Multiphase Flows: Analytical Solutions and Stability Analysis

Prof. S. Pushpavanam
Department of Chemical Engineering
Indian Institute of Technology – Madras

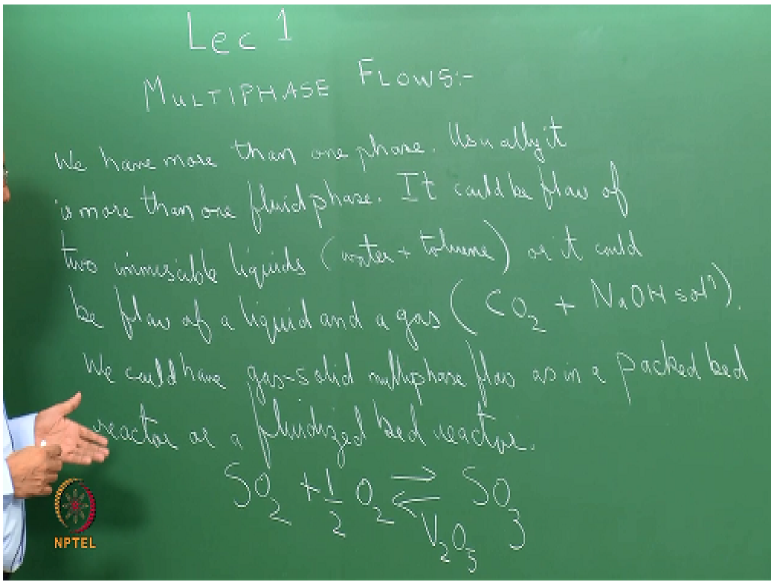
Lecture - 01
Introduction and Overview of the Course: Multiphase Flows

I would like to welcome all of you to the very first lecture of this course on Multiphase Flows and what I like to do today is basically give you an overview of the entire course so that you know exactly what you are in store for. Now this is an advance level course and I have been teaching this course for the last 4 years or 5 years. The contents of this course have actually evolved over these 4 years primarily with inputs from students.

And I think now the course is in a shape where there is a lot of fundamental scientific approach which is being emphasized. The very first time I offered this course it was following the classical book of (()) (01:37) and there the emphasis was on using some kind of empirical correlations to calculate quantities like pressure drop when you have a Multiphase Flow pattern.

Now there is hardly any of that we are going to be doing in this course. The idea is that those are concepts which you can quite easily pick up on your own. Should you have very strong fundamentals and keeping that in mind this course has gone through a complete over whole.

(Refer Slide Time: 02:38)



So this is the first lecture on this course Multiphase Flows. Clearly when you talk about

Multiphase Flow System you have a system where there is more than one phase. And so we have more than one phase usually it is more than one fluid phase. So for example it could be the flow of two immiscible liquids and organic liquid and aqueous liquids. So for example it would be water + toluene or it could be flow of liquid and a gas.

For example, it could be carbon dioxide and gas stream containing carbon dioxide and may be an aqueous stream containing sodium hydroxide solution. So these days you know carbon dioxide (()) (04:55) is talked about and so you have a situation where people want to remove carbon dioxide coming from process plants and one way to do that is to observe it in a solution of sodium hydroxide.

So you have a gas stream and a liquid stream which have to be in contact for this to occur. You can have other situations where you are talking about extraction of maybe some impurities from 1 liquid waste to another and in that case you have 2 liquid phases flowing together and that could be an example of water and toluene mixture flowing for example. You will of course have a solid phase as also be one of the phases.

So we could have a gas solid Multiphase Flow as in a packed bed reactor or a fluidized bed reactor. So here one of the things that happens is the solid particles can be for example catalytic and they are the sides over which the chemical reaction takes place. The reactants are usually in the gaseous space. So if you are talking about a chemical reaction of the kind where sulfur dioxide gets converted to sulfur trioxide.

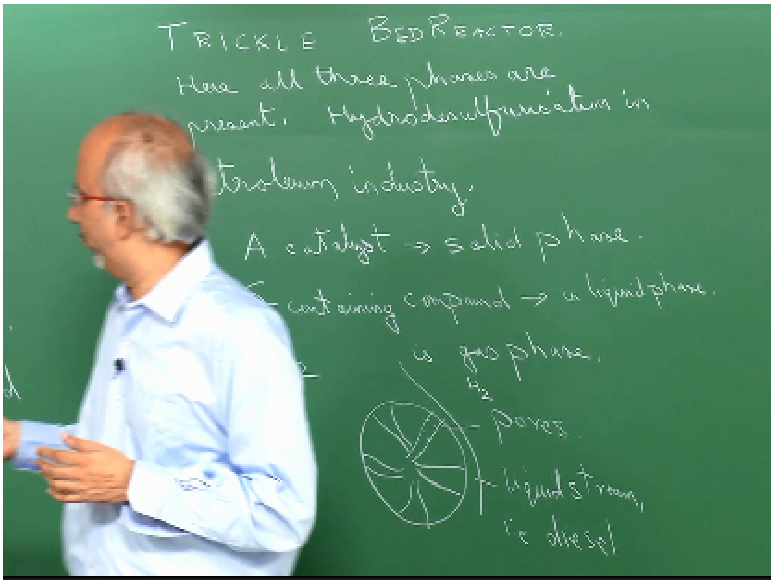
This is a reversible reaction and this takes place on a catalyst of vanadium pentoxide that is my catalyst. So this occurs in a packed bed reactor where you have solid particles on which vanadium pentoxide is deposited. These particles are pores and the vanadium pentoxide is deposited on this inner surfaces of the pores. The reaction takes place on the site of the catalyst and sulfur dioxide and (()) (07:38) are actually latent then diffuse into the particle reaction takes place and the product is formed.

So here is an example of Multiphase Flow where you have a solid particle and you have a gas phase. There are other examples for example in the field of catalytic cracking where fluidized bed reactors are used where the solid particles are not stationary, but they are also moving in my reactor. So the entire reactor is in a fluidized state. The fluid phase is moving and the solid phase is also moving.

So that would be an example of a fluidized bed reactor. So what I am just trying to tell you is that there are several situations in the chemical processing industry where in Multiphase Flows do occur and that is basically by way of trying to motivate you for going through with this course and seeing how the concepts that you are going to be learning in this course can be used for analyzing some of these processes and some of these systems.

And see if we can get a better understanding so that we can possibly improve the performance of these systems. So these are example where Multiphase Flows occur and you could also have a system where you have all the 3 phases present a gas phase, a liquid phase and a solid phase.

(Refer Slide Time: 09:28)



And an example of that would be actually a Trickle Bed Reactor. Here all 3 phases are present. And one industrial process in which a trickle bed reactor is used today is the hydrodesulphurization in the petroleum industry. So as the name suggests what you want to do is you want to remove the sulfur compounds present in your crude oil and one way to do this is to react this with hydrogen.

So the hydrogen reacts with the sulfur compounds and produces Hydrogen sulfide. Now the sulfur compounds are therefore eliminated from the feed stream of the crude and what you get is petrol after the refining step with a very, very low sulfur content. The reason why this becomes important is tomorrow when you are driving your car or your scooter or your motorcycle and you are burning all this petrol, diesel etcetera.

The sulfur oxide emissions that you are going to let out and you are going to contribute to the atmosphere is going to be minimal. So you do not want to pollute the atmosphere by burning these sulfur containing compounds the thiophene, the Mercaptan etcetera. So you want to remove them like other refinery stage itself and the way this occurs is you have again a catalyst pellet.

We have a catalyst and this is the solid phase. The sulfur containing compound this is the liquid phase and the hydrogen is the gas phase. Again the reaction has to take place in a pores solid particle. So I am just drawing a random network of pores so these are my pores and that is my catalyst particle. The catalyst is deposited like I told you earlier on the surface of these pores which we can assume to be cylindrical.

The sulfur containing compound a crude oil is the fraction of crude oil, diesel. Sometimes you are only processing diesel or sometimes you are only processing petrol is going to be flowing around this catalyst particle. So this is your liquid stream and the hydrogen gas is going to be present outside. So this is your liquid stream and this could be diesel for example. What we want is both diesel and hydrogen should reach the site of the catalyst for the reaction to occur and here in order for that to take place.

These pores have to be very small so that the liquid can enter the pore through basically (()) (14:25) reaction. I want the concentration of hydrogen to be as high as possible at the site of the catalyst. In order for the concentration of hydrogen to be very high I must have a very low resistance to the path of the hydrogen so that the concentration difference is very low. So one way to ensure that the path of the resistance to the flow of hydrogen is very low is to make sure that this film of liquid which is flowing around the catalyst particle is very thin.

If the film is very thick then hydrogen would have to diffuse to a longer distance and therefore the resistance would be much higher. So what is done in industry is you make sure that you have a very thin film. In other words, the liquid is basically trickling over the surface of the catalyst particle. So the tricking over the surface of the catalyst particle ensures that the concentration of the hydrogen is very high at a surface of the pore and have an effective reaction.

So this is an example of a gas, liquid, solid system where a very important process takes place. This basic insight of wanting to have a very high concentration of hydrogen in the surface of the catalyst particle has told us what should be the relative flow rates that we need for the liquid phase and the gas phase. Now one of the objectives which an engineer has is to understand questions like what is going to be the pressure drop in a system where there is more than 1 fluid phase flowing.

Now if you want a quick answer you can possibly use the ideas from the single phase flow that you have studied and try to adopt them to the Multiphase Flow context. For instance, if you have a 2 phase flow of a liquid-liquid system you may be able to calculate things like an effective density and an effective viscosity for the 2 phase mixture. So you may take a weighted average of the density and the viscosity.

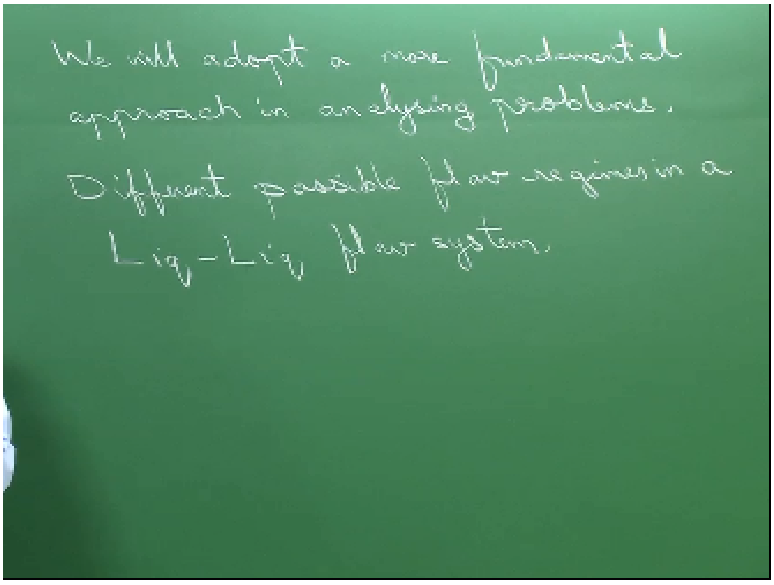
And treat it as a pseudo-homogeneous fluid as a single phase fluid and then use whatever knowledge you have about calculating pressure drops for single phase systems use these average properties and estimate the pressure drop. Now this is one way by which you can try to solve the problem of course the basic assumption of whether the weighted average is going to work or not is not something which we can know a priori.

So what you would do is you would get an estimate and then you are actually do an experiment and see if this estimate is accurate to what you are actually finding in an experiment there could be a mismatch or that could be a match, but there is no guarantee that there would be a match because you have made approximations. So what we want to do is we want to see if we can get away from this kind of an ad hoc approach, this kind of semi empirical approach.

So the semi empiricism comes before of this adhocism that has basically been used in addressing this question and can we go for something more fundamental. Let me just go back one second and to this problem of this packed bed reactors and maybe even to the problem of the trickle bed reactor where this solid is in stationary phase. So typically if you want to calculate the pressure drop you would calculate the pressure drop using the argon relationship in a packed bed reactor.

Now but that only gives you an idea of the microscopic phenomena the pressure drop versus the flow rate or the velocity of the system. What you want to do is possibly get a more detailed insight about what is exactly happening in the system and that is going to be the focus of this course. So by way of motivation for processes where Multiphase flow systems exist. I have just given you these examples.

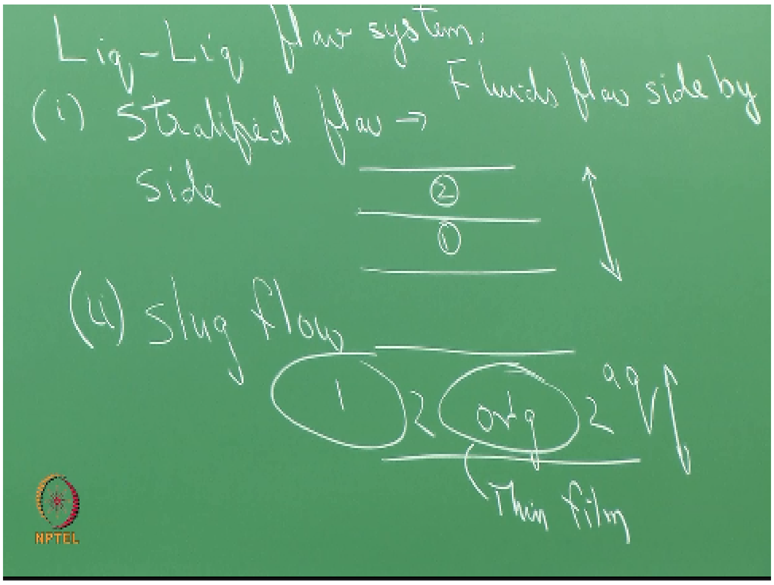
(Refer Slide Time: 20:27)



And what I am going to tell you now is as far as this course is concerned we will adopt a more fundamental approach in analyzing problems. In particular, we are not going to be using any correlation at all in this course. Although we have a very complex problem with Multiphase systems we will not be using any correlations. So for example to give you an idea of the liquid-liquid flow problem that we mentioned earlier in the context of extraction there are different possible flow regimes in a liquid-liquid flow system.

So for example you could have the liquids in a cube flowing side by side or you could have drops of one liquid suspended in another liquid or you could have big slugs of one liquid flowing in a periodic manner with the other phase in between these slugs.

(Refer Slide Time: 22:35)



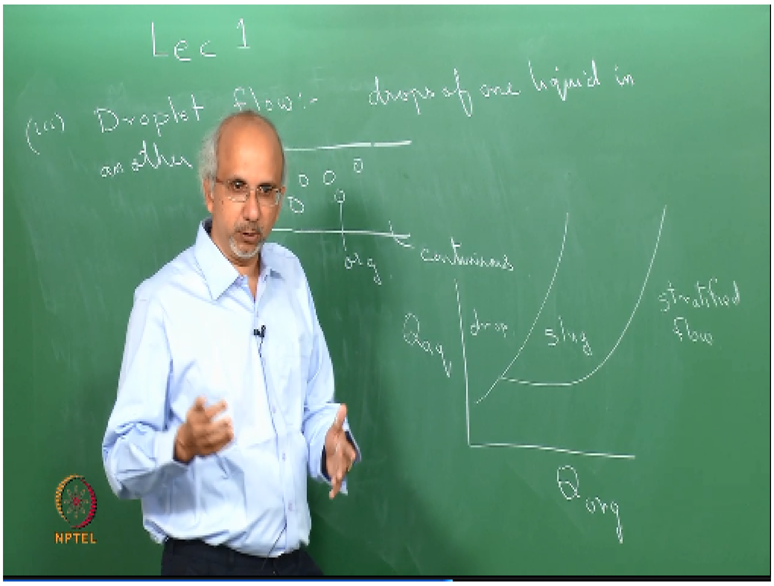
So there are different possible flow regimes. One is I am just giving you some examples one is a stratified flow. Here the fluids flow side by side. So we have a pipe or a channel you will have phase 1 here and you will have phase 2 here that is an example of stratified flow. You could have a slug flow. Here you would have slugs of one fluids and this is the second fluid and then you have another slug of the first fluid and this is the second fluid. So the one represents one of the fluid phases could be toluene two represents another fluid phase could be water.

And you have these slugs which are all of uniform size surrounded by the aqueous phase which is a continuous phase. So basically this is a dispersed phase this is a discontinuous phase because one slug of let us say that the organic phase and this is the aqueous phase. The organic phase is the discontinuous phase. The aqueous phase is a continuous phase and then this basically depends upon the material of your pipe.

So if you have glass then glass is weighted by water and so there is a possibility for the continuous phase to be aqueous phase. When I talk about slug what I mean is that the slug occupies almost the entire cross-section of the pipe. So this is my pipe, this is the cross-section of my pipe and here this is my cross-section of my pipe. The slug occupies entire cross-section of the pipe and there is going to be a very thin film which is going to exist between the wall of the slug.

And the wall of the pipe so this is a thin film.

(Refer Slide Time: 25:22)



And I think I will just talk about one more flow regime the droplet flow regime where we have drops of 1 liquid in another. So here we have this pipe and we have drops and this is your maybe an organic phase and this is your continuous or aqueous phase. The different between the droplet flow and the slug flow is that the size of the dispersed phase. Here the size of the dispersed phase is much smaller than the size of the channel.

The size of the dispersed phase in the slug flow occupies the comparable to the size of a channel. Here the size of the dispersed phase the organic phase is much smaller than the size of the channel. Now one of the questions which we want to know is we want to be able to predict on the basis of the properties and the operating conditions things like the flow rate what is going to be the flow regime present in my system, in my channel, in my pipe.

In the past what people have done is they have done extensive experimental investigations and they have depicted the different flow regimes in something like a 2 dimensional parameter space. So for example for a given combination of liquids and for a given pipe diameter you could find out for different combinations of the flow rates what is going to be the pattern of the flow.

So if you have a very, very low flow rate of the organic phase and high flow rate of the aqueous phase you might expect something like a droplet flow. So this is low organic (()) (27:59) as it keeps increasing the organic phase flow rate you might want to go towards or you might observe a transition from a droplet flow regime to a slug flow regime. And if you want to increase the flow rates further you would get a stratified flow regime.

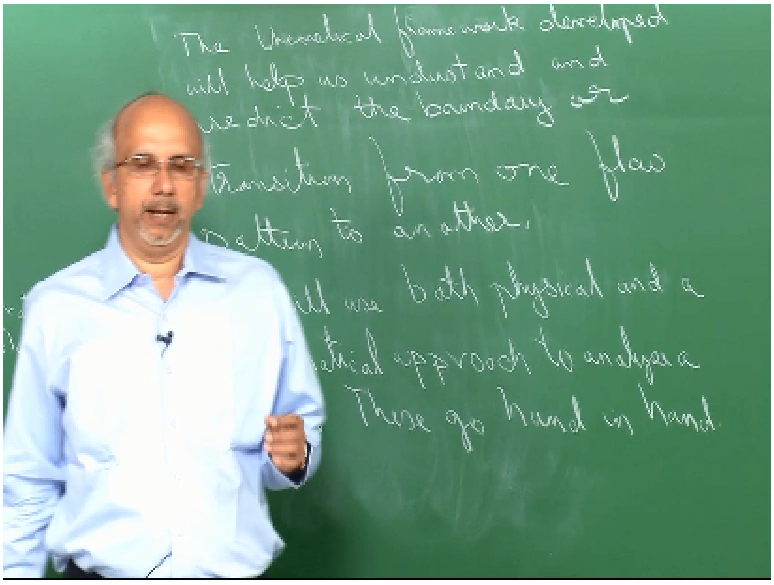
So basically what I am trying to tell you is for a given combination of liquids this means the properties of a liquid affects the density and the viscosity. Depending upon the flow rates you are going to see these different flow patterns. And of course one way to do this study is by just doing a bunch of experiments, making observations and then coming up with things like some kind of a boundary wherein you would say this is the region where I am going to get a droplet flow regime.

This is the region where I am going to get a slug flow regime, this is the region where I am going to get a stratified flow regime. Remember this is just to illustrate the idea. So please do not take this figure for being actually observe an experiments. This is just to tell you that there are boundaries in this parameter space which demarked the different flow regimes. So the question now is I can do that experimentally is it possible for me to determine these boundaries.

Determine these changes in these flow patterns using some kind of a theoretical framework and the answer is yes and what you are going to learn in this course is basically this theoretical framework which helps you understand not only the answer to this question, but a whole bunch of questions which are going to be of a similar nature. So our objective is to see if I can predict these boundaries where the transition occurs from one flow regime to another.

And just to tell you how we are going to do it we are going to do this by looking at what is called the stability of a particular flow.

(Refer Slide Time: 30:52)



So the theoretical framework developed will help us understand and predict the boundary or transition from one flow pattern to another. So as far as this course is concerned our focus is mainly on getting both a physical understanding as well as the applying mathematics to get a quantitative understanding of the particular system. So when we talk about physical understanding.

I am talking about analyzing a specific problem. I am talking about the Physics of that problem. The mathematical framework that we are going to be establishing is sufficiently general so that you can actually apply those mathematical frameworks to any system and only thing is the inferences are going to be different for different systems because the Physics is going to be different.

So one of the things we are going to focus on in this course is both the mathematical approach as well as the physical understanding you need to have both. And normally what happens when you do your math courses you learn a lot of the mathematical techniques you do not see what the relevant physics is when you do a physical problem you possibly are applying a very specific technique and you do not realize how general it is the mathematical technique that you are using.

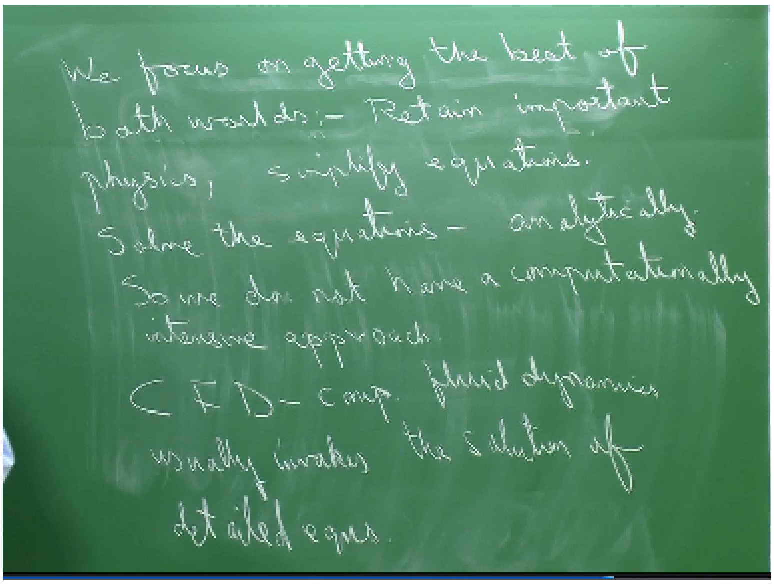
So you are going to learn not only mathematical techniques which are sufficiently general. but you are also going to be applying these techniques to a host of problems which will arise in the context of engineering and so that you can actually see applications of whatever you are learning. So we will use both a physical and a mathematical approach to analyze a system. So these go hand in hand.

As a result, since we want to do both mathematics as well as physics. We are going to be focusing mainly on trying to keep the model as simple as possible, retain all the important physics so that we can develop a much better understanding of the problem, physical understanding of the problem and look for analytical or semi analytical solutions which are not computationally intensive.

So I am trying to get the best of both worlds in the sense I am trying to get quantitative information by doing mathematics. I am trying to deepen my understanding of the physical problem by retaining important physics throwing out all the not important effects and this helps me focus on what I think is important physically and a mathematics gives me quantitative results.

And since I have thrown out all the unwanted physics I can possibly aspire to get analytical or semi analytical solutions. So that my reliance on the computers on computationally intensive method is actually reduced. So there is a completely different approach here as compared to what people in classical computational fluid mechanic use.

(Refer Slide Time: 36:12)



We focus on getting the best of both worlds retain the important physics simplify the equations then solve the equations analytically. So we do not have a computationally intensive approach. This is as opposed to computational fluid dynamics because Multiphase Flows can also be analyzed using CFD techniques. CFD computational fluid dynamics usually invokes the solution of detailed equations.

So this is a computationally intensive approach as a result of which the physical understanding that we may get is lost. So usually when people are using computationally intensive approaches they may be going towards software packages. And the software packages if used properly would give you accurate results. If you do not use them properly you may get erroneous results and it would be very difficult for you to find out if the results coming from this packages are indeed right.

One of the things you are going to do in the course is come up with solutions for physical problems under some circumstances under some limiting circumstances. So these solutions are going to be analytical what you can do is you can verify this analytical solutions using the computational packages that are available and see if the results coming from the computational packages are indeed the same as what you get from your analytical solutions.

So what I am trying to advocate here is that the computational packages, computational fluid dynamic packages like fluent definitely have their virtues that are important, but one should be very careful in using these packages because if you do not use these packages carefully you are going to get erroneous results. Definitely you would get results okay you would get some velocity field, you would get some pressure fields, but you need to develop the ability to make sure that the results that are coming are right.

You need to be able to figure out if those results are (()) (40:13) accurate. Whether they are actually worth their weight involved. To do this you may want to run the same package under some simplifying situations like may be very low Reynolds number so that the turbulence is eliminated. Then you would be able to possibly get an analytical solution like your laminar flow.

The parabolic velocity profile in a pipe. See if the computational packages give you the same parabolic velocity profile so under low Reynolds number. If it does, then when you actually use the same package for high Reynolds number flows you have more confidence. So that of course was a very simplistic example, but you would have situations which are more complicated.

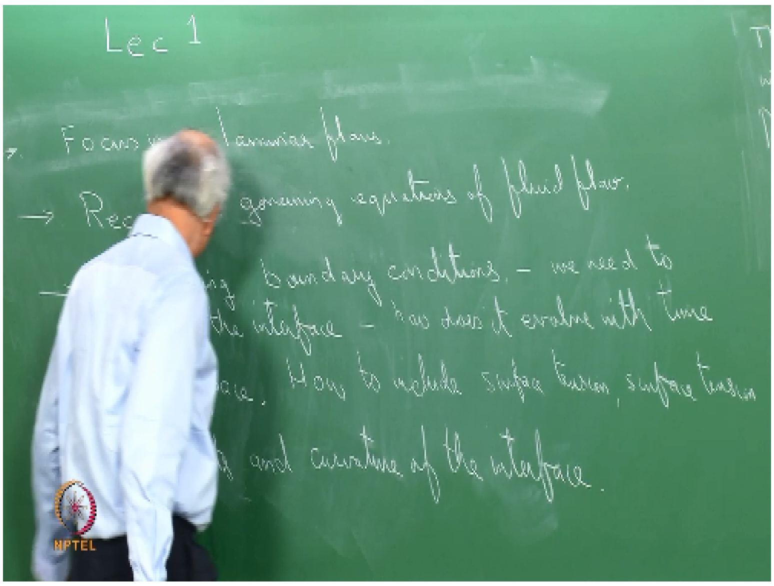
Wherein getting an analytical solution something analogous to your parabolic velocity profile is very important to validate results coming from computational packages. So and that is the reason why we in this course are going to actually use this kind of (()) (41:27) trying to simplify equation concentrate on important physics try to get the simplified equation solve them in an analytical manner and if you cannot solve them analytically at least solve them numerically, but not very computationally intensive and get results.

So you would therefore in this course necessarily get to a point where you will have to solve some of the problems, writing programs in some of these packages like MATLAB or Mathematica or Maple or Fortran whatever suits you. So as far as this particular course is concerned this is what we are going to be doing. We are going to be looking at situations simplifying the equations and retain the important physics.

And try to answer important questions which are of engineering interest like can you predict the transition from one flow regime to another flow regime. Are you in a position where you can predict how you can go from the droplet flow to the slug flow or from the slug flow to the stratified flow. If you can predict this, then you can actually validate that using experiments. If you are not in a position to do experiments, then you can possibly predict validate this particular prediction of this boundary by doing computations on a package like fluent.

And see if the results of your prediction are indeed validated by the computational experiments on the computer.

(Refer Slide Time: 43:22)



So coming to a little bit of the details of this course. We would in fact mainly be talking about laminar flows focus is on laminar flows and we would look at doing a small recap just to make sure that this course is self consistent so we would recap the governing equations of the fluid flow. And I am going to do this in the context of using things like the Reynolds transport theorem and see how we can get the Navier Stokes Equations.

One of the important challenges that you are going to face when you are solving multiphase flow problems is in analyzing boundary conditions. When you have a change from a stratified flow to a slug flow the boundary itself is going to change. Normally what we are use to solving problems where the boundary is fixed, the boundary does not change. Now we want to be in a position to be able to track how the interface the boundary between 2 liquids is actually going to change with time and position.

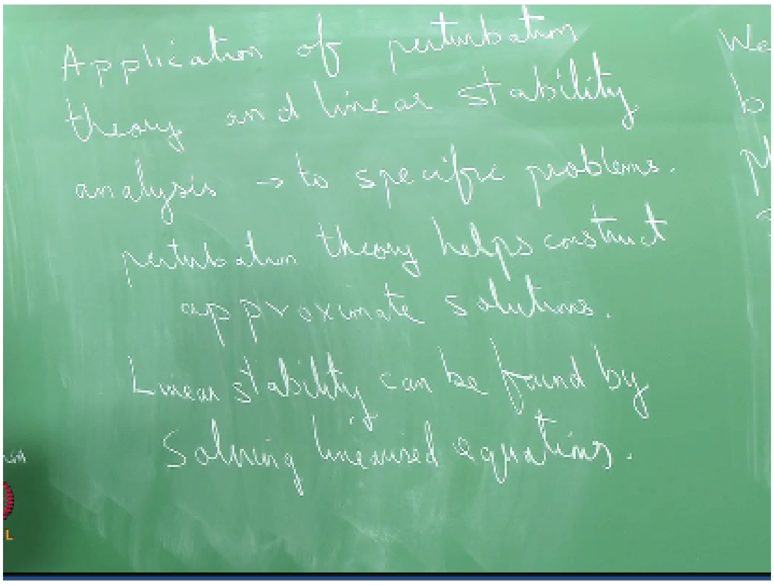
So we need to track the interface how does it evolve with time and space. And you are used to using boundary conditions of a particular kind continuity of velocity, continuity of tangential stresses and in the past what you have done is you have not worried too much of the existence of the surface tension along the interface. Now the presence of the surface tension is actually going to modify these classical boundary conditions that you are used to.

For example, if you have a curved interface there is going to be a pressure gradient the normal stresses are going to be balanced because the surface tension force is present at the interface. If there is a concentration gradient of let us say surfactant along an interface. The surface tension is going to vary along the interface. There is going to be gradient of the surface tension/

And this is going to also cause inequality between of the tangential stresses across the interface. So these generalizations to the existence of the surface tension, to the existence of a surface tension gradient, to the existence of the curvature of the interface and how the boundary conditions are formulated in this scenario is what we are going to talk about and that is extremely important.

So how to include surface tension, surface tension gradients and the curvature of the interface.

(Refer Slide Time: 47:50)



Finally, and this is possibly the application part where we are going to talk about the application of methods like perturbation theory and linear stability analysis to specific problems. Now I spoke about you know getting analytical solutions and what we are doing when you are talking about analytical solution. We use these simplified equations here, we use these general equations which we simplify depending upon a specific problem.

And use this perturbation approach and the linear stability approach where you actually convert a bunch of non-linear equations to linear equations and then try to analyze them. The idea is that liner equations is something which we all know how to analyze using things like Laplace transform using things like Fourier transform. And we can hope to get analytical solutions and get insight.

So that is basically the focus of the course what the perturbation theory tells you is it helps you construct approximate solutions which are analytical and what we do is we exploit the presence of things like small parameters in the space in the operating conditions governing equations and I can actually construct solutions to this the perturbation theory helps construct approximate solutions.

And the linear stability can be found by solving the linearized equations. So I think at the end of the course what I expect is that the students would have actually got an enough knowledge that when they go to the library and pick up a journal and look at a paper they will be in a position to be able to derive those equations and to be able to understand what this paper is talking about to understand the physics explain in the paper, to understand the mathematics explain the paper and to be able to do research in this area.

I think the course itself will consist of in class exams where we will be testing your fundamental knowledge that you gained in this course, will have computational assignments where we will make you solve your problems on the computer and of course a final exam which would again test your fundamental knowledge. Thank you very much.