

Multiphase Flows: Analytical Solutions and Stability Analysis
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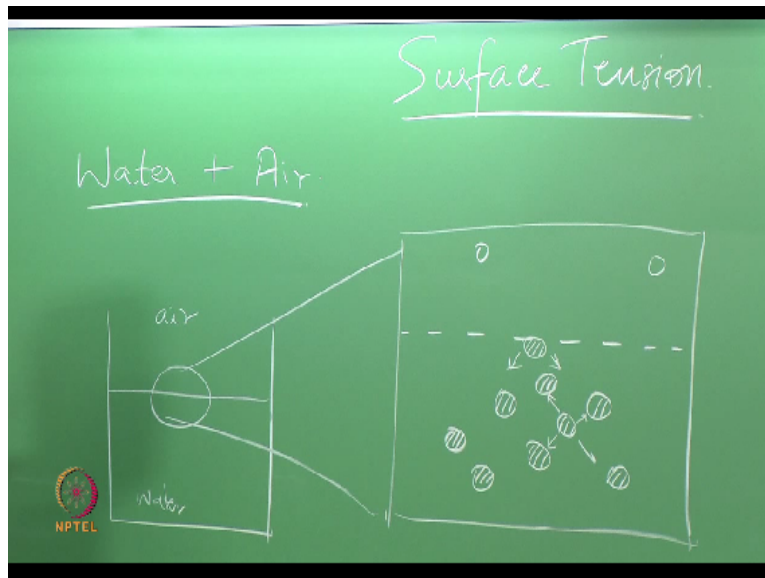
Lecture – 06
Interfacial Tension and its Role in Multiphase flows

So welcome to the 6th lecture of multiphase systems and today we will be looking at a very interesting feature of 2-face flows and multiphase flows in general which is what happens at the interface between the 2 fluids. So the interesting thing about fluid mechanics is that it is a subject that transcends entirely different scales. So essentially a fluid is made up of molecules. So you have phenomena going on the molecular level.

Then we are mostly interested in flows that happen on our scale which means micro to scale of a few meters. So on that scale you can sort of forget about the molecules and look at the fluid as a single condensed matter and that is where continuum fluid mechanics comes in which is what most of us work with and what the Navier-Stokes equations are about and then of course those equations themselves behave differently on very large scales which is on planetary geophysical flows and even astronomical flows where things again, you still use the same equations.

But again you are looking at different scales. Now in 2-face flows, the interesting thing is that even though you may be on a macroscopic scale of, may be micrometers or meters, you still cannot avoid confronting molecular forces and that is because when you look at what is going on at the interface between the 2 fluids, you are actually forced to deal with the fact that the molecules and the interaction between those molecules are different.

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So for example if you look at the classic 2-face system, water and air, so water and air is a 2-face system that we deal with constantly. So even if you have a small beaker and you have got some water here and you have air on the top and I were to, so there is some sort of difference between the 2 that you can clearly make out if you look at it from some distance and if I zoom in at what is going on over here, at the interface between the water and the air.

I will see an interesting principal where for, I mean things are much more dynamic and complicated than what I will draw here but just to give you the picture. You have some water molecules which I will sort of shade and somewhere at the interface, that again is not a straight line because of the molecular level, you will just have more water here and suddenly you will have more air on top.

But the point is that here that are some air molecules at much larger distance from each other. So what happens to this molecular water that is exactly at the surface. That is the question? So if you look at the water inside, inside the bulk, say this guy, you will realise that it is being tugged around by intermolecular forces in different directions and in fact, the way it is arranged inside so that it will experience forces in all directions and they sort of cancel each other out and they will keep bouncing around and so.

But the idea is that it gets tugged in many different directions and it sits nicely inside the bulk of

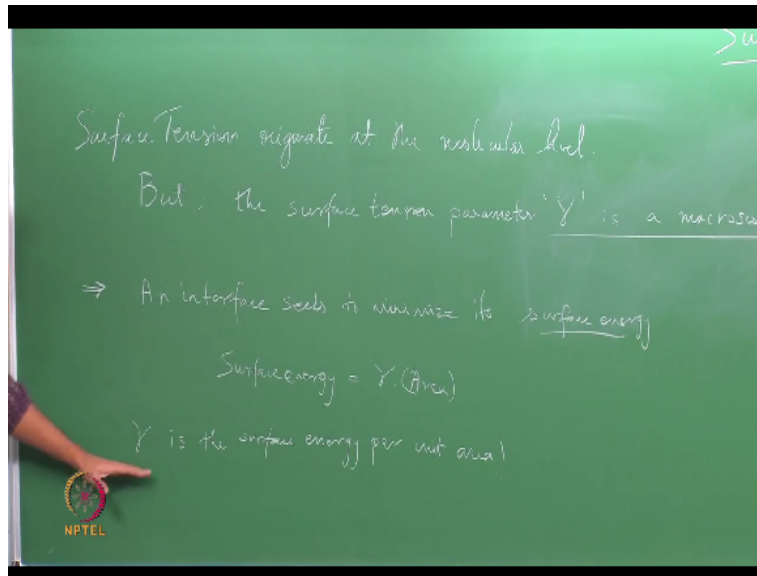
the fluid. On the other hand, the molecules that are on the surface only get tugged downwards in to the fluid because on top, there is nothing to pull it out. In the case of air, there are very few air molecules. So it hardly sees anything on the other side but behind it, it still has a lot of water molecules pulling it in different directions.

The same thing would apply if he even had a water-oil interface because now even though you have, it is a liquid here, that could be the liquid with different molecular properties and so the water-water interactions are different from the water-oil interactions. In this case, water-air interactions. So what happens is that all these molecules that are sitting on the interface feel a differential pull towards the bulk of its own liquid and that sort of is the molecular origin for what we call broadly surface tension and that is what we will be talking about today in detail.

So this is a sort of heuristic picture of why surface tension exists and how we can picture it at the molecular level. But now if you want to actually understand why the water assumes a flat configuration or if I take the same glass and threw it into the air, why the water will form drops and why does not it form triangles or why even if I throw it, it will not form a single big drop. It forms many small drops and so on.

So to understand these types of interface configurations, we need to couple the equations of fluid mechanics for the fluid that is water and air and as well as having equation to describe how the interface behaves. That is to account for surface tension at the interface. So we will see how that is traditionally done today.

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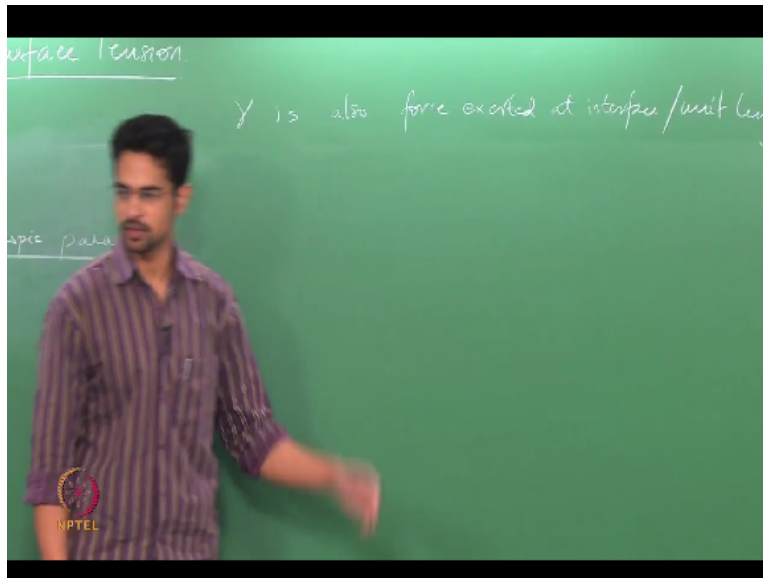


So essentially surface tension originates at the molecular level but the surface tension parameter itself which we use in our equations that is a macroscopic parameter and this is the key idea here that this surface tension that is used in our calculation when we want to understand how water becomes like a drop or what is the configuration of stratified flow through a micro channel or why the interface is flat and when is it not flat.

We use this parameter surface tension very often and that although it is coming from the molecular scale, it is actually defined only at the macroscopic level and the idea for its definition originates from the very simple idea that you would see at the molecular level as well that basically an interface seeks to minimise its surface energy and this surface energy is given by the surface tension parameter times the area and that is essentially how surface tension can be defined which is the surface tension therefore becomes...

So gamma, which is the surface tension, is basically the surface energy per unit area of this 2-face interface. So basically as scientist we see that these interfaces keep trying to minimise their area in certain situations so that is why when you take a drop in even vacuum, it will just form a sphere immediately that is because that has actually minimised its surface area. So it is continually seeking to minimise the surface energy locally and that is how we can characterise that behaviour using surface tension.

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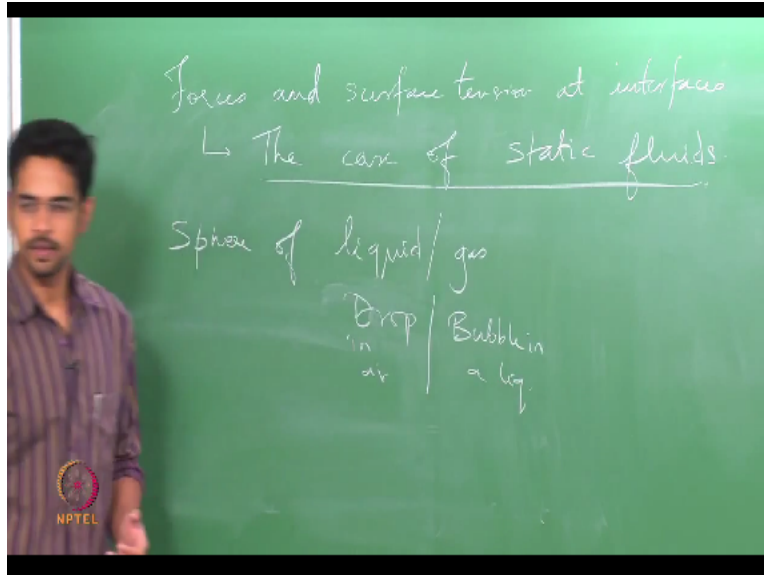


An alternative way of looking at surface tension is that surface tension is also the force exerted at the interface per unit length and that you can see works well with this definition as well. So it is energy per unit area and energy is nothing but the force into displacement. So it is force per unit length. The exact nature of surface tension and whether it depends on the fluid motion itself or whether it is purely thermodynamic property depending only on temperature and whether it depends on pressure and so on.

These types of questions are still being investigated where you need to go to molecular level and see actually what is going on but if you want to describe some simple problems at the macroscopic scale, then what we do is just work with surface tension as a parameter, like how I just laid it out that is just the surface energy per unit area and that can be measured in different ways.

And once we have this parameter gamma, we can then incorporate it into our equations and do our calculations. So while it works well for us and that is what we will be doing throughout this course, is good to bear in mind that surface tension is still an extremely interesting and open area of research. So now we will get down to looking at forces at interfaces which have surface tension.

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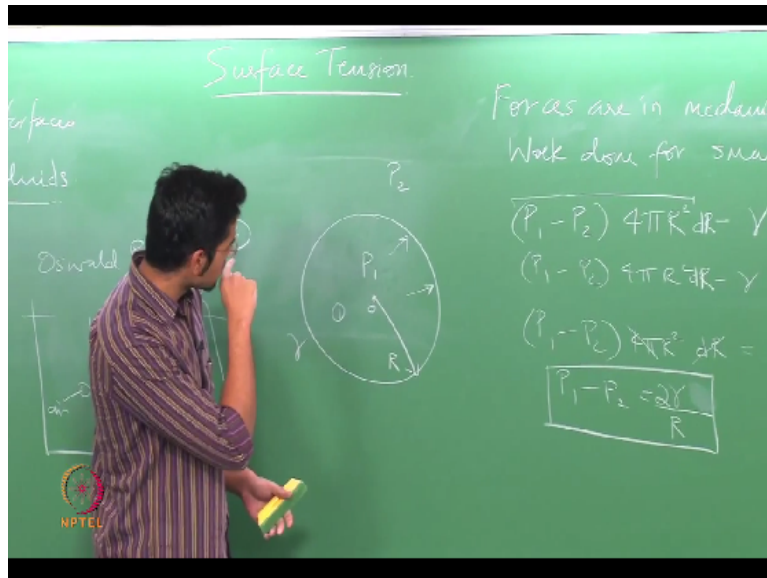


And in this class, we will focus on the relatively simple situation of the case of static fluids. So what I mean by static fluids is we will look at configurations where the fluids are not in motion. So simply a drop suspended in air is static and yet that drop has a certain configuration of its interface, that depends on the surface tension or a glass of water sitting on the table, again the water is static.

And so is the air on the macroscopic scale and in that case again, we have 2 static fluids with some interface configuration. So for these types of problems, we can work out simply how surface tension effects and what are the forces at the interface. Now later on in the course, we will look at situations where the fluid start to move either because we have applied some stress or some pressure gradient.

And then things get little complicated because surface tension has to counteract the forces of the moving fluid itself. So before we go to that stage, in this class we will just look at some of the very simple yet interesting effects of surface tension in static cases. So very obvious first case that anyone would want to deal with is the case of a sphere of liquid or it could be gas, so it is either liquid in the bubble basically. So either a drop of liquid in air or a bubble in a liquid. So schematically, we can think of it like this.

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So I have this big round drop and let us just call it fluid 1 for generality and then outside, I have another fluid that is suspended throughout the domain and I can look at this drop and I know that it is going to be spherical, experimentally I can look at it and it will be a sphere and because that is the only situation in which everything is in equilibrium which means that fluid 1 is static, there is no motion inside the drop of bubble and everywhere around it also, everything is static.

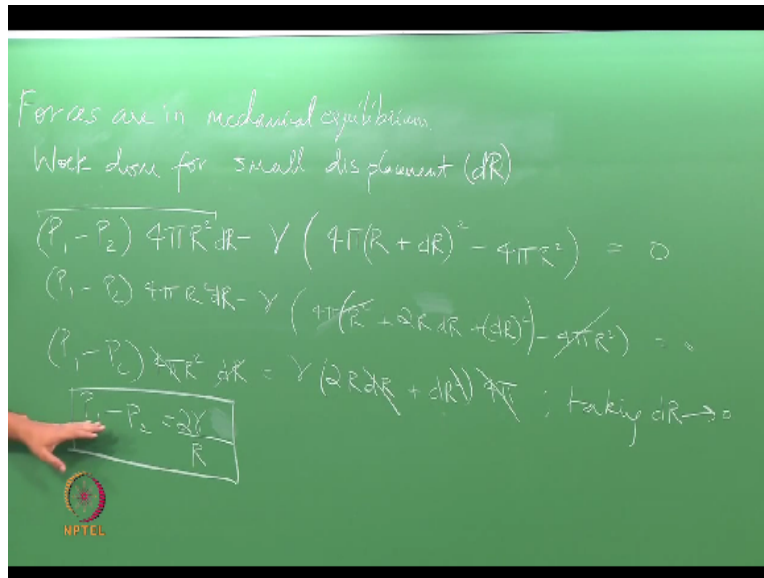
And the question we want to answer is, there is a pressure that is felt inside this bubble, P_1 and there is a pressure in the outside fluid as well, P_2 . Now we know that the reason the fluid is assuming the spherical shape is because it is time to minimise its surface area or the molecules are all getting pulled. So naturally there will be some forces at the interface at a time to compress it into a compact shape.

So then the question arises, why does not it just keep collapsing and keep getting smaller and smaller and keep reducing its surface area, right naturally. So what will happen eventually is that as it keeps getting condensed into a smaller packed volume, the pressure will rise within the drop or bubble and at that point you reach an equilibrium. So clearly $P_1 \neq P_2$. So there has to be some force that is preventing the bubble from shrinking all the more or some counter pressure is there that is preventing the bubble from shrinking any further.

And that is why we have an equilibrium shift and these bubbles can be really very stable and

hard to break, that is a big problem in any industrial processes where you have these bubbles form and you would like to break them but it can be really difficult to do because of this stable equilibrium that it reaches. So we want to find out how gamma which acts at this interface, effects this P1-P2. So to do this since the fluids are entirely static, we can adopt a very simple approach.

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So as pointed out the forces are in mechanical equilibrium, right. So the force exerted by the pressure which is going to be normal to the interface, is counteracted by the force exerted by surface tension. Now the problem here is we are not yet in a position in this course to say what the force is because of surface tension and we will develop that rigorously later on but right now we will assume that we have not heard of Young-Laplace law as yet which I am sure some of you have had.

And we just looked at how we can reach their heuristically at least by using some energy arguments. So instead of looking at the forces directly, we will do what Laplace himself probably did at his time and look at the work done for a small infinitesimal displacement. So let us say the radius from the origin is R and what I want to do is we are saying that this is an equilibrium configuration.

So there is no net force anymore at the interface because it is at equilibrium. So then any small

displacement you give and calculate the work that will also turn out to be 0 because the work has just force times the displacement and if the force is 0, then any small displacements will give you 0 work essentially. So we will compute the work done by the pressure forces, the work done by the interfacial forces and equate them to 0 and see what that tells us about the pressure jump.

So we can start working that out. Let us look at the pressure forces first silver. So the work done by the pressure will be basically the pressure force at the interface $P_1 - P_2$ multiplied by the total surface area. So that is just $4\pi R^2$ on the sphere. Now surface tension will also do its own work and that is why we use the definition of γ as the surface energy per unit area.

So we need to multiply this surface energy per unit area into the increase of the area that we would achieve if we make the small dR . So that just comes out to be $4\pi R + dR$ the whole square- $4\pi R^2$ and the total work done because of this small displacement will be $= 0$. So now we can proceed to simplify this and see what that tells us about $P_1 - P_2$. **“Professor - student conversation starts”** (()) (19:50) right, I have a problem.

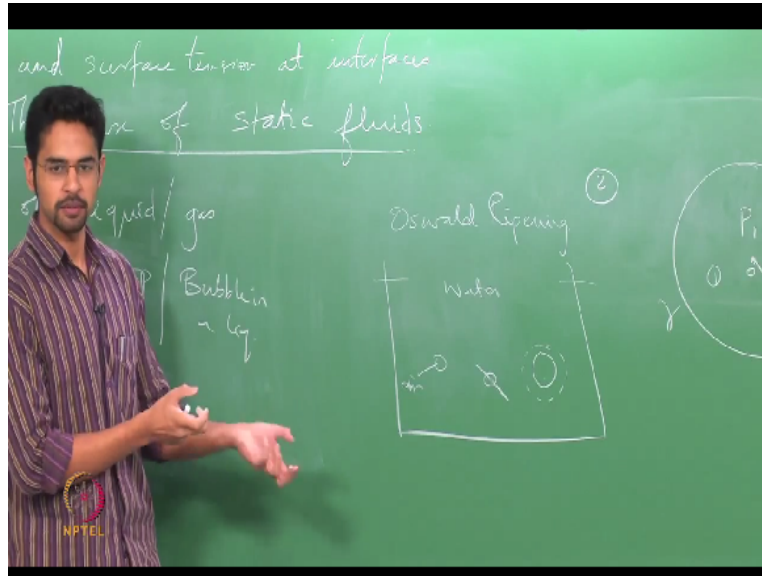
The energy (()) (19:57) sorry, yes, yes, I forgot to multiply the force with the displacement, right. **“Professor - student conversation ends”** So for those of you all just to follow this, this was the force on the surface but naturally the work was the force into the small displacement dR but this directly gave us the energy because we interpreted γ as surface tension or the energy per unit area, right. So now that makes sense as well because I needed an infinitesimal here. So what am I left with? I have $P_1 - P_2$ take it to the other side.

“Professor - student conversation starts” (()) (20:56) Yes, yes, I kept it out here. (()) (21:06) right, okay, okay, yes, yes, I missed that, right. **“Professor - student conversation ends”** So then it just falls out and if I take dR down and knock this off, it is left with dR but that is not a problem. So after cancelling off the terms and then taken the limit of dR going to 0 or recognising that it is an infinitesimal, I land up with $P_1 - P_2$ as $2\gamma/R$.

So what we have here is a relationship for the pressure jump across the bubble when the surface tension is γ . So clearly you will see that for 2-face systems with higher interfacial tension,

the over pressure inside the bubble can be much higher, that is the first point. Second point of course is that bubbles which have smaller area, also have a higher internal pressure.

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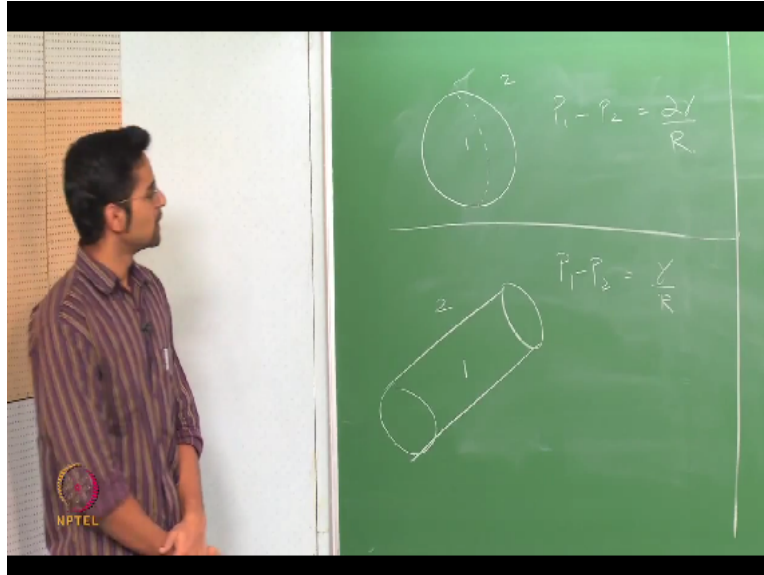
So this simple expression explains a lot of interesting phenomena, like if you have a beaker full of water and you have bubbles of air, then you will find that if you have a big bubble and a small bubble, you may think that the big bubble will like to share some of its air with the small bubble but that does not turn out to be the case.

In fact, after sometime, the small bubble will disappear entirely and give you a slightly bigger bubble and that seems a little funny but the reason that happens is because the pressure inside this is much higher than the pressure over here and so if there is some way for the air to transfer, maybe by dissolution, mass transfer and then absorption, that is what is going to happen because of this overpressure being greater in the small bubble.

And this is called Ostwald ripening and it seemed in many other processes apart from this simple system where you have emulsions and small bubbles and things like that. The important thing to note here is that the pressure inside the bubble is basically larger or if you want to look at it in a slightly heuristic way, we can say that, if you look at the surface, the surface is curved essentially.

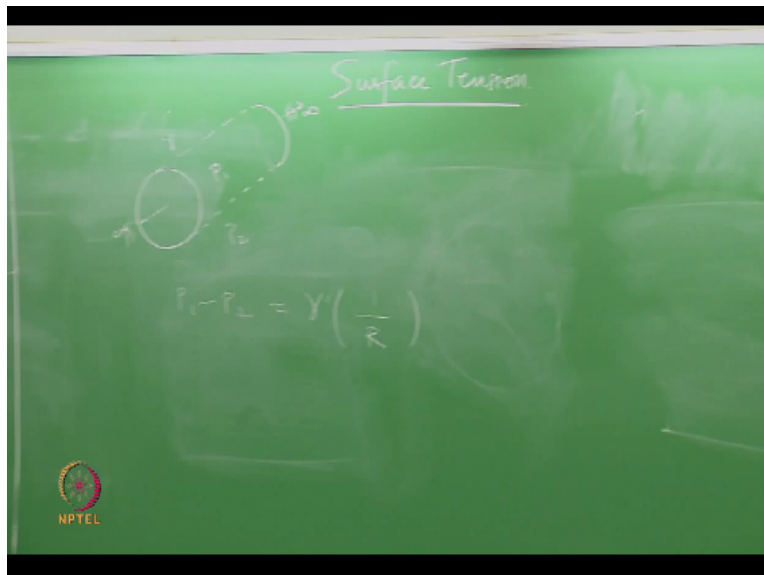
And the idea is that any curved surface will have some surface tension force acting on it. So the pressure which is on the inside of the surface which means that if I have a circle, basically the surface itself is a circle now and the center of the circle is in the fluid of higher pressure whereas on the other side, the pressure will be low.

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So let us store the result we have for a sphere and now very quickly having worked through this problem, let us look at another case and see whether we can generalise from there to a more general expression for the interfacial force. So first we looked at a sphere and I will just draw something like this, it you remember that it was a 3-dimensional sphere.

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And now let us look at another situation which may seem a little bit more artificial but it in fact is encountered in many practical applications as the starting point for a calculation and that is the case of a cylindrical body of fluid which extends to infinity in either direction. So just extremely long cylinder and let us see what the difference would be in the pressure on the inside P_1 and the pressure on the outside P_2 .

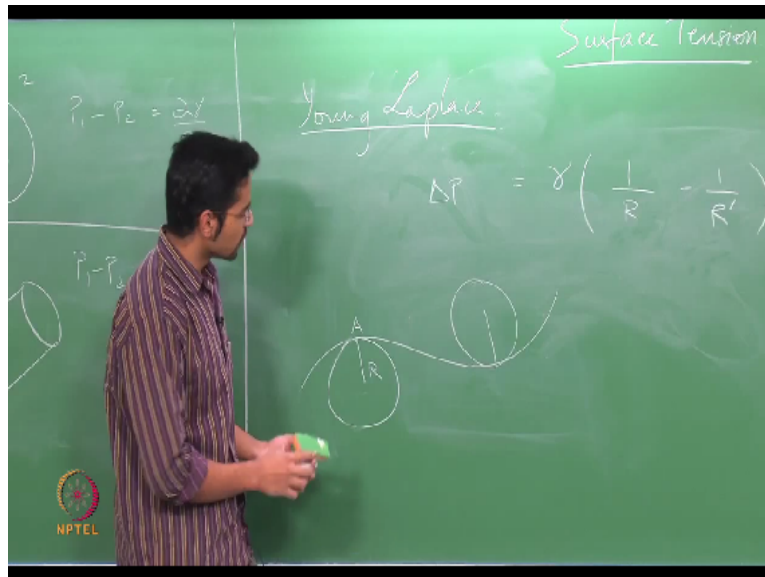
Where the surface tension acting everywhere on the surface again is γ . So let us do the same equilibrium calculation and let us see what we get. Naturally I am going to have γ over there but I want you to fill in the remaining factor. We will calculate $P_1 - P_2$ multiplied by the surface area here is just the curved surface of the cylinder $2\pi R$, you can take it per unit length. So just $2\pi R$ and that will be $= 2\pi R \cdot \gamma$ the displacement of course dR and that is equal to the change in the surface area.

So that is just $2\pi R + dR - 2\pi R$ and equating that, should be able to tell me what the pressure jump will be. So any answers? $1/R$. Right. So the answer is $1/R$ and you will see that is exactly easier than the previous problem because here is just 1 surface to look at, just the curved surface. So now we have the case for a cylinder as well, okay. So now with these 2 cases, let us see if we can figure out what is going on here because we could continue this way with various imaginary configurations of fluids at infinitive but would like to get a formula for this.

So in the case of a sphere we had $\gamma \cdot 2/R$. Now in the case of the cylinder, we just have $1/R$. So if you look at the origin of the surface tension is because there is the curved surface and in the case of the cylinder, there is only 1 curved surface because here it is basically flat. If I sit on the cylinder and look straight, I will not know whether I am on a cylinder or on a flat sheet. It is only if I look in this direction, I will say oops it is curved.

So there is curvature on only 1 direction whereas if I stand on the top of the sphere, whether I look in this direction or in this direction, I will see the same curvature which is $1/R$. So clearly the sphere is in some sense more curved than the cylinder which accounts for the higher overpressure and from this, we can sort of guess at a general law which is basically the Young–Laplace relationship.

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And what this relationship is again only for static fluid configurations that $P_1 - P_2 = \gamma \dots$, right. So the pressure jump or let me get rid of this $P_1 - P_2$ itself. Just the $\Delta P = \gamma \left(\frac{1}{R} - \frac{1}{R'} \right)$, this R' is not the derivative, I am just using this to differentiate 2 different radii of curvature as they called. So I will explain what this means. Suppose you have a curved surface, just a 2-dimensional curved surface that is going like this extending outside in this direction.

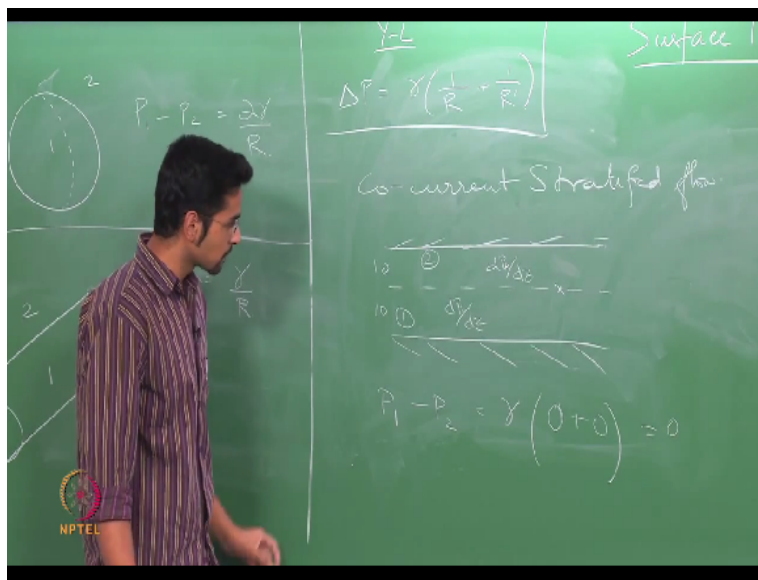
So in this direction, the surface is kind of flat. It is only curved here. So the way I can define this radius of curvature is by say at this point A , what I will say is that at this point, tangentially I can have a circle and that circle will have a radius R . So this is the radii of curvature in 1 direction and in the other direction, similarly I can find if the surface is curved say downwards, then I can fit another circle which lies in the perpendicular plane and that will be the second radii of curvature R' .

And we represent the curvature with a positive sign if the centre of the circle lies within the fluid of interest. So if we take ΔP in such a way that we are looking at the jump from fluid 1 to fluid 2 and if the circle lies in fluid 1, then we say the curvature is positive. In this case, the curvature is negative because the circle would lie outside. So R itself changes from positive to negative here.

And in this case, the R prime is 0 because it is just flat. So if we apply this law to the sphere, you will see that this $2\gamma/R$ is nothing but $\gamma \cdot 1/R$ which is the positive curvature in one direction and then when I look in the other perpendicular direction, I still have $1/R$ because it is still is just a sphere, so I will get $1/R+1/R$, then I should not put a minus, sorry. Plus $1/R$ and then I will get $2\gamma/R$.

So the sign comes out of whether the center of the circle lies inside the fluid or outside and naturally for the cylinder, you had curvature in only one direction which was again positive because the circle center lies within the fluid 1, lay within fluid 1 and the other radius of curvature was 0. Basically in the z direction, things were flat. Sorry the radius of curvature was infinity. The curvature was 0. Are there any doubts here?

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Alright, so now for the last 10 minutes of the lecture, I would like to engage in a few thought experiments and I mean these experiments have been done in reality as well but right now, we will just try to look at what we can deduce by using the Young–Laplace relationship about the qualitative nature of some surface and to make things more interesting, even though this law is still valid only for static fluids, we will look at some Quasi static situations.

So what I mean by that is even though you have flow, that flow is not really affecting the interface. So a very typical situation is what we have already studied, the case of co-current

stratified flow and here we thought of breeze through it without thinking too much about it but now really we can revisit the situation and see what is going on. So I said I have a fluid 1 at the bottom and I have a fluid 2 at the top.

And at that point, we said they are okay the interface was going to be flat and that is totally fine when you look at the problem for the first time to assume, okay let us study the case of a flat interface but what we should make sure is that this flat interface does not create any problems in terms of the force balance around the interface and within the fluids. So what we need to do is in this problem of course we looked at the balance of the tangential stresses.

But when we came to the balance of the normal stresses, we need to see whether the normal forces are also in balance. So what happens at this interface of course is that P_1 , which is the pressure here, $-P_2$ and they are in a normal stress because of the fluid motion because the fluid is only moving in this direction and the only stress exerting is tangential shearing stress. Only if there was flow happening upwards with the fluid pushed the interface.

But since that is not happening in this version of the problem, that is why it is a Quasi static thing and I can look at the Young–Laplace relationship. So then I have $P_1 - P_2 = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ but in this case, both those terms are 0 because the radius of curvature in this direction is infinity in the sense that if I try to put a circle, I will have to make the circle infinitely large so that it practically it becomes flat surface.

In the other direction also, we know things are extending out uniformly. So in this case, $P_1 - P_2$ was actually = 0 and that is what we assumed when we did the problem. So when we had the fluid driven by pressure gradients dP/dZ and dP/dZ here as well, you could have asked the question how do I know that dP_1/dZ is the same as dP_2/dZ but that is what we did when we solved the problem and the reason is that it falls out from this.

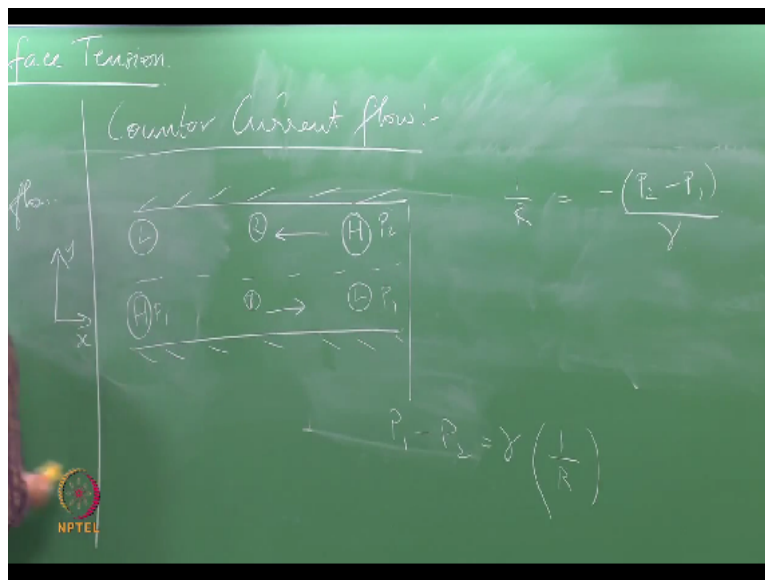
Since the pressures have to be equal across the interface, there is no way that the pressure drops in each fluid can be unequal because say example I will start here at 1 atmosphere each at the beginning and I am pumping the fluids, or let us say 10 atmospheres. Now if the pressure drop in

fluid 1 is different, then at fluid 2, at some point here, I might have 5 atmospheres and here I will have 6 if this drops faster, say because it has the high viscosity.

Then naturally I will have a problem because now the interface will not remain flat but I know that I have to have $P_1 = P_2$. So it is on this that we concluded really that the pressure drops are also the same and then it solved the problem. What this also tells you is that if the interface were not flat for some reason, then you will not have the same pressure terms. If you look at it in the inverse way.

So the only way my interface can be flat is my pressure drops are equal and if my interface is not flat for some reason, immediately the pressure drops cannot be equal in this situation.

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So that was pretty straightforward but now let us look at the interesting system which we have not considered as yet that of counter current flow. Now the counter current problem is substantially more difficult to deal with and all I want to do here is to try and see how it relates to the co-current problem and whether just by thinking about the problem itself and using Young-Laplace, we can reach some conclusions about what assumptions will hold and what will not hold if we try to move from here to here.

So once again let us begin with a schematic and now I have to be little bit careful about what I

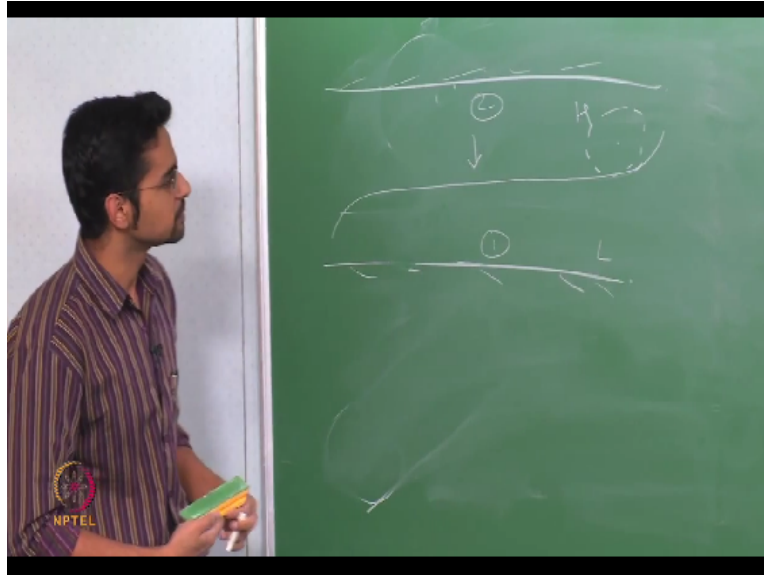
say next. Will the interface be flat or will it be curved? So let us assume since I am just sort of reading in the dark right now that there is some interface, maybe it is flat. So there is fluid 1 here and fluid 2 here and the fact that its counter current as I wanted to flow in opposite directions. So for that to happen naturally, the pressure P_2 and P_1 will have to drop in opposite directions as well.

So if the pressure P_2 is, this is the high-pressure zone in this fluid but it reaches a lower pressure zone here for P_2 . P_1 on the other hand has to have its high-pressure zone here to drive the fluid and reach a low pressure zone at the other end. So now assuming for the moment that the only velocity is in the X direction, just for the case of assumption, then we can apply once again the Young–Laplace relation and see whether the interface can be flat as it was in this problem.

So if we come say to this end of the channel and try to apply it, we get $P_1 - P_2 = \gamma$, the surface tension, \times the curvature. So I will have only one curvature because the other guy is going to extend out to infinity. So in this problem, we saw that, okay, if $P_1 - P_2$ is equal, the interface will be flat, I mean $P_1 - P_2$ is 0, the interface is flat but here we can see that there is absolutely no way I am going to have P_1 and P_2 equal at this point, because otherwise I needed to be unequal for that to be a counter current flow.

So in that case if I were to plug this in, I will see that $P_1 - P_2$ is actually some negative quantity, right. So therefore this $1/R$ or the radius of curvature relationship has to be..., right or this curvature in this situation is actually going to be negative which means that if I look at fluid 1 at the centre of the circle, will be actually fluid 2. So now if I redraw the situation remembering that I actually have a high-pressure zone here and a low-pressure zone here. Then I would realise that I cannot actually have a flat interface at all.

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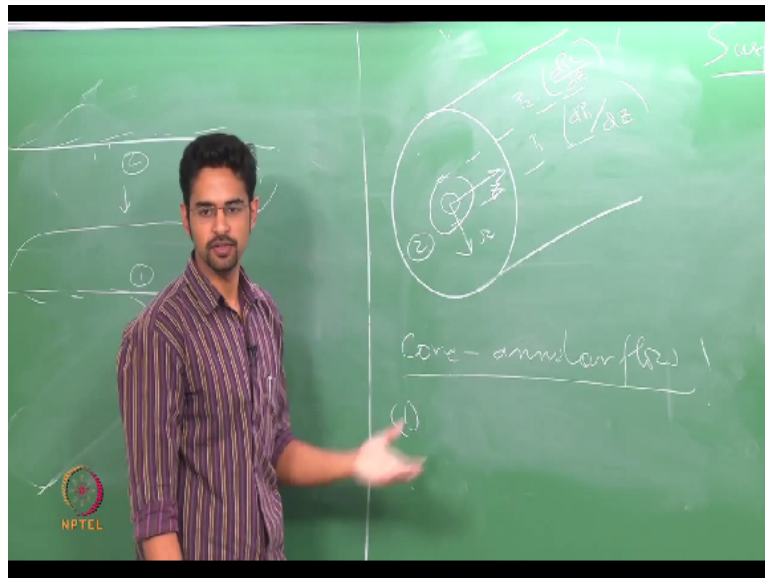
This is 2, this is 1. So now in the beginning of the channel, I need to have my curvature something like this. So that at any point if I draw a circle, it lies in fluid 2, that is because the pressure was high here and the pressure was low here. And do the same thing on the other end and I realise that where the pressure has switch now, so the pressure is actually higher in fluid 1. So now again the curvature will actually have to reverse and eventually will have to go the other way round.

And in fact in this type of a problem, this is what interface shape you would expect simply by using the Young–Laplace type of idea. You realise that the interface cannot be flat now, it has to deform in this sort of direction but to make any more progress, we have to solve the full problem because you will see that if my interface is actually curving up, my area for flow is increasing and then the continuity equation itself will tell me that there has to be a downward flow.

I mean the fluid is going out at some velocity and the area is increasing, it will spread out which means that there is a component of velocity now perpendicular to the interface. So to exactly solve this problem, might have to go back and solve the full equation with the force balance that will come when I include normal stresses perpendicular to the interface but the point here was, you can still use the Young–Laplace law to get some initial idea for how the interface will behave.

And what are the assumptions you need to make at the beginning of the problem and even these flows have been seen in micro channels. Maybe we can show you some experimental work done by people in Europe I think. There is a lab where they have done this counter current flow problem nicely and the chemical engineers would know that counter currents processing is usually more efficient than co-current. Alright so as a final point before I wind up, we will look at another interesting flow situation and this will probably be coming in assignment in one form or the other sooner or later.

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So we are looking at, very briefly I will do this so my class is nearly over. I have a core annular flow problem. Core annular flow is essentially involves one cylindrical pipe and one fluid coats the wall of the pipe entirely and the other fluid is sent through the center and these core annular flows are a lot of practical significance especially in the transport of oil because what happens is unlike the stratified flow problem, you can have one fluid completely contact the wall in core annular flow.

In stratified flow, both fluids are in touch with the wall. So regardless of if you have a very small viscosity also, you will still have a lot of shear stress generated by the other fluid but in this situation, you can actually put the more viscous fluid in the centre and have a thin layer of the less viscous fluid touching the wall and the entire viscous fluid will just lubricate itself through the pipe.

So what they do in oil transport is that they mix oil and water and the water actually coats the wall and you can pump through a lot more oil at the same pressure drop than you would have done just oil itself. So that is the motivation behind setting these problems and there is a lot of work being done. The question that we want to address in this class which is what the assignment will be is to calculate the velocity profile for the core annular case mirroring what we have done with the stratified flow.

So of course in this case, you will adopt the cylindrical coordinate system because that fits much better with the geometry of this problem than as the Cartesian during stratified. So if you take cylindrical coordinates, R and Z , and assume things are symmetric in (θ) (45:58) and then go ahead with the usual assumption, say the velocity is only in the Z direction and so on. You can simplify the cylindrical coordinates in Navier-Stokes and that should be straightforward.

Then the point where I want you to have to pay some attention to is dealing with the interface pressure jump. Essentially P_1 and P_2 . So the question is now in this problem what would be the jump across the interface because the core is essentially a cylinder and then whether the possibility of a purely cylindrical core, whether that can hold the interface of this pressure jump and the fact that there will be a pressure drop in each fluid.

So I have a dP_1/dZ and the dP_2/dZ . So the question is, are these 2 pressure drops equal when you have a cylindrical core and based on that answer that you could get from the Young-Laplace relationship, we should be able to see if we can get the full velocity profile. Thanks. I will take any questions if there are?

“Professor - student conversation starts” (θ) (47:11) that curvature is 0 or and then in counter current, then you say that is only in one direction you will have that curvature. Yes, yes, in the counter current problem, what I am saying is that I am looking at the problem only in these 2 dimensions. In the third dimension, everything is extending to infinity. So it is like a big wall going straight.

The interface is also curved like a wave but in this direction, the wave just gets repeated. So if I were to stand at any point and look straight, I will not see any curvature at all. It will just be straight. Just like the cylinder, there is curvature in one direction but not in the other. But of course if you take a finite box, then also it could be flat in this direction.

Because if there is no flow perpendicular to the box. So that is still possible but then you will have some end effects and stuff. So practically what I am saying is that the actual box is sort of wide. So if I look at the center of the box, I will not see the walls and then it will look something like that. **“Professor - student conversation ends.”**