Computational Techniques

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Module No. # 09

Lecture No. # 02

Partial Differential Equations

Hello and welcome to lecture 2 of this last module in the computational techniques course. What we are doing in this particular module is to look at numerical methods to solve partial differential equations.

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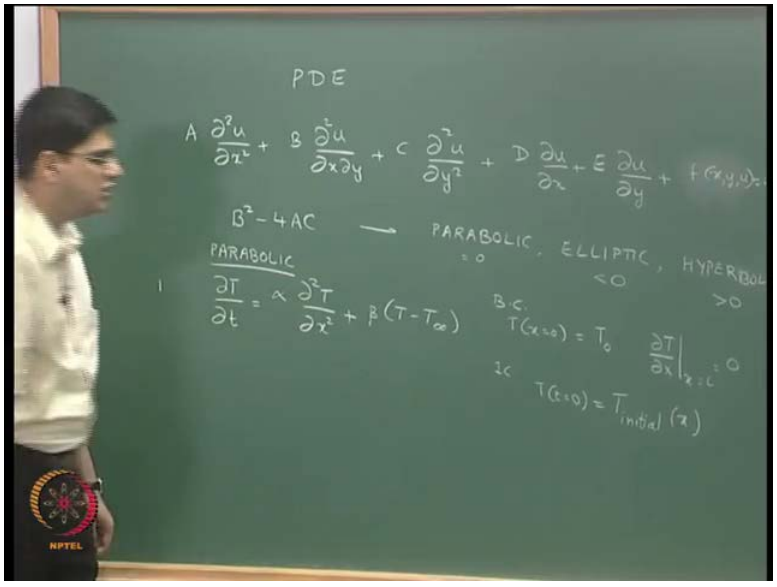

So, I will just recap what we did in the previous lecture. For P D E's, we said that we are really interested in solving first order and second order P D E's. And general second order P D E can be written in the form A dou square u by dou x square plus B dou square u by dou x dou y plus C dou square u by dou y square plus D dou u by dou x plus E dou u by dou y plus f u plus G equal to 0.

Now, if in this case A B C D E F and G are all constants or they are functions of x and y only, and not functions of u. Then what we have is a linear second order P D E, instead

of that we can write this particular guy if we if we write this as some say small f of x y and u, and if either of A B C D E and small f, if either of them are if sorry if A B C D and E are functions of u or if f is a non-linear function of u; then what we have is a non-linear P D E that is what we discussed and then, we talked about classification of P D E's. And P D E's were classified based on what the value of B square minus 4 A C is, and based on this value we had classified them into parabolic, elliptic and hyperbolic. Parabolic was when B square minus 4 AC was equal to 0; elliptic when this guy was less than 0; hyperbolic when this guy was greater than 0, and the three examples that we took off, one each of parabolic, elliptic and hyperbolic equations.

For the parabolic equation, the example we took was a transient plug flow reactor and sorry transient heat transfer problem in the rod, and that would end up being the equation would be partial T by partial time is going to be equal to alpha partial T square by partial x square plus beta multiplied by T minus T infinity, where alpha and beta are two constants or they can be function of temperature and space also. Alpha is nothing but the thermal diffusivity and beta is going to be the ratio of the heat transfer coefficient to the thermal inertia of the overall system.

We require initial conditions at time T equal to 0 and boundary conditions at various locations- at two different locations of of x. And typically the boundary conditions in this particular case or going to be T at x equal to 0 is some T 0, and partial T by partial x at x equal to L, can either be some T 1 or some T dash one or it could be 0. So, that is one example of the boundary conditions, and the initial condition for this particular system is going to be say T, at T equal to 0 is some profile T initial.

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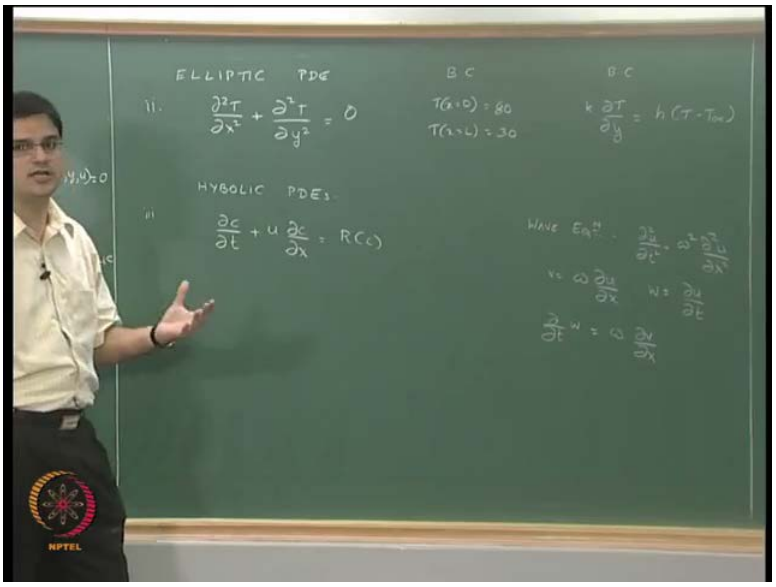

So, given these boundary conditions and these initial conditions, we can solve this parabolic equation. The second example we took was of an elliptic P D E and that was a heat conduction problem, which gave us into a Laplacian type of an equation - partial squared by T by partial x squared plus partial squared T by partial y squared equal to 0.

This would be heat conduction in rectangular slab of a solid and there is going to be a thermal conductivity term that gets canceled on both sides and we are left with this type of an equation, and that would be subject to boundary conditions in x as well as boundary conditions in y. And for example, one type of boundary conditions in x could be that temperature at x equal to 0, was specified to be say 80 degrees; temperature at x equal to L, was specified to be 30 degrees.

And the boundary conditions in y was that, there were heat losses taking place to the surroundings and that we got in terms of k d T by d y equal to h T minus T infinity and the sign would be positive or negative based on the direction of the heat transfer. So, this is the kind of boundary conditions that we will get to solve this elliptic P D E's well.

And the third example would be hyperbolic P D E's. And the typical example of a hyperbolic P D E is the wave equation, and the wave equation is written in the form partial square u by partial t squared equal to omega square. This is how a typical wave equation is written as and then, we can substitute a new variable say v equal to omega d

u by d x, and w equal to d u by d t and then we will get and we can then substitute that; so we can write say v equal to omega partial u by partial x; w equal to partial u by partial t; we can substitute in this equation and we will get dou by dou t of dou u by dou t, and dou u by dou t is w.

So, dou by dou t of w is going to be equal to omega multiplied by dou by dou x of omega dou u by dou x; so that is going to be omega dou v by dou x that is going to be one equation, and the other equation was going to be dou v by dou t is going to be equal to omega dou w by dou x. And so, we will have two equations and two unknowns, and these two equations are essentially the linear first order P D E's, and linear first order P D E's written in this form have a parabolic hyperbolic nature. Based on this and again I am not using mathematically correct analogies over here, but approximate analogies just to give you an overview of why we call the equations of the type of transient p f r as hyperbolic equations, although though are those are first order P D E's. So, the transient p f r equation is going to be of the form dou c by dou t plus u dou c by dou x equal to some rate of reaction, which is a function of the concentration c.

So, we have dou c by dou t plus some velocity multiplied by dou c by dou x, and this kind of, has the form has a form like this and where u as well as omega can either be positive or negative numbers; if the flow is from left to right, u we say by convention is positive or from right to left, u by convention is negative.

Likewise, we have positive or negative omegas. So, if you if you kind of compare these two equations that is where we get this, that is why we get the statement, that a P D E of this type has a hyperbolic like qualities. And here, we will need initial conditions in time t as well as initial conditions in spatial location x.

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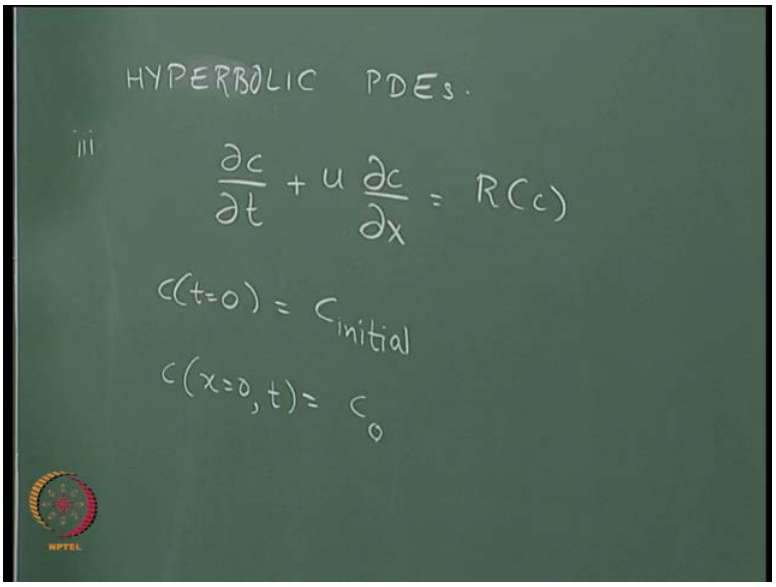

So c at t equal to 0 is going to be some c initial; and c at x equal to 0 and at all times is going to be c 0, which is the concentration at the inlet. This in time both parabolic and hyperbolic P D E's are going to evolve in one direction, starting from t equal to 0,

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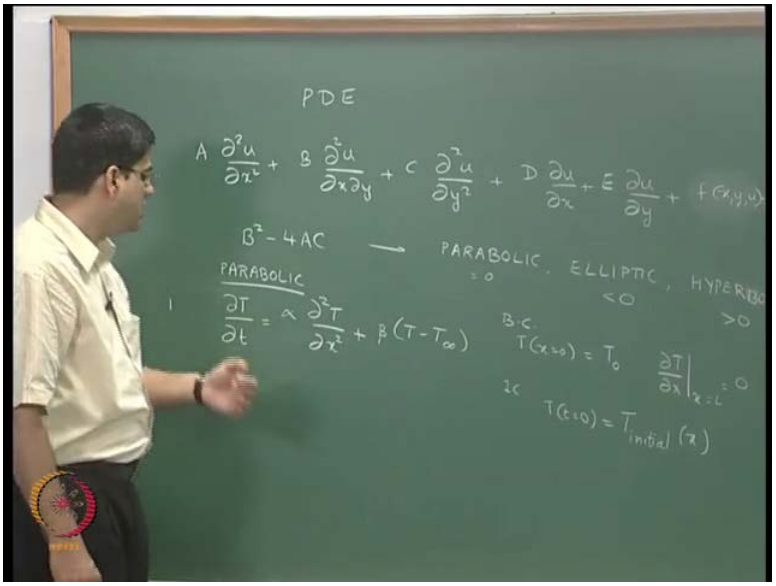

We will evolve these equations in the future times in both, hyperbolic or parabolic P D E's.

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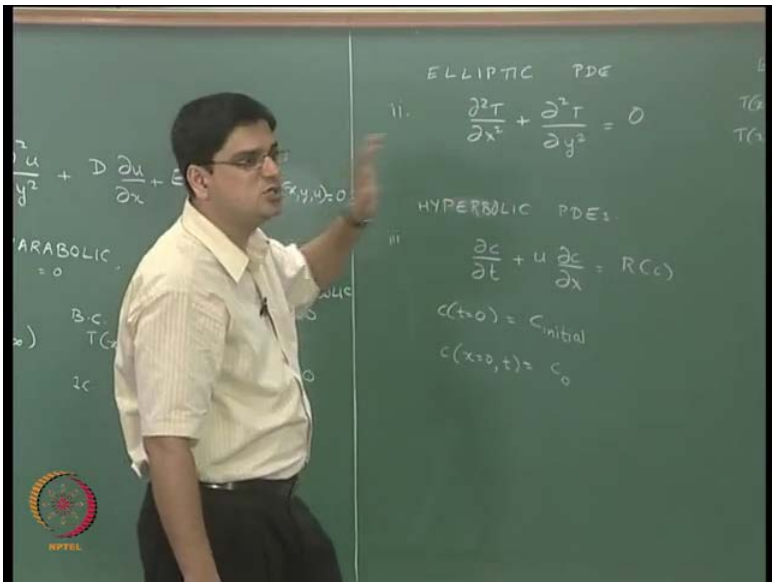

The difference between hyperbolic P D E's and parabolic P D E's is that, in the parabolic P D E we have this diffusion term. What we mean by the presence of diffusion term? When we try to solve this equations numerically, is that the overall solution in the x direction is going to be determined by the overall solution in the x direction is going to be determined by two boundary conditions; whereas in this particular case, the overall solution x direction is going to be determined by a single initial condition.

So that is the difference between hyperbolic and parabolic P D E's; it does lead to some difference in solving hyperbolic and parabolic P D E's numerically. On the other hand, elliptic P D E's do not evolve either in x or y direction, but the solution or the temperature T at any location x, y is determined by all the four boundaries in this particular plane. So that is the again recap of what we have done with elliptic hyperbolic and parabolic P D E's.

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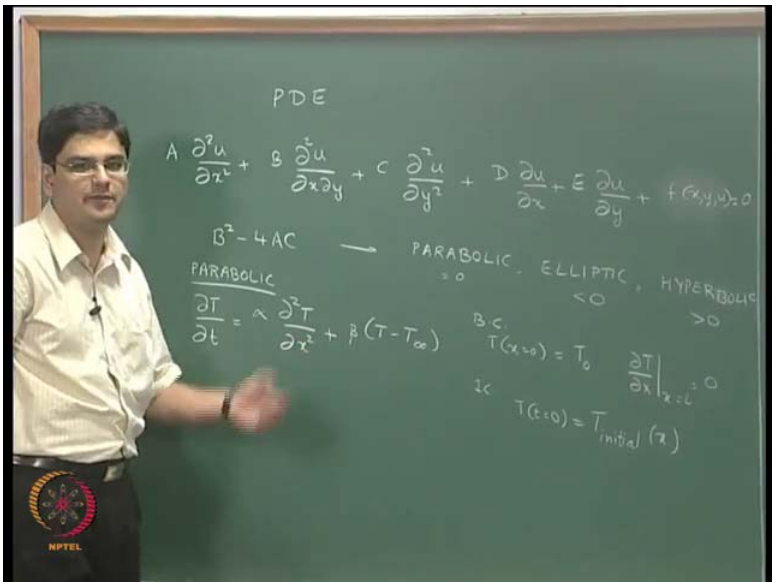

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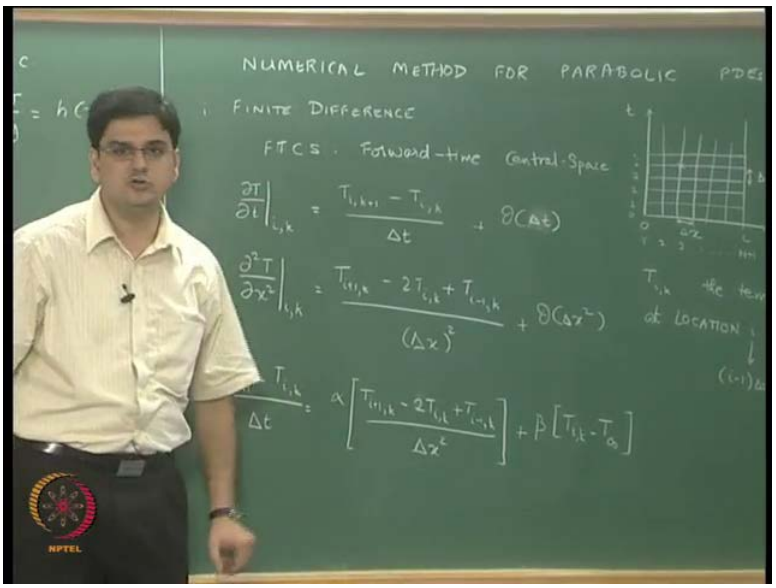

We will now take up the example of parabolic P D E's and will start with this example and show different methods that we can use to numerically solve this parabolic P D E. The finite difference method - so what we have is that the temperature in this rod is going to evolve with both time and space.

So, let us put the special direction on the x axis and the time direction on the y axis; and the spatial direction is limited from 0 to length L and this is the overall domain in which

the temperature t is going to evolve in time the temperature t is going to evolve in time. anything sorry.

So, now, that we have this domain; in finite difference, whatever we have done in O D E solving techniques, we are we going to use the same ideas and so using this in the P D E solving techniques also. In O D E's, we had done finite difference only in one dimension; we had done finite difference either in space that was in case of the heat conduction problem or in time that was in case of the reaction - reacting system problem.

However in this particular case, we need to discretize this in both time and space. So, let us say this is how we discretize in time and space. So, we have location x 1, x 2, x 3 and so on up to say n plus 1 locations, and the time- we will start the time, where time t 0,1, 2, 3 and the time goes on up to various number of sets.

What we will assume for now is that in spatial dimension as well as in the time dimension, these guys are equally spaced. So, the spacing between location 3 and 4 is the same as the locate spacing between location 1 and 2, that means, the intervals are of the same size; and the interval we will call this as delta x in case of spatial location, and delta t in case of the time location.

And let us look at the point, which is at time; let us take this particular location. So, this is the point at time 4, and the time index is 4 and the spatial index is 3. Now, we have this temperature, which is represented at each of these points. Let us call T i , k is the temperature at location i and time k. So, i is the location index and if you want to get the actual x at this particular location, the x is going to be equal to i minus 1 multiplied by delta x.

So the location So, for i equal to 1, we have x equal to 0; for i equal to 2, x is equal to delta x; for i equal to 3, x equal to 2 delta x so on and so forth. For i it is i minus 1 multiplied by delta x; and at time k, that actual time is k multiplied by delta t. So, now, we are going to use forward difference in time and central difference in space, and we will get at strategy known as F T C S -Forward in Time Central in Space. (Refer Slide Time: 15:26) So, what we had is dou t by dou t, which is going to be computed at location i and time k.

Now, we are going to use a forward difference in time; so this is going to be T i, k plus 1 minus T i, k divided by delta t; that is going to be our partial T by partial t plus, we will have the error, which is going to be of the order of of the order of delta t. And now, we have dou square T by dou x square, again at location i, k and dou square t by dou x square is what we are going to represent this as the central difference in space.

So, its T i plus 1 minus 2 T i plus T i minus 1; so T i plus 1 minus 2 T i plus T i minus 1, each of them are computed at the time t equal to k, and this thing is going to be divided by delta x squared. So, we have this expression for d T by d t; this expression by d square T by d x square, we substitute this and this is going to be order of delta x squared accurate. Now, we substitute this in our original equation and we are we are essentially going to get then, T i, k plus 1 minus T i, k divided by delta t is going to be equal to alpha multiplied by d square T by d x square.

So, alpha multiplied by T i plus 1 plus beta multiplied by T i, k minus T infinity. This is going to be our overall expression for the scheme, which is forward in time central in space finite difference scheme. Now, if we go back to the equation that we had written earlier, which I have this over here.

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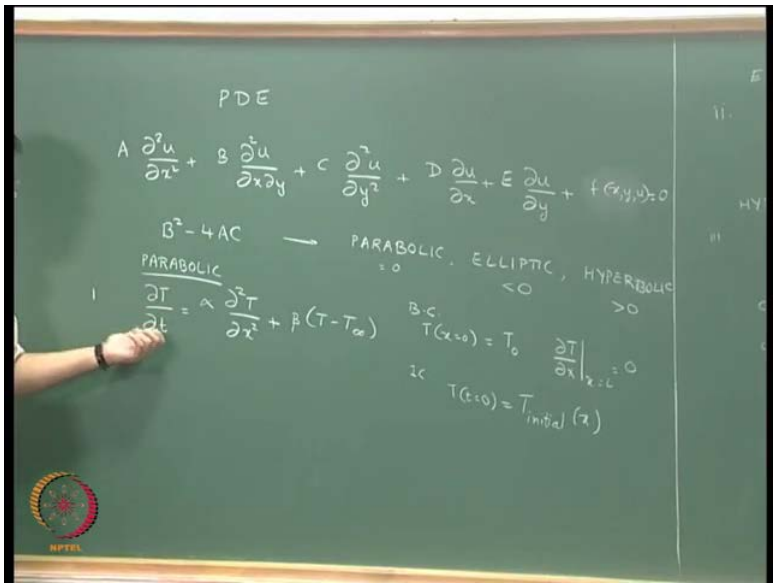

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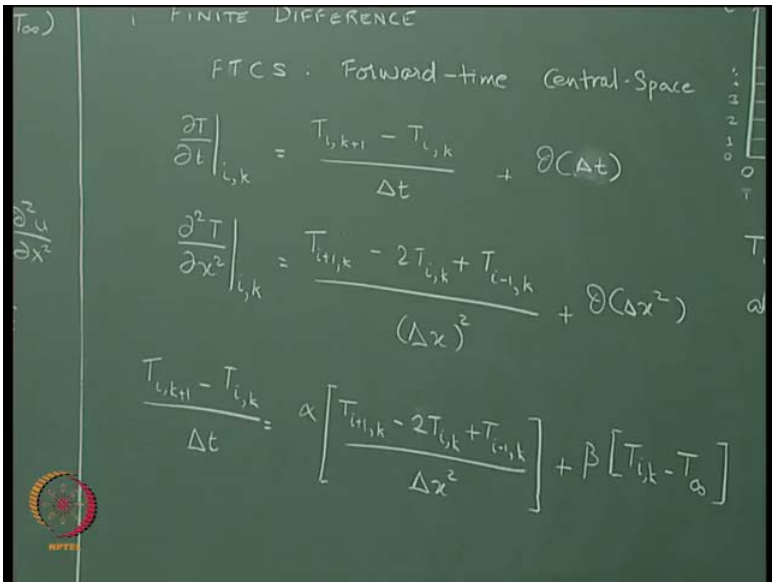

Just to recap what we have done; we have obtained partial T by partial t, at time t equal to k using the forward difference scheme; partial square T by partial x squared, at x equal to i using the central difference scheme. And we have substituted the value of T at location i and at time k, and that is what we have done and finally, when we do that, we will get this kind of an expression.

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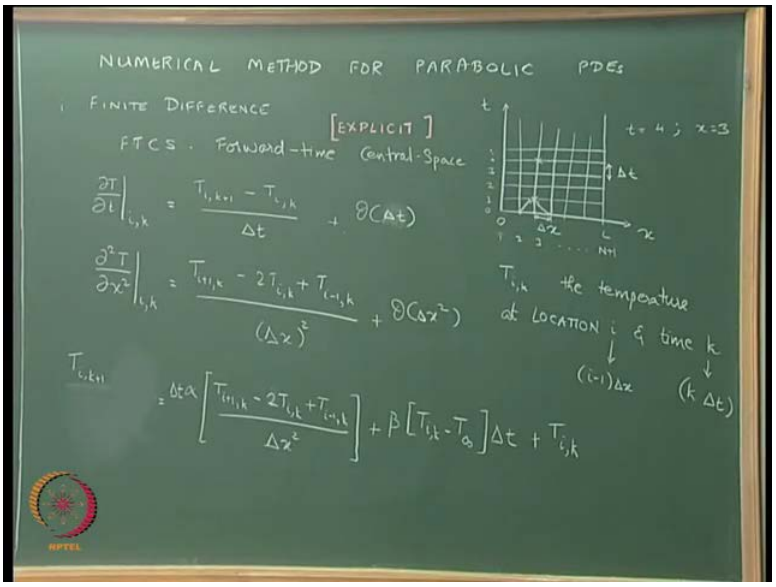

Now, what we can do is multiply by delta t throughout and take T i, k on to the other side and we will get the final expression as which I will write down here. So, we multiply

throughout by delta t, what we will get is delta t multiplied by alpha and delta t over here and then, take T i, k on to other side and will get i, k. And this is going to be the expression that we will use in order to get the value of at T i, k plus 1.

So, what we do is, we will start with the values of temperatures, specified at time t equal to 0 at each of these locations. So, we have for example, the initial temperature t is specified along the entire length of the rod. For any domain point over here, this is at k plus 1; the value at k plus 1 is going to depend only on the values of temperature at time k. So, when k is going to be equal to 0, we have all the values of temperature known at this point.

So, let say we are interested in finding temperature at this particular location; so T at k plus 1, i is going to depend on this guy, this guy and this guy, and we will then be able to compute T i, k plus 1 using this particular expression. And then, we continue doing that in a step by step manner, because this is an explicit expression in time t along the time direction t as a result, we do not have to do any kind of solving of linear equations or non-linear equations. This is just an expression, where we substitute the known values of the temperature and we will get the value of temperature at time k plus 1.

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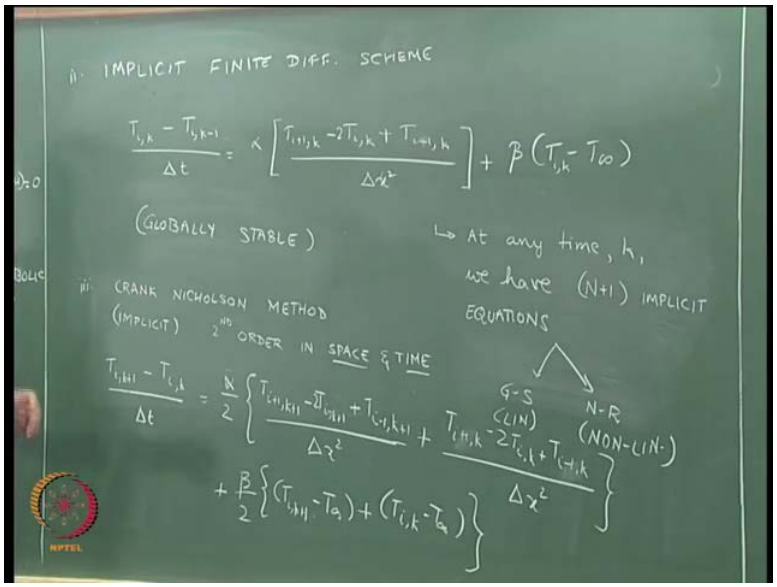

The forward in time central in space is an explicit method. And in a similar fashion, we can get an implicit method and that implicit finite difference scheme is going to be T i, k

minus T i, k minus 1 divided by delta t is going to be equal to alpha multiplied by T i plus 1, k minus T i, k sorry minus 2 T i, k plus T i minus 1, k divided by delta x squared plus beta T i, k minus T infinity. Now, we have this particular equation is going to be not an explicit equation, but the T i, k is given implicitly by this equation.

So, at any time at any time k, we have n plus 1 implicit equations and these n plus 1 implicit equations can be solved either using a Gauss Siedel method or Gauss elimination method, if these are linear equation or using a Newton Raphson's or fixed-point iteration method, if they are non-linear equations.

So, we will have n plus 1 implicit equations in n plus 1 unknown and they can be they can be solved using either the Gauss Siedel method or the Newton Raphson method; this is for linear case and this is for non-linear case. And this example is an example of a linear equation, and this linear equation in n plus 1 unknown we can solve it using any of the linear techniques, such as the Gauss Siedel or the Gauss elimination method.

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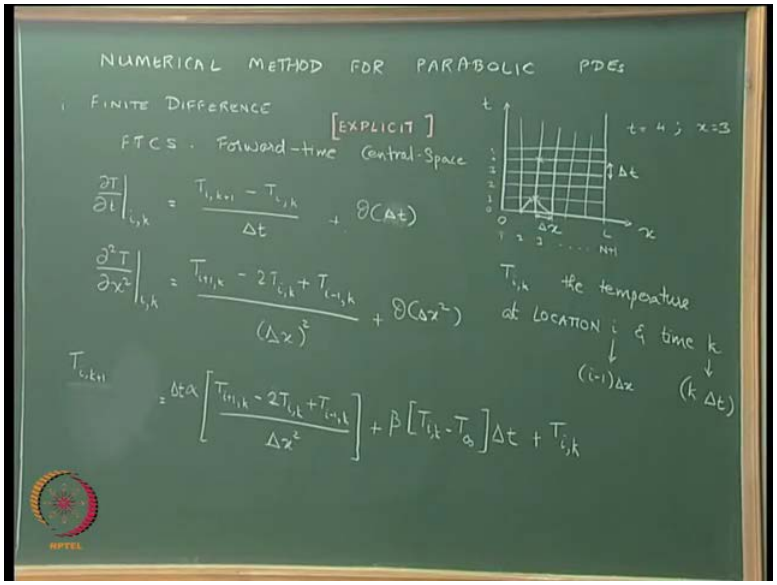

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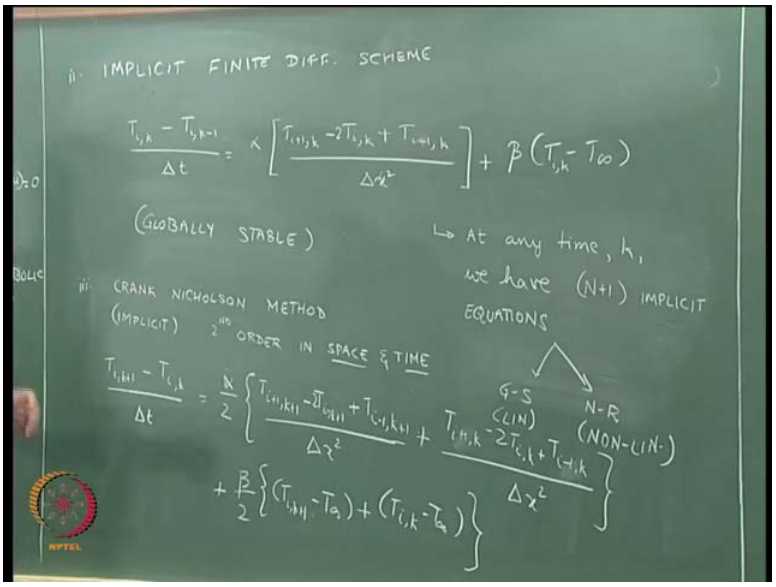

So, these are the two different finite difference method; these are fully explicit and a fully implicit scheme. Now, the advantage of fully implicit scheme is that it is a globally stable method and we will come to implications of globally stable method in the next lecture. In the next lecture, we are going to take up a couple of numerical examples and solve them using both, implicit as well as using explicit scheme, and we will show that the explicit scheme does not result in a stable method for certain conditions, whereas the implicit scheme is always going to result in a globally stable method.

So, we have this forward in time central in space and we have backward in time central in space, and the third and the final method; third method that we are going to talk about is what is known as the Crank Nicholson method. A Crank Nicholson method is an implicit method, but unlike the fully implicit finite difference scheme, Crank Nicholson method is second order accurate second order accurate in space.

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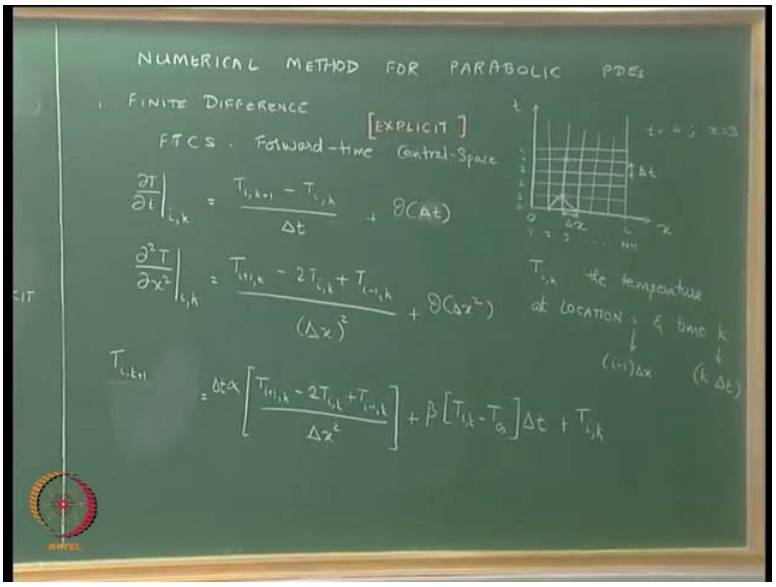

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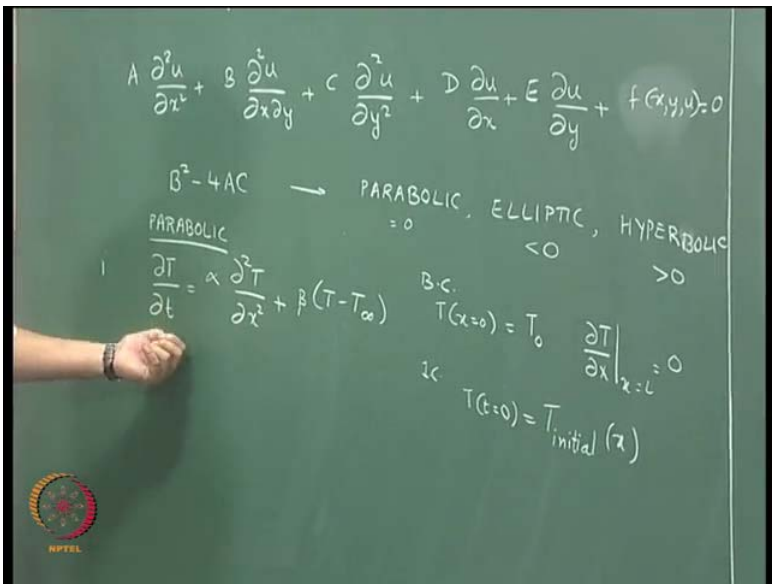

The idea behind Crank Nicholson method is that we will use a mixture of the forward in time central in space ideas and backward in time central in space ideas. So, what we are going to do is partial T by partial t is, we are going to represent this as T i plus 1 minus T i divided by delta t, however this we are going to take average at the time k and k plus 1.

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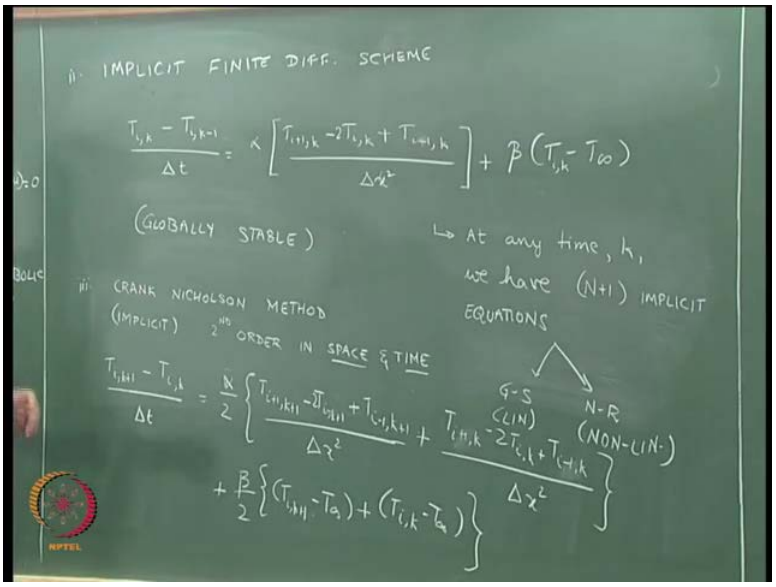

So, dT by d t will be represented as k at t k minus k plus 1 minus T k divided by delta t, whereas this will be represented as average of the two finite difference schemes. So, what I mean by that is, we will write T i, k plus 1 minus T i, k divided by delta t is going to be written as before that is going to be equal to 1 by 2 or alpha by 2 multiplied by this guy computed at T k plus 1 and this guy overall computed at T k.

So that is that is what we are going to do minus 2 T i, k plus 1 plus T i minus 1, k plus 1 divided by delta x squared plus we will have T i , k minus sorry T i plus 1, k minus 2T I , k plus T i, T i minus 1, k sorry divided by delta x squared plus beta by 2 T i, k plus 1 minus T infinity plus T i, k minus T infinity.

This is going to be the Crank Nicholson expression. So, d T by d t we are writing at it as T i , k plus 1 minus T i , k divided by delta t and this right hand side expression, we are going to write this as an average of this expression computed at time k plus 1 plus and the expression computed at time k.

So, for example, this particular term we have this written as an average of that term computed at k plus 1 and time k; likewise, for this term, we have this as an average computed at k plus 1 and time k. And this overall method is going to lead us to a second order accurate method in not space, the method was second order accurate in space, this is second order accurate in both space and time.

So, the Crank Nicholson method is a semi implicit method, which is second order accurate in space and time. The advantage over the explicit method is that this Crank Nicholson method is going to be a globally convergent method, globally stable method just like the implicit finite difference scheme. However, the advantage over the implicit finite difference scheme is that it is second order accurate in time as well; whereas implicit finite difference scheme is second order accurate in space, but first order accurate in time.

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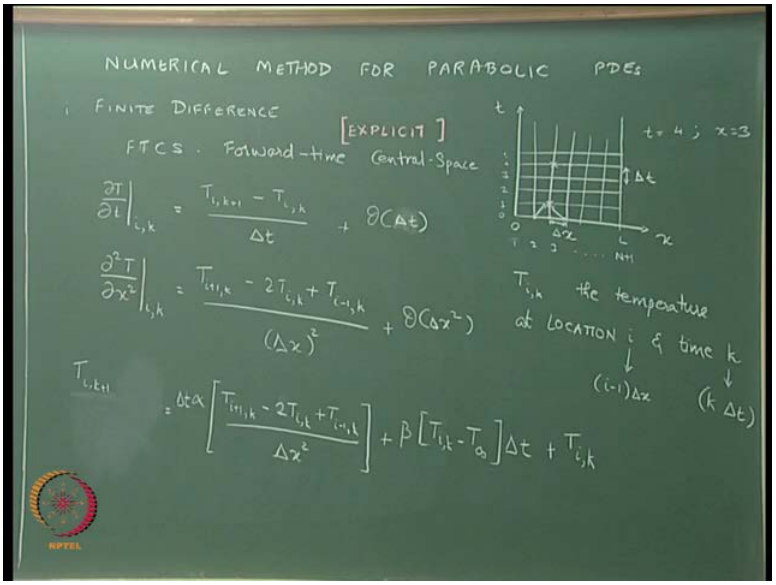

So, these are the numerical methods that are based on the finite differences that can be used for solving a parabolic P D E.

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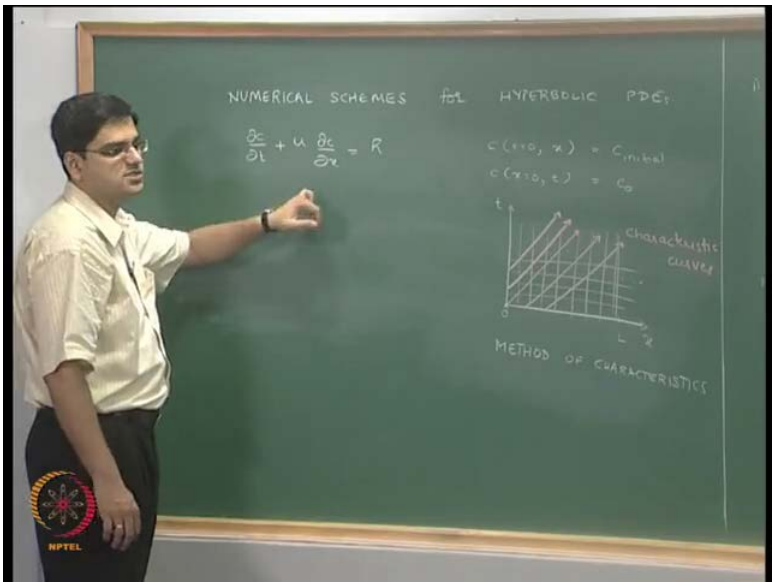

Numerical method - now the hyperbolic P D E we had written it in the form partial c by partial t plus u partial c by partial x equal to some r, which is the function of concentration c. This was our hyperbolic P D E, subject to the initial conditions, c at t equal to 0 and all x is going to be c initial; c at x equal to 0 and all t is going to be equal to c naught.

These were the conditions for which we are going to solve this particular this particular equation, as we have done in the parabolic P D E's. Now, the difference between parabolic and hyperbolic P D E is going to be now, we have this in space and this in time. In in case of parabolic P D E's what happened was the solution in the x direction or in the spatial direction was fixed at time 0 sorry at for first fixed at location 0 as well as at location l; whereas in case of hyperbolic P D E's, it is fixed only at the location 0 along all times and it is fixed at time 0 along all locations. So, if we draw the grid as we had drawn previously, we will get a similar grid as we had we had done previously; however, there is no reason for us to necessarily stop at length L.

For example, if the reactor length, in this particular case is length L, the overall equation based on the overall equation it is not predicated that we need to stop solving these equations; when we reach L, we can indeed continuous solving this equation well beyond L as well, whereas when it came to the parabolic P D E's,

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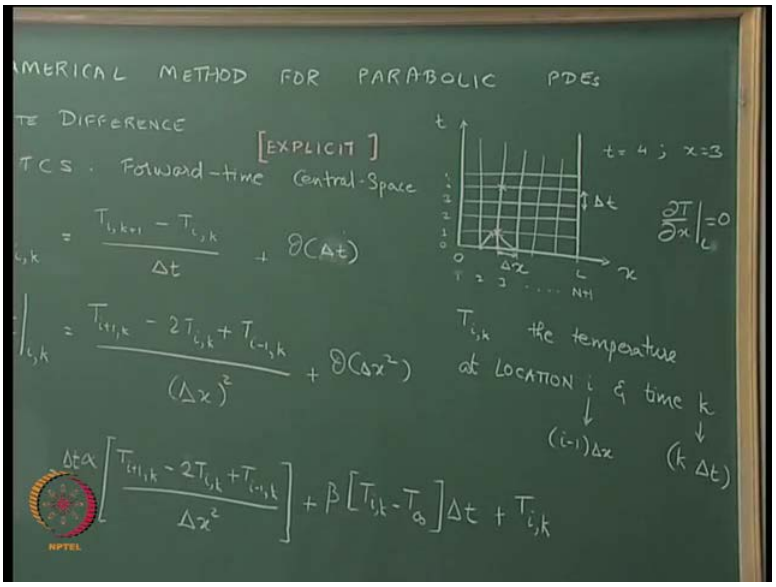

when we had the parabolic P D E's of this sort, there the solution was fixed at l as well, because at L we had another boundary condition and that boundary condition was for example, d t by d x at x equal to L was going to be equal to 0.

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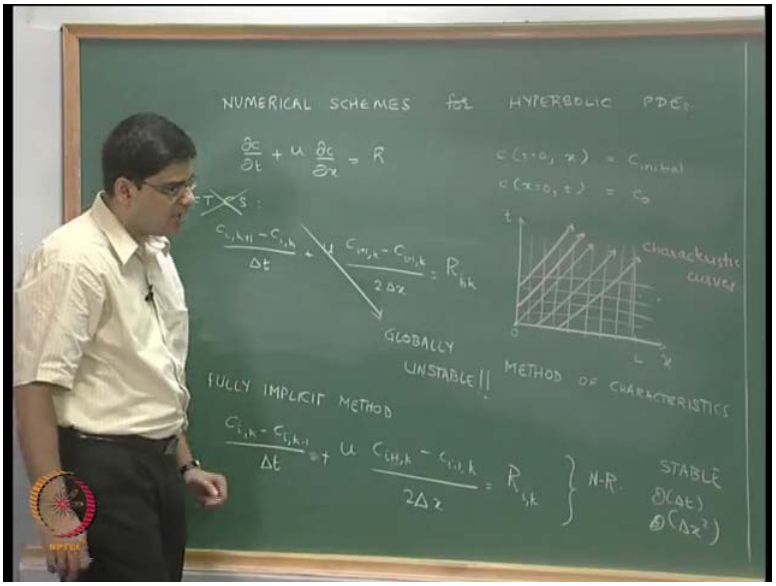

Because of this particular boundary condition when we solve this problem, the solution stops at L and the solution is only going to evolve along the time t. On the other hand in case of the hyperbolic equations, the solution can evolve either along time t or it can evolve around length L. And indeed, if you recall the method of characteristics that you

had used and I hope this was something that was covered in probably the first semester in the first semester math course.

If we were to solve this equation, if we were to solve this system using method of characteristics what we would get is, we will get characteristic curves, which are basically straight lines with slope equal to the velocity u. So, these red lines are the characteristic curves; the solution along the characteristics is to the right of this particular characteristic curve that goes passes through the origin.

These solutions are determined by the inlet condition at sorry at the initial condition at t equal to 0 and various excess. And the solutions at these particular lines, which are to the left of the characteristic curve that passes through the origin are going to be determined by the inlet condition at x equal to 0 and various times.

So, this is how we looked at when we try to solve this particular equation using the method of characteristics. What we mean by that is essentially when it comes to the numerical methods of solution, we can either start off with the solution at initial time or move ahead in space or we can start off the solution at the initial time and move ahead in time.

So, at various spatial locations, the solution is given at initial time and we start moving at a time. Now, because of the similarity between the physical nature of parabolic systems and the hyperbolic system, keep in mind the difference between the parabolic systems and hyperbolic system is that you do not have a d square c by d x square term, and if you recall d square c by d x square term comes in because of the actual diffusion or actual dispersion within the reactor. We had done this and when we talked about O D E boundary value problems.

So, in absence of that particular term, we result we get hyperbolic P D E's. These hyperbolic P D E's although… numerically there is nothing that stops us from solving this particular P D E beyond the length L. Physically, length L is the length of the reactor, beyond that the reactor does not really exist. So, there is a physical restriction even though there is not a mathematical restriction for these particular equations to evolve in space. As a result, just as we did in the parabolic equations, we are going to

discretize in space and we are going to discretize in time and we are going to march forward in time, rather than matching forward in space.

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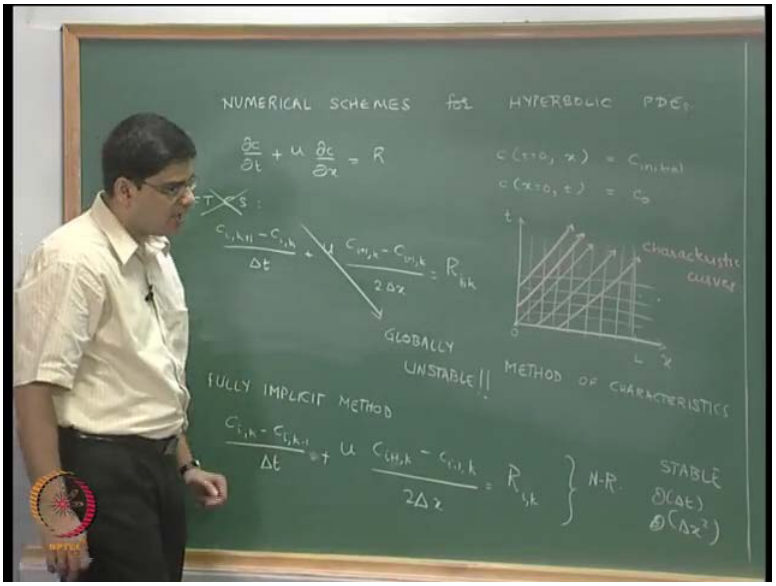

However, numerically there is nothing that stops us from marching forward in space as well. So, one thing we can do is the forward in time central in space idea in which case we are going to get c i, k plus 1 minus c i, k divided by delta t plus u multiplied by c i plus 1, k minus c i minus 1, k divided by 2 delta x equal to r computed at i, k.

So, here the concentration is to be found at time k plus 1. We know all the values just as before, we know all the values at time k. So, you can move this particular equation to the right hand side multiplied by delta t and you will get an expression for c i, k plus 1.

So, you can conceive to use this particular method in order to solve hyperbolic P D E. However, the problem is that this particular method is globally unstable. What we mean by globally unstable is that, we cannot use this particular equation at all.

Because if we use this equation as the time progresses as we march forward in time, the solution is going to go either to plus infinity or minus infinity and it will grow unbounded, as a result the forward in time central in space method is not going to be applicable for hyperbolic P D E's. Forward in time central in space is going to be applicable under certain conditions only for parabolic P D E's, not for the hyperbolic P D E's.

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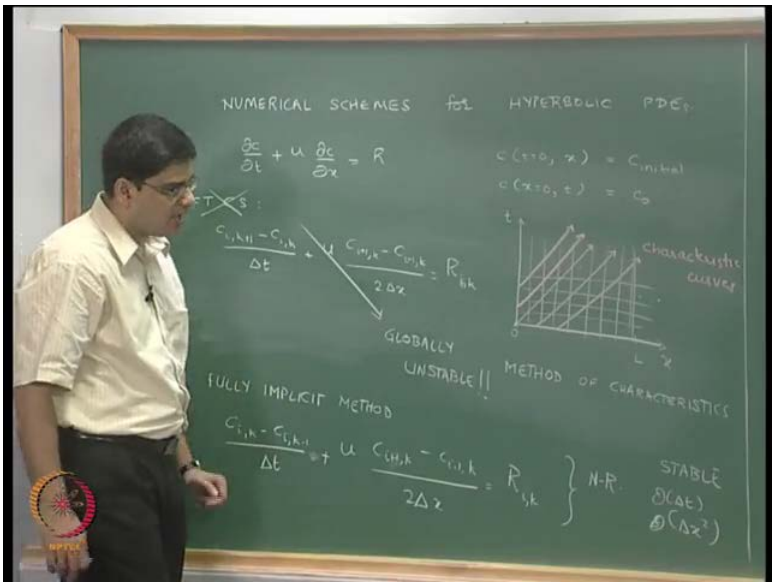

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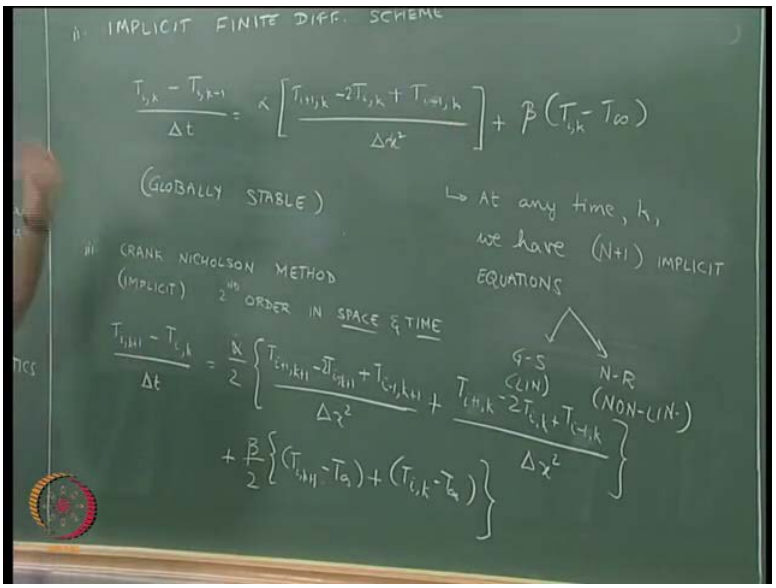

So, the second option is going to be a fully implicit method. In fully implicit method, exactly in the way we solved it earlier, we are going to get a globally stable method again for a hyperbolic P D E's.

So, for hyperbolic P D E we will be able to use this particular equation appropriately modified for our new P D E that we have. We can use the same equation the same type of discretization technique and the resulting solution is going to be a globally stable solution.

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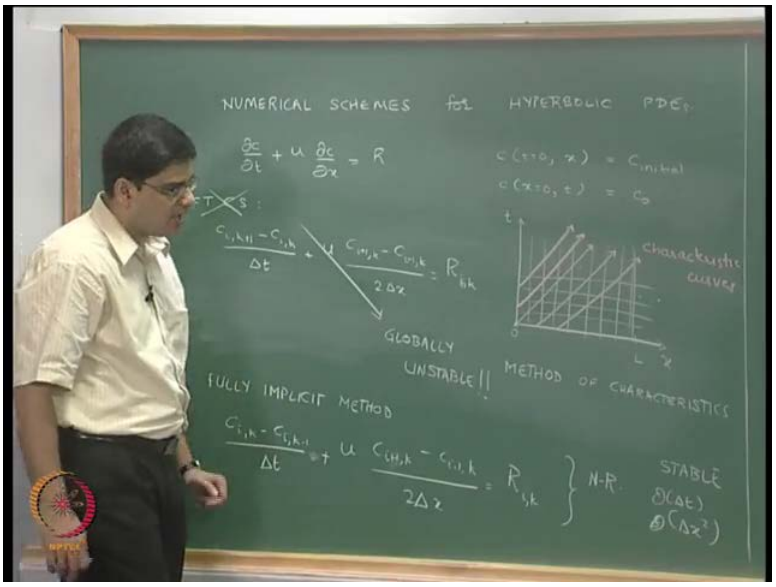

So, the fully implicit method for hyperbolic P D E is going to be c i, k minus c i, k minus 1 divided by delta t is going to be equal to or plus u multiplied by c i sorry c i plus 1, k minus c i minus 1, k divided by 2 delta x equal to r computed at i, k.

Now, if r is a non-linear expression, we are going to have non-linear equations; n non-linear equations at every time k will have n non-linear equation-sorry n plus 1 non-linear equations in c 1 c 2 up to c n plus 1, and we will need a non-linear equation solving technique, such as Newton Raphson's method in order to solve this equations.

The fully implicit scheme is globally stable; its order of delta t accurate and order of delta x square accurate that is the advantage of the advantage of the fully implicit method is that it is stable; the disadvantage is that it is only delta t accurate. So, what is the way that we address this this aspect? The way to address this aspect is again to use the Crank Nicholson scheme.

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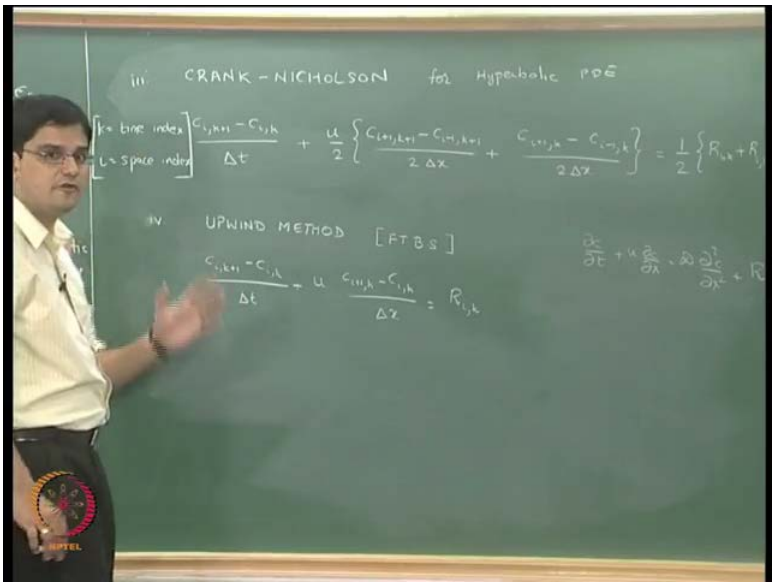

So, the third method is going to be the Crank Nicholson method, and Crank Nicholson is going to be exactly same as the way we had used it for the parabolic P D E. In case of hyperbolic P D E, we will have c i, k plus 1 minus c i, k divided by delta t is going to be equal to… Now, what we have is going to be (Refer Slide Time: 40:40) So, u multiplied by instead of this guy, we will have this as an average of the value computed at k and k plus 1 so that is what we are going to have so not equal to plus u by 2 multiplied by c i plus 1, k plus 1 minus c i minus 1, k plus 1 divided by 2 delta x plus c i plus 1, k minus c i minus 1, k divided by 2 delta x is going to be equal to half of r i, k plus r i, k plus 1.

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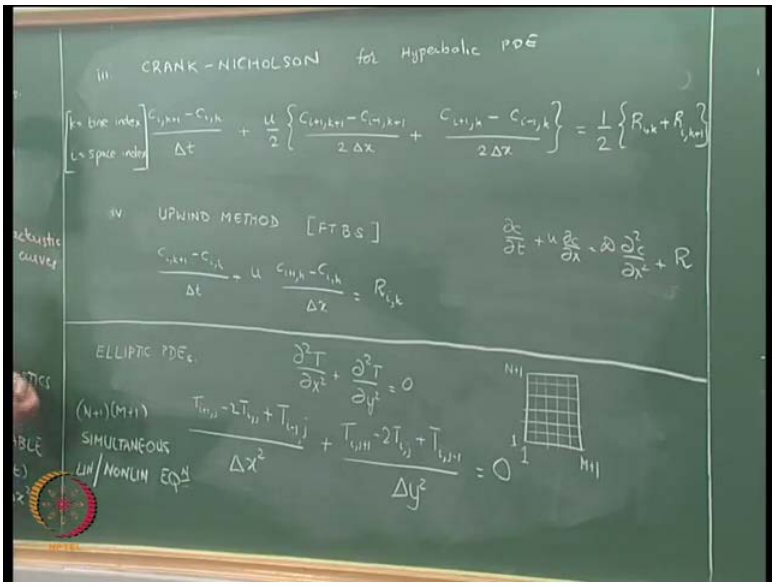

This is going to be our crank Nicholson scheme for the parabolic P D E. keep in mind k is the time index; i is the spatial index; so this is the Crank Nicholson method for hyperbolic P D E. And the final method that I want to talk about for hyperbolic P D E is what is known as an upwind method or for our purpose if velocity is positive, let us assume that the velocity or the flow rate flow takes place from left to the right.

In case of flow taking place from left to the right, this method is going to be forward in time and it is going to be backward in space method. And in this particular case, what we are going to have is c i , k plus 1 minus c i , k divided by delta t plus u multiplied by c i plus 1, k minus c i , k divided by delta x is going to be equal to r computed at i , k.

This is going to be upwind method for solving the P D E's. So, this is a method that is actually not implemented when it comes to the parabolic P D E's; this is a method that is actually use only for hyperbolic P D E's or in case of parabolic P D E's when the velocity is much greater than the diffusivity.

For example, we can have an axial dispersion p f r of the form d c by d t plus u d c by d x equal to d d square c by d x squared plus R and when we non dimensionalize this particular equation, we will essentially get a peculiar number term; and the peculiar number is ratio of the convective to the diffusive fluxes and when the peculiar number is

very large, under those conditions we will have to use an upwind method even for parabolic P D E's.

But for the purely diffusive problem an upwind method is not used, instead we can use the forward in time and central in space method. The forward in time central in space method is not applicable for the hyperbolic P D E's. And the Crank Nicholson method is an implicit method and therefore, it is a globally stable method that is second order accurate in both space and time. So, that is the overall overview of the finite difference methods that we can use for hyperbolic and parabolic P D E's. We will finish off with the finite difference method for elliptic P D E's. Our options for elliptic P D E's are rather limited and we do not have to worry so much. In case of elliptic P D E's, we do not have to worry so much about the stability of the P D E's for the most part.

So, from a conceptual point of view solving elliptic P D E's are in some ways simpler than solving parabolic or hyperbolic P D E's, but the actual implementation usually is significantly more tougher, because the solution is determined by the boundary conditions at all of those boundaries.

So, to take the example of partial square T by partial x squared plus partial square T by partial y squared equal to 0. This where i is going to be is the spatial index in x; j is going to be the spatial index in y. So, if this is the domain, we are going to split the domain in both x and y - in x we will go from 1 to m plus 1; in y we will go to from 1 to n plus 1. And for all of the interior points, we will apply this particular equation; and for all the interior points, we will get T i plus 1, j minus 2 T i, j plus T i minus 1, j divided by delta x squared plus T i, j plus 1 minus 2 T i, j plus T i, j minus 1 divided by delta y squared equal to 0, and this is going to be the overall equation.

Now, here the equation neither evolves in i directions nor does not evolve in j direction. As a result when you write all these equations, you have written the equations for i equal to 1 to m plus 1, and j equal to 1 to n plus 1. As a result, we will get n plus 1 multiplied by m plus 1 simultaneous equation, which could be linear or non-linear equations.

So, you will have to solve n plus 1 multiplied by m plus n equations simultaneously in order to get the overall solution. The difference between this method and the fully implicit or the Crank Nicholson method for hyperbolic or the parabolic P D E's is that, in

hyperbolic or parabolic P D E's you only have to solve n plus 1 equations in the spatial domain at one time and then we move on to the next time.

We again have to solve n plus 1 equation, we move on further and further; so it evolves in one direction and time, whereas elliptic P D E's you have to solve the entire n plus 1 multiplied by n plus 1 m plus 1 equations simultaneously.

In general, the amount of effort required for solving m plus 1 multiplied by n plus 1 equation in one go is going to be much greater than the amount of effort required in solving n plus 1 simultaneously equations n plus 1 number of times.

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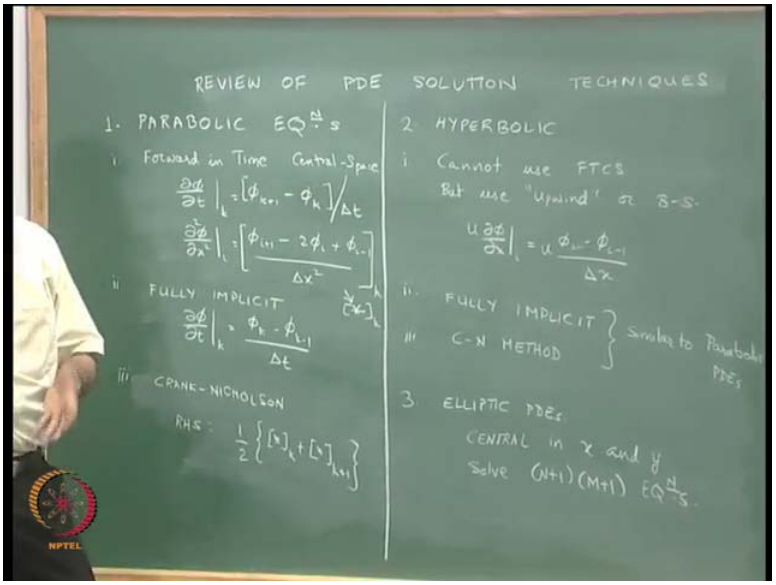

So that is the overview or that is the various finite difference methods that can be used for solving elliptic parabolic and hyperbolic. First we looked at parabolic equations; the first method was forward in time central in space. Forward in time central in space, involved writing d phi by d t at time k as phi k plus 1 minus phi k divided by delta t, and writing d phi by r d square phi by d x square in i as phi i plus 1 minus 2 phi i plus phi i minus 1 divided by delta x squared.

This guy was computed at the previous time k. This was the forward in time central in space method, which can be used for parabolic equations. The second alternative for parabolic equations is to use a fully implicit method, and in the fully implicit method we write d phi by d t at time k as phi k minus phi k minus 1 divided by delta t, instead of the

sorry i miss the delta t over here instead of phi k plus 1 minus phi, we write phi k minus phi k minus 1 in the fully implicit method.

The third method we discussed was the Crank Nicholson method. And in the crank Nicholson method, we use the explicit that is the forward in time kind of a discretization in the time domain. However, in the space spatial domain we take d square phi by d x square is going to be average of this guy computed at k and this guy computed at k plus 1.

So, if we write this as say star k, then in the Crank Nicholson method we are going to use - the right hand side is going to be half of star k plus star k plus 1, which basically means this overall derivative as well as all the constant terms have to be computed at k and k plus 1 and we take an average over there.

Those are the parabolic equations. The hyperbolic equations - in the hyperbolic equations we cannot use F T C S, instead we use an upwind method or backward in space method for positive for positive velocities. And in that particular case, u dou phi by dou x, at location i is written as phi i minus phi i minus 1 divided by delta x multiplied by u. And if u is negative, we write the forward difference, instead of the backward difference approximations; I would not i would not really go into that.

The second option is the fully implicit method and the third option is the Crank Nicholson method; these both methods are similar to parabolic P D E's. And finally, for elliptic P D E's , we use central difference in both x and y domain and then solve the resulting n plus 1 multiplied by m plus 1 equations simultaneously. central in x and y and solve the n plus 1 multiplied by m plus 1 equations simultaneously

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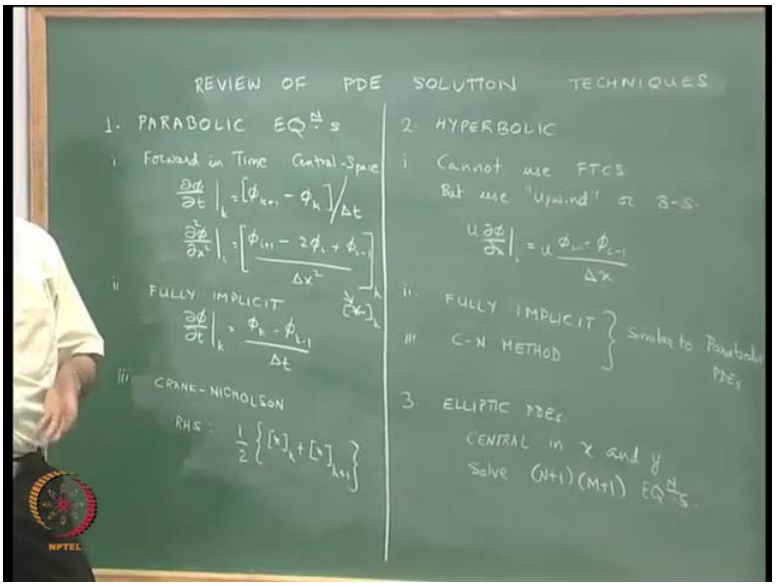

So that is the overview of all the numerical techniques for solving P D E's. In the next lecture what I am going to do is take up a couple of examples and solve those couple of examples using the forward in times central in space for the parabolic equations and the upwind method for hyperbolic equation. And see under what conditions we get the overall solutions to be stable, under what conditions we get the overall conditions equations to be unstable. After that, we will go on to the board and I will state the various conditions, which has for example, in case of parabolic P D E's those are current conditions for stability.

I would not derive those conditions; I will just state those conditions, which have to be met in order to ensure that the P D E's are stable.

So that is our game plan for the next couple of lectures essentially to solve the parabolic hyperbolic and elliptic P D E's, and see what are the pit falls and what are the good methods that we have to incorporate in order to solve these equations.

Thank you and see you in the next lecture.