Computational Techniques

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Module No. #03

Lecture No. #03

Linear equations

Hello, and welcome to this lecture three of module three, what we did in the previous lecture was to consider an a small example, example of two equations or two linear equations in two unknowns, and then three three equations in three unknowns, and solve that using a gauss elimination technique. What we did was try to first motivate the gauss elimination technique by solving the equations naturally; from this point onwards, we will talk about the gauss elimination technique.

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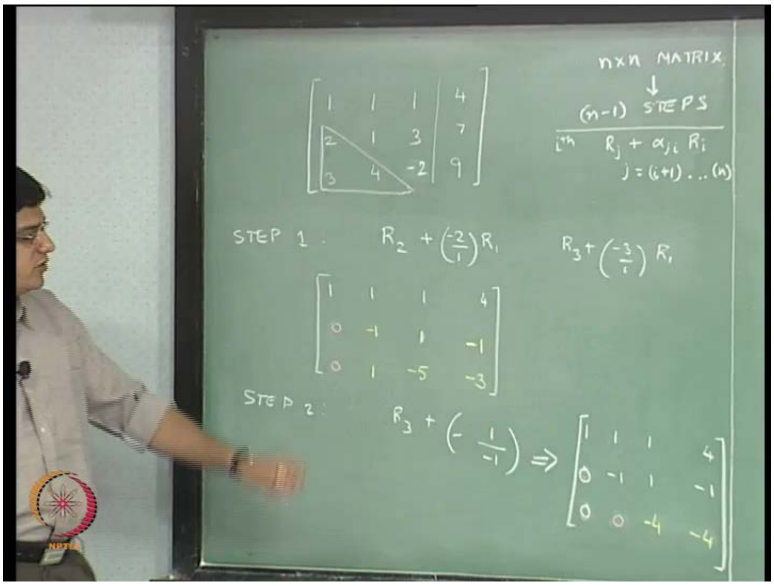

I have written down the matrix A, and the vector b, that we considered in the previous lecture. In this matrix form, I have also shown here, this lower triangle part of this matrix. We are going to make this lower triangle 0, using the gauss elimination process. So, what what we did was use row one in order to get zeros over here, and over here. Next, we use row two in order to get 0 over here, when we ended up doing when we did that we ended up with on upper triangular matrix; that means, the values above at, and above the diagonal may or may be non zeros, where as all the values below the diagonal are definitely going to be equal to 0.

So, that is what the gauss elimination process is going to lead us to, the first step, that we said was R 2 plus minus 2 divided by 1 into R 1, and R 3 plus minus 3 by 1 multiplied by R 1; What that let us to the first row do not change so, we get 1 1 1, and 4; the second row, we definitely know the this term is going to be 0; that is through the choice of our alpha 2 1. So, we do not need to do any computation over there. We will just get 0, at this particular location; this is going to be 1 minus 2 which is minus 1; this is going to be 3 minus 2 which is going to be 1; and this is going to be 7 minus 8 which is minus 1. This term, again we know for a fact that this is going to be 0, we do not do this extra computation. We will just put 0 over here, this is going to be 4 minus 2 which is going to be equal to 2; this is this term sorry 4 minus 3 not 4 minus 2 sorry 4 plus minus 3 multiplied by 1. So, 4 minus 3 which is going to be equal to 1, this is minus 2 minus 3 which is going to be minus 5, and 9 minus 12, which is going to be equal to minus 3 over here.

So, this is what we get at the end of step 1, and in step 2. We will have R 3 plus 1 minus of 1 divided by minus 1 minus of this coefficient divided by this particular coefficient. That is what we will have in step 3, and as a result of step 3, we will get this matrix as 1 1 1 4, that remains unchanged; 0 minus 1 1 minus 1 remains unchanged; and this is the row, that will change this remains unchanged, because this guy 0. So, we would not do anything to it, this we know for sure that this value is going to turn to zero. So, we do not do this computation just put that as 0. So, the only computations, that we will do is minus 5 plus 1 which is minus 4, and minus 3 minus 1 which is also equal to minus 4 .

So, this is the gauss elimination process, what I will do in each of this steps is all the elements, which we have changed through the row operations, I will put them in different colors. So, we have changed this with row operations we have changed this with row operations we have changed this with row operations 1 minus 5 minus 3 we have change with row operations 1 minus 5 minus 3, we have changed with the row operations; this 0, and 0, although we have done the row operations; that row operation was really the computation of this coefficient; we have not done an explicit addition over here. So, I will use a third color, the red color to represent this; likewise, I will use the red color to sorry in this particular operation, these values are not changed. So, I will keep using white over here this will I will use red color, because we know for a fact, that this guy is going to be 0 after the row operation. So, we do not change it, and these have been affected by row operations. So, I will put them as yellow colors. So, a few things that we had noted in the previous lecture as well, is that gauss elimination for n by n matrix; it has n minus 1 steps n minus 1 steps. In this case, it is a 3 by 3 matrix, and in the 3 by 3 matrix, we had two steps 3 minus 1 equal to two steps in it step we do R j plus alpha j i multiplied by R i where j equal to i plus 1 i plus 1 to n.

In step 1, we will change R i plus 1, that is R 2 R i plus 2, that is R 3, and so on up to R n; in this case, n are 3. So, we are changing row two, and row three; in the second step, we will change in the second step i equal to 2. So, j is going from 3 to 3; that means, we will do only one row operation, and that is going to be R 3 plus negative of a 3 2 divided by a 2 2 multiplied by R 2. So, that was the row operations row operation that we did. So, this resulted in the lower triangular matrix 1 1 1 4 0 minus 1 1 minus 1 0 0 minus 4 minus 4.

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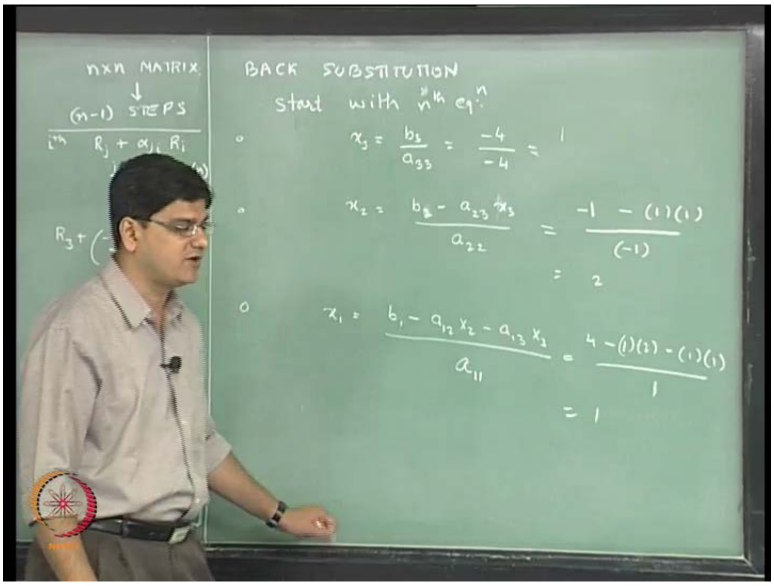

So next, what we do is the back substitution step; in back substitution, what we will first do is, we will use the last equation to get. So, nth equation we will use to get a n n minus 1 the equation two, and the value of a n in order to get a n minus 1 sorry x n minus 1. We will use n minus 2th equation, and the values of x n minus 1, and x n in order to get x n minus 2 so on, and so, forth. We start with nth equation; so, our x n in this case, x 3 is going to be equal to just b 3 divided by a 3 3, which is just minus 4 divided by minus 4 which is equal to 1. So, that is the first step in back substitution; the second step in back substitution is going to be to use x to use the n minus 1th equation, in order to get x n minus 1. So, our x 2 is going to be equal to b 3 minus a 2 3 multiplied by x 3 divided by a 2 2. Why because our equation was a 2 to x 2 plus a 2 3 x 3 equal to b 2 sorry not b 3. So, we take a 2 3 x 3 on to the right hand side, and divide by a 2 2; and what we are going to get is minus 1 minus 1 multiplied by 1 divided by a 2 2 which is minus 1; so, that is minus 2 divided by minus 1 which is equal to 2. So, we will get x 2 equal to 2 in the second step, and in the first step, we will get x 1 equal to bone minus a 1 2 x 2 minus a 1 3 x 3 divided by a 1 1, which is going to be equal to in this particular case, 4 minus 1 into 2 minus 1 into 1 divided by 1 which is 4 minus 2 minus 1 that is 4 minus 3 equal to 1, and that is the value of x 1.

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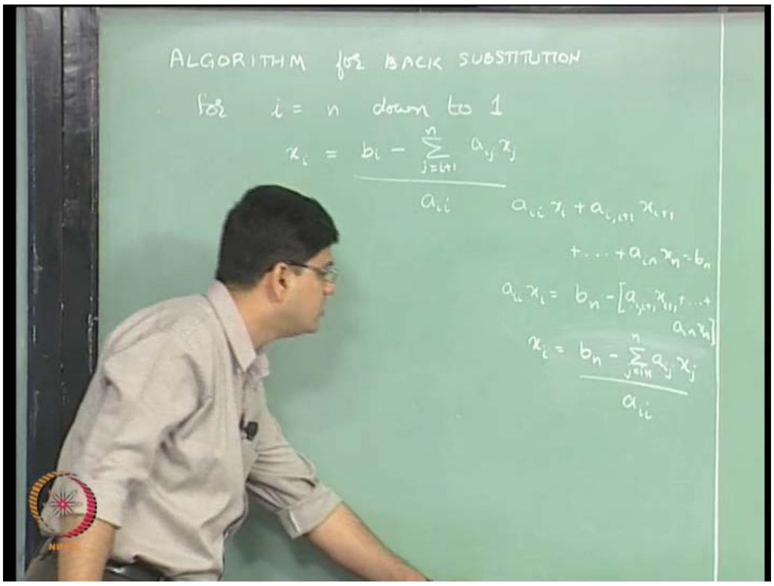

So, that is the overall gauss elimination with back substitution for a general n by n equation. So, writing down the algorithm for back substitution; in this, in the back substitution step, we go, we let the, I go from n to n minus 1, and so on up to 1. So, for i going from n down to 1, we are going to compute x i is, so for the ith equation is going to be a i i x i plus a i, i plus 1 x i plus 1, and so on up to a i n x n equal to b n. So, we take all of these to the right hand side. So, we will get a i i x i is going to be equal to b n minus a i i plus 1 x i plus 1 plus so on up to a i n x n whole divided by a. So, this is going to be this term, and in the shorthand notation, and dividing by x i by a i i; we will get b n minus summation a i j x j j equal to i plus 1 to n divided by a i i.

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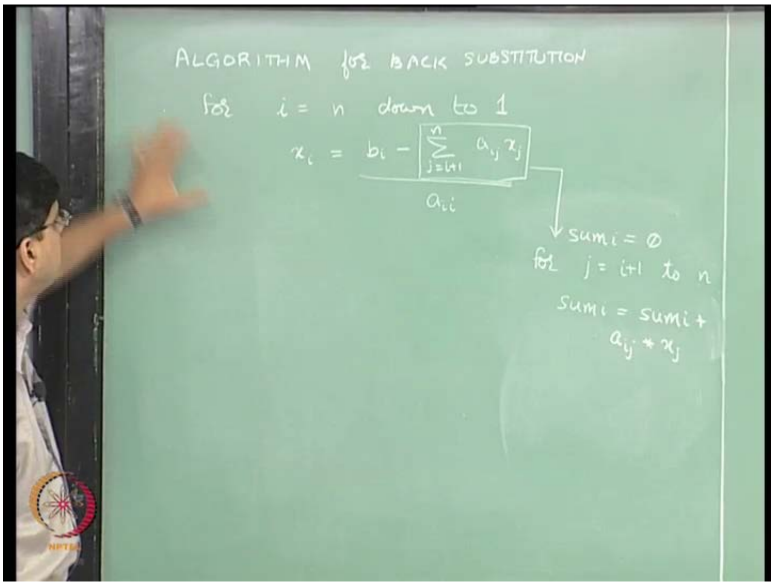

So, this is the value of x i, that we are going to get so, for i equal to n down to one, we are going to compute x i equal to b i minus summation j equal to i plus 1 to n a i j x j divided by a i i. So, this is how we are going to compute the back substitution step; where the summation itself, we are going to compute this as for j equal to i plus 1 to n. Let us, call this particular term as sum term say sum i equal to 0 we will call this, and then we will write this as sum i equal to sum i plus a i j multiplied by x j, and this is how we are going to calculate this particular sum over here. So, we have b i minus this sum i divided by a i i that is going to be our algorithm; again, a rough algorithm for the back substitution step that we are going to follow.

In the next lecture, I will take up an excel example, using Microsoft excel to exactly show, how we are going to the gauss elimination step, and the back substitution step. So this is overall, what I have here for the gauss elimination, and the back substitution. Next, what I am going to do is talk about two other methods, which are the gauss Jordan method, and next method is going to be the L U decomposition method. I will just, give I will just, use this particular example, to show what gauss Jordan method is going to be, and how it is different from the gauss elimination method, and I will you also use the same example to show the L U decomposition method.

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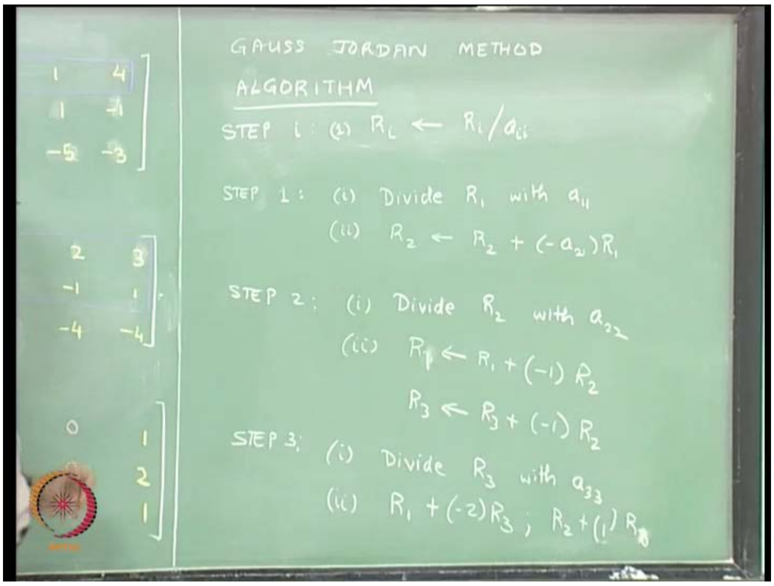

Once, I am done with that I will go, and do one important thing, and that is to figure out, how much computational effort the gauss elimination method requires, and where the L U decomposition method, and where the gauss Jordan method are going to find practical applications. So, now we go on to the gauss Jordan method, what we did in gauss elimination is we use the ith row, in order to get the coefficients in the ith column 0, only for the rows below the ith row. So, what we ended up with at the end of the gauss elimination process, we ended up with an upper triangular matrix.

So, for example, when we started with the original or when we started when we started with this particular original matrix 1 1 1 2 1 3 3 4 minus 2, after at the end of the gauss elimination step, we ended up with the matrix 1 1 1 0 minus 1 minus 1 0 0 minus 4, which basically meant, that we converted essentially these guys into zeros, and only using ith row, in order to change the values of the row i plus 1, i plus 2, and so on up to row n. We did not use ith row to do any changes in any of the upper rows. So, that is what we actually did, when in the gauss elimination process; in the gauss Jordan process, we are going to use the ith row, in order to change all the upper rows as well as the lower rows. So, that we will end up with an identity matrix.

So, the idea of the gauss Jordan method is the idea behind the gauss Jordan method is to convert the matrix A into an identity matrix; using row operations, very similar to the gauss elimination method. So, step 1, the thing that we did in step 1 in gauss elimination method is, we did R 2 minus alpha 2 sorry R 2 plus alpha 2 1 multiplied by R 1 R 3 plus alpha 2 1 multiplied by R 1. Instead in gauss Jordan method, the first part of step 1 is divide row one with a 1 1. So, we divide the entire row with a 1 1; so, that the a 1 1 element, we will make that equal to 1. In this particular example, that we had over here this guy was already equal to 1.

So, we do not do any row operation, we do not do any first step over here explicitly, because we already have this equal to 1. If you did not have this equal to 1, we would have done the division, but when it comes to a computer, we do not know a priory that this particular number is going to be equal to 1. So, rather than doing having a, if statement to check whether this number is 1 or not it is always more convenient to just go ahead, and do this division step, in an in a process that keeps repeating at each and every row. So, in a computer, I implementation we will indeed divide this particular row by a 1 1; then use row 1 to make zeros over here.

So, the in case of gauss elimination, I will write down this overall matrix over here.

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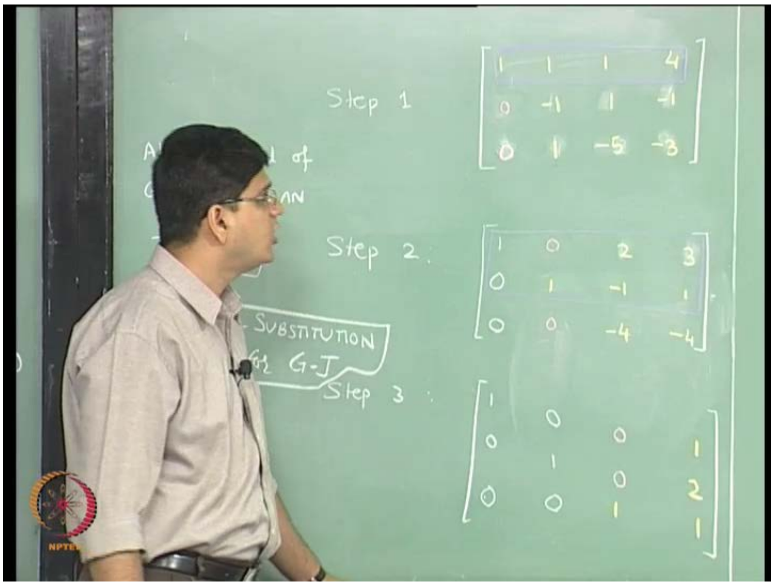

At the end of the first step, the first part of the first step, we have divided R 1 with a 1 1, and when we do that, we will we will change all these guys. So, to a different color, I have changed to yellow color, because we have done the row operation on that particular row also. And then, we will have R 2 as R 2 plus minus a 2 1 multiplied by R 1. So, previously we had this coefficient, as the ratio a 2 1 divided by a 1 1. Now, we do not need that ratio, because a 1 1 has become equal to 1 through the first operation. So, with now what we will do is, this row minus twice this row is the computation that we will do, now this number we know is going to become equal to 0.

So, in a computer implementation, we are not going to do this particular calculation. So, I will do the put this in red color, and we will do rest of these calculations over here. So, this is going to be 1 minus 2 that is going to be make it equal to minus 1, this is going to be 3 minus 2 that will make it equal to 1, this is going to be 7 minus 8 that will make it minus 1. This 3 is going to be automatically being converted to 0. So, we would not do this particular step at all; we will just change that to 0. So, this is what we will get over here, 4 minus 3 we will get this as equal to 1, minus 2 minus 3 we will get this as minus 5, again this is same as before, and 9 minus 12 we will get this equal to minus 3. So, the difference that we find between the gauss elimination step, and the gauss Jordan step; in the step 1 is, what I will show, I will just box it with this purple color, rest everything is the same that we did in the gauss elimination step; what is different in the first operation, in the step 1, is that we have divided the row one with a 1 1.

So, this is the result after step 1; step 2 is going to be 1 is divide R 2 with a 2 2. So, when we do that, now the first row of course, as remain unchanged 1 1 1 4 0 1 or I should actually be using yellow color so, 0 1 minus 1 1, because we have divided throughout by a 2 2. And then the second step, the second part of the step is going to be R 1 should be equal to R 1 plus negative of this coefficient multiplied by this guy. So, negative 1 R 2, and R 3 should be R 3 plus again R 3 was again 1 over here so, plus minus 1 multiplied by R 2. So, this is, these are the steps that we will implement. So, what that means is that, we will we will have once we have this particular guy, we will subtract from this particular equation, we will subtract this equation. So, which means this term is going to become equal 0; this is going to be equal to 2, and this is going to be equal to 3.

And likewise, the row below has also been change, this particular guy has not been affected - this 0 has not been affected; this particular element will become 0, we know for a fact that this will become 0; likewise, we know for a fact that this will become 0. So, we would not do the computation over here. And then, the computation will be done over here, this minus 5 is going to as we had done before this minus 5, again it is going to be equal to minus 4, and then this minus 3 is also going to be equal to minus 4. In general, for the gauss elimination step, our gauss elimination would have ended over here. The difference in gauss elimination, and gauss Jordan, now lie in both these rows, because we have divided throughout by this particular element, and then we have done the same thing over here.

So, the gauss elimination step, if we were to follow, it would have ended at this point; but in gauss Jordan step, we have the nth step also, and n the step so, like gauss elimination, we follow n minus 1 step with a difference that we make the a i ith element equal to 1, but we have the nth step also, and in that n the step again, we will have divide R 3 with a 3 3. So, when we do that, we will have 0, and we divide this by 4 minus 4. So, we will have 0 0 1 1, and then we will do R 1 plus in this case, this guy is 2 over here. So, R 1 plus minus 2 into R 3, and then R 2, and the coefficient over here is minus 1.

So, it is just going to be R 2 plus 1 into R 1. So, sorry R 2 plus 1 into R 3; it is actually not 1, it is minus of negative 1, and therefore, it is 1. So, what change that will make so, this 1, and 0, remain as they are this 0, and 1, remain as they are, we know that these two elements are being made 0. So, we are not going to do explicit computations over here. So, we are putting this in red color chalk, and only things that are going to change are these two elements over here. We are we are essentially, doing R 1 minus 2 times R 3 so, R 3 minus 2, that is going to be equal to 1, this we will have over here and R 2 plus 1 times R 3.

So, in the new R 3, the value over here is 1 the old value of R 2 was 1. So, 1 plus 1 is going to lead us to 2. So, this is, what we get at the end of the gauss Jordan method, the matrix A has been converted into an identity matrix; the matrix b has been converted into a solution matrix. So, what happens at the end at the end of gauss Jordan method, we have basically identity multiplied by x bar equal to b dash bar, where b dash is the b that has been transformed through the row operations, that were perform on the matrix A as well; which basically means, that what we get really on in place of the vector b is the solution, that we are seeking. So, no back substitution is required. So, unlike gauss elimination step, we do not require any back substitution in the gauss Jordan step, and the algorithm for gauss Jordan is that we have step 1, step 2, step 3.

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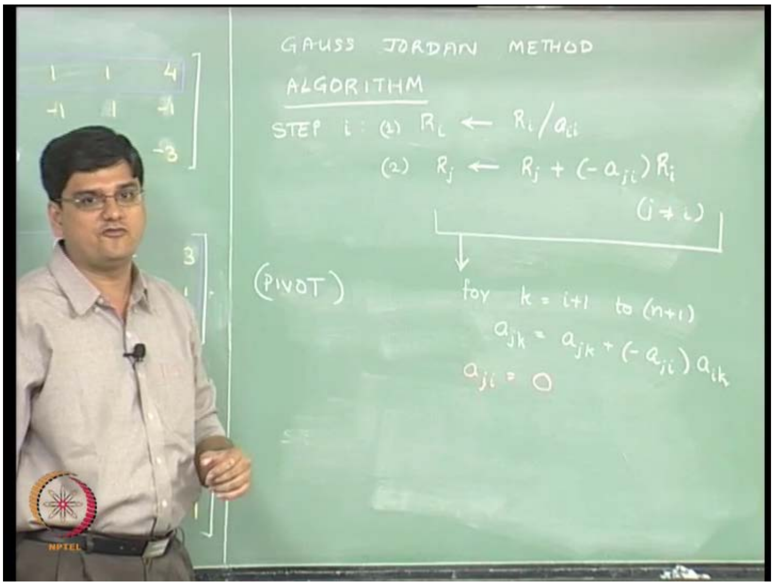

So that means, we have n steps; and in the ith step, we have R i is nothing, but R i divided by a i i. So, R 1 divided by a 1 1; R 2 divided by a 2 2; or R 3 divided by a 3 3. So, for the ith step, the first part of the ith step is, R 1 is this, and the second part of R ith step, consists of n minus 1 row operations; n minus row 1 row operations is because row 1, row 2, up to row n, except the ith row has been changed by ith row. So, the second part is going to be R j is R j plus minus a j i, because it is the ith column, and the jth row multiplied by R I, where j is not equal to i. So, R 1 is R 1 plus negative of a 1 2 multiplied by R 2; R 3 is R 3 plus negative of a 3 2 multiplied by R 2; likewise in step 3, R 1 is negative of a 1 3 multiplied by R 3 a 1 3 multiplied by R 3; R 2 is negative of a 2 3 multiplied R 3. And each of those row operation, itself has multiple operations for each element in that particular row. And let us, look at what we did in thus in the third row operation; in the third row operation, the first element was 0; the second element was 0; as a result, we did not do any computation for the first, and second element; the third element, we knew that, these two guys are going to be equal to 0. So, we directly substituted that those equal to 0, and only did the computation for rest of the steps.

So, what this particular procedure or this particular step sorry expands into or sub steps expands into is that for column j j plus 1 sorry for column i plus 1, i plus 2, i plus 3, and so on up to n, and then one more for the b the step. So, for k equal to i plus 1 to n plus 1 what we are going to do is a k; that means, n sorry a j k, because we are doing the jth row jth row kth column a is going to be equal to that particular element; jth row kth column a plus negative a j i, that remains as a j i multiplied by the element; the kth element in the ith row, which is going to be a i k.

So, this is what this particular step expands into so, this is those yellow things, that I have I have shown over here in this in these particular columns, and the red guys will also have that step, and that is going to be the red guys over here; in the third step are a 1 3 equal to 0 a 2 3 equal to 0; that means, a j 3 equal to 0 for j going from 1 and 2. So, that is going to be a j 3, which is in this in the third step, i equal to 3; in the second step i equal to 2, so on, and so forth. So, a j i are going to be equal to 0. So that is, this step, we are we are going to implement, if we implement this step over here. There is going to be a problem that will be caused over here, as a result this step will be implemented after this procedure is done.

So, I will erase this from over here, and I will put this at the end of it. And again, explicit computation of alpha j i is not needed, because alpha j i is nothing, but a j i, what because what we could have done over here is alpha j i equal to a j i, and then put a j i equal to 0; an easier way to do is just replace this a j i from over here to over here. So, this is the overall procedural steps, for the gauss Jordan elimination. So, the algorithm is in the ith step, we will first divide the ith row with a i i. So, a i i is the diagonal element on the ith row; the diagonal element on the ith row is also known as the pivot element. So, we will divide the ith row by the diagonal element on the sorry we will divide the ith row by the diagonal element on the ith row, and then use ith row to make ith column zeros in row 1, row 2, up to row n, except ith row.

So, j going given given by j plus minus a j i R j where j not equal to i, and this expands itself - expands into these sets of equations. So, that is the algorithm, overall for the gauss Jordan method. The difference between gauss Jordan, and the gauss elimination method is that in gauss elimination method, this step is not there; the second difference is that, this particular coefficient is a j i divided by a i i, and we have represented that as alpha j i, and it is not for j not equal to i, but it will be done for j equal to i plus 1, i plus 2, up to n. So, that is the difference between gauss Jordan, and the gauss elimination method. So, what we have done so, far is look at gauss Jordan, gauss elimination methods.

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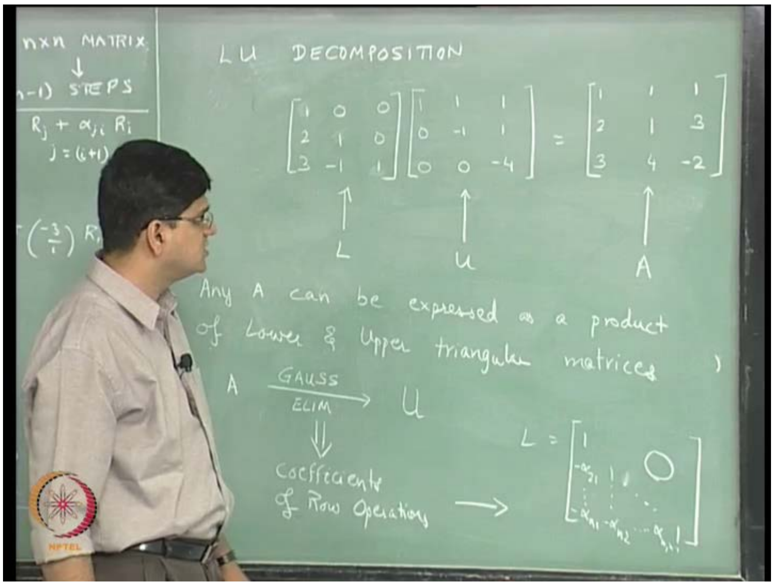

I will try, I will now, next talked about another method, which is known as the L U decomposition method. And in order to talk about L U decomposition method, I will use the results that, we had in when we did the gauss elimination method. So, this was our gauss elimination result; from the gauss elimination method, we got this as the upper triangular matrix. So, let me write this upper triangular matrix as 1 1 1 0 minus 1 1, and 0 0 minus 4. Now, what I will do is, I will look at these coefficients, this was alpha 2 1; this was alpha 3 1; and this was alpha 3 2; and I will create a matrix, which is going to be a lower triangular matrix, containing this alphas in the lower triangular elements, which I will write this as negative of alpha 2 1; negative of alpha 3 1; negative of alpha 3 2.

So, what we get over here, when we do that is 0 0 0 0 0 0, and alpha 2 1 was minus 2 so, negative of alpha 2 1 is plus 2; alpha 3 1 was minus 3 so, negative of alpha 3 1 is plus 3; and alpha 3 2 was 1 so, negative of 1 is what we write over here. So, this is one matrix; this is the other matrix; let us, take a product of these two matrices oh sorry not 0 0 0. I should be putting 1 1 1 over here my mistake. So, we have this 1 1 1; we have put zeros in all the upper triangular elements, and the lower triangular elements, we have put that as negative of the coefficients that we have used in the gauss elimination step.

Let see, what product we get this is going to be equal to 1 1, and 1; this will be 2 plus 0 plus 0 that is 2, that is 2 minus 1 that is 1, 2 plus 1 equal to 3, and this is 3 plus 1 equal to 4, and 3 minus 1 that is 2, minus 4 equal to so, 2 minus 4 equal to minus 2. So let see, what matrix this is, if we compare this with matrix a 1 1 1 2 1 3 3 4 minus 2; that is exactly, what we have got by multiplying these two matrices 1 1 1 2 1 3 3 4 minus 2. So, this is a lower triangular matrix L; this is an upper triangular matrix U; and this is the matrix A. So, L U decomposition is nothing, but any matrix A can be expressed; the choice of the lower, and upper triangular matrix is not unique. This is just one L, and U matrices that excuse me, this is just one L, and U matrix that we can get through the gauss elimination process.

So, we start with A, and perform the gauss elimination. The gauss elimination will give us the matrix U, which is the upper triangular matrix, and it will also give us coefficients of row operations. We take these coefficients of the row operations negate them, and put them in a lower triangular matrix, and we will get that lower triangular matrix L, and that lower triangular matrix L is just 1 1 or in general, for an n-dimensional case; the diagonal elements are all 1; the upper triangular elements are 0; and the non diagonal elements are minus alpha 2 1; and so on up to minus alpha n 1 minus alpha n 2, and so on up to minus alpha n comma n minus 1. This is, what our lower triangular matrix is going to be, I will re write this lower triangular matrix over here in the more general form.

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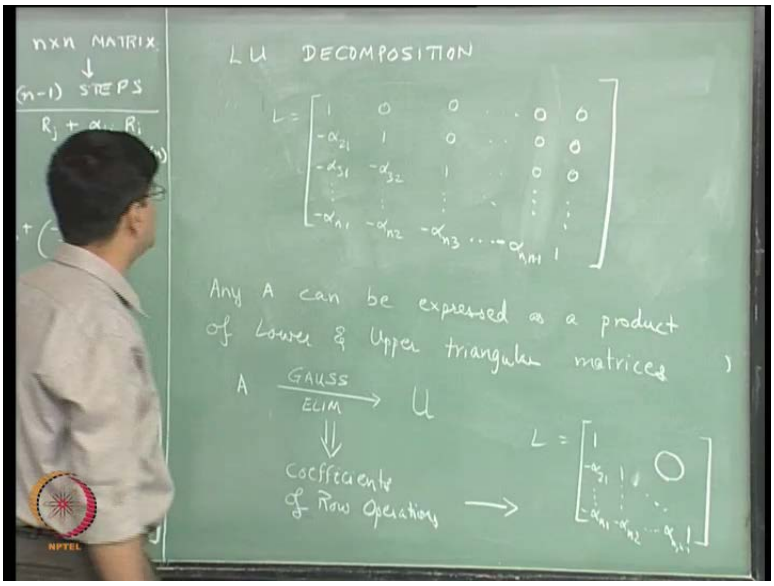

So, this is what, we will get 1 0 0 0 minus alpha 2 1 1 0 0 0 minus alpha 3 1 minus alpha 3 2 1 0 0 0 and so on, up to minus alpha n 1, minus alpha n 2, minus alpha n 3, up to minus alpha n n minus 1, and ending with the number 1. So, this is the lower triangular matrix, the upper triangular matrix is what we get as a result of the gauss elimination step, and the product of the two matrices is going to be the matrix A. So, that is the idea behind the L U decomposition. Now question is, where L U decomposition is useful. So, fine we have now, gotten the matrix L; and we have now gotten the matrix U; what do we do with the matrix L, and the matrix U.

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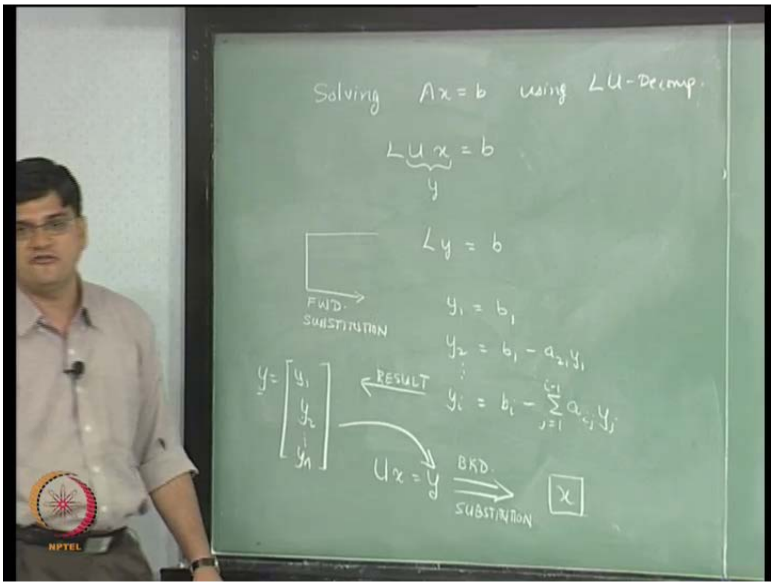

And they answer to that question is, that we can use this L U matrices, that we obtain in order to get this solution of Ax equal to b. So, if we have performed L U decomposition; and obtain the matrix L; and the matrix U will be able to write this particular equation as L U x equal to b. Let us, write U multiplied by x or equal to some other variable say y; if U x is going, we are going to write this equal to y, we can write L y equal to b; where the matrix L has a form of this type. What we get in this particular form is that the diagonal elements are all 1, and what we get is the upper diagonal elements are all 0. So, instead of a back substitution step, we use a forward substitution step to get L, not to get L sorry to get y. So, the forward substitution step, again this is the original b matrix; the unchanged b matrix in the L U decomposition case, we apply the gauss elimination only to the matrix a not to the matrix compose of A and b together.

So, you just apply the L U decomposition to matrix A. And then with combination of forward, and backward substitution, we try to solve this particular problem, and the forward substitution is going to be y 1 equal to b 1, y 2 equal to b 1 minus a 2 1 y 1, and so on y i equal to b i minus summation of a i j y j, j equal to 1 to i minus 1 and so on, up to n. So, this is what we will do in the forward substitution step? When we perform the forward substitution step, we will get the matrix y. So L y equal to b, we are solving in order to get matrix y.

So, result of the forward substitution step is, y bar equal to y 1 y 2 up to y n; y 1 y 2 up to y n is nothing, but this the solution of the equation U x equal to y; remember, we have substituted U x, and replaced it with y. So, the second equation is going to be U x equal to y, where y is obtained from over here, and you can solve this using backward substitution. And when we solve this using the backward substitution step, we will indeed get x as a result of the backward substitution step. The backward substitution step is exactly, the same as the backward substitution steps; that we have considered, we have discussed, when we talked about the gauss elimination method. So, that is how we solve the equations using the L U decomposition method.

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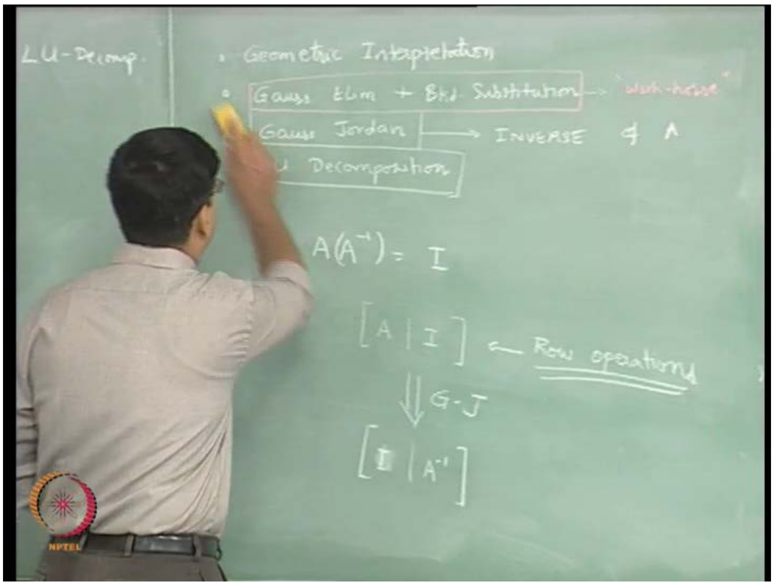

So to recap, what we have done so, far is the following: started with two equations in two unknowns, two linear equations in two unknowns, and saw the geometric interpretation, which we said was intersection of hyper surfaces, we took a small aside into the whole idea of vector spaces, and try to motivate a different geometric interpretation as the linear combination of various vectors. So, the two geometric interpretations, we are we initially considered. After that, we started talking about the gauss elimination method, and also we talked about backward substitution; we specifically took a 2 by 2, and then a 3 by 3 example, to talk about the gauss elimination method, but tried to expanded to general n dimensional systems system in n unknowns; that is what we talked about that; that is what we did when we talked about the gauss elimination method; then we took two other methods, which is the gauss Jordan method, and then the L U decomposition method. So, this is what we have done so far. Now the question is, why would you use gauss Jordan method or why would you use gauss elimination method or is it actually useful to learn about an L U decomposition method, when we already have gauss Jordan method as a way of solving.

So, to the answer to that question is that the gauss elimination plus backward substitution method is really the workhorse that is the that is the method, that is typically used in most example. This is the workhorse or it is the most popular method the reason, why it is most popular method is, because it compared to the other methods. It has the lowest amount of computational effort required. So, when n is going to be of an order of one million, a small factor also becomes very large number, because as we will see in the next lecture.

These method scale as n to the power 3, which basically means that the computational effort of a matrix of the size ten to the power of the size is one million, which is ten to the power of 6 the computational effort is of the order of ten to the power eighteen operations on that particular matrix. So, it becomes a really a big issue, how much computational time you are going to spend in order to solve this problems; as a result this gauss elimination plus backward substitution is the workhorse; there is there are a couple of modifications that, I am going to talk about in the next lecture, that are that are very essential which we have not yet covered.

The gauss Jordan method is very popular also lot of people actually, prefer to use gauss Jordan over gauss elimination as well, but it is especially useful in finding inverse of a matrix. And finally, the L U decomposition method is useful, when we have to solve repetitively the equations of the form A x equal to b m where m, when b keeps changing with every iteration. So, we will see examples of this type n module 9, module 8, and module 9, when we talk about basically the O D E's ordinary differential equation boundary value problems, and partial differential equations problems, where those problems get converted into linear problems of this sort where the matrix A does not change often the matrix A remains constant; it is the matrix b that keep changing. So, by removing the gauss elimination step, and replacing it with forward and backward substitution step, we can get significant improvement in the computational accuracy.

Please, keep this in mind when we go to module 8, and module 9, where I will quick, I will spend a few minutes may be 15 minutes to 20 minutes talking about, where we encounter problems of the nature A x equal to b m. And in what cases methods, such as this L U this modified L U decomposition method can be used, and to finish of this lecture, I will talk about to Jordan decomposition to find the inverse of matrix A, and what we do over that is, we write the equation in the form A A inverse equal to i.

So, what we have done is, we have just replaced x bar with A inverse, and we have replaced b bar with i. So, if we were to then get a matrix of the form A i and do row operations. And then, we were to do row operations on the matrix A i and use using the gauss Jordan method at the end of these row operations, what we will be left with is the identity matrix on to on to this part, and what we get on the right hand part the last n equations is going to be A inverse, because what we are solving by doing this particular set of row operations is, we are solving the equation A A inverse equal to i.

So, by using a gauss Jordan decomposition on a matrix form by concatenating A with an identity matrix of n by n dimensions, what we will end up getting is A inverse at after gauss Jordan. This is nothing, but saying i times A inverse equal to A inverse; that is what this particular thing represents, and the result that we get over here is the inverse of the matrix A and that is the gauss Jordan method. And finally, just to leave you off with a small question, when we started off with this particular example, this matrix A had the form 1 1 1, 2 1 3, 3 4 minus 2, which was equal to 4 7 9.

So, instead of this particular form of matrix A, what if the matrix A looked like 2 2 3, and we had to do gauss elimination. Because now, when we do R 2 minus 2 R 1, we will get this as 0, this as 0, and 1, our a 2 2 has become 0; in that case, how do we proceed forward with the gauss elimination. So, I leave you off in this lecture, sign you off in this lecture, with that particular question in lecture 4, we will pickup with this question, and an talk about a more appropriate may way of many doing gauss elimination, and that is gauss elimination with partial pivoting. So, see you all in lecture 4. Thanks.