Computational Techniques

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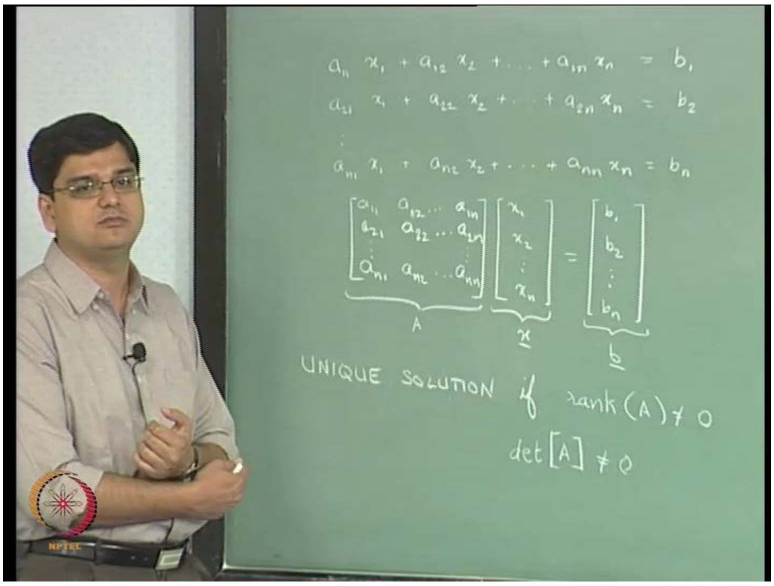
Module No. # 03

Lecture No. # 02

Linear Equations

Hello, and welcome to this lecture two of module three; yesterday, we were discussing linear equations, and discussed about vector spaces, talk took as small detour to talk about condition number, and other things. What we left out in the in the previous lecture is what happens, when we go from 2 by 2 system to an n-dimensional system; where we have n equations, and n unknowns. (Refer Slide Time: 00:42)



And a typical equation in that particular case, will be written in the form of a 1 1 multiplied by x 1 plus a 1 2 multiplied by x 2, and so on, up to a 1 n multiplied by x n equal to some right hand side term b 1. Where a 1 1, a 1 2, up to a 1 n are the coefficients; and x 1, x 2, up to x n are the variables, that we are interested in solving the equation form. Likewise, we will have the second equation as a 2 1 x 1 plus a 2 2 x 2, and so on, up to a 2 n x n equal to b 2, and the n th equation of course, would be a n 1 x 1 plus a n 2 x 2, and so on, up to a n n x n equal to b n.

As, we have done, before the unknown - all the unknowns will be put in a vector form, and that vector is going to be a vector x 1, x 2, and so on, up to x n; this is what we will call as the vector x bar. That is going to be equal to the right hand side; again, it is straight forward that will get a vector b 1, b 2, up to b n. And we will get a matrix A on to the left hand side. This matrix A, the dimension of this matrix A have, it has to have n number of columns, because there are n variables. So, it has to have n number of columns, and the number of rows will be equal to the number of equations. In this particular example, we have taken n equations in n unknowns. If there were the number of equations were not equal to n, if there were say m number of equations, we will have m number of rows, and n number of columns in this matrix A. Now, how do we populate this matrix is this first element your 1 1 element over here is going to multiply with the first variable; over here, the second element will multiply with the second variable; the third element will multiply with the third variable, and so on.

So, first row will contain all the coefficients in the first equation; the second row will contain the coefficients in the second equation, and so on; and the ith column will represent the coefficient of the ith variable in this particular vector. So, we will have a 1 1, a 1 2, and so on up to a 1 n, as the first row; a 2 1, a 2 2, and so on up to a 2 n, as the second row; and a n 1, a n 2, and so on up to a n n, as the as the nth row. This becomes our n by n matrix A, and this is our matrix representation of the n equations in n unknowns; of course, when n is going to be equal to 2, we have just these 4 terms, a 1 1, a 1 2, a 2 1, a 2 2, and we just have b 1, and b 2, over here on to the right hand side.

So, this is the overall matrix representation for an n by n system; the next question that, we asked ourselves is when do this system of equations have a unique solution; when does it have no solution; and when does it have multiple solutions; and what we had seen in the previous lecture is, it has the system of linear equations has a unique solution, if rank of A is not equal to 0, what do we mean by the other way of putting rank of A is not equal to 0 is that the determinant of A is non 0.

Both these terms are equivalent, but it is much easier to evaluate a rank of a matrix rather than to evaluate a determinant of a matrix for a larger - large size systems. And In fact, that evaluation of a determinant is a very computationally intensive, and that is the reason, why crammers rule although, it is looks fairly straight forward to implement; it is actually, not very useful for any type of practical examples. And that is, because the amount of computational effort required in computing a determinant is much larger than that is required for computing say a rank or for solving this set of equations; using the numerical methods, that we are going to cover in this particular module. So, there will be a unique solution, if rank of A is not equal to 0; if rank of A is equal to 0, we can either have no solutions or we can have infinite number of solutions.

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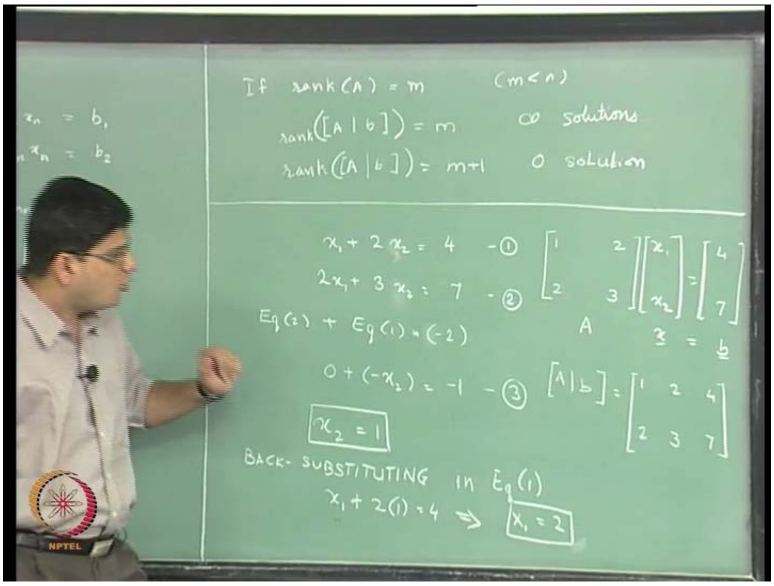

If rank of A equal to m, then we will have either 0 solutions or infinite number of solutions. In that particular case, what we had seen in the previous lecture is is, if each of the columns of the matrix A, if we represent them as vector say, we represent this guy as vector v 1, this guy as vector v 2, and so on up to this guy as vector v n. Then if b bar can be expressed as a linear combination of these vectors, then there will be infinite number of solutions, if it cannot be represented as a linear combination of these vectors, there will not be any solution. What that translated into in the previous in the previous lecture of for a two by two system was that the rank of matrix obtained by putting a, and b together; what we mean over by this is that the first n columns contain the n columns of a, and the n plus 1 the column contain the vector b over over here.

So, this is a new matrix; that is formed by horizontally concatenating the two the matrix with the vector. Now, if we create a matrix of this form, if the rank of this matrix is going to be equal to m as, which is the same as, what we had over here? Then we will have infinite number solution rank equal to m, then we have infinite solutions, and if rank of this matrix equal to m plus 1, we have 0 solutions. Why m plus 1, because we have just added 1 more column. So, the rank cannot actually be greater than m plus 1, the largest rank, that this can reach is indeed m plus 1. So, if these are the conditions, for this the system of equations to have one solution infinite number of solutions or 0 solutions. And in this context would that small detour, that we took in the previous lecture, talking about vectors, and vector spaces, and solving the equations, in terms of the in terms of vectors rather than intersection of lines is very instructive, because all these results directly fallout from very lucidly, and very simply fallout from explanation in that particular form.

So, what I will do? Next is, take up the two dimensional example 2 equations, and two variables; that we saw in the previous lecture, try to see how we will naturally solve that example, and try to derives certain points or certain hints on how we solved it, and why that particular solution technique worked. And then, we will extend it to an n dimensional system. So, the two dimensional example, that we took in pretty much the last minute of the previous lecture was, x plus 2 y equal to 4; and 2 x plus 3 y equal to 7. And for this particular example, the matrix notation of this equation is going to be, I will put it in x 1, and x 2, just to make it more consistent with the notations, we are going to use later.

So, the two variables are going to be x 1, and x 2, instead of x, and y; nothing really changes in this equation by changing the variables from x, and y to x 1, and x 2, and I have changed from x, and y to x 1, and x 2. So, that extension to an n dimensional system, becomes very clear; when we actually do that e extension, and then we have the right hand side, right hand side is going to be 4, and 7, and the a matrix will be 1 2 on the first row, and 2 3 on the second row. So, we have 1 2, 2 3 on the second row, and the matrix over here or the vector over here is 4 7. So, this is matrix A, this is x; this is b; and we will to define concatenated matrix A bar b as nothing, but 1 2 4, 2 3 7. So, these are our definitions follow; now, the question is, how we go about solving it, and this is perhaps, then that we have been trying solving from may be the seventh or the eighth class

So, this type of equation is not very difficult for us to really solve; what we will actually do is, we will first try to eliminate x 1 or x 2 from this equation. In this particular case, it is very clear that eliminating x 1 is, what we ought to be doing. So, we will write this as equation 2 minus equation 1 multiplied by 2. So, this is what we are going to do, but instead, I will write it in this form, plus equation 2 multiplied by sorry plus equation 1 multiplied by minus 2.

So, 2 x 1 minus 2 x 1 will give us 0 3 x 2 minus 4 x 2 will give us plus negative x 2, and 7 minus 8 2 into 4 is going to give us minus 1, and so, this will be our new equation. So, this was our equation 1; this was our equation 2; this we call as equation 3. Now, using equation 3, and equation 1, together; we can then solve these equations, in order to get the solution. So, of course, the 1 1 solution over here is x 2 equal to 1, and then back substituting in 1. So, we put x 2 equal to 1 over here. So, x 1 plus 2 equal to 4, and x 1 equal to 2. And this is, how in a fairly straight forward manner, we have been solving these equations.

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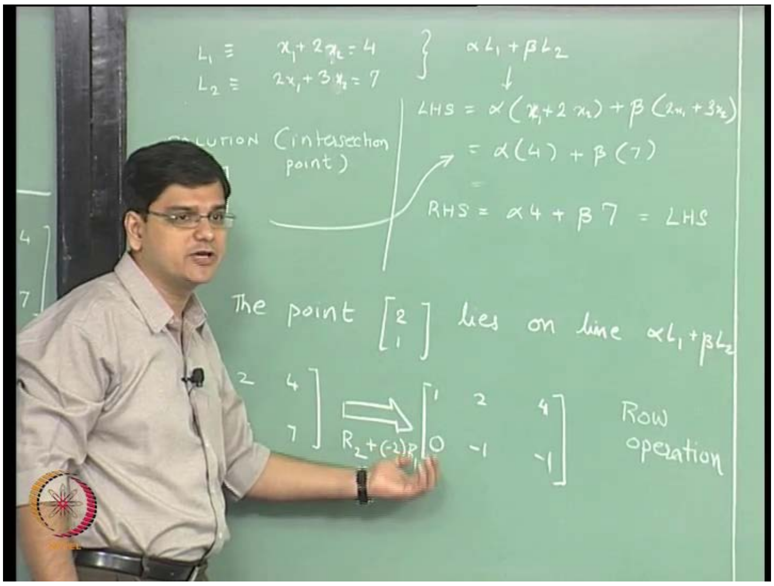

So, what we actually did was this, we had equation 1; which let us again go back, and think about it, from a geometrical perspective equation 1 represented a line. So, let me call that l 1, which corresponds to x plus 2 y equal to 4, and let us, talk about line l 2, which corresponds to 2 x plus 3 y equal to 7. Now, let us take a linear combination of the line l 1 with line l 2, and the linear combination basically means, we are going to take alpha l 1 plus beta l 2 where alpha, and beta, are two scalars.

So, let us look at the left hand side of this equation is going to be equal to alpha multiplied by x plus 2 y or rather, I should use x 1, and x 2; rather than x plus 2 x; and y plus beta multiplied by 2 x 1 plus 3 x 2. Now, the solution of these two equations was the point 2, 1. So, the solution was, let us substitute this particular value; in this equation, we will get alpha multiplied by 2 plus 2, that is 4 plus beta multiplied by 4 plus 3, that is 7.

So, we have 4 alpha plus 7 beta or alpha multiplied by 4 plus beta multiplied by 7; that is nothing, but the R H S of alpha l 1 plus beta l 2. So, the right hand side of alpha l 1 plus beta l 2 is going to be alpha times 4 plus beta times beta times 7, which is the same as L H S. So, what does it mean what it means is that the 0.2, 1. It satisfies the equation l 1, and it satisfies the equation l 2; that means, this particular point lies on the line l 1 as well as, it lies on the line l 2, but it also satisfies the equation alpha l 1 plus beta l 2, which means that for any scalar values alpha, and beta, including alpha equal to 0 or beta equal to 0 the point 2, 1 is going to lie on that particular line.

So, we are interested in finding out this point, what this point actually is, and since this point also lies on a linear combination of these two line segments as a result of this, we can actually do any type of manipulation of this sort; that means, equation 2 plus minus 2 multiplied by equation 1. And the result, that we get is still going to pass through the solution of those two lines, and this is not just true for a two dimensional system; this will be true for any n dimensional system. If we have n equations in n unknowns; that is going to represent basically, it is going to represents n hyper surfaces. And if we take linear combination of any hyper surfaces among those n hyper surfaces the point of intersection of those hyper surfaces is also going to lie on the linear combination of those hyper surfaces. This linear combination, and this property of the vector spaces is really the reason, why this idea works of taking equation 2, and subtracting 2 times equation 1 from this equation.

So this is, how we did when we tried to solve this set of equations, using the methods, that we were familiar with, now let us, look at what we actually did. So, that we can learn some lessons from it, and go ahead, and try to implement that. In the gauss elimination process, the first part, what we really did is something that we will try to relate to gauss elimination the second part as we did is something that, we will relate to what is known as back substitution. So, what we did in gauss elimination is, we took the matrix A b; sorry we took the matrix A b over here, and then we subtracted from the second equation, 2 times the first equation.

So, from this particular guy, we subtracted 2 multiplied by this guy. So, the A b matrix we had was 1 2 4, and 2 3 7, from this the 2 equations that, we were left with were these. So, first we had equation 1, and 2; we did equation 2 plus minus 2 multiplied by equation 1, and we ended up with this equation. If this equation, we have to write this in basically a Victoria notation in a matrix notation; it is going to be 0 multiplied by x 1 plus minus 1 multiplied by x 2 equal to minus 1. So, I will erase this equation, and write it in a different way; it is 0 multiplied by x 1 plus minus 1 multiplied by x 2. So, now, what we are left with is, we are left with equation number 1, and equation number 3. And we will try to solve equation 1, and 3, together; that is exactly, what we did in the back substitution step.

So, equation 1 is 1 2 4 in the matrix A b, and equation 2, if we again go back to this equation number sorry equation number 3 is 0 minus 1 minus 1; our matrix A is going to consist of 1 2 0 minus 1; our matrix b will consist of 4 minus 1, and when we put those 2 matrices together, the first 2 will contain 1 2 4; the second row will contain 0 minus 1 minus 1. So, what happened is we went from this matrix A b to this new matrix A b, and how did we go ahead, and get this particular matrix; this particular matrix, we got by taking row two, and adding minus 2 multiplied by row onesss as row operation.

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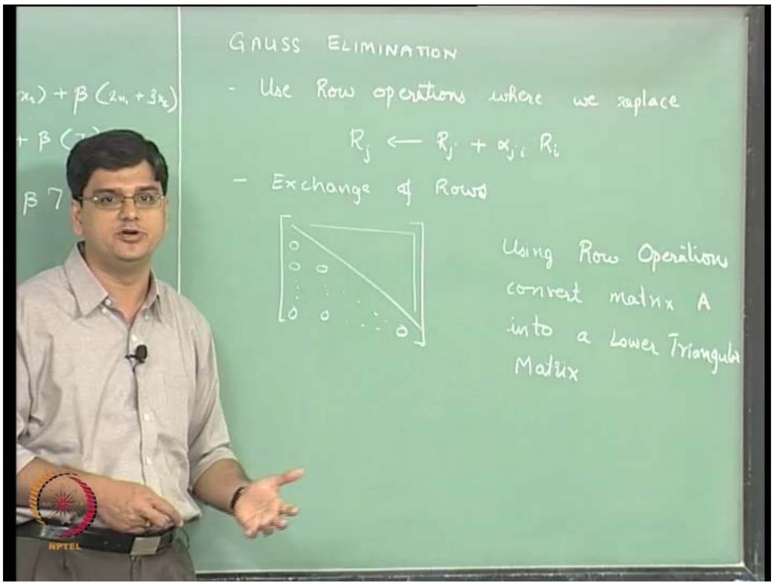

So, what we are going to do in gauss elimination is, to do the following: in general, what we are going to do is, we will take the jth row in this particular case; the second row, and add to it some coefficient alpha multiplied by the ith row; i th row, in this particular case was first row. So specifically, what we did over here is, we did R 2 where the j is equal to 2 plus minus 2 multiplied by R 1, where i equal to 1. So this is what we will do as a row operation in gauss elimination step. So, what row operation does it replaces one of those equations from n equations with a linear combination; for example, in the problem that we just solved, what we did was we replaced l 2 the line l 2 with the line l 2 minus 2 times l.

So, we replaced the one of the lines with a linear combination of the two lines that we had in the previous equation. So, this is one type of row operation; the second type of row operation is, we know that it does not matter, whether we call one particular line l 1, and l 2. Instead of calling, this line as l 1, and this line as l 2, we could have very well called this line as l 2; and this line as l 1; and it would not have made any difference; that means, that by exchanging the two rows the solution of the overall linear equations also remains the same. So, the other type of row operation possible is exchange of rows. So, these are the two row operations, that we will use such that we get an upper triangular matrix for the matrix A.

So, what do we mean by upper triangular matrix for matrix A is that all the values on the diagonal, and above diagonal, may or may not be zeros, but all the values below the diagonal have to be 0. So, we will try to get zeros in all the equations, below equation number 1 will get zeros for the variable x 1, below equation number 2 for all equations we will try to get 0 for the variable x 2, so on, and so far. And when we go back and check what we have over here, this is A part of our matrix, and this is the new b part. So, what we have done over here is A itself, we have got this as an upper triangular matrix; all the values below the diagonal are 0 in this particular case, and this is our new vector b. So, that is our gauss elimination step; using row operations, convert matrix A into a lower triangular matrix, and the same row operations have to be implemented on the vector b as well.

So, this is what, we will do for doing the gauss elimination. Let us, look at another example three dimensional example, and then we will extended to an n dimensional system. I am taking the three dimensional example, and I will use to explain both gauss elimination as well as another method called the L U decomposition method.

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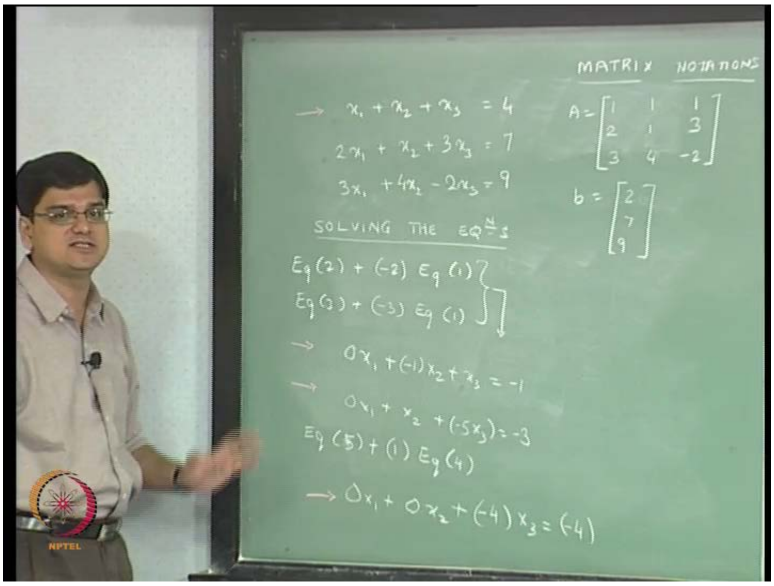

Let us consider, these three equations - these three linear equations; then 3 unknowns; the unknowns are x 1, x 2, and x 3; the coefficients 1, 1, 1, 2, 1, 3, 3, 4, and minus 2. So, these are three equations, and three unknowns. Let us, give our self about half a minute to think about, how to have a matrix representation of this equation, and how to have, how to solve these particular equations. So, let us talk about the matrix notation for these particular equations. So, we are going to write at n form of a x equal to b the matrix A is going to be 1 1 1 in the first row; in the second row, we will have 2 1 3; and in the third row, we will have 3 4 minus 2; for matrix for the vector b; we will have 4 7 9 as the vector b. So, now we have the matrix A, and the vector b, and when will try to solve these equations; using the gauss elimination process, before that will try to solve it using the algebraic method that we typically use.

So, the first step that, we will do is take equation number 2 and add minus 2 multiplied by equation number 1, and we will take equation number 3, and do minus 3 multiplied by equation number 1, and they will give 0 x 1 plus 1 minus 2, that is minus 1 x 2 plus 3 minus 2, that is x 3 plus 1 x 3 equal to 7 minus 4, that is 7 minus 8, that is minus 1; we will again have 0 x 1 plus 4 minus 3, that is 1 x 2 plus minus 2 minus 3, that is plus minus 5 x 3 equal to 7 minus 7 minus sorry 9 minus 12; that is going to be equal to minus 3. So, now, one equation is going to be equation number 1; this is going to be our new equation number 2; this is going to be our new equation number 3.

So, next what we will do is, we will add these two equations, and replace the last equation with that particular sum. So, we will do this as equation 4 sorry equation number 5 plus 1 multiplied by equation 4, and when we actually do that, we will get this as 0 x 1 plus 0 x 2 plus minus 5 plus 1, that is minus 4 x 3 equal to minus 3 minus 1, that is going be equal to minus 4. So, now we have a three equations in three unknowns; the first equation is plus 0 x 1 plus 0 x 2 minus 4 x 3 equal to minus 4, back substituting will get x 3 equal to 1, next we will substitute x 3 equal to 1 in this particular equation, and we will be able to get x 2 equal to 2 over here, and x 3 equal to 1, and x 2 equal to 2 will actually give us x 1 into this equation, and that actually give us x 1 equal to minus x 1 equal to 1 in this particular equation .

This is how we go about solving, what I have done. So, far are these steps that will be involved in getting a lower triangular matrix for the matrix A; keep in mind, the same row operations, that we use for the matrix A; we are applying the same row operations for the matrix b as well.

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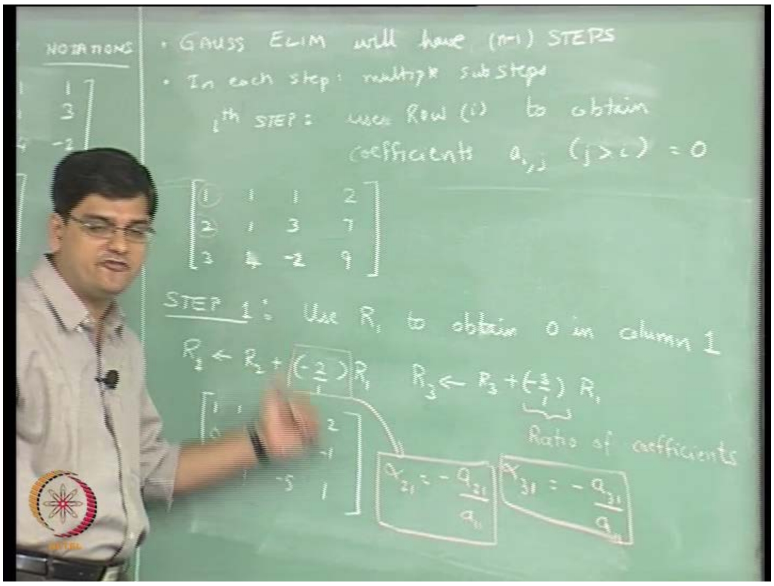

So, the first point is that gauss elimination step, when we have n equations in n unknowns will involve n minus 1 steps. In each step, we have multiple sub steps. What we will do in ith step, we will use the ith row we will use i the row to make the coefficients of x i in all equations in row i plus 1 up to row n equal to 0 to obtain, we will use ith row to obtain the coefficients a i comma j for where j is greater than i quall to 0. So, what we mean by that is the first step is to construct or the 0 x step is to construct the matrix a b which will be 1 1 1, 2 1 3, 3 2 minus 2 sorry 3 4 minus 2, and our v is 2 7 9; step 1 will involve making this, and this element as 0.

So, step 1 is use R 1, to obtain zeros in column 1 in row 2, row 3, and so on, up to row n. So, what we will do is R 1 or sorry our R 2 is going to be our original R 2 plus some coefficient alpha 2 1 multiplied by R 1, and likewise, we will have R 3 as R 3 plus some coefficient multiplied by R 1; I will write it in this particular form. So what did we do over here, if we go back to this equation, what we did is we multiplied it with minus 2 and in the equation 3 we multiplied it with minus 3, and we have multiplied equation 1 with minus 3, and added that to equation 3.

So, what we do over here is we multiply the row one with negative 2, which is which I will write this as minus 2 divided by 1, and this I will write this as minus 3 divided by 1. And when we do that the next matrix a b, that we will get is we will get 1 0 0, 1 1 2 over here. So, this will be negative 2, this term will be 1, this term will be equal to negative 1, and this of course is 0, 4, minus 3, is 1; minus 2, minus 3, is minus 5; and then 9, minus 2, is 1; and if we compare it with the equations, that we got the new equation was 0 minus 1 1 minus 1 0 1 minus 5 minus 3.

So, we actually do oh sorry this should be minus 1 0 minus 1 1 minus 1 0 1 minus 5 1 is actually what we get as our matrix A after the first row operation; keep in mind, what we have done over here is multiplied this R 1 with ratio of coefficients. So, alpha 3 1, because we are changing the row three with by using row one alpha 3 1 is going to be equal to negative a 3 1 divided by a 1 1; likewise, what we have over here is alpha 2 1 is going to be negative of 2 divided by 1, which is a 2 1 divided by a 1 1. So, this is what we did as step 1; in step 1, we use row 1 in order to make zeros in all the in the first column below the first row.

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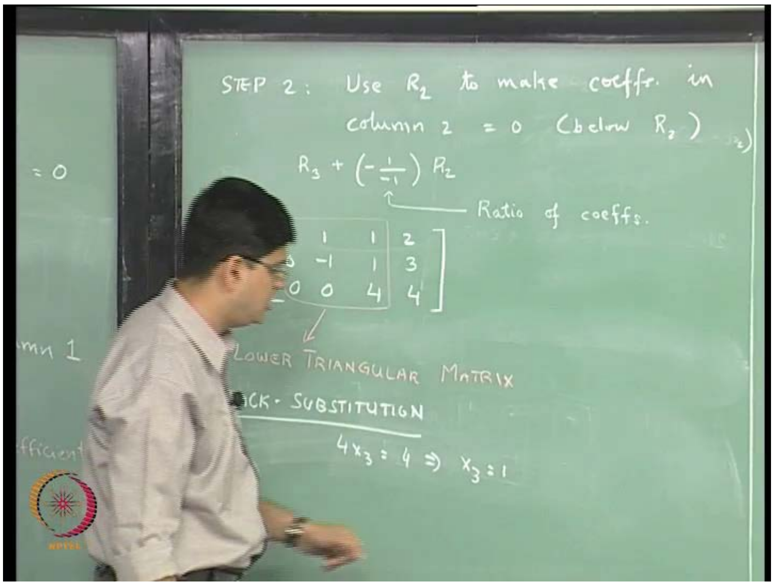

So in step 2, we will use the row two, in order to make the coefficients in column two equal to 0 below row two, using the row operations. So, the result of step 1, we had this particular matrix, and in order to make 0 over here; we have to multiply with negative of the ratio of the coefficients. So, minus 1 divided by minus 1 is what we have to multiply it with; in other words, we just have to add those 2 equations. So, what we have to do is we have to do R 3 plus minus 1 divided by minus 1 multiplied by R 2; keep in mind this is ratio of coefficients.

So, when we do that, what we will get is, we will get 1 1 1 2 in the first row; the first row is unchanged 0 minus 1 1 minus 1; the second row is unchanged, we will get this as 0 0 minus 4, and 0 over here. So, that is what we are actually going to get when we do addition of these two rows. I think I have probably made a mistake over here. So, when we do this row operation, this will be 0 minus 1 minus 2 is minus 1, 3 minus 2 is plus 1, 7 minus 4 is plus 3, and 0 1 minus 5, and 3 times 1. So, this is, what we have we ought to be getting. So, we will get 0 minus 1 1 3, and 0 0 4 4.

So these, this is what we will get. So, in terms of this matrix A that, we have over here. This has now, reduced to a lower triangular matrix. And then, we will use the back substitution, and in the back substitution step, we will x 3 going to be equal to 1; we will substitute this x 3 equal to 1 over here x 2 equal to minus 2; we substitute x 2 equal to minus 2, and x 3 equal to 1 over here, and we will get x 1 equal to 1, in this particular equation. So, this is, how we will we will actually get the final solution. So, in back substitution, we will use 4 x 3 equal to 4 which will give us x 3 equal to 1, and then we will substitute x 3 into this equation, and then we substitute x 2, and x 3, into this equation, and we get the overall solution.

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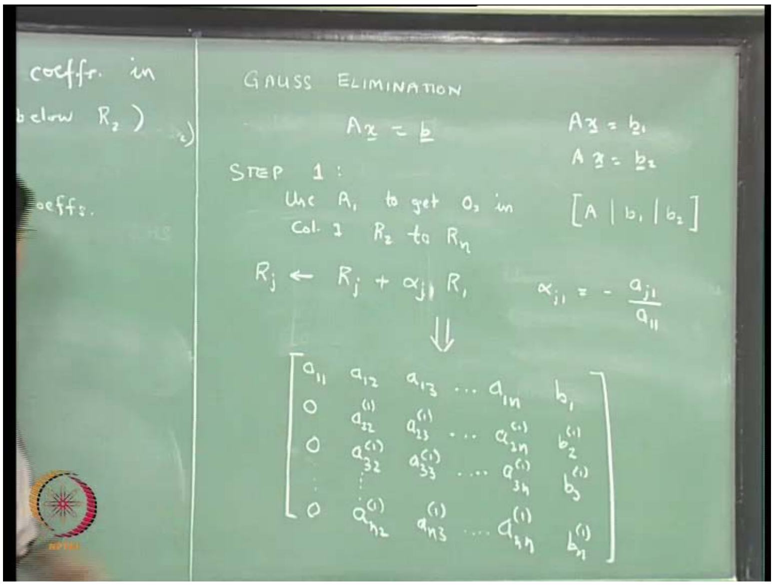

So, gauss elimination will consists of n minus n minus 1 step. So, we let us say, that we have n by n equations in n unknowns. So, we have the matrix A, and the vector b bar; And if instead of one equation, if we had n or more, if we had had more sets of equation, in terms of A x bar is equal to b 1 bar, and a x bar equal to b 2 bar. We can solve both these equations simultaneously by creating a matrix A b 1 b 2.

If we had a x bar equal to b 1 bar as one set of equations, and a x bar equal to b 2 bar as other set of equations. And we had to solve this set of equation independently of this set of equations; the way we can go about solving this is create matrix A b 1 b 2. And then apply, all the row transformations to this particular matrix, and we will be able to indeed solve these two sets of equations simultaneously. This is going to be very useful, when we are going to consider inversion of a matrix that, I am going to consider in the next lecture in this particular module.

So, we have A x equal to b; step 1 is going to be use R 1 to get zeros in column one row two to n, and how we do that is R j is going to be equal to R j plus alpha j, i or alpha j, 1 R 1, where alpha j 1 is the ratio of the coefficients, and if we go back, and look over here the ratio of the… So, we had R 2 given as R 2 plus negative of alpha a sorry a 2 2 a 2 1 divided by a 1 1; alpha 2 1 was negative of a 2 1 divided by a 1 1; likewise, over here alpha 3 1 was equal to negative of a 3 1 divided by a n a 1 1; in general alpha j 1 is going to be equal to negative of a j 1 divided by a 1 1. So, this is what we are going to write a j 1 is negative of a j 1 divided by a 1 1.

So, once we have this particular step, as a result of this step, we are going to get. So, we will start off with the original matrix a 1 1, a 1 2, a 1 3, up to a 1 n, and then b 1. So, on and. So, forth. So, our first row remains unchanged. So, we will have a 1 1, a 1 2, a 1 3. so on, up to a 1 n b 1; our second row is going to be we will get 0 at this location; we do not have to compute that simply by using a j 1 divided by a 1 1, we are going to get 0 over 3. So, we will have 0 over here, this will be a 2 2 after the first operation a 2 3 after the first operation and so on, up to a 2 n; after the first operation b 2 after the first operation; likewise we will change our row 3 or row 4 or row 5 and so on. So, we will immediately get 0 over here.

We will have a 3 2 after the first operation, a 3 3 after the first operation and so on up to a 3 n after the first operation, and b 3 after the first operation, and so on up to the nth row. Again in the nth row, we are going to get 0 over here, and over here we will get a n 2 after the first operation, a n 3 after the first operation, and so on, up to a n n after the first operation followed by b n after the first operation. So, this is going to be the step 1 of the gauss elimination step. In step 2 of the gauss elimination step, we will use row two get zeros in column two. So, that is going to be our next step.

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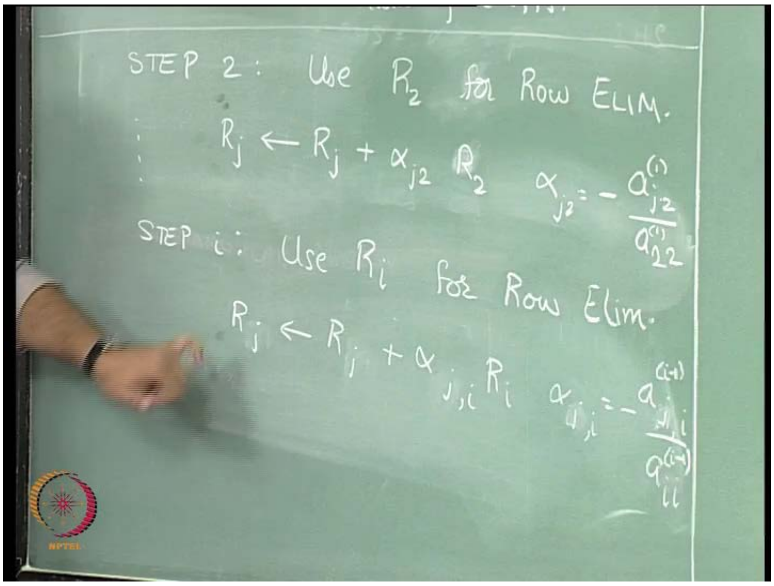

So, the step 2 is going to be use row two for row elimination. So, we want to get 0 over here and so on, up to this point, and therefore we are going to multiply R j with an appropriate coefficient multiplied by R 2. So, R j will be, R j plus alpha j 2 multiplied by R 2, and in this particular case alpha j 2 is going to be for example, over here it is going to be negative of a 3 2 1 divided by a 2 2 1 negative of a 4 2 1 divided by a 2 2 1, and so on, up to negative of a n 2 1 divided by a 2 2 1 Or in general, alpha j 2 is going to be equal to negative a j 2 divided by a 2 2, and so on, up to step i is going to be using ith row for row elimination. So, R i plus 1 is going to be equal to R i plus 1 plus alpha i plus 1, i R i where alpha i plus 1, i is going to be equal to minus a i minus 1 i plus 1, i divided by a i minus 1 i, I; because this has been obtained after i minus 1 row operations. In fact, we will have 1 as a subscript, superscript over here. And we do this for i plus 1, we do this for i plus 2, and so on. So, we can replace this i plus 1 with just j over here and so, all the i plus one, we will replace by j. So, this is what we end up doing now, if we consider this what we do in step 2, and what we do in step I, we have R j over here where j is going to be greater than 2; in this case, j is going to be greater than i.

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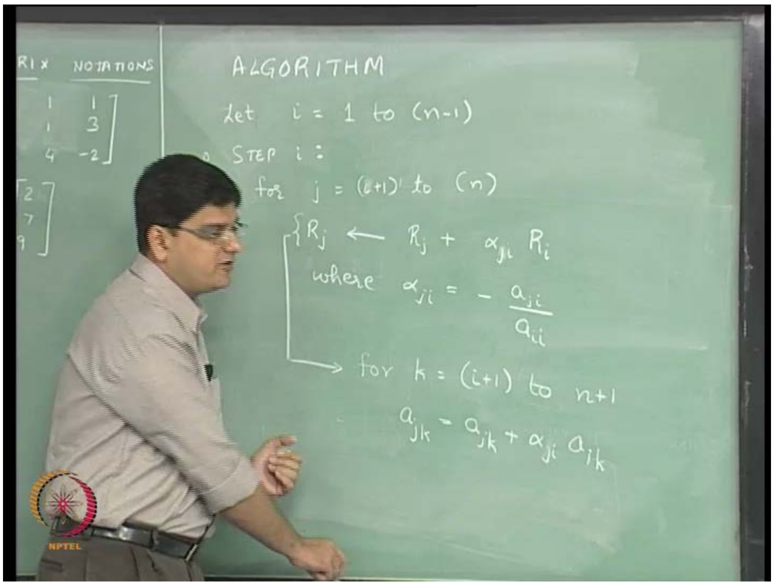

So, let me write the algorithm, and again I am going to very roughly write the algorithm not in a standard form. Let us so, we will do let, I go from 1 to n minus 1, because we are doing this row operations using row 1, row 2, row 3, up to row n minus 1. We are not using the row n minus 1, the row to do any row operation. So, which means in step i for is going to involve for j going from 1 sorry from j going i plus 1 to n; that means, the row i plus 1, row i plus 2, row i plus 3, and so on. We need to get R j or we need to replace R j with R j plus alpha j, i R i where alpha j, i is going to be equal to negative a j, i divided by a i i. So, that is what we have in the ith step; keep in mind, this is not one single operation, but this it involves multiple operations.

So, if you were to expand, this operation; we will have so, R j is the jth row; in the jth row for the first column, we will have a 0; second column, we will have 0; and so on up to the j minus 1 column; we will have 0 in the j the column, we will definitely get 0, because we are using R j to make that particular in sorry in the ith column, we will definitely get 0, because we are using R i in order to get 0 in that column. So, we will have for k going from i plus 1 to n plus 1, where the n plus 1 is for the bth for the vector b; in this particular case, we will have a so, we are looking at the jth row, and all the columns after the column i. So, we will go for a j, k is going to be equal to a j k plus alpha j i multiplied by i multiplied by a.

So, it is the ith row so, we will have sorry it is the jth row. So, we will have… so, I am sorry we are going to use the ith row; in order to make zeros in the jth row ith column, and this will change all the other columns as well. So, this is going to be a i comma k, because this is going to be the column k; the k th column, and we are going to use the row i, in order to change that particular variable. So, this is very roughly, what algorithm we are going to follow. So, what we have covered so, far essentially is the back ground behind the gauss elimination, and very briefly look at the algorithm for gauss elimination.

In the next lecture, I will take off from the algorithm for gauss gauss elimination, and try to compute how much amount of computational efforts, the gauss elimination step is going to require. Next, I am going to talk about the back substitution itself; and finally, I will finish off with two different two other methods; specifically, the gauss Jordan method, and LU decomposition. So, that is basically, what we will cover in the next lecture of this module.Thank you.