Introduction to Time-Frequency Analysis and Wavelet Transforms
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Lecture - 5.4
Closing Remarks

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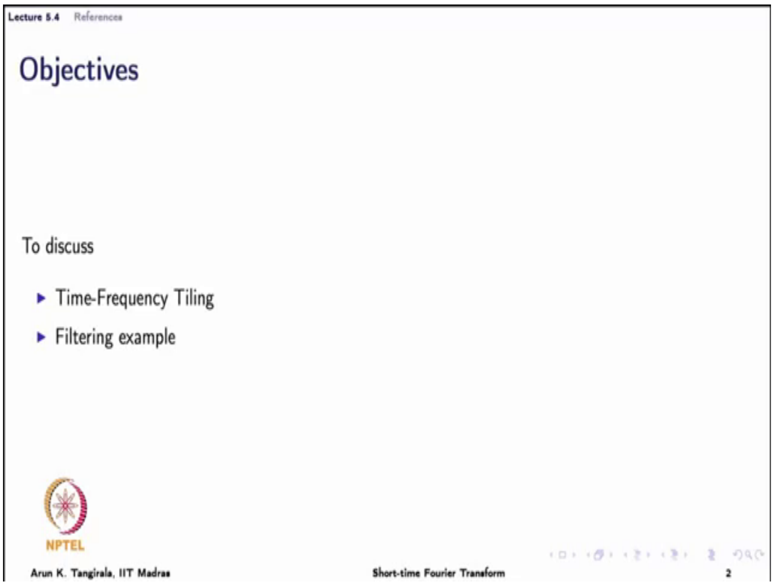

Hello friends, welcome to the last lecture on the topic of short time fourier transforms in the course on introduction to time frequency analysis and wavelet transforms. As per our convention this lecture is numbered lecture 5.4. And, in this lecture I am going to illustrate essentially the idea of filtering using short time fourier transform which will pave the way for wavelet filtering later on.

Before we do that, I would like to briefly talk about time frequency tiling. Again, this is keeping in mind that we will look at wavelet transforms shortly. So, both the topics of interest in this lecture are a kind of futuristic, in the sense, in our lecture it is futuristic. And then we will close the lecture with the few remarks; it is more or less a summary of what we have learnt.

We have looked at the theoretical aspects of short time fourier transform. In the previous lecture we examined how the window length and window type can affect the nature of short time fourier transform. We also discussed the descrete short time fourier transform which is the most practically implemented version of the short time fourier transform.

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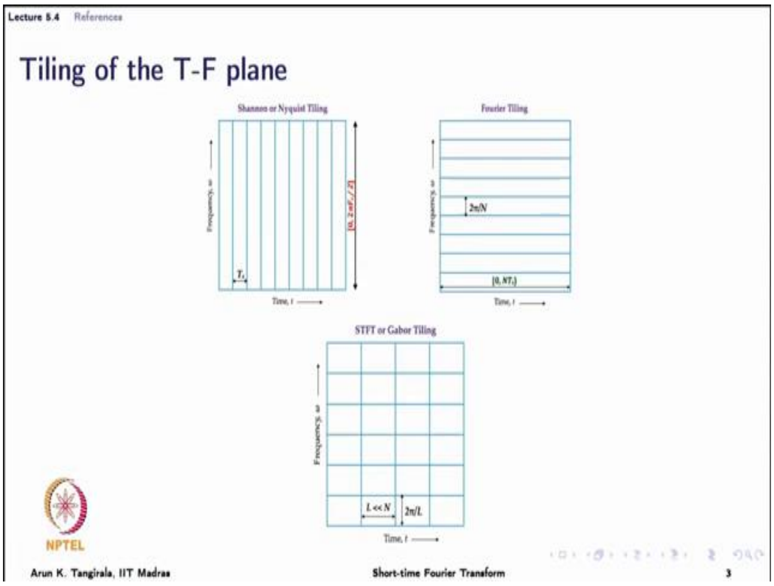

Now, what I would like to do is, with the help of this time frequency tiling idea that is commonly used, reinforce a concept of short time fourier transform in the context of time frequency analysis. Now, let me explain these 3 figures to you which will help you understand what short time fourier transforms do. And, later on, we will use this tiling again to illustrate the idea or the concept of discrete wavelet transforms, or in general wavelet transforms.

So, let us begin with the top left figure here that I have for you. The idea behind each of this, in fact, each of the figures that you see here is, the main idea is how you are actually tiling the time frequency plane, how each method tiles the time frequency plane because we are working in this time frequency, 2 dimensional time frequency plane and I want to be able to localize the signals, features or energy in this 2 dimensional plane; that is the basic objective in time frequency analysis.

And, each method differs in the way it arranges this tiles. So, imagine that you have a floor and you want to lay down some tiles, or you want to lay down the tiles for this room or floor of that room, and what the regular sampling or the shannon’s tiling does for you is, it tiles this room, that is this 2 dimensional floor, in such a way that you have vertical tiles; in which direction? Well, the vertical direction is in the direction of the frequency, but of course, the sampling is in time.

What you have here by virtue of shannon tiling is a very fine spacing in the time direction, but poor localization in the frequency direction. Why we say poor localization is, when I obtain a sample signal, then I have no idea what frequency it contains; all I know is that the signal contains frequencies in the range 0 to 2 pi f s by 2; why f s by 2, because recall f s by 2 is a nyquist frequency; it is a maximum frequency that I can detect; maximum frequency of the underlined continuous time signal that I can detect at a sampling frequency of f s. So, this 2 pi is just to account for the angular frequency units.

So, the tiling here is in such a way that I have very good localization of the signal features in the time domain; I cannot get a better localization in this; which means I cannot get more information than the resolution or localization at which I have the time domain signal itself. And, that resolution is often specified by the sampling interval.

What fourier tiling does for you is it actually does the reverse for you; it gets you a very fine localization of a signal features in the frequency domain, but looses out on what is happening in the time domain. So, it is just completely orthogonal to what is happening in the Shannon tiling.

The fourier comes along and says, I am going to tile the floor this way, in such a way that I have very fine localization frequency with the spacing given by 2 pi by n. Well, why 1 by n, because, again recall the theory of d f t, discrete fourier transform, the grit spacing in a frequency domain is dictated by the number of observations that you have.

And, if you think of n as a number of observations then I have a frequency spacing of 1 by n in cyclic frequency, or 2 pi n in angular frequency. But I do not know when this frequency exists in time and therefore, I have lost out on the time localization of the signal features. This frequency could have existed in time, from the start to end of my observation interval. So, whatever you see for the frequency in Shannon tiling is what you observe in the fourier tiling.

Now, these are good by themselves individually if I am looking a time domain analysis or fourier frequency domain analysis. However, in time frequency analysis I would like to have 1 foot in the time domain and another foot in the frequency domain.

So, now, Gabor comes along and says, I am going to tile the floor this way; I am going to tile in such a way that neither am I going to have a fine spacing in the time direction as I get from Shannon’s tiling nor am I going to get as fine as a localization in fourier tiling in the frequency direction, but trade off. Because, fourier does not get me the time localization, Shannon does not get me the frequency localization, here I am trying to get you an intermediate solution.

Now, the spacing here, so there is some time localization, unlike fourier now I have improvement in short time fourier transform. I have some time localization. Of course, not as fine as in Shannon's tiling. The localization here is determined by the length of the window l. As the window width l becomes larger and larger, I, the Gabor's tiling moves on to fourier's tiling. Or, and, as the window width becomes narrow and narrower, the short time fourier transform tiling specializes to, or simplifies to Shannon's tiling, right.

So, why this relation exits? The relation exits because of the duration band width; so, that is what is the underlined principle that connects all these tiling together. So, behind these tiles, or you can say, under the floor there is this duration band width principle that is trying to tell you, you cannot tile in whichever way you want; you cannot choose a very fine tile, very tile which is very narrow in time and frequency domain; there is a thread underneath this floor which is telling you what you cannot do and what you can do.

So, I start with Shannon tiling and I let go the time localization from t s to l, so that means, I have a loss of resolution here, but what I gain is only frequency localization. Or, you can say I can start with fourier tiling and I let go the frequency localization, and what I have is some time localization. Now, what you should observe here is all the rectangles in this tiling are of the same dimension, right. They have the same breadth and length. This is nothing but a representation of what we have seen earlier, the spreads in time and frequency of the energy localization due to short time fourier transform is the same across the time frequency plain.

Stated otherwise, whether I am looking at extracting high frequency components or low frequency components from the signal, I am going to use the same, filter with same band width, right. But, that is not necessarily the best thing to do when you have the signals where high frequency components may persist only for very short period of times, high frequency components would mean sudden jumps, abrupt changes and blurbs and so on. They usually do not persist for a long time.

Whereas, low frequency components take time to evolve. So, this is the typical scenario, not necessarily this scenario always; lot of time this is a scenario. In that situation short time fourier transform is not the best option that you have. Because, what is happening in short time fourier transform is that you are using the same duration and bandwidth for all frequency components; whereas, I have a situation where the high frequency component persist for a very small interval in time. Therefore, I should capture the time localization of the high frequency component very nicely which means I have to let go the frequency localization for high frequencies; on the same note low frequency components exists for a long period of time.

So, there is no point in searching for low frequency components with a narrow window if I use, so I have to use wide window; when I use a wide window I will only search for low frequency components because the moment I use a wide window I have lost a time localization. Therefore, there is no point in searching for the high frequency components. This is the key difference between short time fourier transform and wavelet transform. It is just the way you search for the frequency components; and, that is the way you capture the frequency and time localizations of the features, is what makes the wavelet transform different from short time fourier transform.

So, when we study wavelet transform we will look at how wavelet transform tiles the 2 dimensional plane. For the moment I leave it to you to imagine; you can actually sit and draw based on what we have discussed. Wavelets get you very good frequency localization for low frequency components and very good time localization for high frequency components. So, you can imagine now how wavelets would tile the time frequency plane.

And, in passing I also want to mention the tiling that you see for short time fourier transform is for the discrete short fourier, short time fourier transform with orthogonal representations. That is, you are marching in such a way; that is, you are marching a window in search a way that you are generating orthogonal family. Remember there is no, that means, there is no overlapping between the windowed segments.

That is the simplest to understand; and therefore, we have drawn the tiling with that perspective in mind. Of course, you can draw the tiling for the other one, the redundant one as well, but that is not necessary or useful for discussion. So, we will return to this tiling when we talk about wavelet transforms.

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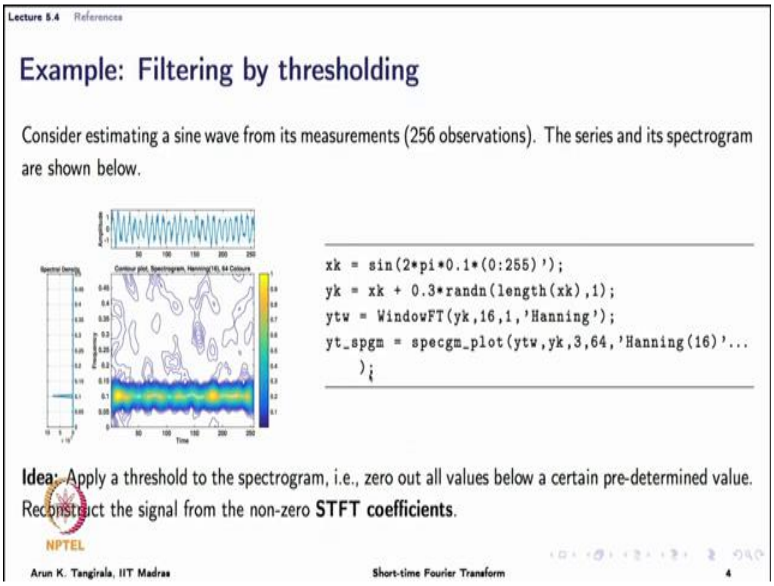

The next topic in the agenda for this lecture is to show you how you could use short time fourier transform for filtering. You have already seen how you can use the spectrogram for time frequency analysis where you plot the energy density in a 2 dimensional plane, search for features, oscillatory features or other features of interest, and then extract those features.

Here, I just want to illustrate to you the idea of thresholding on a very simple example. This is actually not necessary; in the sense, it is not necessary to you short time fourier transform, the classic venue filtering based on the spectrum, but on the same idea that we are going to talk about does a very good job for you. But, anyway, the purpose is to illustrate how one could filter the signal with the help of spectrogram.

So, let us consider a simple example. I want to estimate the sine wave from its measurements. I am showing the measurements on top of this figure here. I have 256 observations. And, its spectrogram is shown here. I am using the wave gab to generate the spectrogram. I have the windowFT command here. And, I am using a Hanning window of width 16. And, I am marching ahead in lengths of one sample which means I am generating a highly redundant representation. And, to generate the measurements I have added some amount of noise here. I leave it to you to figure out what is the signal to noise ratio; it is not too high.

What I have shown to you as usual is a spectrum. Of course, a spectrum tells me what is the frequency content of the signal. Now, let me tell you that it is a good idea always to go through this analysis systematically; that is, you look at the time domain measurement and then you look at the spectral density; that is, you carry out the fourier transform, construct the spectrum. Then, it will give you an id of the global features. So, before you zoom in into the time frequency plane it is always good to have an idea of the global features which means what frequency components are present and then you can choose your window length accordingly, alright.

Here, of course, I have chosen a window length based on some prior knowledge of the signal, but prior first look at the spectrum and then determine what windows you should be using, alright. So, what you see here as this blue line contuse according to the color bar is essentially it is a weak intensity at these points and you should guess this is due to noise. If you were to plot, if you have to obtain this spectrogram plot using t f r STFT that is the routine that I have used earlier to illustrate the ideas of continuous time short time fourier transform.

Then you would see on top, value called threshold. That threshold essentially will suppress some of these features and it will only display features that are greater than some value. Just because it believes that in practice there is always going to be some amount of noise in your data and there is no point in displaying the intensity which is very small in magnitude.

Here, I have not suppressed anything, but I have plotted a contour for you. You could also obtain an image plot. Now, this spectrogram underscore plot, a routine that I have written, takes the short time fourier transform from window f t, plots the spectrogram for you. I will of course, apply this mat lab routine to you. There is also another routine that I have written which plots an image lot of the spectrogram, but normally I prefer the contour.

So, let us get back to our objective. Our objective is to obtain a cleaner version of this measurement by filtering out the noise. As I said venam filtering is very good at doing this when frequency components are present throughout and when there are no transience in the signal. But our idea is to illustrate the use of short time fourier transform.

Now, in venam filtering the idea is again to cut out or to 0 out the spectral values below a certain threshold in the spectrum and then reconstruct the signal using the phase because using spectrum alone you cannot construct the signal. The idea is same here that I am demonstrating for filtering, I will cut out all the values of the spectrogram lesser than a predetermined threshold and then reconstruct the cleaner signal by, from the remaining non zero short time fourier transform coefficients.

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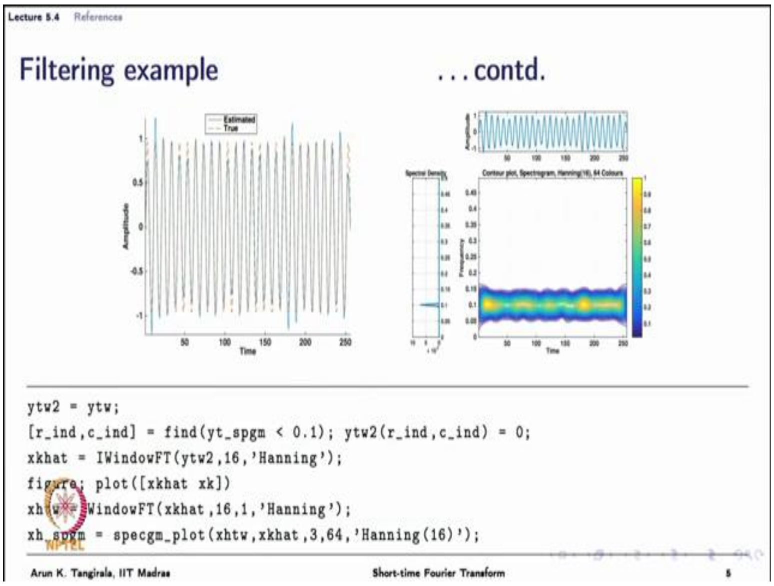

So, here is the result of that. The mat lab code for doing this is given below. What I am doing here is in the second row, line of this code and finding out the time and frequency indices of those values of the spectrogram that are less than 0.1 in magnitude. Of course, you can ask what is the sanctity about the point from. There is nothing, no sanctity about it.

But, the threshold, this value here is largely determined by apiary knowledge of the noise variants that you may have. Or, you can actually estimate this noise levels, there are methods of estimating that, from this spectrogram itself based on some assumptions that you make on the noise itself. This is also the idea in wavelet based estimation, signal estimation. There you perform it wavelet transform threshold and you reconstruct. There are, again in wavelet transform, methods for determining the threshold automatically from data statistically.

So, I have found those indices, zeroed out the short time fourier transform coefficients at those locations, performed an inverse short time fourier transform. Note that you how to use a same window that you used for analysis; you cannot use the different window for synthesis. And, now I am showing to you the cleaner version of the measurement which is in the blue here, solid blue, and the original sine wave itself which is the dash red color. Of course, you cannot expect a perfect match; if you does then there is a match fixing there, right. So, that is not correct. There will be some difference between the reconstructed sine wave and the true one, but it is pretty close. Of course, you can come up with measures of what is meant by closeness and so on.

What I show you as well is a spectrogram of the clean signal on the right side. You can compare this with the spectrogram of the measurement. Effectively I have cleaned up the noise in these areas. Now, this estimation of signal by cleaning the noise in transform domain is a classic idea that has been there for very long since a time of venam filtering and so on.

The basic idea is like washing our cloths; when our cloths get dirty then I have a measurement that is a noisy measurement. Now I want to wash this cloth, what do I do? I just, I cannot wipe it and get rid of the dirt; I have to soak it in water and also add some detergent, right. Soaking in water takes the dirty cloth into a transformed domain, and then adding the detergent is equivalent to performing some kind of thresholding operation, including the spinning and so on, everything.

And then, I, once the water is in stowed with the dirt, the dirt and cloth are nicely separated in the water plus detergent domain, I get rid of the dirty water and the dirt. I get a cleaner cloth, I dry it, then I have a clean cloth which is obviously not the same as the original shirt that I, cloth that I purchased from the cloth store; there is going to be some difference.

Now, what would be the difference? Maybe the detergent has eaten away a bit of the cloth, depending on the power of the detergent. If it is a very powerful detergent then it can eat away a lot of the cloth; and it can claim that you have a cleaner cloth, but what you have is a cleaner, but lesser cloth. So, the same story here, in signal estimation, you can be very aggressive, you can be conservative. If you choose a larger threshold then you will lose a signal as well; if you choose a smaller value of threshold you may end up retaining more noise, right. So, that is the basic idea.

So, what you are doing is you are laundering the data and getting a cleaner signal. So, exactly the same idea in wavelet transforms as well. When we discuss the signal estimation v w t you will find very strong similarities, and that is why I am laying the foundations for you right now. So, you can play around with the thresholds or you can try other signals and so on, I leave that to you, but this is the basic idea, alright.

You can also use this idea to extract certain frequency components; when we say signal estimation, it need not be always that I want get rid of noise, it could be always, it could be also that I want to only selectively extract certain features of the signal. There instead of applying a certain threshold you can completely zero out the value of the short time fourier transform coefficients in a band that you do not want the frequencies or you can retain only those short time fourier transform coefficients in the band that is of interest to you and then reconstruct the corresponding signal.

So, that is the generalization of this thresholding idea. Again that is what we will also do in wavelet transforms. With this I would like to close the lecture as well as the topic of short time fourier transform.

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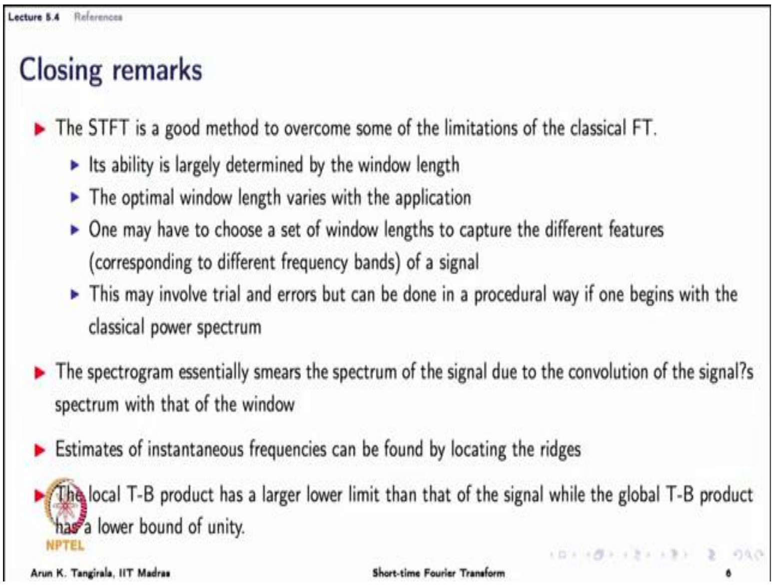

So, this is the summary of what we have learnt until now. I just want to highlight main points. One - that the short time fourier transform is definitely a significant improvement over the fourier transform in getting the time localization, but it is largely dependent on the window properties such as window length and window type; mostly window length governs the ability of the short time fourier transform to get the time frequency local features for you.

And, there is no universal answer for choosing the optimal length. It will depend on the application. So, the best thing is to plot the spectrum first and then determine the number of windows and the window length that you want to use. The good news is that you can estimate instantly frequencies by locating the ridges, right, which has the points of local maxima in the spectrogram, and we have demonstrated that as well on chaps.

And, importantly, the local duration band width product, when I say T-B it is duration bandwidth product. It has a larger lower limit than that of the signal which means exactly what we have seen in this tiling here; it, the time localization of the short time fourier transform is worse than that of the signal, and the frequency localization is also worse than that of the fourier.

Therefore, the upper limit as of, the lower limit for the duration bandwidth is larger than that of the signal itself. But, the local one is bounded. It is actually bounded above by the signal itself, that gives some good, that is that gives me some good localization properties of the short time fourier transform. So, it is somewhere, essentially the summary of the, the message of the last point is that the short time fourier transform gives me a solution that is intermediate between the signal, the shannon tiling and fourier tiling, that is what it means.

When we, the another point that I want to mention which is not listed here is, when we talk of winger distribution, winger ville distribution, we will learn that spectrogram is a special case of smooth winger ville distribution. And, there we will also briefly talk about what is known as reassignment where I will redraw the spectrogram by using the center of, mass of center of gravity concepts. The idea is to remove that smearing that window brings with it when I compute the spectrogram.

Remember, the energy of the signal is scrambled with the energy of the window as a result of which I have of lost smearings. I can get read of that smearing to a large extent by what is known as a reassignment and that is a very, it is more of a modern idea relatively and gives some beautiful results; we will talk about it when, once we talk of wigner ville distributions. So, hope you enjoyed the lecture, please go through all the lectures in this topic once again; feel free to ask any questions; my teaching assistance and I are available to answer your questions, and enjoy the learning.

Thanks. Bye.