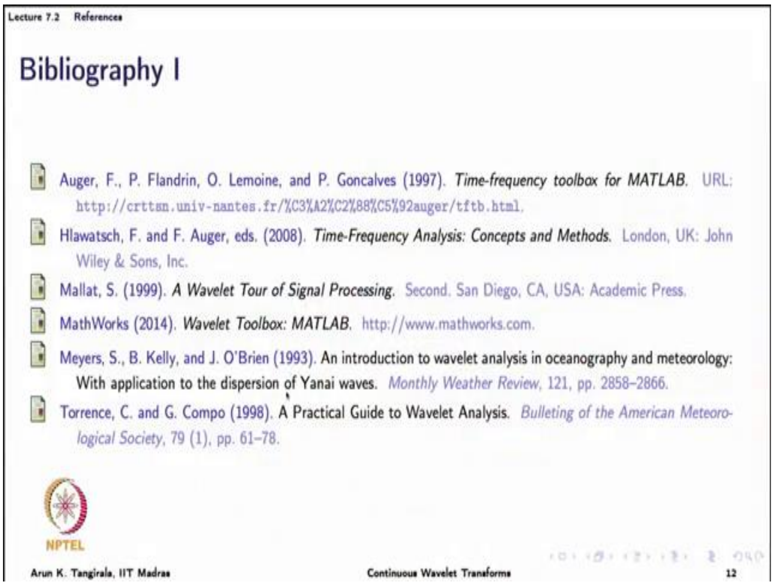

So, that is the summary for you. Of course, we have also shown MATLAB demonstrations and from the discussion we had it should be clear by now, that trying out things by yourself, that is, coding things by yourself will bring in the lot more clarity to your understanding and reinforcement of the concepts rather than using some built in routine. So, if you are using built in routines better be aware of what exactly it is doing. With those words I will conclude the lecture and here are few references.

So, let me draw your attention to these two references that we have pointed out earlier, the one by Meyers and Torrence. Read through these, they are very nicely written articles and they give you lot more insides into, of course,, usage of CWT besides the scale to frequency conversion. In the next lecture we will look at computational aspects of continuous wavelet transform. See you then.