Multiphase Flows: Analytical Solutions and Stability Analysis

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Lecture - 03B
Flow Regimes in Microchannels: Modeling and Experiments

Alright, so what I want to do for the next 10 minutes to show up this lecture is to show you a few videos that hopefully will better illustrates some of the things we have been talking about.

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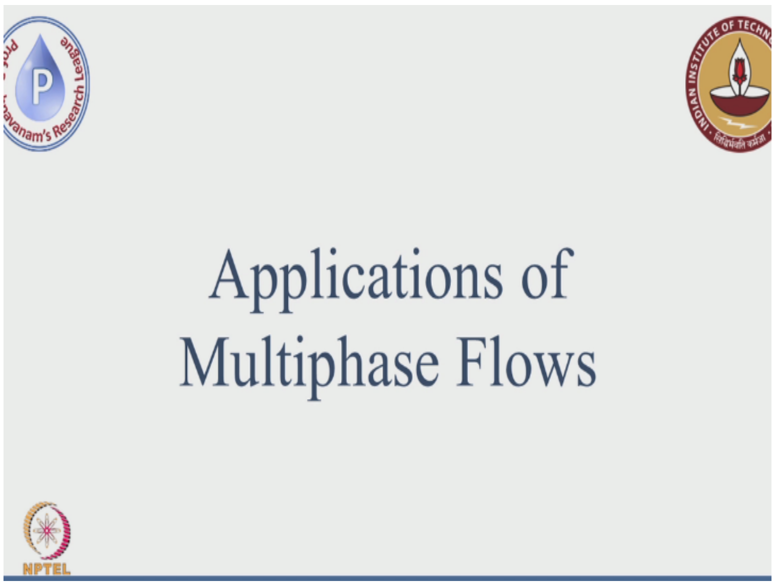

See you just have to look at the screen for a while. So as the title suggest, we look at a few Applications of Multiphase Flows and see exactly how some of the things we will be looking at in this lecture translates in the real world.

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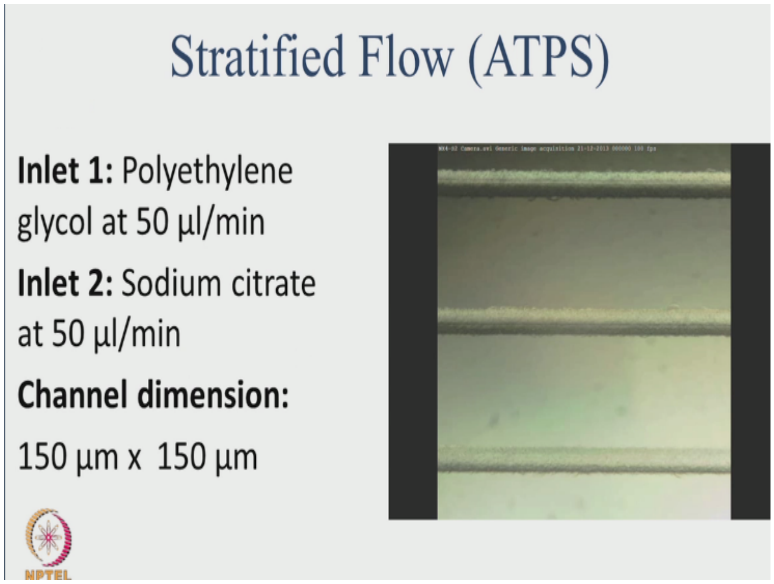

So this is an image in a microchannel of Stratified Flow in a Aqueous 2-phase system. So what you will have to do if you look carefully at those bends that you see each bend is in fact a microchannel. And the dimensions as you can see are about 150 micrometers. So that is extremely small width maybe a few have it. Then if you look closely in each width you will see that about half the channel is dark in black color and the other half is sort of a lighter white.

And it is that white portion which is one fluid in the dark portion which is another fluid. And you have a perfectly flat interface between 2 of them. And the 2 different fluids are given here at 50 micro liters per minute, Polyethylene glycol and sodium citrate. And this was used in some extraction experiments. So there is a video as well. And (()) (02:05) much because there is no interface deflection at all, so the fluid just flows straight through.

And the-- what we need to look at then we look at this video to see were the assumptions that we made in class actually hold, so for example if the interface really steady and flat or is you know fluctuating all the time. So you see this experiment at least in the interface is flat it is steady and we can possibly be confident that at least that assumption was right.

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So we can look at the models that we used.

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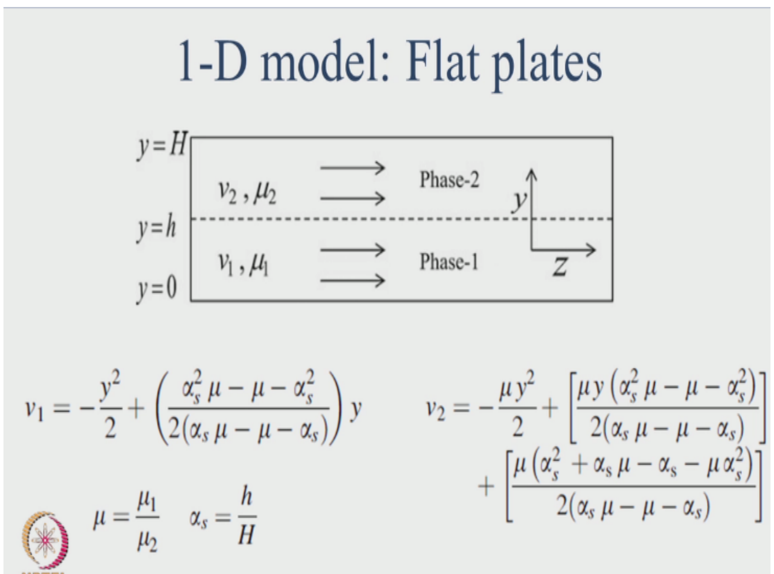

This was the model that we looked at in class where we had 2 infinite plates in the x direction in this case flow in the z direction and we were looking at how the velocity profile was in the y direction and we have 2 fluids Phase-1 and Phase-2 and then applying simplification to the Navier-Stokes and the boundary conditions you would have been able to drive the velocity profiles which I given here. Note that the viscosity ratio here is Mu and alpha S is the Holdup or the thickness ratio.

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Flow rate fraction and Holdup-1D

$$Q_1 = \int_0^{\alpha_s} v_1 \, dy \quad \text{and} \quad Q_2 = \int_{\alpha_s}^{1} v_2 \, dy$$

$$\Phi_s = \frac{Q_1}{Q_1 + Q_2}$$

$$\Phi_s = \frac{\alpha_s^2 \left[3\mu - 2\mu\alpha_s - \alpha_s^2(\mu-1)\right]}{\alpha_s^4(\mu-1)^2 - 4\mu\alpha_s(\mu-1) + 6\alpha_s^2\mu(\mu-1) - 4\alpha_s^3\mu(\mu-1) + \mu^2}$$

So then after getting the velocity profiles we discussed about calculating the flow rates that is Q1 and Q2 by integration and then to find the flow rate fraction which is the ratio of the flow rate to the total flow that was sending to the channel. So here in fact we show the relationship between flow rate fraction and the holdup which is other thickness ratio, and you see that the viscosity ratio plays a role in this whole relationship.

Of course, as we expected delta p is no longer to be found because that was simple magnitude parameter that should have been lost when we took ratios of the flow rate. And you remember I said that it is only when the viscosity is 1 or the viscosity ratio is 1 and the holdup is 0.5 then I would expect the flow rate fraction to be 0.5. But what happens when that does not hold is what we can look at now in this slide quickly.

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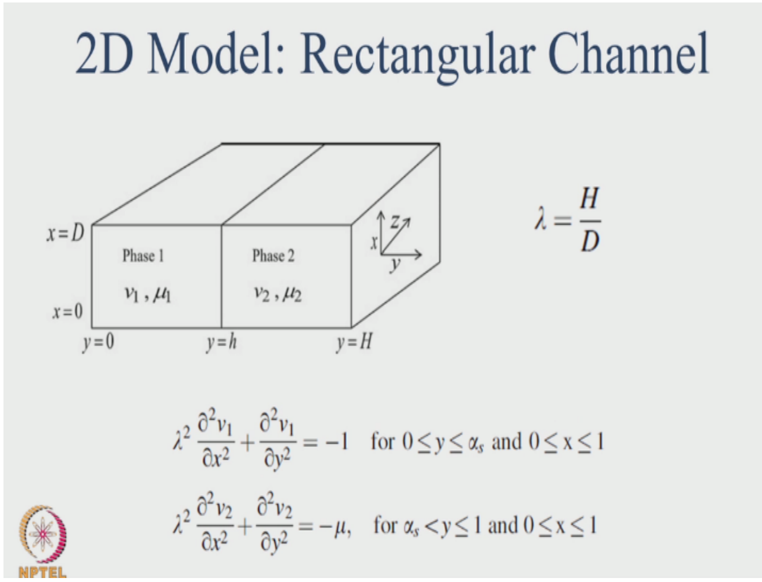

Before that the model that we considered one of the biggest assumption was it was an infinite plate or that we did not look at the third dimension which most obviously is going to be there in the micro channel and that third dimension will in many practical situation is be of the same order as the height of the channel that we were looking at. So in this work, some of the work we have done and we looked at the 2D Model as well so that we could you know relax the assumption and the infinite and one of the direction.

So we got an aspect ratio naturally which was the width of the channel in the stratified direction that is in the y direction which we previously had, and d which was the now new added finite dimension in the problem so that give me an aspect ratio. And you would expect that if H much, much smaller than D then I should be able to work with the assumption that I had before.

In other words, if D was much larger then it would tend towards the infinite problem that we have solved. So if lambda is 0 essentially it goes back to the previous problem for any other finite lambda we would have to use the rectangular channel. So you see that here we have partial differential equations and these are not reducible any further. So this is clearly why we had looked at the 1D problem in the first place.

Because it gave you significant mathematical simplicity while retaining considerable physics. But in some of the problems you might have to come to this stage where there you have to deal with the mathematical complexity as you may. And then of course but good thing is you already have some insight from this simpler problem.

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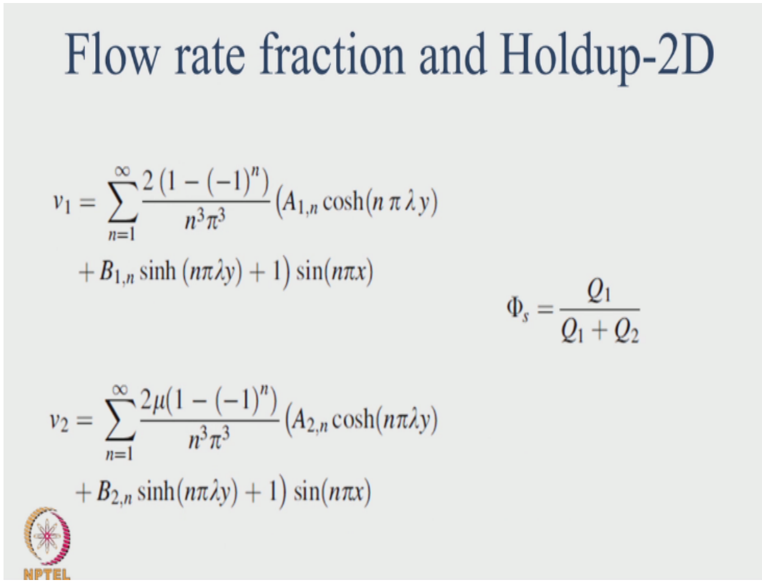

In terms of that you can get analytical solution and then once again calculate the flow rate fraction.

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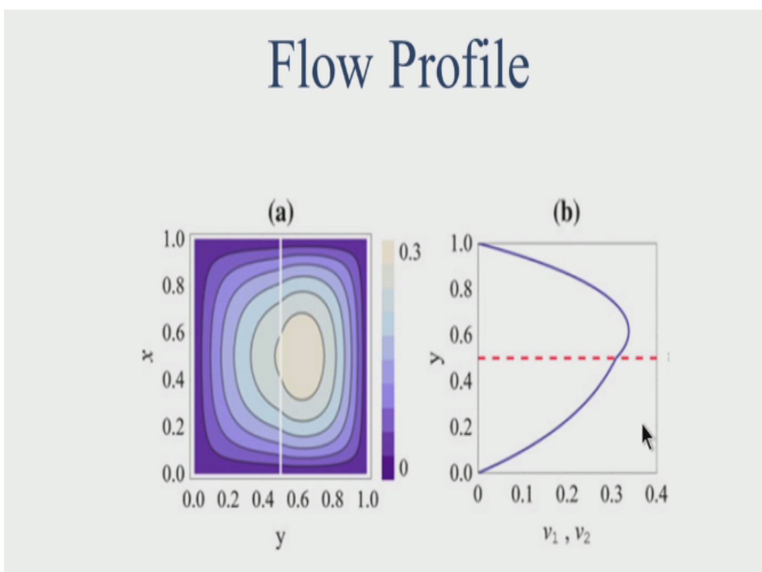

So here you see a Flow Profile. This is part of your assignment to calculate these different profiles flow profiles. You can look first at the most familiar figure, a figure B where you see that the maxima are, in fact, in fluid 2and it turns out that though not shown here that the viscosity of fluid 1 is twice fluid 2 so it makes sense that maxima is there. On the Figure A you see the crosssectional view of the velocity contours so each colored surface represents a velocity of a particular value. And the max velocity is on the white color and 0 is the blue color.

So you see that the maximum velocity is located somewhere within fluid 2 near the center of the channel and it just a slice along the center that is shown here as a velocity profile.

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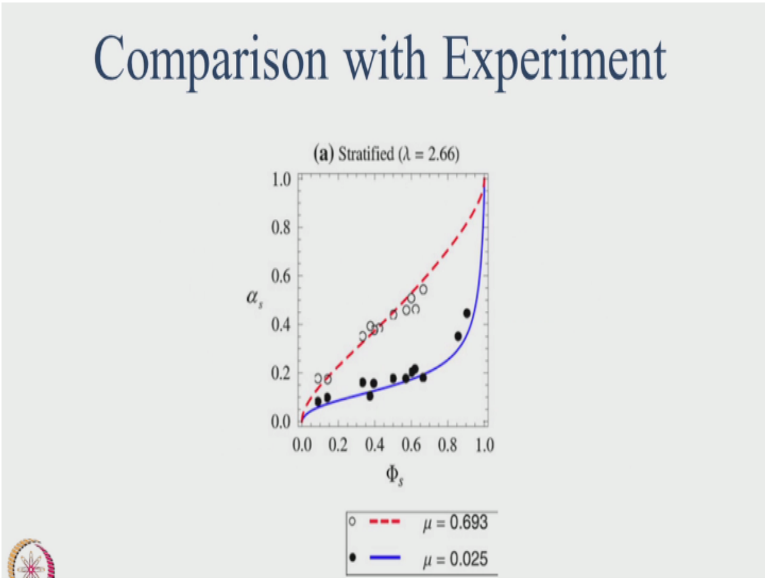

All right, so now of course what every modeling exercise has to deal with is how were the comparison with experiment and that is what we shown here so simply in this figure where you can see the data points from 2 experiments. One these experiments been carried out by (()) (07:36) and I can give you his reference later. So he did some need experiment where he looked at exactly what we are plotting here, he pumped in different flow rate fractions of 2 fluids.

And then he using a images track where the interface is located or the thickness ratio. And what he found was the when the viscosity was somewhat similar about half each other the relationship move more or less a diagonal but then the viscosity ratio is very different he got large deviations from a diagonal. And the diagonal is not important because it just means that Phi S = Alpha S. Or in other words the-- if you pump 2 fluids in a certain fraction you would expect the thickness to have the same fraction.

But we see whether that is totally false if they have very different viscosities. And that make sense because the problem favors the fluid with lower viscosity because that will give a lower pressure drop if you want to think of it in terms of (()) (08:38) imagination. So this becomes very important when you are trying to estimate profiles of the flow, estimate its pressure drop using empirical relations where you want to know what the holdup is.

And there is no simple way to estimate it from the flow rate fraction unless you do the kind of calculations they are doing. And of course what we were able to see is that the model or the simplified model, you are looking at predict this experiment will data very well. So you see the blue case flows it set of data and the red dash line follow with set. And there is a considerable difference between the 2.

And the fact that the model captures is tells us that the using for it difference really is in the viscosity ratio and the physical mechanism of that viscosity ratio in fact is what a model is captured which basically involves the shear stress at the wall that is about the only think that the viscosity is doing that makes the fluid difference. So both fluids are moving along their own walls and the more viscous fluid exerts must greater drag so naturally it has difficulty to flow the less viscous guy pump through in easier way.

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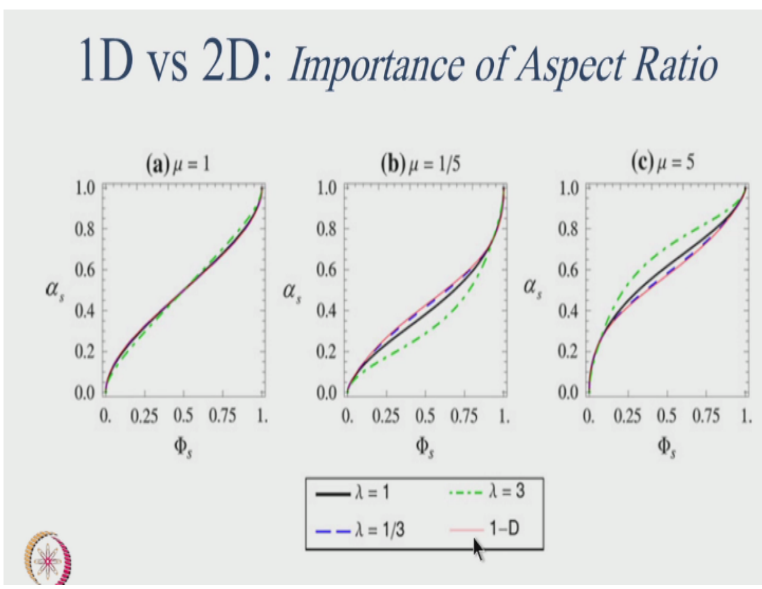

Finally, here we can look at 1D versus 2D cases and see what the importance of the aspect ratios. So let us just look at the first where I have viscosity ratio 1 to equal viscosity fluid for simplicity. And I have shown different figures different relationships for different aspect ratio. So this is the square channel and then this lambda=3 is where my depth is smaller than the width between the 2 plates. This case is closer to my 1-dimensional approximation where D is larger than H. And this is the 1D calculation. It is a light, pick light.

So in the first case you hardly see much difference between these 4 different results the 4 aspect ratios. And from the 1D case. So I guess I could say that the 1-dimensional case so us as a good model here and I do not really need to go to the square channel because no matter what the aspect ratio is, it does not much of the affect on this relationship. But when you come once again the different viscosities you start seeing that the aspect ratio has a significant role to play.

So the especially along flow rate fractions of 0.5 you have some considerable difference between the thickness ratios depending on what my aspect ratio of the channels. And importantly what we should verify is that whatever be these cases they should all tend towards the 1-dimensional case is my aspect ratio goes towards 0 which means as D becomes much larger than the width between the 2 plates H.

And that is what we see here that when lambda is P it is the further away from the line pick line which is the 1D case and as my aspect ratio is become smaller I approach it (()) (11:43). So the same thing you can see Mu=5 is just that the relationship has sort of reversed. So what this tells us also is that if my channel is long and rectangular or all standing up I can have different thickness ratios and nearly by orienting my channel or orienting the fluids in that channel in the right way I could have some control over what my holdup is for the same flow rate fraction.

It also tells us when we would be able to use the 1- dimensional model with some better accuracy. So we will stop here for today and we look some of the other applications of multiphase flows in the next class.