Multiphase Flows: Analytical Solutions and Stability Analysis
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Lecture – 07
Eulerian and Lagrangian approaches

So, welcome to the next lecture on multi-phase flows. What I want to do is before I start, I just want to summarize what we have done so far over the last 4 lectures or so in this class and to tell you what is going to happen at least in the next 4 lectures okay and you will have some idea about what is in store for you. So, what we have done is used the fact that all of you have some basic knowledge of fluid mechanics okay.

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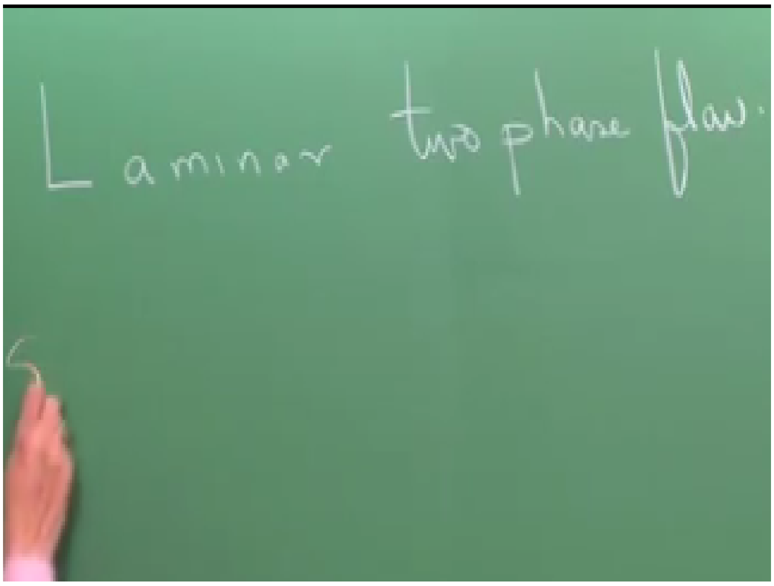

And we have used that and we did not bother to know derive the Navier stokes equations or anything, we just said that we all know fluid mechanics and then we decided to analyse a specific 2 phase flow problem, okay and the thing that we respected ourselves to is laminar 2 phase flow. Now, I just want to emphasize that as far as the entire course is concerned, we are going to mainly be looking at laminar flows.

We are not going to be looking at turbulence at all because there is a completely different ballgame, you need to analyse situations using computational intensive methods. The focus here is not so much on computationally intensive methods but on getting a good physical understanding. So, if you were interested in doing a turbulent flow analysis of a 2 phase flow problem.

What you would be doing is looking at doing direct numerical simulations, doing computational fluid dynamics okay, so we are not going to be doing all that. So, the reason I am telling you this is tomorrow, when you are reading papers, you will see that there are people who are doing fluid mechanics, who are using those kinds of principles, you will see people who are not using CFD but they are using the kind of methods that we are talking about.

So, but the idea is for you to know the differences and see if we can learn from one technique, one approach and how we can apply it to another. Similarly, if you are doing turbulence, how I can use ideas from that method to this, so that is something which you should be clear about and that is something which not only as far as this course is concerned for anything that you are doing.

If you are going to be looking at; working on a particular problem, you are working on an area, which is, maybe reaction engineering or fluid mechanics whatever, there would be so many things in the same area, you should be able to know what the similarities are and dissimilarities are, between what you are doing and what somebody else is doing because that is the only way you can make a more clear in your concepts okay.

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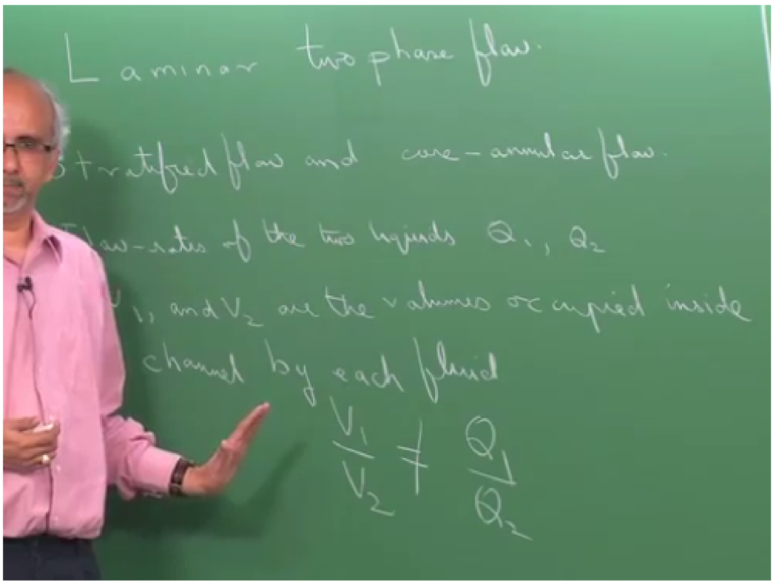

So, what I am trying to tell you now is that what we are doing is mainly non CFD approach okay but then that gives you a lot of insight tomorrow, when you want to do computational fluid dynamics, so that is the way you have to take this course okay. So, what we did is; we looked at stratified flow and towards the end, we were to do the core annular flow and we want you to basically work out a couple of problems.

The idea is normally, what you would think, when I am pumping 2 different liquids in a tube, if I pump 2 liquids with; let us say the flow rates of the 2 liquids is Q1 and Q2, the volume fraction, which is going to be occupied by each of the liquids okay, it is not going to be given by the ratio of Q1/Q2 or the volume ratio okay. So, that is one of the things which we wanted you to understand by the 2 examples that we did; the satisfied flow example as well as the core annual flow example, okay.

So, that is the reason why we wanted you to calculate the velocity profile, we wanted you to calculate the holdup, the holdup is the volume ratio. So, let us say V1 and V2 are the volumes occupied inside the channel by each fluid. We want to you to be clear that V1/V2 is != Q1/Q2 and that would be something, which you normally think. I am pumping in 2 liquids with the same flow rate, so you would expect the half of the tube is occupied by one fluid, half by the other.

But those of you who have done the assignment will realize that this is going to be decided by the ratio of the viscosities okay and the reason why the other parameters do not come into the picture is because we are assuming that we are in the low Reynolds number regime, so that the density does not show up okay. So, that is one thing, which I wanted you take from the lectures that we have seen so far okay.

Now, what we want to do is; we want to do things in a slightly more formal setting, we do not extend, we assume that the interface is not changing in size, in shape as well as the position is concerned. So, we want to get to the position, where we are able to incorporate the changes of the interface. So, to make this entire course self-contained, we have to go back a little bit and I am going to derive.

In the next 3 or 4 lectures, we will do some theory and then we will do some problems. So, the theory is going to basically focus on deriving the Euler's acceleration formula, which have already seen, which then we talked about the Reynolds transport theorem, which is basically a conservation principle okay and then we will see how we can in generalise some of the boundary conditions that you saw earlier regarding the pressure jump across a curved interface.

When you have a curved interface in a core annular flow, the surface tension basically, it helps you; you know evaluate what the change in the pressure is; you need to know the curvature, you need to know the surface tension and you can estimate what the pressure difference is. So, now what we want to do is; we want to generalize that to the case where there is flow, so that is what we are going to do and then we start working out some problems okay.

So, next 3, 4 lectures are going to be basically on trying to establish the thing on a firm theoretical footing. Some of it may be repetitive to what you already see but it is always good to; you know revise and revisit some things and then go forward.

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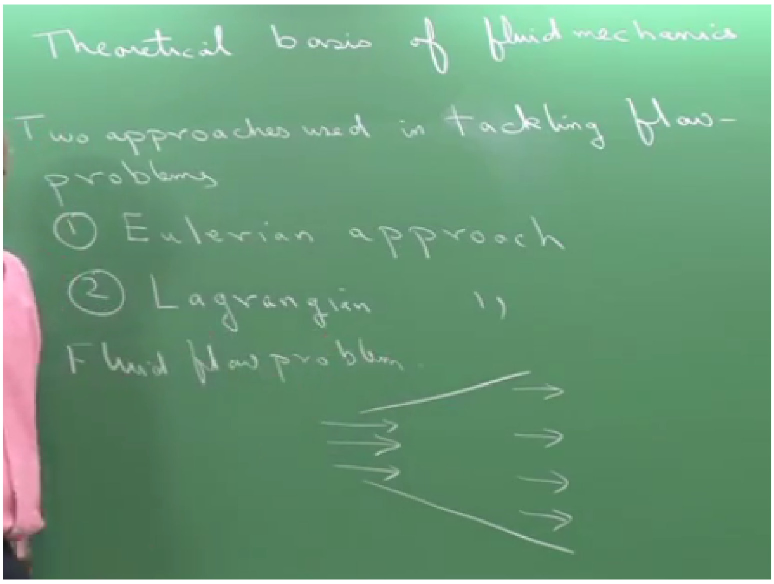

So, let us look at the theoretical basis of; let us say a fluid mechanics okay, not being very careful with the English here but it is okay. So, normally 2 approaches; you are already familiar with okay, which you have seen in the past and the 2 approaches in tackling flow problems. The first one is the Eulerian approach and the second one is the Lagrangian approach okay. So, what is the difference between these 2 approaches?

What we do in the Eulerian approach is; we are focusing our attention on a fixed region in space okay and we are trying to understand how things; parameters like velocity temperature etc change in that fixed region in space. So, now this region in space can be very infinitesimal, it can be very small, it could be just a point and supposing you actually were to measure temperature at a particular point (()) (9:32).

It is going to measure change in temperature, so you get a temperature as a function of time but that is at a fixed position. So, what you are doing is; you are basically using an Eulerian approach, whereas the Lagrangian approach is one where what we are doing is; we are not looking at a fixed region in space but we are looking at a fixed particle or a fixed collection of molecules or a fixed molecule.

Because it is a fluid flow problem, the molecule is going to move, so in order for you to be able to track the change in the temperature, you need to be able to have a probe, which is tagged on to the molecule okay. If you have a probe, if you have a thermocouple which is able to sense the temperature of the molecule, as the molecule moves, you will be able to actually measure the temperature, so then you are actually doing a Lagrangian approach okay.

But now, what is happening is; this probe is going to occupy different regions in space but it is tracking the same molecule, okay or same particle. So, there is a difference between these 2 approaches. Now, the reason why we need to be clear about this is because most of your fundamental laws that we have come across earlier like your conservation of mass, conservation of energy, you have; are used to those laws in the framework of the Lagrangian approach okay.

Whereas, because when you are talk about mass cannot be created, mass cannot be destroyed, you are talking about you know, this chalk piece; same set of molecules is going to remain, so you are talking about a fixed set of particles fixed, you know object. Whereas, what you are talking about here is fluid mechanics but things are going to be flowing okay, it does not really; and what you are interested in this, how is the velocity in a particular point.

So, you need to understand the difference between these 2 approaches because what we want to do is; we want to see how we can extend the fundamental laws like Newton's law of motion, where you say force is equals, the rate of change of momentum okay. We want to see how I; because that is for a fixed particle, we want to see how I can extend that to a fixed region in space because that is what we are interested in.

We want to find out rate of change of momentum and how that depends upon the forces etc. in a fixed region in space okay. In fluid mechanics, since you have a fluid flow problem, we are; so, normally what happens here is in a fluid flow problem, you know I am just put a diverging channel because I am going to use this later on, you will have, you know let us say fluid moving from left to right.

If you focus on a particular point here, this particular point is going to be occupied by different molecules at different times because there is a continuous flow. So, if you want to measure the temperature here, you will be measuring the temperature of different molecules but at the fixed point okay.

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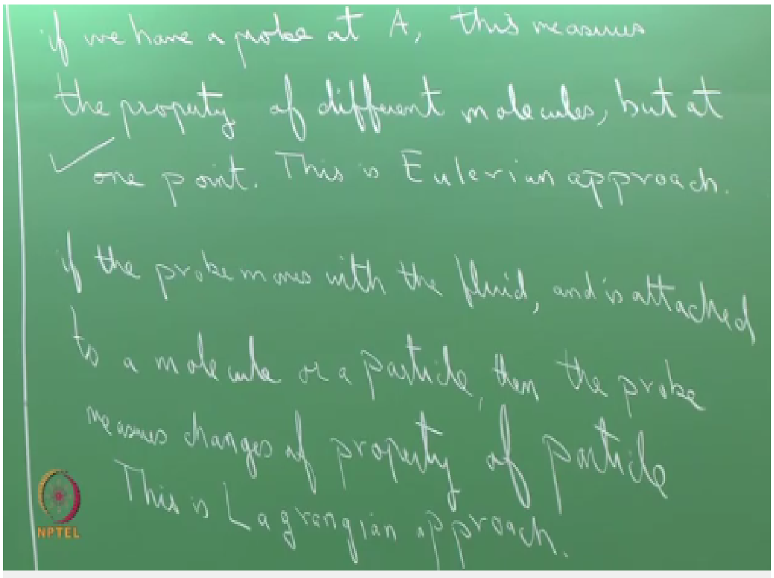

So, if we have a probe at; let us say A, okay, this measures the property of different molecules but at one point. So, because the probe is here, my thermocouple is right here okay because of a fluid flow, it measures the average temperature of all these particles there. Now, this is; what approach is this, this is the Eulerian approach and this is what as an experimentalist, you would be doing.

You will be measuring at a fixed point in your reactor or your heat exchanger or whatever it is, whereas, if the probe moves with the fluid and is attached to a molecule okay or a particle okay, then the probe measures changes of the property of the particle, okay, this moves around in the fluid and this is your Lagrangian approach and the fact is; we are really not interested in what is going to happen to the temperature of the particle.

So, if you want to have a probe here, stuck to this particle, maybe some kind of a radioactive dye or something and you want to try to find out how the concentration is changing, it moves. We really want to find out what is happening inside my system, once it goes outside my system, it may not be of interest to us, so there is no point in trying to talk about changes of the material particle.

Because you are only interested in what is happening inside your heat exchanger, distillation column, reactor whatever it is okay. So, we need to therefore go ahead to using only the Eulerian approach okay but what is so special about the Eulerian approach, the fact that the repeat; most of the laws; conservation laws are in the framework of the Lagrangian approach and we want to know how to relate changes in the Lagrangian approach to changes in the Eulerian approach.

That is the idea okay and once we know; once we are able to do that then, we can go back to solving the problems. In fact, we have already seen this but maybe in a different context or in a different framework, so we will just try to relate that.

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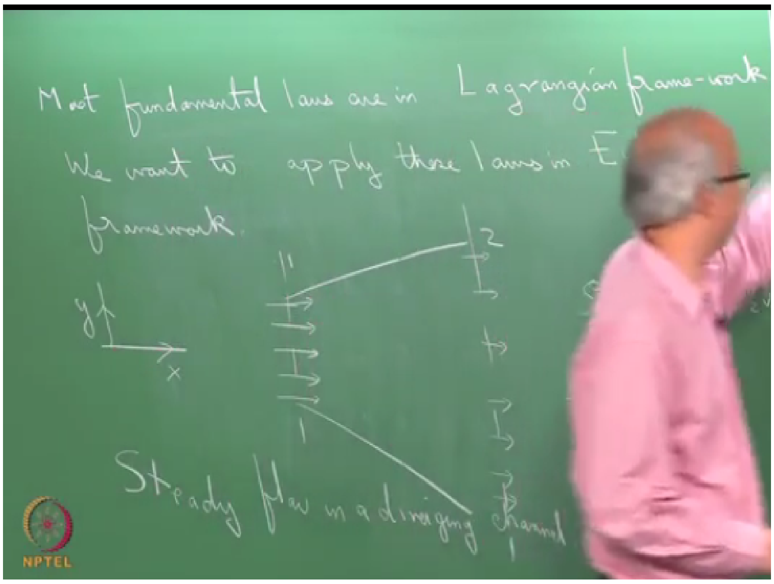

So, just to summarize most fundamental laws are in the Lagrangian framework okay and we want to apply these laws in the Eulerian framework. Now, I will go back to this problem of flow in a diverging channel okay to basically point out what the difference is between these 2 approaches okay, just to clarify things. So, let us go back to this problem in a diverging channel, let us look at steady flow.

And to make my life simple what I am going to do is; I am going to assume that the flow is uniform across the cross section okay, so this is a rectangular channel which is extending to infinity outside the plane of my board and just to illustrate the idea okay, so I am going to say and I am going to push slightly smaller arrows here, so the length of the arrow basically is reflecting the magnitude of the velocity okay.

And what I am trying to tell you here is that the magnitude of the velocity is uniform here, so the velocity is not changing in the directions perpendicular to the flow. So, if this is my flow direction in x and this is y, direction does not change in the y direction; the velocity does not change in the y direction everywhere. So, since the channel is diverging, one of the things we do expect is that the velocity is going to decrease, as you go along the flow, okay.

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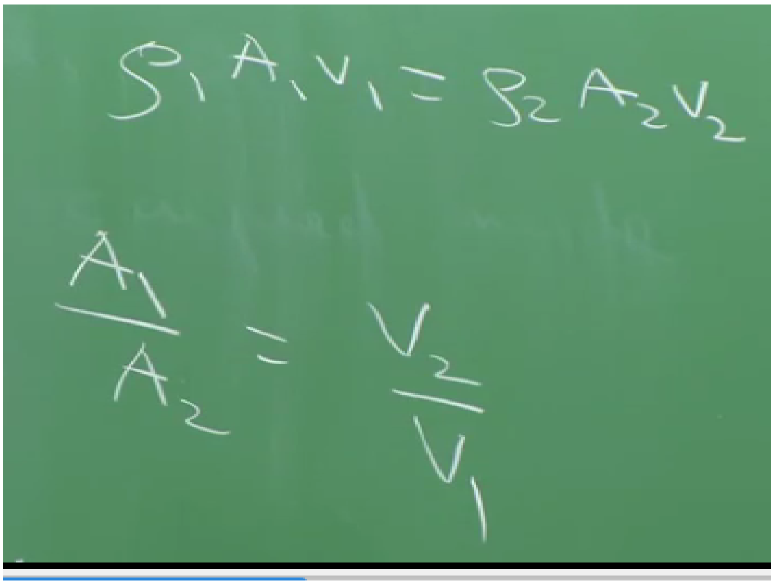

So, you have a uniform velocity here, you have uniform here but the value of the velocity here is going to be different from the value of the velocity here. So, if you want to call this point; this section 1 and this section 2 and if you are talking about liquids, then we all know from the macroscopic continuity equation that rho 1 A1 v1 equals rho 2 A2 v2 okay and we are talking about liquids, the densities do not change, you will have A1/A2 equals v2/v1.

So, if A1 is lower than A2, v2 has to be lower than v1 and that is the basically is going to decelerate. So, now I am going to ask you the following question. Supposing, I were to have a probe which is going to measure the velocity at this point okay, you keep your pitot tube or you have some device by which you are trying to measure the velocity and if you are going to measure the velocity here, what would be the value of the velocity which your probe is going to measure.

It is going to be V1 and this V1 is not going to change with time, right because I am basically assuming that my flow is steady; steady means at a fixed point in space, the V1 does not change with time okay so as far as my fixed probe is concerned.

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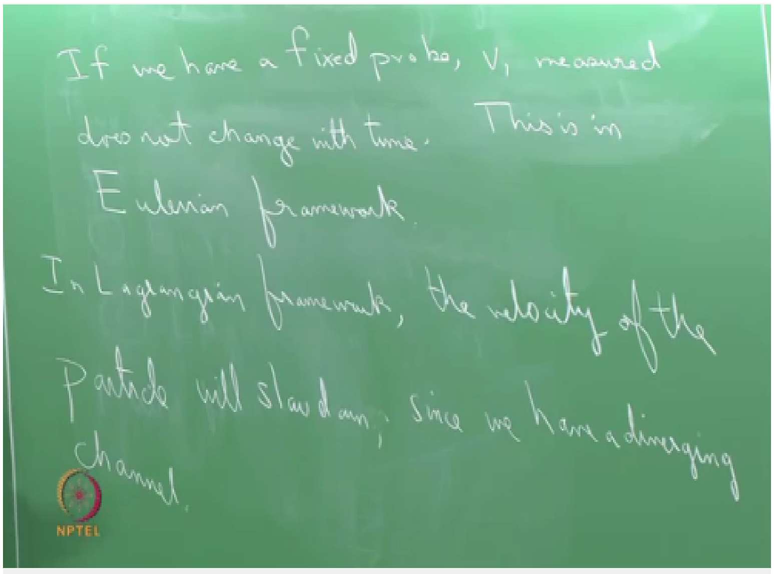

So, if we have a fixed probe v1 measured does not change with time and this is my Eulerian approach that is what it will tell us right, this is in the Eulerian framework but now what is happening to the actual liquid particles, which is moving from here to here? Liquid particles supposing, you were to actually track a liquid molecule, it has a high velocity here, it has a low velocity here. So, now the molecule is actually decelerating, okay.

So, the particle per say is having a change in the velocity, so if you want to actually sit to a particular part; sit on a particular particle, you will find that you are slowing down. Clearly because v2 is lower than v1, okay. So, in the Lagrangian framework, the velocity of the particle will slow down okay, since yeah; we have a diverging channel okay. So, it may appear; do you like, there is some kind of an inconsistency.

You are looking at a particular point, things are steady, things are not changing and whereas, if you were to actually go but the idea is that you are using 2 different frameworks, 2 different reference frames for doing your analysis, so depending upon the frame of analysis that you are working in, you will have 2 different observations. So, what we want to do is; we want to see if we can actually relate these 2 things, okay.

And that is basically our thing and we are going to extend this to a macroscopic region and that is what Reynolds transport theorem is. So, first we will do it for a very simple case, which is for an infinitesimal.

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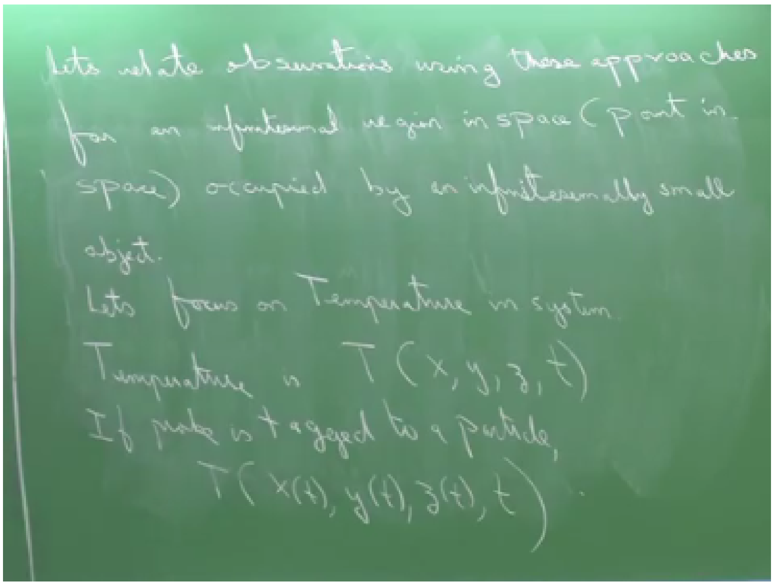

So, let us relate the observations using these approaches just for and infinitesimal region in space. Basically, what I mean is point in space occupied by an infinitesimally small object, okay. So, basically I am not talking about; I am talking about points, I am just trying to make relate changes in points to changes in infinitesimally small particles that is the idea, okay. Now, just you are trying to make the things clear, let us talk about a scalar quantity like temperature.

So, now we have temperature is something which you can measure using a thermocouple okay. So, now let us focus on the temperature in the system. So, now, if clearly in the most general situation, the temperature in that particular system is going to vary with the spatial coordinates; x, y, z okay and also time, if you have an unsteady state situation okay. So, now temperature is can be written as temperature of x, y, z, and t okay.

This is the most general variation of temperature that you can see, spatially as well as temporally okay. Suppose, have your probe, suppose you can think of an ingenious probe, which is tagged onto a particular particle okay and it is moving around the fluid, this particular probe is going to change its position, it is going to move around, right. So, what this means, the spatial coordinates x y and z are not fixed anymore they are also going to be changing.

Because now, if you want to use a Lagrangian approach, if you were to actually try to change to find out how the temperature is changing of this particle, you are going to measure the temperatures change not only as a function of time but also as a function of position okay. So, in the Lagrangian approach, if the probe is tagged to a particle, the temperature that is measured is going to be depending on x y and z with change with time.

Because the particle keeps moving and it also has retains this explicit dependency on time, okay. So, what we want to do is; we want to basically talk in terms of rates of change of the temperature that you are going to observe from your reading, okay. If you have a temperature probe, which is giving you data signals, you will be getting temperature as a function of time and position, okay but of the same particle.

So, this particular change of temperature with time; the rate of change of temperature with time is going to be what is called a material derivative because I am focusing on a single particle okay.

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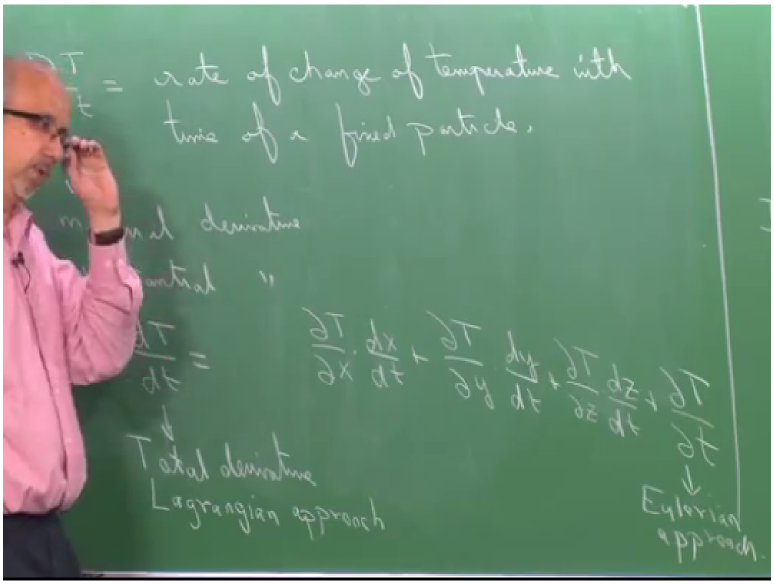

So, DT/ Dt is nothing but the total derivative, so this is rate of change of temperature with time of a fixed particle okay and I am using this particular symbol capital D, you will hear this kind of nomenclature, we were reading books, some people call this material derivative, some people call it substantial derivative, okay and this is basically taking into account the changes of temperature with respect to both position and time okay.

So, in this framework DT/Dt is nothing but DT/Dt, which is the total derivative, in mathematics you have come across a partial derivative, total derivative okay. So, this is the total derivative, I am just going to go back to calculus and establish this relationship between Eulerian and Lagrangian frameworks okay without really complicating in the issue too much. This tells me how for the particle, the temperature changes with time.

And now, I need only differentiate that expression, this functional dependency okay and keep in mind that the spatial position also are the functions of time. So, now when I want to differentiate this thing, find the total derivative, I will get DT/Dt as partial derivative of T with respect to x times dx/dt + dT/dy times dy/dt + dT/dz multiplied by dz/dt + dT/dt, okay. So, all I have done is I have just differentiated that expression and I am differentiating that with respect to time.

And I am telling you that I am incorporating the dependency of x on time explicitly, to take into account for the fact that the spatial position is changing with time and when you are actually using the Eulerian frame of reference, what are you doing? You are fixing yourself at a point and you are trying to measure the rate of change of temperature at a fixed point okay and that is the partial derivative with respect to time of the object that you are interested in, which in this case is temperature, okay.

So, what I am trying to tell you is that this last term here represents the Eulerian derivative and that is how; that is the rate of change of temperature with the fixed probe would measure, this is the rate of change of temperature, which are variable probe which is moving with a particular particle is going to measure and basically, this equation which is nothing but we just come from calculus okay, which is a relationship between total derivative and partial derivative is basically the relationship between my Lagrangian framework and my Eulerian framework okay.

So, what I am trying to tell you here is that this total derivative is what I measured in the Lagrangian approach and this is what I measure in the Eulerian approach and all I have done is just use calculus nothing else okay.

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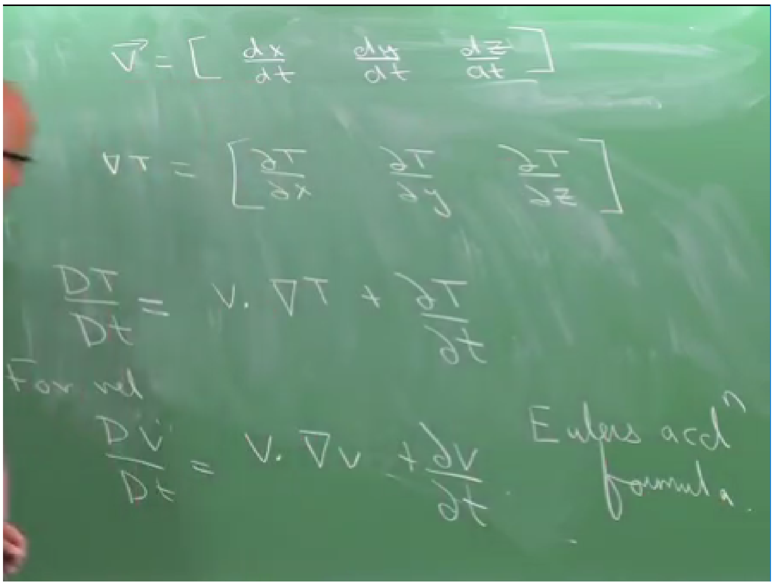

So, now what I would like to do is; write this in a slightly more general form, which is this velocity vector is nothing but dx/dt, dy/dt and dz/dt, right, the 3 components of velocity and I can write gradient of temperature as dT/dx, this is also a vector, dT/dy, dT/dz, I am going to write these first 3 terms on the right hand side in a compact way as V dot del T okay and what that relationship gives me now is DT/Dt equals V dot del T + dT/dt, okay.

So, this is the Lagrangian derivative or the substantial derivative or the material derivative, how does temperature of a particular particle or a particular material change with time, this is how does temperature at a fixed point changes with time okay and the relationship between these 2 is actually given by this, okay and I can write this for any quantity that I want. In fact, I can write this for velocity.

I can write this for velocity, this becomes Dv/Dt equals V dot del v + dv/dt, I did not do this for velocity at the beginning because velocity is a vector and then some of you may not have been comfortable with gradient of a vector, so I just did it for temperature and having done it for temperature, I have just written it for velocity okay. We will see how gradients or vectors are defined later on in the course.

And this is something, which we are all familiar with and this is called the Eulers acceleration formula okay. Now, going back to this problem of diverging channel, where we spoke about the fact that the velocity is actually going to decrease as the particle moves, one of the things we want to do is; we want to see how we can apply the acceleration formula here to that particular system okay.

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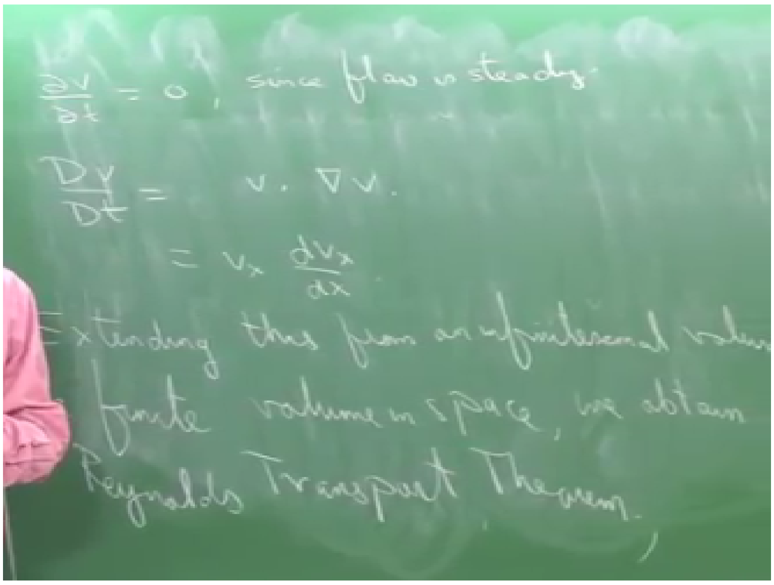

And our flow was steady, so our flow is steady. What that means is the partial derivative with respect to time is going to be 0, okay, so this is 0, so dv/dt is 0, since flow is steady and what about Dv/Dt? Dv/Dt is the rate of change of velocity of a particle and we know that it is not 0 because it is decelerating, the particle is actually slowing down, so this is not 0 but what is the value of this Dv/ Dt, it is v dot del v.

And if you; for a very simple system that we have, where we have velocity only in one direction in the flow direction vx, which is actually changing in x direction, if you want to evaluate this expression, this is going to be given by vx dvx/dx, so that tells you how the acceleration is for the particle okay and you can find out the; if you know how the area is changing, you can actually calculate how this velocity is changing with x and you can actually calculate this.

So, the deceleration of the particle, so I just wanted to illustrate clearly that this is the relationship between the Lagrangian approach and the Eulerian approach, this is the time derivative which you measure at a fixed point, which is Eulerian framework. This is the time derivative you measure when you are tracking a particle and this is how they are related okay. What I have done here is for a small at a particular point.

I am just assuming that temperature is a function of x, y, z, I am trying to relate changes of temperature at a point to it changes of temperature occupied by a molecule at that point, an infinitesimal mass, infinitesimal volume, our objective next is to generalize this to a macroscopic system okay. So, the same conservation; so when I do this, when I actually try to relate the Eulerian and the Lagrangian frameworks for a macroscopic region, I get what is called the Reynolds transport theorem okay.

So, extending this from an infinitesimal volume to a finite volume in space, we obtain the Reynolds transport theorem or I will do is; tomorrow and the next class is talk about the derivation of the Reynolds transport theorem, try and show to you that we have already seen the Reynolds transport theorem in a different form, it is just that we are having a slightly different way of looking at the problem, okay.

So, you all seen Reynolds transport theorem in some other avatar, we are going to see this in new avatar now, then we will apply it to deriving the continuity equation, the Navier stroke equation that you are all familiar with which you have been using. So, once that is established then we go back to the formulating the boundary conditions and stuff like that but let me, since I do seem to have a little bit of time, let me just add a few more things.

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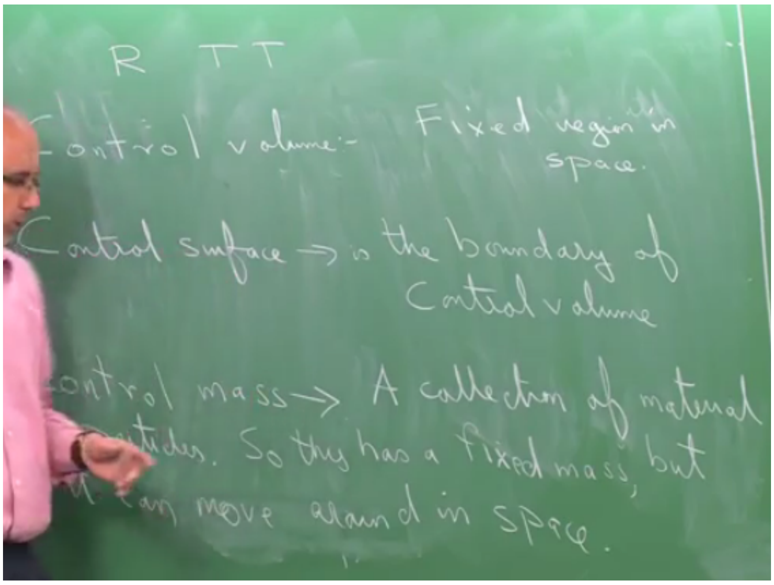

So that I can do things in a more relaxed way tomorrow, so just to; I am just going to call this RTT; Reynolds transport theorem. I want to just introduce 3 concepts, which are very simple and so makes my life easy tomorrow, control volume and control volume is nothing but a fixed region in space okay, it can be of any arbitrary step but it is not changing in the fixed region. It is analogous to my infinitesimal point, okay.

This is space; the boundary of this control volume, which is going to actually demarcate the volume from the environment, is what I call the control surface, okay. Surface is the boundary of the control volume okay and then I am going to talk about this other thing, which is control mass; control mass is a collection of particles, collection of molecules, which have the same mass, okay.

So, this is a collection of material particles, hence, so this has a fixed mass but it can move around in space okay. So, just to tell you what the analogues are what we have just seen, this is have a fixed point okay that is the point, when you are looking at a fixed volume, you are doing Eulerian approach, when you are looking; tracking a control mass which is moving around, you are actually looking at the Lagrangian approach, okay.

So, the mass can change shape okay, it can change size but then it is not a fixed region in space, it gets moving around with the flow. So, what we want to do is; we want to try and extend this particular relationship that we have got for an infinitesimal particle okay, for occupying an infinitesimal region in space to a finite size control volume and that is going to be the topic tomorrow and that would basically be the basis.

If you understand Reynolds transport theorem, if you can actually apply Reynolds, what you would be doing as an engineer is you would be applying Reynolds transport theorem to different systems and depending on the system, the final form of the Reynolds transport theorem is going to be different, so what we will do is just give you the general form of the Reynolds transport theorem and then depending on the particular system, you are going to be analysing, you will be looking at different final versions of the Reynolds transport theorem.