**Introduction to Time-Frequency Analysis and Wavelet Transforms**
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**Lecture - 6.3**
**Properties of WVD**

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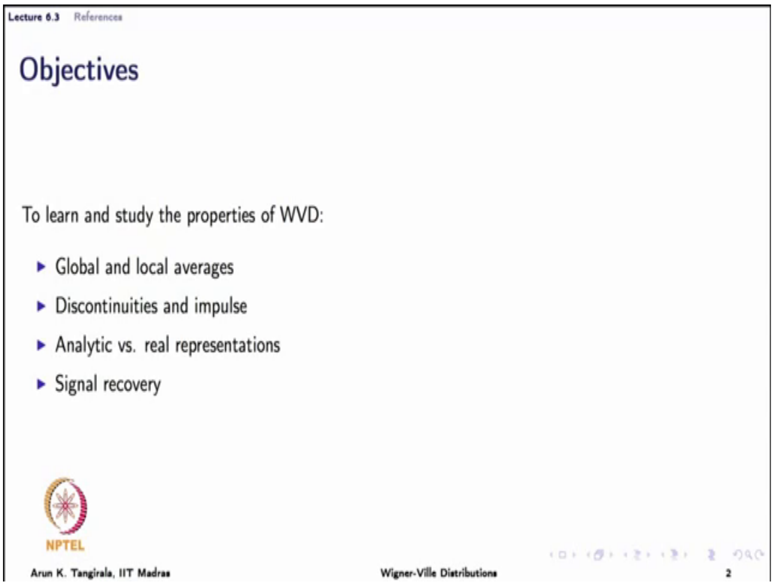


Hello friends. Welcome to lecture 6.3 where we continue to discuss the properties of Wigner-Ville distribution. So, I hope that you have already thoroughly viewed lecture 6.2. The objective of this lecture is to examine the other properties that we have not discussed in lecture 6.2. Namely the global and local averages that are we want to know what conditional averages will give us. And whether the duration and bandwidth and the mean time and frequency that I compute globally, do they yield the same one as the signal in the spectrogram they (Refer Time: 00:53)? But does the Wigner-Ville give me nice results in the respect. We will also look at how discontinuity and impulse manifest in the Wigner-Ville. In particular, will talk about the analytic versus real representations and talk about signal recovery towards the end I have a, very short video for you talking about interferences. With this lecture, we will actually conclude talking about the properties of Wigner-Ville. In a next lecture, we will ask how to deal with some of the properties that we do not like about Wigner-Ville distribution.

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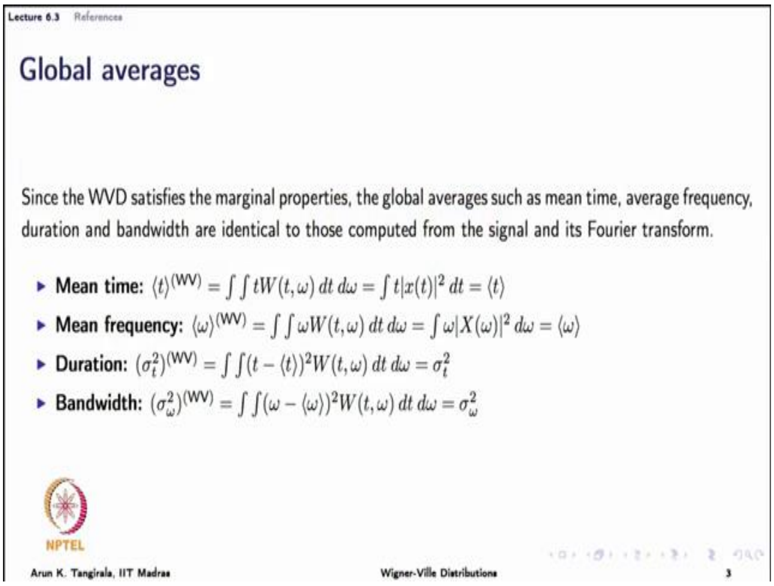


So, the global averages, it turns out coincide with the ones that we compute from the signal, why does this happen? Because Wigner-Ville satisfies the marginal properties. So, whether I calculate from the signal or whether I calculate from the Wigner-Ville; obviously, they should yield the same thing that is a very nice thing about the WVD. And the proofs of this fairly obvious you can just walk through the integrals here. This is the expression for computing the global time where you calculate the first moment of the joint energy density. And use the marginality property of the Wigner-Ville and you will recover the mean time that you recover from the signal itself. And likewise for mean frequency and duration and bandwidth what this means is, that unlike spectrogram the duration product of the global duration and bandwidth is the same as that of the signal. So, WVD does not make it worse than unlike what this spectrogram does remember. The spectrogram has the worst global property than the Fourier transform or the regular signal and its Fourier transforms that see it is a very nice property of the Wigner-Ville.

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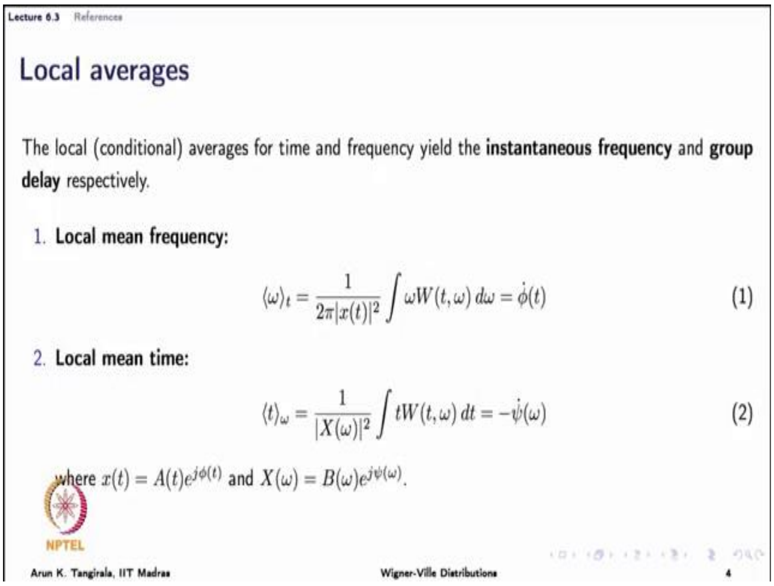


The other interesting and beneficial or favorable property of the Wigner-Ville is that the local frequency and local mean time. Local mean frequency and mean time correspond to the instantaneous frequency and the group delay respectively. This is beautiful all I need to do is compute the local averages for example, if I want to compute instantaneous frequency. I just locally compute the average frequency and that gets me the instantaneous frequency again the proof is fairly obvious. This is the definition of the local average frequency we are standing at a time t and asking what is the average frequency. Therefore, I need to use the conditional density and the conditional density is W of t of omega which is joint density divided by its marginal. Marginal is 2 pi times x of t square this 2 pi again appears, because of the angular frequency. And then the first moment of the conditional density will give me the average frequency at that particular time t. And it turns out when you substitute for the x of t and X of omega whichever way you work with using the standard representations that we have been using the complex representations.

You will end up with phi dot of t which is nothing, but your instantaneous frequency likewise for the local the mean time. Remember the definition of group delay is the average time spent by a frequency component or frequency of omega. So, once again the same arguments here I have to locally evaluate the average time at a given frequency use the definition. Here, the 2 pi does not appear and I end up with the group delay where I have see dot of omega unlike a phi dot of t that is it. So, contrast this with what we have in spectrogram. To compute the instantaneous frequency I need to locally evaluate the maxima essentially I have to compute the ridges,, but I do not have to now evaluate any maxima at all this need to compute locally the averages. There is one major drawback nevertheless that the conditional spread, local spread of let us say of the in frequency domain that is the local bandwidth. Unfortunately it is negative for can be negative for Wigner-Ville which is again the reflection of the lack of non-negativity property. So, all the Wigner-Ville gives you very nice meaningful local averages the local bandwidths do not make any sense anyway.

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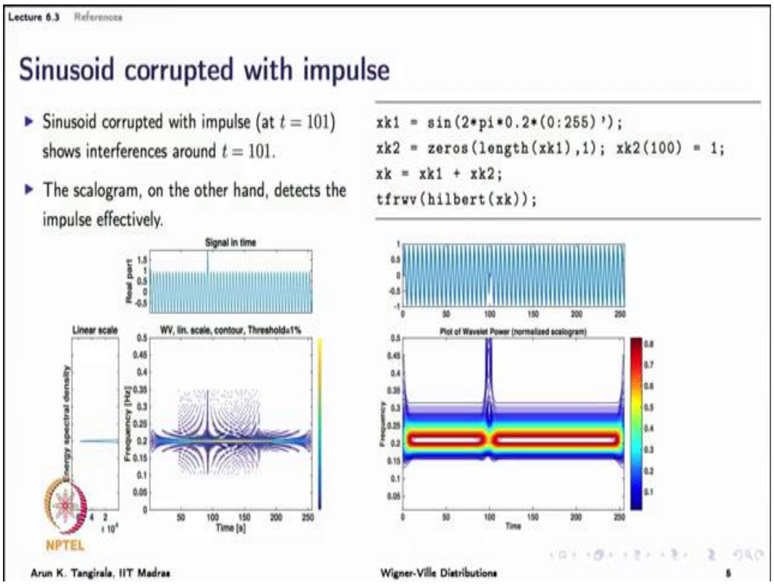


So, let us move on and talk about the case of sinusoids corrupted with an impulse. So, we want to ask if I have this signal which is experiencing two extremes scales of times one which exist all over and one which exists only at a single time. So, these are the two extremes scales time scales that we are looking at. Then the Wigner-Ville is shown which is shown here on the left bottom plot you have the Wigner-Ville it shows this presence of discontinuity,, but there is a lot of ambiguity as well. That is again due to the presence of interferences remember it does not have this finite nice finite support property as well as mainly the problem is that of interferences. In contrast the scalogram nicely detects the discontinuity. I have computed scaleogram in fact, this is the normalized scalogram. We will talk about normalized scalogram later on using we have computed this using a morlet wavelet nicely picks the discontinuity which is present at this instant here. However, it has lost out on the frequency localization the energy of this sine wave in frequency domain is smeared heavily compared to that in the Wigner-Ville.

So, you should now expect what we may have to sacrifice when we want to improve the

Wigner-Ville. In all of this at some point in time a question such as why am I studying Wigner-Ville when it does not have this nice properties or very use important properties such as positivity or lack of interferences why am I studying Wigner-Ville? Well, the idea is this you start with Wigner-Ville and then you stipulate the requirement that you want. Then you may be able to derive energy densities that are satisfying your requirement for the given application which probably the scalogram or the spectrogram are unable to fulfill. So, it is not you should not be content with having only scalogram and spectrogram; you should ideally be equipped to derive the joint energy density that suits the certain class of signals.

Remember the scalogram suits one class of signals which is those signal where the high frequencies exist only for a short period of time and low frequency persist for long time But not always an application will produce or a process will produce signals with these characteristics then what you do. You cannot use Wigner-Ville the original one,, but you can then go and stipulate the requirements and start modifying the Wigner-Ville to arrive at a joint energy density that performs much better than either the spectrogram or the scaleogram. This is the very important point that you should remember and we will be able to stipulate the requirements by placing certain requirements on the way we performs smoothing are the modification of the Wigner-Ville. And will talk about it in the next 2 lectures.

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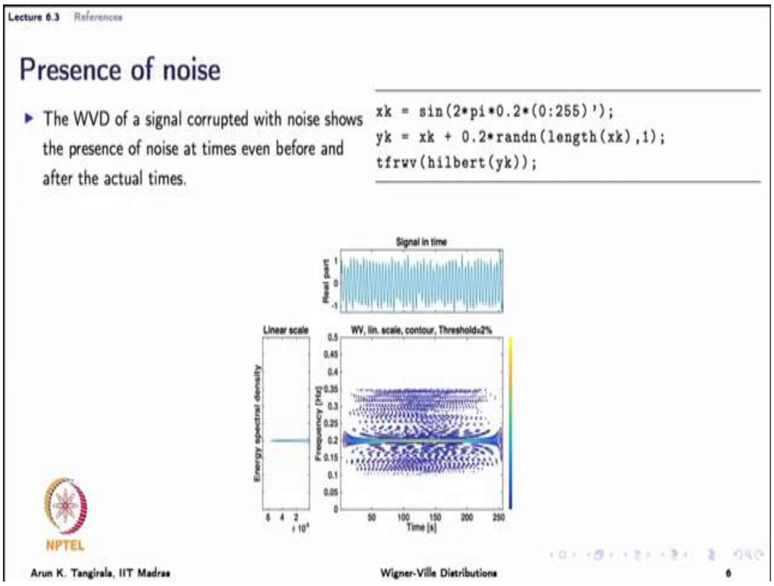


So, let us proceed what happens when I have noise? Well, the same story we should expect of course, the energy joint energy density to the spread over the entire time frequency plane. But visibly in the frequencies where the deterministic part is present here in this example I have a sine wave corrupted by noise. The main deterministic signal is the sine wave of frequency 0.2. And I should expect maximum energy there with the energy of the noise spread all over the frequencies, because noise is a broad band has a broad bond band energy density. Now, I have set the threshold here 2 percent,, but if you lower the threshold for display then you will be able to see the entire spread of the energy density. And what I say here on the top is that the Wigner-Ville of the signal corrupted with noise chose the presence of noise at times even before and after the actual times.

Of course, the simulation is not really simulating noise over a finite duration. It is assuming that the noise exists forever or as long as signal exists. But you should go and simulate another signal where you have noise only over a finite interval and you will able to see that the Wigner-Ville will show you as if the noise was present even before and after that is again, because of the interferences. And the lack of this strong finite support property. So, the basic message is again the Wigner-Ville is not. So, good at handling noise. But in any case we are working with deterministic signals there exist a definition of Wigner-Ville for noisy signals where again the end result is that you will get the densities in time and frequency. And you will work with auto covariance functions of the random signals and. So, on. Anyway when we improve the Wigner-Ville distribution hopefully these properties are also going to get back.

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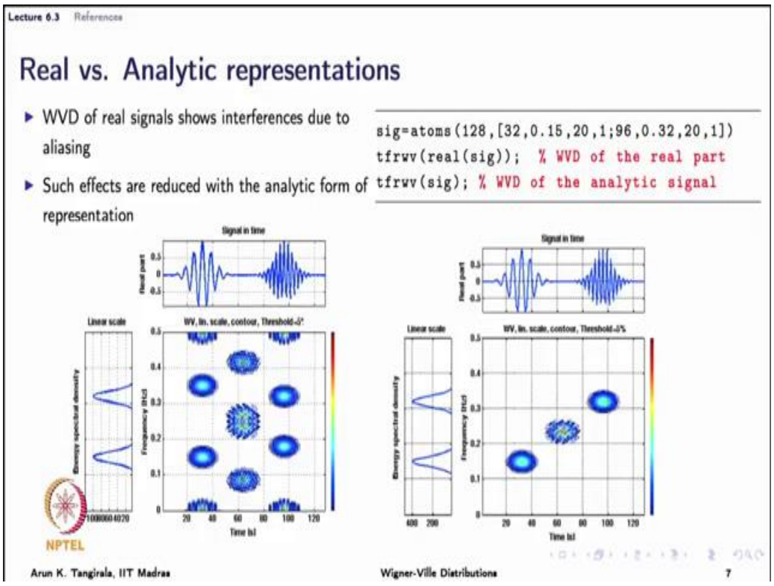


Let us look at this very important aspect of the use of Wigner-Ville or the way we use

Wigner-Ville. Until now, you have being saying in all the course that I have been using analytic representations of these signals. Although we generate and simulate real value signals when we calculate the Wigner-Ville distribution we use the analytical version of those real valued signals. So, why this necessary. Well, is let us look at this example and understand why this is necessary. So, I have simulated a signal which has 2 Gaussian modulated waves separated by a zero activity region when I just compute the Wigner-Ville of this real valued signal. Of course, in time frequency tool box the atoms routine will give you analytic version themselves. So, I have to deliberately take in or consciously take the real part of that and compute the Wigner-Ville. You can see that there are just too many unidentified flying objects here. That is this interference here all over the joint energy density.

Ideally I should have had only 2 regions of joint energy density to active regions in the joint energy density,, but I have this too many and why is this? Because number one any real signal even if I had only 1 Gaussian modulated wave and not the second one. The real value signal can be written as a sum and addition of 2 complex value numbers. For example, if I had a sine wave for a short duration sine theta is e to j theta plus e to the minus j theta by 2 j. The moment I have addition of signals I will have interferences. So, even if I had a single wave here single atom even then I would see interferences. So, now, for a single value real finite duration real wave I will have interferences. And I have 2 such as atoms and therefore, I have. So, many combinations of interferences.

The moment I work with analytic versions the interferences terms associated with the respective atoms will disappear. And even the cross terms between the cross terms of the individual ones will disappear in the end I have only a single interference term appearing which the lot is easier to get rid off. So, this should tell you why analytic representations are very important in Wigner-Ville. There is also another reason which we will learn later on when we talk about the sample version of Wigner-Ville. Until now you have been looking at theoretical Wigner-Ville when I look at the sample version then I will understand why the analytic representation is necessary. So, there are 2 reasons; one, because we want minimum number of interference terms. And secondly, I want to avoid so, call aliasing when I compute the discrete time sample Wigner-Ville which we will talk about in the next lecture.

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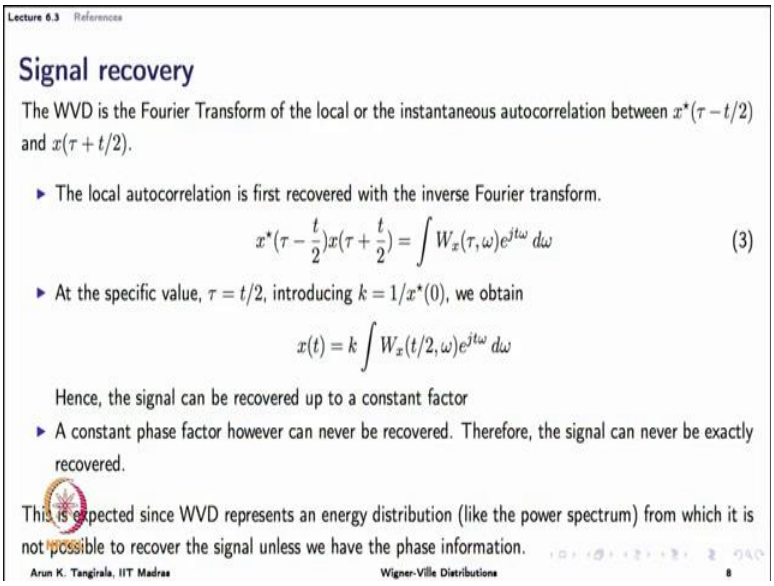
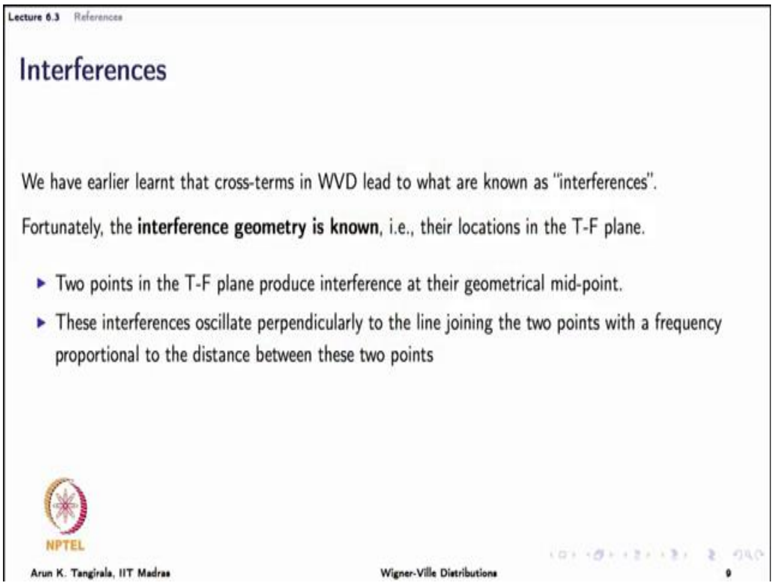



So, let us quickly look at whether I can recover the signal, because this is the question that we had also asked when I am using the time frequency analysis tool if I can use it as a filtering tool. With respect to the Wigner-Ville unfortunately you cannot although there is some application of using Wigner-Ville for filtering. The main reason is that the phase of the signal is lost, because you are computing this product. So, look at this the first step in recovering the signal is to recover the local or the instantaneous auto correlation by using an inverse Fourier transform. So, I first construct this product given Wigner-Ville by applying the inverse Fourier transform and then I said tau equals t by 2. So, that I have x of tau equals one over x star of zero time see integral evaluated at tau equals t by 2. I just rewritten this integral in terms of t that is all.

So, I am able to recover the signal fine,, but there is this k that I do not know which is x star of a zero unless I recover the signal how would I know value of the signal. So, this ambiguity will always exist this is, because I have lost the phase. You can never recover the signal uniquelygiven the autocorrelation function. It does not matter Wigner-Ville or whatever it maybe the case. It is like saying I will give you the spectrum can you recover the signal? Yes,, but only up to a phase factor, because spectrum does not have the phase information that is the main problem. But where phase is not really a requirement and or may be phase randomization is alright then you can use the Wigner-Ville for filtering the signal. So, there are certain applications in image analysis. I welcome I invite you literature review and find out what those applications are.




Finally will close this lecture, with a revisit of this concept of interference, because it has been interfering in many of the properties that the useful properties that we want. And we have already seen this theoretically why this interference term occurs. This interference terminology that we use is only a mathematical one there is no physical interference of any signal like we see in optics and. So, on. It is just a mathematical term. So, it is an only artifact of the technique. The good news, so, the bad news is interferences the good news is that interference geometries is known we know exactly where the interference occur. So, recall this one of these slides where I have the interference occurring exactly mid way in time and frequency this is always the case. So, let me actually show you a video very quickly on that,, but before I do that also remember that the interference oscillate perpendicularly to the line joining the 2 points.

And with the frequency proportional to the distance between those 2 points and you will understand what I mean when you look at the video. So, let me play this video for you where I have the same kind of signal 2 atoms separated by the zero activity regions. Let us see how the interferences geometry changes when I move or shift one of the atoms. So, you saw what happened right. Let us play it again. The interference term also moves exactly the mid way location does not change in time or in frequency all right. So, again this is just an animation to show you that the fact that interference term occurs at a particular location that is a mid way is not a coincidence for a particular signal it is true for all signals we will make use of this property to get rid of interference. If I know exactly where it is interfering it is lot easier,, but if it keeps changing with the signal then only we have the problem. So, we will exploit this property to get rid of the interferences.

In the next lecture, we will talk about this in fact; will talk about the sample Wigner-Ville distribution that is the one that we use practically. And then will talk about the pseudo Wigner-Ville where we will apply window function. The reason for applying the window function we discussed very briefly in the definition in the first lecture on this topic. The Wigner-Ville looks at the entire time horizon therefore; it does not really get me the local truly local property applying window function will help in improving it. But we may lose out on the nice time frequency resolution that we have. Then will talk about how to enforce positivity connections between spectrogram and scalogram all of this in the next lecture.

Thank you.