**Introduction to Time-Frequency Analysis and Wavelet Transforms**

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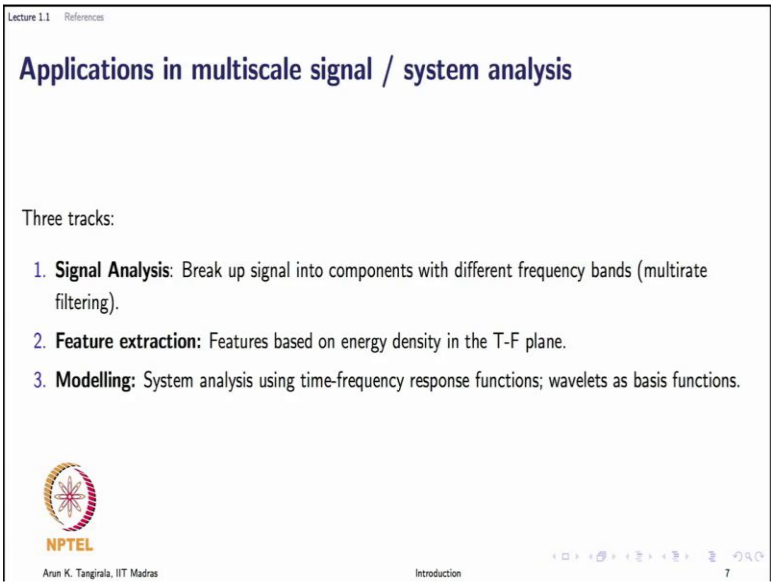


Now, as you are doing a literature review, you will find that there are three tracks in which, at least three different tracks in which the multi-scale analysis can be categorized into: one is a signal analysis itself like, where you are looking at, what is a frequency content and all of that. Then you have, what are known as feature extraction, where looking features in the time frequency (), is there a particular oscillation that I am looking at, or is there any particular shape in case of image analysis and. So, on. Again in the time scale or the time frequency plane. As the third one being systems analysis; no signal comes out without any system..

So, there is always a system that is generating the signal. If I am looking at more than one signal; let us a two signals, then there is a system that relates probably these two signals, and I am interested in modeling that system which connects these two signals or two or more signals. So, that is where you will find multi-scale systems theory, predominantly where people talk of wavelets and all of that. So, there are these three tracks; of course,, there are other kinds of tracks which have left out, as I said we have predominantly in signal analysis. And in image analysis you will find a few more tracks system. We are primarily going to focus on signal analysis and feature extraction, and leave out the modeling part, because that requires mode a the topic advanced foundations and. So, on.

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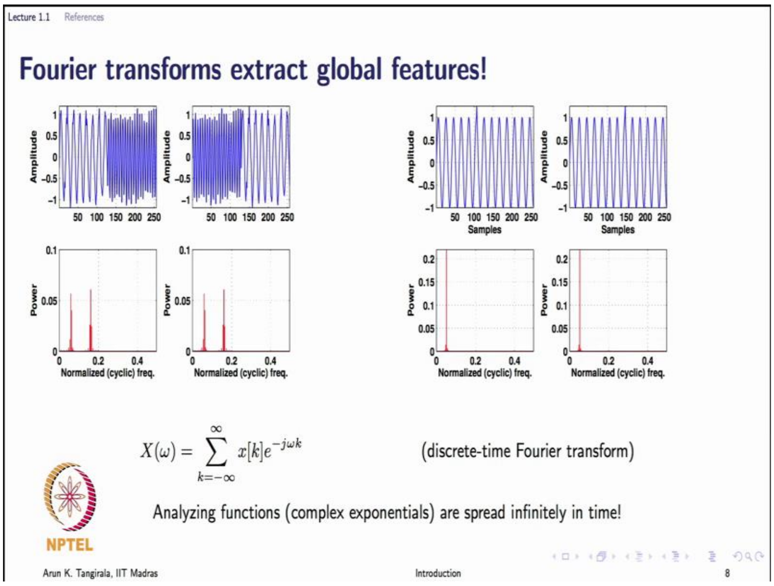


So, let me quickly give you a feel of what Fourier transform, why the tools are necessary, and what we have qualitatively talk about earlier that Fourier transforms are unable to give you the local features. I have a simple example for you here. On the left you see two signals, where I have two sin waves of two different frequency's,, but switched in terms there occurring in series,, but the order is switched to give you a feel of what Fourier transform cannot do..

So, on the top view have the time series, and in the bottom panel you have the spectra, which again, which has frequency on the x axis and power on the y axis, which essentially telling you how much each frequency component has contributed to the overall power of the signal. We keep using this terms power energy, will give definitions later on. So, if I look at the spectral plots, we are not able to distinguish between these two signals, because what has happen is your thrown out, what is known as a phase information..

And typically it is hard to use the phase information, because of noise and other reasons. Normally we work with the spectra, and the spectral density is unable to get you the time features of the signal; that is, it is unable to tell you which frequency occurred first, and which occurred later. There is another signal, there is another example that I would like to give you, which is a very classic example that is use, in testing the ability of multi-scale analysis tools, which is a sin wave corrected with an impulse. And here I have the impulse location being changed. First of all the Fourier analysis will not able to tell you anywhere, when the impulse occurred,, but just show you the point, I just shifted the impulse in both these signals, and you can see the spectrum remains more or less invariant to the location of the impulse, and that is what it is a spectrum, is invariant to the occurrence of the features in the signal..

In fact, this an acid test for most multi-scale analysis tools, where you have sin wave; that is the persistent throughout in time,, but highly localized in frequency, there is a signal peak in frequency. Whereas, you have an impulse which is highly localized in time,, but we know from the Fourier analysis that its spectrum is broad band, its spread throughout. So, they are there actually to extremes that are present in a signal, and if your tool is able to pick out these two extremes without much user intervention, then it is a very nice tool. Now, the prime reason why Fourier analysis is a able to, I use unable to extract the local feature is, the building blocks, or you can say the analyzing functions which are the complex sinusoids are spread throughout the time. So, that is the main point, and the main message; therefore, we should take from here is, if I want to capture local features in time, I need basis functions that are highly localized.

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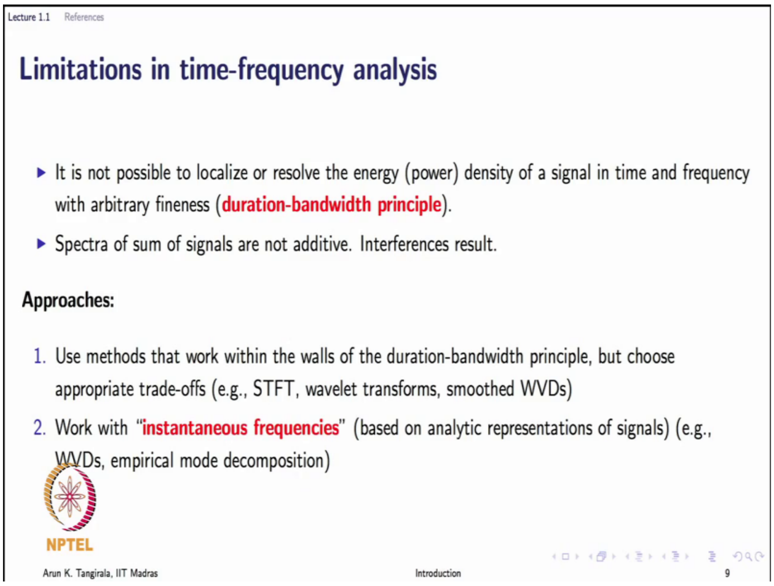


And that is the basic effort in sort time Fourier transforms,, but before even talk about any technique we should understand, that there are fundamental limitations in time frequency analysis, because of the duration band with principle, which says that I cannot obtain the local information in time and frequency, with arbitrary fineness. If I try to capture the local features in time, I will have to sacrifice the information on the local features in frequency, and that is the trade of in which that all these tools work. Fourier analysis actually takes extreme trade of, and time series that you have, is an one end and the other end, where you have very nice information in time by virtual sampling. Whereas, with Fourier analysis, you have very fine information, or localization in frequency by virtual of the transform itself,, but it loses out on the local information in time. Short time Fourier transforms, and wavelet analysis, and even Wigner Ville they are all, in some sense of the other govern by this duration bandwidth principle. The other limitation is, when I add up spectra. So, earlier you saw that, the signal was made up of two components at least, when you look at the spectrum of this added signals, they are not going to be, this spectrum is not going to be the sum of the, spectrum of the individual. Once there are going to be interferences terms that I am going to generate spurious features in your spectrum, and that is another limitation time frequency analysis.

So, predominantly there are two approaches; one is a classical Fourier root, and then you make changes to that; that is a sort time Fourier transform and. So, on. Then you have the multi-scale analysis root, where you recall the relation between scale and frequency. What you do is, you break up the signal into different scales, from course to fine scales, using wavelet analysis, and that is what wavelet transforms do for you. There is a third approach, which is although I said two here; the scale approach and frequency approach are more or less same. So, the second approach. So, to speak, is approach of the instantaneous frequency, where you think of the function of frequency that changes with time. So, instead of analyzing with the basis function that a fixed frequency at all time, which is what Fourier does. You can think of breaking up this signal into components that have frequency is changing with time, and this was basic idea in Ville’s analysis and that followed Wigner’s work. And then more recently the empirical mode decomposition also follows this approach. We will study both this approaches fairly at length.

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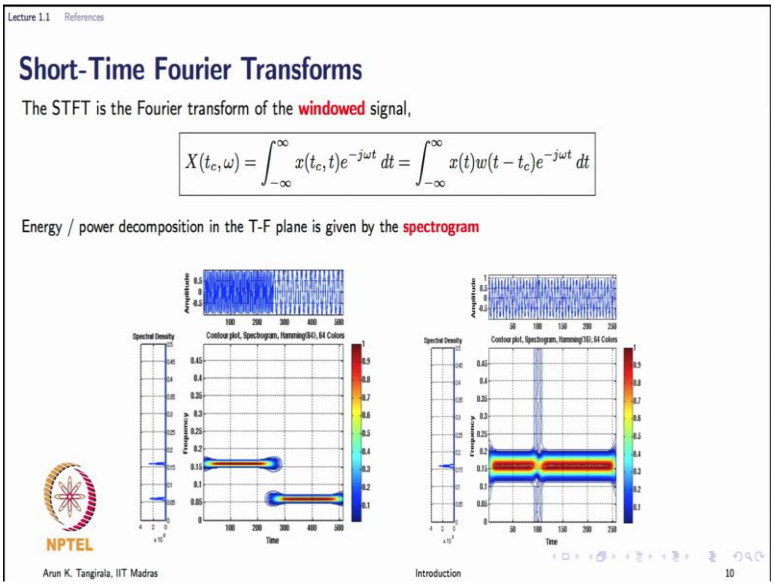


So, just to conclude the talk, I will give you feel of what sort time Fourier transform and Wigner Ville distribution do for you. In the second part we look at wavelet analysis. So, the short time Fourier transform is nothing,, but a variant of the Fourier transform, it is a natural idea. We have identified the problem with Fourier analysis. Analyzing function exits over the entire time plane. Whereas if I want to get local features I need this analyzing functions to be of finite length. So, what I am going to do is, I am going to click the sinusoid, or what I am going to do is, I am going to segment the signal. Either way, either perspective is all right. And the expression here which you are not going to discuss too much in detailed, tells you that either that the signal, is being clipped with the help of this. So, called the window function. Or we can say that signal is being analyzed by what is known as a clip sinusoid. This window function is responsible for clipping either the signal or the sinusoid itself, whichever way you look at it. And t c refers to the center of the window in time, and omega also refers to the center frequency.

So, let us look at what I get from. So, called spectrogram. The definition that I have given in the top is for the short time Fourier transform, spectrogram is simply computed as magnitude square. Now, just for the illustration I have taken of the same signal that we have seen earlier. This shot, this spectrogram is able to clearly tell me that there are two frequencies in the signal located over different time intervals. Fourier analysis tells me simply that there are two frequencies,, but now this spectrogram is able to get me the local information in time,, but of course,, there are certain limitations to which to this technique,, but by enlarge its a major improvement over the Fourier analysis of this spectral analysis. Likewise here, I am able to locate the existence of an impulse, that is buried in the sine wave, more or less appropriate location. Notice that we are unable to pin point the location of the impulse, because of the duration bandwidth principle. And likewise for the left plot, I am not able to say confidently that these are the two frequencies that are present in the signal; there is some swearing of the energy.

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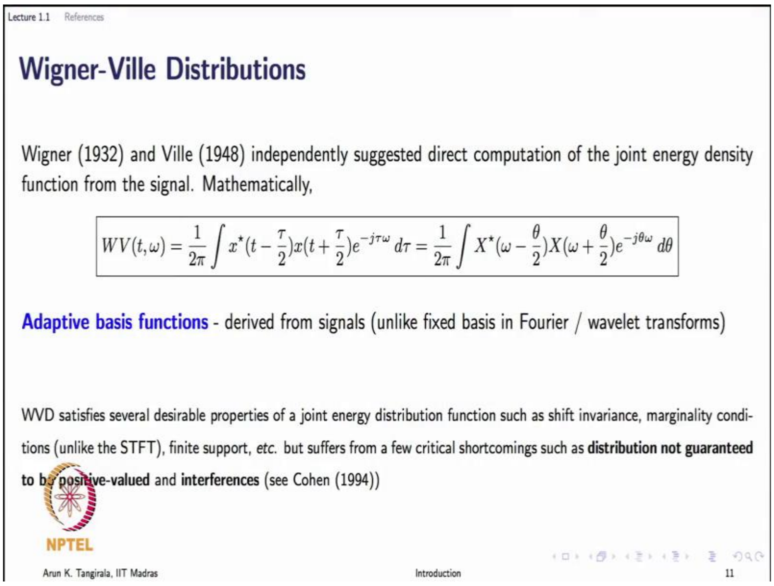


We say that there is this localization in frequency is lost, because I am trying to get the local features in time, that is the duration bandwidth principal working for you behind the curtains. And the Wigner Ville distribution takes, as I said different root, where it does not work with the transform directly computes the energy density, and the expression, mathematical expression right now, the details of it are not. So, important,, but the only point that I want to make with this expression is, you can thing of Wigner Ville distribution, as being working with what is known as an adoptive basis. In Fourier analysis and wavelet analysis we use what are known as fix basis, fixed analyzing functions. I know what is analyzing functions are, and therefore, I know the properties of the broken of the segments. I know exactly what the properties of the individual segments are, as a consequence,, but there is also another root to signal analysis where, I do not choose the analyzing functions at priory, I will wait for the signal to arrive, and then determine how to break up this signal. That approach is called an adaptive basis approach, which has numerous advantages of fix basis, because in fix basis you are always imagining signals to be made up of components that you have pre imagine; that is not necessarily the case.

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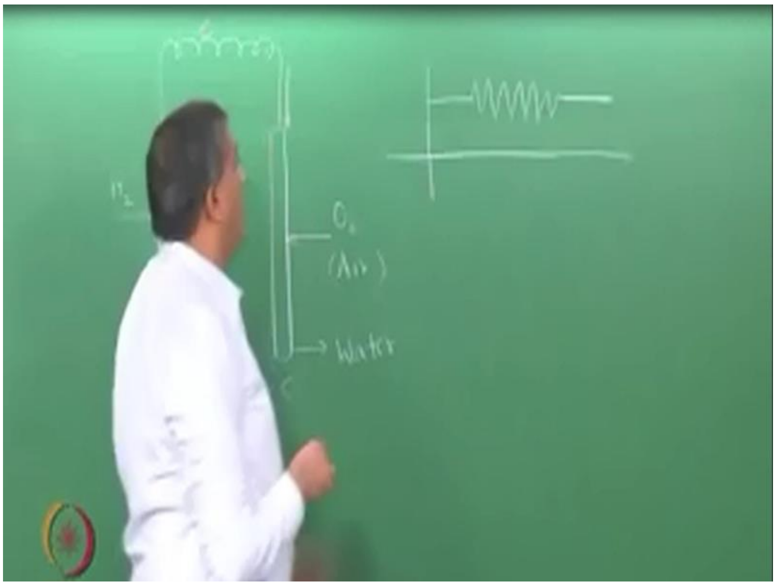


For example, if I have a signal such as this, where it is inactive for a while, and then it oscillate for a while, and then goes back to zero. Suppose this is the signal that you are analyzing, and you did not know that exactly this is what exited. Then what would happen is in if your performing a Fourier analysis or even a wavelet analysis. Well, let us talk about Fourier analysis, what Fourier assumes is that this zero; that is no activity region, is made up of several sine waves, which are the analyzing functions, adding and canceling each other, and that is not the case, because simply there is no activity. You cannot imagine that this is how the signal would have been generated. It is simply like saying zero, can be one minus one, can be two minus two, can be hundred minus hundred or zero itself. So, zero has to be treated as zero rather than imagine it to be one minus one and. So, on, and that is what happens when you have fixed basis, because you have already biased yourself, towards the certain wave the signal has been synthesized. Whereas, with the adoptive basis, you look at the signal say no, that these must have been the basis that would have generated the signal, and that is what exactly Wigner does for you.

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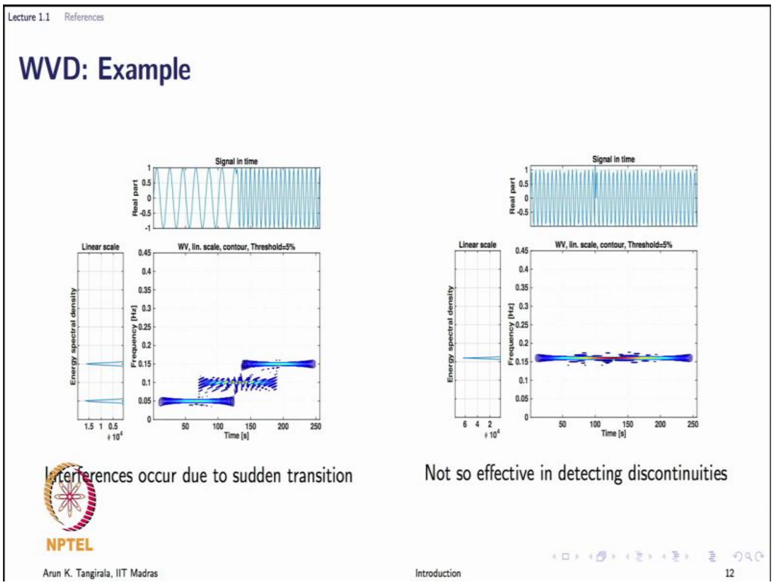


So, to give a feel of what Wigner Ville distribution does for you. Again I have the same signal here. Now, if you just compare this plot with the spectrogram, there is a very fine localization of the energy in frequency. Once again we have frequency on the y axis, and of course,, time as well,, but there is a certain middle term that is appearing, which is called the interference term, and that is one of the main drawbacks of Wigner Ville distribution. And I should tell you that here, you do not have to really set any parameters, like the way you do in spectrogram. In spectrogram you have to choose a window type window length particularly. Here there is no such choice as required. So, in that respect it superior, and Wigner Ville is superior in many other respects as well. But this interference term is the one that causes a headache. And then for the sine with impulse, you get very fine localization of the sine,, but unfortunately the location of the impulse is last. Last in the sense; of course,, you can still get it, there is a certain thresholding that I have done, that it displays only features above certain intensity, and the intensity of the impulse is. So, weak that you are unable to find it. Strictly speaking you should be able to find a contour..

So, that is not the main headache, the main headache is the presence of interference, as a results of which numerous effects of grown into improving Wigner Ville, because Wigner Ville has certain nice properties. So, there were efforts to retain some of the nice properties and also remove interferences. Now, it terms out that you cannot retain all the nice properties, you have to scarifies one for the other. And the nice properties that has to be scarifies is the localization of energy in frequency, and that is exactly what you see here. But the localization in frequency is good, the localization frequency has become worst in the previous one,, but the interferences are vanished. These interferences are the ones that have to go away, because then they introduce a lot of headache, or artificial features in signal analysis. So, that is the trade of now you can see, in time frequency analysis. On the other hand for chirps, the original Wigner Ville distribution without any modification works beautifully. It is in fact, the best suited tool for analysis of chirp like features. Now, this should tell you that you have to pick your tool based on the signal. This is the massages that you should take with you.

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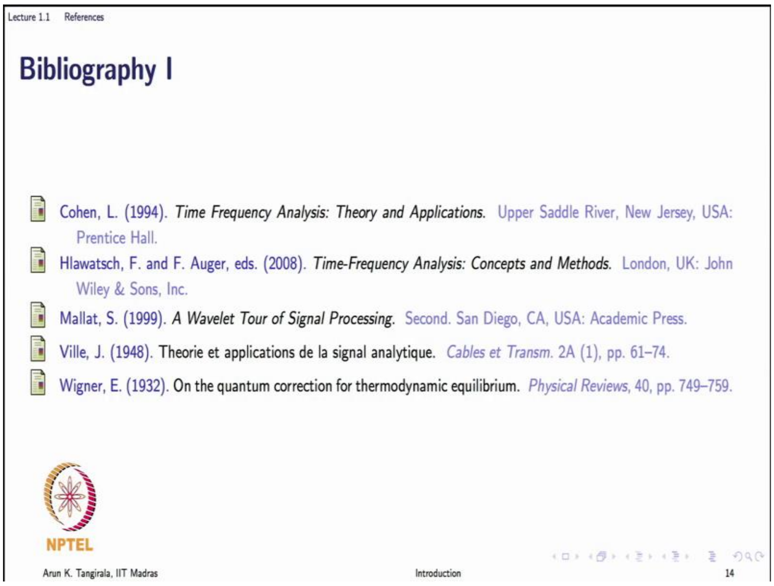


With this we will come to a close for this lecture; that is, this is part one of the introduction. Now, you see that there is this lecture is title number, lecture one point one on the left top, that is the convention that we are going to follow. The course itself is divided into several units or modules as you may call. And in each unit, you will have different certain number of lectures, and this is the introduction unit or module that you are looking at, and this is the first lecture in that unit, and therefore, we have this lecture number one point one. These are the some of the references for this part of the lecture, I will give you references for each lecture towards the end,. So, that you are able to refer to the specific references. Of course, some of these references will run through the entire course. There are some excellent researches, survey articles set have been written, tutorial set have been written. I recommend that you go through them first, before taking up any text book..

Of course, there are the books that I have sited, or also the most widely referred books.

The book by Cohens book; for example, will serve as a good reference, the book by Mallat as well, and the book edited by two gentlemen, there the book title time frequency analysis, is also going to search as a good reference for our course. And then there are some other articles somewhere we will draw some, will draw some material. Notation is something that you have to leave it notation differences. Each text book, each author would like to use as a certain notations, and Cohens is a physicist,. So, therefore, , there is a certain notation; whereas, Mallat is an engineer,. So, there is certain other notation and. So, on. And I use a notation that is different from all the text books,, but it is not two different; therefore, when you read of this text book, please look beyond the notation, and do not get what done by the notation differences. This is just a general word of advice. So, we will meet in the next lecture.

Thank you.

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