Introduction to Time-Frequency Analysis and Wavelet Transforms
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Hello friends, welcome to the introductory lecture on the course to Introduction to time frequency analysis and wavelet transforms. I hope you have gone through the introductory video, which briefly described what this course is about, what we are going to do in this and the second lecture in introduction. So, the introduction is divided into two parts. We are going to gain an overview of the course, what is this course, what is this course about and how this course is being organized and the specific topics, that we are going to talk about. We will keep the introduction almost free of the mathematics or the equations, but occasionally we may use an equation or two to illustrate the point. In the detailed lectures we will go through the mathematical part carefully.

Although this course is titled introduction to time frequency analysis and wavelet transforms, predominantly what we are going to look at are multiscale processes; that is because there is a connection between scale and frequency and time frequency analysis is basically meant for these processes called multiscale processes.

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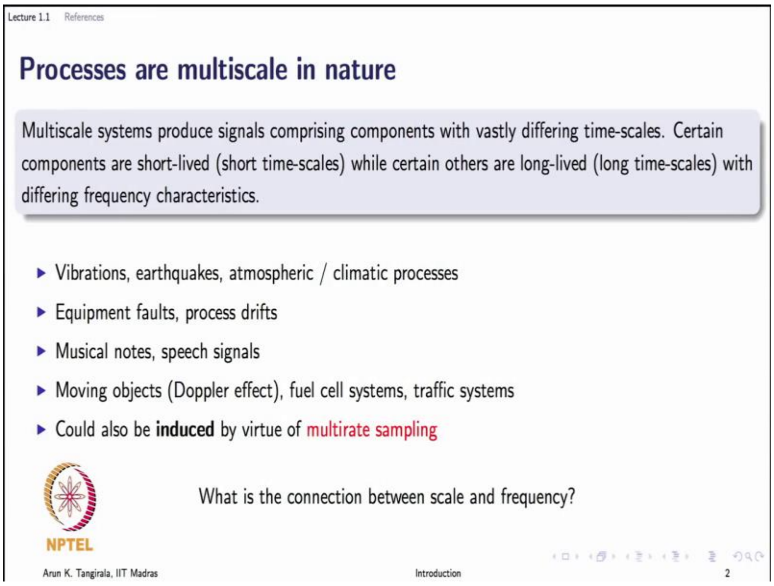

So, what are multiscale processes? Well, any process, which has phenomena that are occurring at multiple time scales is called a multiscale process. And there are many processes out there: natural, man-made, or may be even induced by virtual sampling and so on, which are multiscale in nature. There are several examples that one can give for multiscale processes, I have listed quite a few here, but we can at least talk about two or three examples.

An example that we can easily relate to is the traffic that we see on the road. If I consider the traffic system, I have vehicles driving at different speeds, may be all reaching the same destination. So some take longer time and some take shorter time. There are situations where of course, each person is going to a different destination. So, if you look, if you travel along the road, some vehicles stop driving after a while, some vehicles continues to drive and so on.

Likewise, any system, in any system you will find signals, components of signals that are short lived, that is, they only exist for a short period of time and there are certain components of signal that persist throughout the signal and so on. So, we say then, that the system or this signal has multiscale nature to it. And a classic engineering example, a modern engineering example is a fuel cell system where we have, I hope you are familiar with the fuel cell system, if not, a fuel cell is made up of two electrodes with the membranes separating the electrode. The basic idea is to electrochemically combust the fuel, which is hydrogen at one end and then, that is, at the anode with the help of a catalyst and then, the hydrogen is split there into proton and electron. The electron travels from external circuit and the proton travels through a membrane. So, let me quickly draw this schematic for you.

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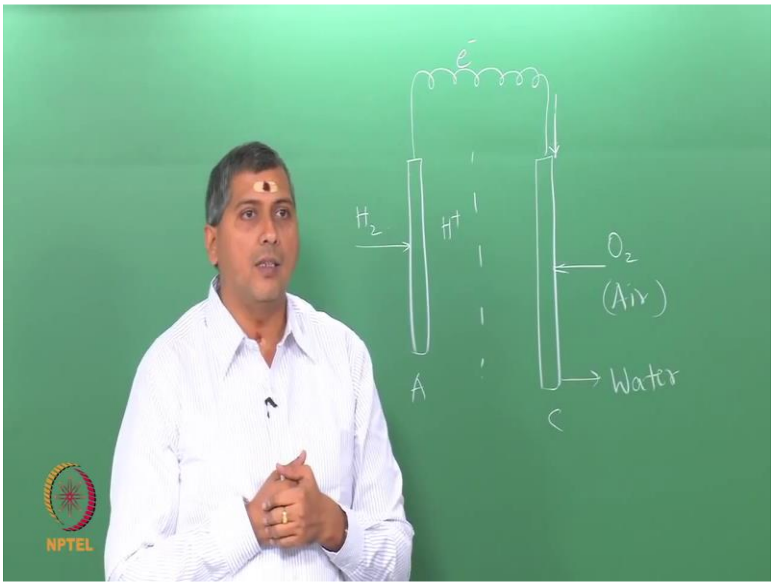

So, I have here the anode and then, I have here the cathode and then there is a membrane here. Hydrogen is supplied at the anode side and this is split into proton and electron. The membrane selectively allows only the proton to go through. And then, there is an external circuit here through which the electrons pass and reach this end here where I pump in air. Primarily, I pump in oxygen; since air is a good resource for oxygen, we pump in air. So, this electron and the proton that travels to the membrane, reach the cathode where all these three react to form water. So, in the end I get here water. Now, this is the very basic operation of what is known as a proton exchange membrane fuel cell, right.

The single cell, typically a single cell with, you can see, that the single cell is able to generate electricity by virtue of this operation. With the single cell, probably, I can generate enough power to light a bulb of may be, 50 watts or 100 watts and so on. It is not enough to really supply power to an entire household. So, what we do is, we stack several cells together depending on the requirements.

Now, coming to the point. What is happening, either in a single cell or the stack of cells are several phenomena. One, you have an electrochemical reaction occurring at the anode and then, there is a transfer of protons and of course, as well as electrons. There are this fuel supplies that are happening at a certain rate and at the highest of the macro scale. You have heat being generated, which basically raises the temperature of the cell or the stack. So, you have phenomena occurring at different scales. What we mean by scales again here, the speeds at which they are happening and they finish, that is, if you take the generation of electron through the electrochemical reaction here, it just occurs in a flash, right, in the order of milliseconds. But if you take the generation of, or the increase in temperature of the cell or the stack, it obviously, does not occur even at the range of seconds, but it can take a few minutes to hours depending on whether you are looking at a cell or a stack. So, you have a range of phenomena happening here at different time scales. So, this is another example of a multiscale system.

When I measure signals from these systems, these signals will naturally have this multiscale component of course, depending on the sampling rate at which I measure in. If the sampling rate is only commensurate with the most a macro scale signal, then you will miss out on the phenomena occurring at finer scales. We say, finer scale would mean basically finer time scales. Normally, we sample in such a way that at least a few scales are captured and therefore, any signal in practice, any system you can imagine to consist of multiple scales. So, far any signal out there in reality is a, is a multiscale signal.

And the issue is when I use traditional techniques, which work at a single scale and I tell you what we mean by single scale, then you are not really extracting the multiple scale features of the signal. So, you will only get what is known as a global feature. Of course, these are technical terms, we will be understanding these shortly. Before we move on it is important to understand, that there is a connection between scale and frequency, alright.

Now, it is not so obvious because this term multiscale is used very widely, even in computer vision, in image analysis and so on, right, where there many may not be need to even use the term frequency. Here, we use a term frequency because primarily, in this course, we are going to look at multiscale analysis of signals rather than looking at purely multiscale analysis of images, right. So, it is important to understand the connection between scale and frequency. In fact, you can find numerous examples of multiscale systems in images.

So, let us comeback and ask, what is a connection between scale and frequency? Well, this is something that we are going to elaborate more in the second part of this talk. But very quickly let me tell you, that scale and frequency are inversely related, which means, if I am looking at low scales, right, then I am looking at what are known as high frequency signals. And if I am looking at high scales, a signal at high scales, then I am essentially, searching for what are known as low frequency signals. So, this inverse relation is what is to be remembered for.

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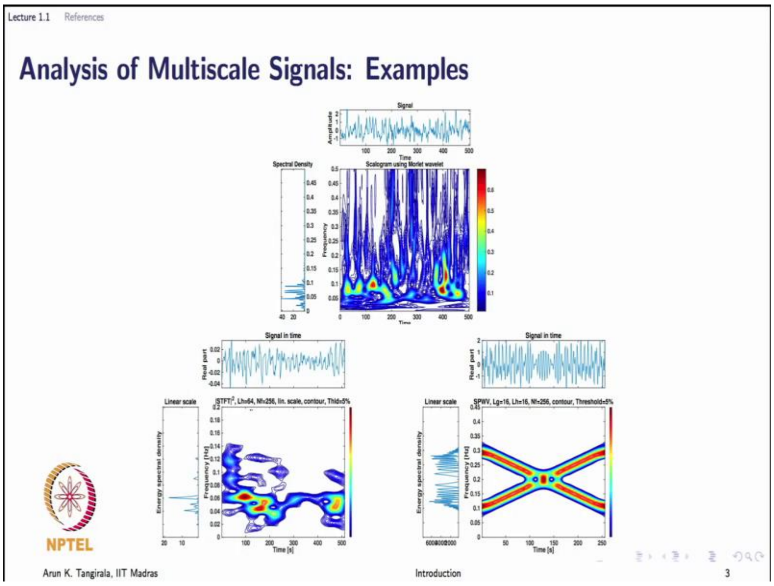

Now, when we talk about wavelets, then we will give more examples on how scale is related to frequency and so on. And then, we talk about wavelet themselves. Then, we will give a quantitative relation between scale and frequency. So to give you a feel of what is involved in analysis of multiscaling signals particularly those, that you will see in this course, I have three examples for you, which basically, which will also, which also highlight the tools, that we are going to use in this course. Primarily, the tools, that we are going to look at are the short-time Fourier transform, the Wigner Ville distribution, the wavelet transforms and just peripherally talk about empirical mode decomposition.

So, the three predominant tools that we are going to consider here are illustrated for you in the context of analysis of multiscale signals. So, what you have in each of this plots is three panels where the top is the time signal itself, that is, the time series itself and on the left panel you have the spectral density giving you the frequency content. This is obtained through Fourier analysis. And in the center you have, what is known as, the energy density in the time frequency frame.

Now, this is what is a crux of our analysis. We would like to find out how energies in the signal are, how the energy in the signal is, is, a net is coming out as a net contribution of components in different frequency bands over different time points, time intervals. So, the Fourier analysis gives me the frequency content. So, it gives me an energy decomposition or a power decomposition of the signal with respect to each frequency. So, it exactly tells me how much each frequency has contributed to the power and energy of the signal, but it has no clue as to when this frequency contributed, that is, in, at what time or over what time interval this particular frequency component existed and how much it contributed, right.

And when we are dealing with multiscale signals, we said certain components are short lived, certain components are long lived and there could be different frequencies occurring at different time and so on. Therefore, this spectral analysis is not suited for such signals. This is what we mean by saying single scale techniques are not suited for analysis of multiscale signals. So, let us look at this plot here.

The top plot, which is generated by means of wavelet analysis, this is called a Scaleogram. What does it tell me? It tells me, on the x-axis I have time and on the y-axis I have frequency. It essentially tells me, that there are, this pockets of time intervals, the red color corresponds to the maximum intensity. There are this pockets of time intervals where those frequencies are contributing maximum to the energy of the signal over that interval of course. Whereas, there are other intervals where nothing much is happening, right.

Without really going into further details the main point here is this plot here, which is called as Scaleogram, as I said earlier, tells me in how much each frequency band the components in each frequency band have contributed to the energy of the signal over a certain time interval ideally. Of course, what we want to have, what we would like to know is, at what time, what frequency component existed, but unfortunately that is not going to be possible by virtue of, what is known as, a duration bandwidth principle, which I will talk about briefly bit later.

Likewise, here I have another signal. By the way, this signal, that we are looking at the top is the, it is, it is not a synthetic signal, it is a surface, sea, surface temperature on the sea in a certain geographical region. So, by analyzing this Scaleogram, I am able to find out the periodicities and when this periodicities occurred, whether the periodicities are changing with time.

Now, coming to the left panel here at the bottom, the left figure at the bottom, I have once again three panels and I have explained to you what the top and the left panel are in each figure. What we have here the two dimensional plot. Once again, we have all these are contour plots by the colours are actually representing the intensity. This plot is generated by means of a short-time Fourier transform and it is called as spectrogram.

Once again, this tells me how much each frequency band has contributed to the, components in each frequency band have contributed to the energy of the signal over that time interval. So, we are able to obtain local information in time that is a key, that is, a keyword that you have to remember.

Now, the signal here is different from the one that you see there. This signal is coming, is out of an industrial control loop of large refinery plan and this kind of analysis helps me in figuring out whether there are oscillatory features in the signal. Of course, spectral analysis can be viewed that, but more importantly it tells me when these oscillations existed, whether the controller was fighting with it, whether it managed to suppress the oscillations or whether these oscillations really existed throughout the signal and so on, which is useful for diagnosis.

Now, the third figure that I have is a synthetic signal, but it is representative of many natural situations that you may see where you could have two sources, sound sources of increasing frequency coming in opposite directions and meeting each other. So, we have what are, what is known as a mixed chirp signal and this chirp signals are very common in many applications and the joint energy density plot that you see here. The two dimensional contour plot that you see is coming out of a smooth pseudo Wigner Ville distribution. It is a variant of the Wigner Ville distribution and once again it tells me what frequencies existed over what time intervals and it has a very nice localization ability.

The original Wigner Ville distribution is the, is the mother of these kinds of distributions, but it has certain drawbacks and therefore, some modifications are performed. And the particular modification that we are looking at is a smooth pseudo Wigner Ville distribution. Of course, when the time interval over which these two sources meet, the frequencies are very, very closely spaced, that may be your tool is unable to resolve the closeness between the two frequencies and that happens with all the tools.

So, this is to give you a feel now of the different tools that we will be using. The reason for showing different energy density plots coming out of wavelets short-time Fourier transform and Wigner Ville is to tell you, that there are many tools out there and it is not necessary, that always the wavelets transforms, which are very popular will offer the best solution. You have to remember, that each application in, for each application you have to really ask what you want and then pick the tool rather than picking the tool and then searching for an application.

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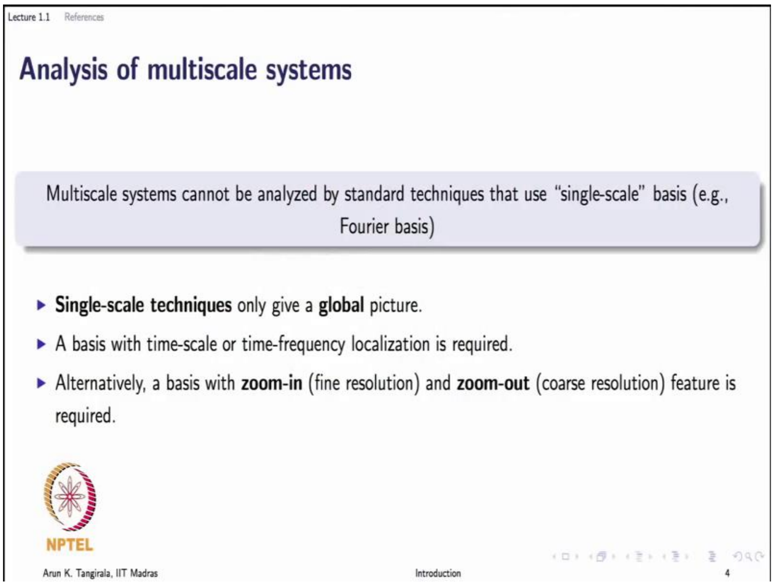

In general, so now, from this very quick brief analysis of the figures we should understand, that multiscale analysis, the multiscale signal analysis cannot be done by single scale techniques such as Fourier analysis. Why do we call the Fourier analysis a single scale techniques, I will tell you shortly. But what do we want, what can tool do we want for the analysis of multiscale signals? Well, we want a tool that is able to break up the signal into multiple time scales, that is, I should be able to zoom in.

The most important thing, that I require in the analysis of multiscale signals is to be able to localize, get the local information in both time and in frequency, whereas Fourier analysis gives me very nice local information about the frequency, that is, it will tell me exactly what frequencies are present. But it loses out on the local information in time. That is why, we say it extracts the global features. It assumes, that the components, that it is, that it is searching for their present throughout the signal, whereas the tools, that we use for multiscale analysis will not assume that. They assume, that the there are going to be components that are short lived, that are going to be long lived, that may be persistent throughout and so on.

And the main point is if you use a multiscale analysis tool for analyzing a signal, you will be able to get the global information. Of course, not as finely, as Fourier analysis, but not the other way around it. If you use a single scale technique, you will not be able to get the multiscale information. So, we say, that we need, we need what is known as a zoom in and zoom out feature of the basis functions and we will understand this better when we talk about wavelet transforms and so on, right.

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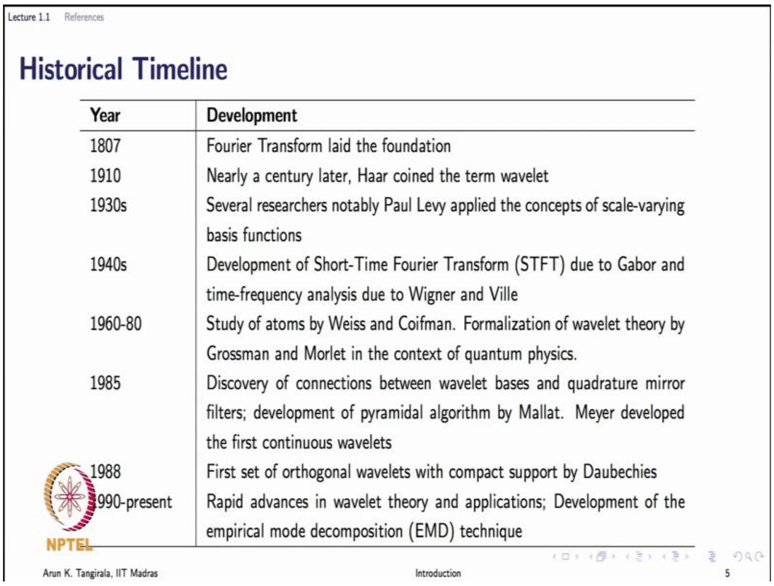

Year	Development
1807	Fourier Transform laid the foundation
1910	Nearly a century later, Haar coined the term wavelet
1930s	Several researchers notably Paul Levy applied the concepts of scale-varying basis functions
1940s	Development of Short-Time Fourier Transform (STFT) due to Gabor and time-frequency analysis due to Wigner and Ville
1960-80	Study of atoms by Weiss and Coifman. Formalization of wavelet theory by Grossman and Morlet in the context of quantum physics.
1985	Discovery of connections between wavelet bases and quadrature mirror filters; development of pyramidal algorithm by Mallat. Meyer developed the first continuous wavelets
1988	First set of orthogonal wavelets with compact support by Daubechies
1990-present	Rapid advances in wavelet theory and applications; Development of the empirical mode decomposition (EMD) technique

So, to give a quick historical timeline, the origins of signal analysis in frequency domain itself starts off from Fourier's work more than two centuries ago. And then, gradually people found out the, of course, the utility of Fourier transform. But more importantly, about 100 and, 100 or 120 years later they figured out, that there are certain limitations of Fourier transforms when it comes to the analysis of the so called multiscale systems.

And then the efforts for devising improvements to Fourier transform began and the prominent improvement is the short-time Fourier transform due to Gabor and others. And once the short-time Fourier transform was proposed, it was studied comprehensively. But then people founded there are certain limitations to short-time use of short-time Fourier transform, that there are, it is a better way of doing things for last class of signals and that is one wavelet transforms came along.

But in parallel what happened is, well, people were looking at the transform techniques for the analysis of signals. There, there was an effort to directly compute the energy without taking the transform root and that those one such effort was led by the Wigner and Ville in different contexts. Wigner was working in physics and Ville was working in signal analysis, but their efforts were completely different. They avoided the transform root because the transform root pose a certain limitations in the analysis of signals and therefore, they came up with this Wigner Ville energy distributions, which directly computes the energy unlike the transform techniques, which take the transform and then compute the energy density.

So, now what we have today are a course of techniques including, what is known as, a empirical mode decompositions, which takes a different stand point. In fact, it is closer to what Wigner Ville tries to, Wigner Ville distribution tries to perform in, in terms of what is known as an instantaneous frequency and I will talk about it very briefly later on.

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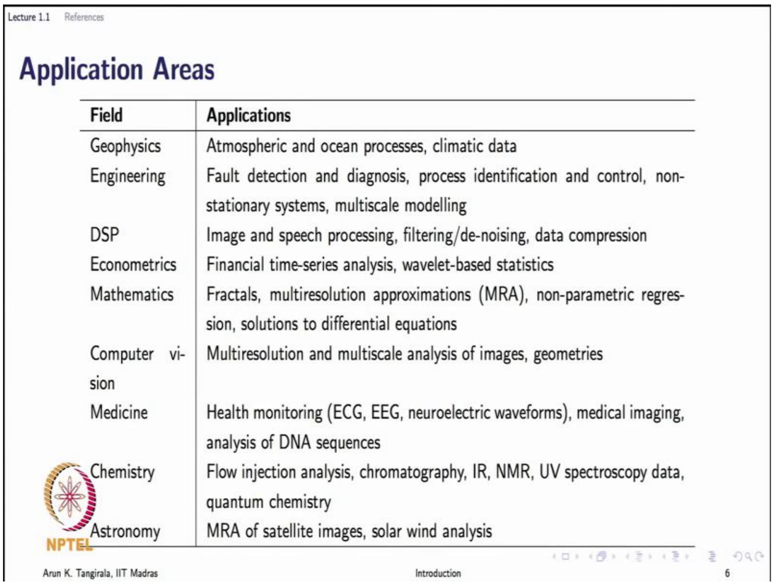

Field	Applications
Geophysics	Atmospheric and ocean processes, climatic data
Engineering	Fault detection and diagnosis, process identification and control, non-stationary systems, multiscale modelling
DSP	Image and speech processing, filtering/de-noising, data compression
Econometrics	Financial time-series analysis, wavelet-based statistics
Mathematics	Fractals, multiresolution approximations (MRA), non-parametric regression, solutions to differential equations
Computer vision	Multiresolution and multiscale analysis of images, geometries
Medicine	Health monitoring (ECG, EEG, neuroelectric waveforms), medical imaging, analysis of DNA sequences
Chemistry	Flow injection analysis, chromatography, IR, NMR, UV spectroscopy data, quantum chemistry
Astronomy	MRA of satellite images, solar wind analysis

So, there are several applications and application areas in which you will find multiscale analysis. In fact, if you look at the table it is fairly clear, that there is not a single field, that has, that is devoid of this multiscale analysis. Everywhere you need the multiscale analysis tools and defining on your area, you can pick your favorite area and go about doing a quickly literature review to appreciate the breadth of this particular topic of multiscale analysis.