Computational Techniques

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Module No. # 06

Lecture No. # 03

Differentiation and Integration

Welcome to lecture 3 of the module on numerical differentiation and integration. What we have done in the previous module is motivating the idea of numerical integration and used a geometric interpretation of what numerical integration means. Specifically, we considered two methods - one was the Euler's method and the second one was trapezoidal method.

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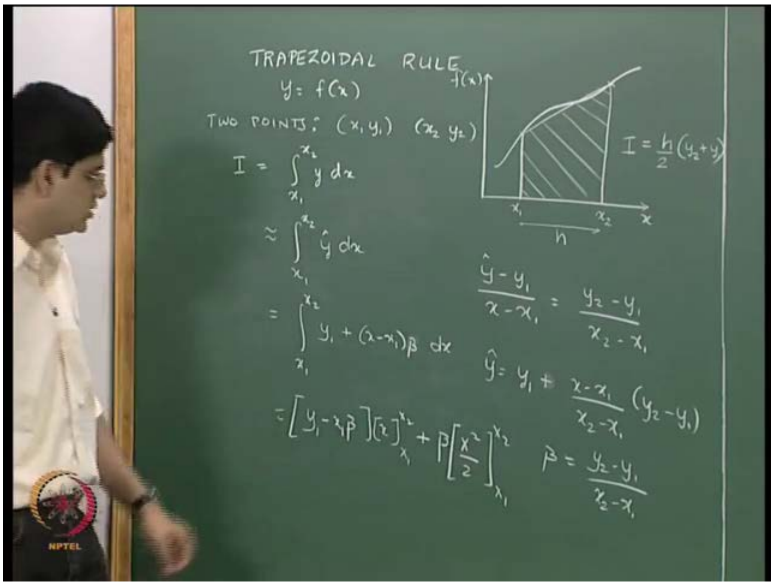

In trapezoidal method what we did was, we tried to approximate the area under the curve as approximately equal to the area under a trapezoid. So, what the geometric interpretation of the trapezoidal method was is, if we plot f of x against x and if we get say some kind of an arbitrary curve of this sort, and we want to integrate from let say x 1 to x 2, what we do is we just connect these two points by a straight line and now, area under the trapezoid that is formed over here is the area under the curve.

So, that is what we studied in the previous lecture. This is really the geometric interpretation of trapezoidal rule. Based on this particular geometric interpretation, we said that the integral i using the trapezoidal rule was equal to half multiplied by x two minus x 1; we call this x 2 minus x 1 as h. (Refer Time: 02:00) So, h by 2 multiplied by f of x2 plus f of x 1 or we will I will just write that as y 2 plus y 1 and this is the area under the trapezoidal; its half multiplied by the base multiply multiplied by sum of the two heights that was the geometric interpretation of the trapezoidal rule .

So, what we are going to do today is try to formalize a couple more ways of deriving the trapezoidal rule, and the first one is again directly motivated by this geometric interpretation. So, what we are doing is we are joining a line that passes through two points; the two points are (x 1, y 1) and (x 2, y 2).

What I am assuming over here is y equals f of x. So, rather than writing every time f in brackets x, f in bracket x and some things like that I will just write it as y. And now, we are interested in finding integral of y d x from x 1 to x 2 or in general from a to b, that is the problem that we are we are trying solve. So, our integral I is going to be equal to integral from x 1 to x 2 of y d x.

So, this is the exact integral that we intend to find. The approximate integral based on the trapezoidal rule is really the area under the trapezoid. So, we will we will write that as approximately equal to x 1 to x 2 and just to use some other notation, I will use say y hat d x, where y hat is nothing but the equation for this particular line.

Now, this particular line, the equation for that is going to be essentially y minus y 1 divided by x minus x 1 equal to y 2 minus y 1 divided by x 2 minus x 1 or in other words, we take x minus x 1 and multiply throughout, and we just rearrange that we will be able to write y equal to y 1 plus x minus x 1 divided by x 2 minus x 1 multiplied by y 2 minus y 1, and because we are representing this as y hat, I will just put the hat signs over here.

So, now, what we the area under the trapezoid is nothing but integral of this particular part d x from x 1 to x 2; so, we will just substitute that over here. So, we have integral from x 1 to x 2 y 1 plus x minus x 1 times alpha or rather than using alpha, I will need the term alpha later on. So, I will just call it instead of that something beta.

So, y plus x minus x 1 beta times d x, where beta I have just called this as y 2 minus y 1 divided by x 2 minus x 1. So, when we integrate this we, are going to get y 1 minus x 1 beta that is the constant multiplied by x going from x 1 to x 2 plus beta multiplied by x square by 2 from x 1 to x 2. So, y minus x 1 times beta is a constant integral, y minus x 1 beta d x is y minus x 1 beta multiplied by x the limits of integration are form x 1 to x 2. And likewise, here we have beta multiplied by x this term, when we integrate that it is going to be beta x square divided by 2 going from x 1 to x 2.

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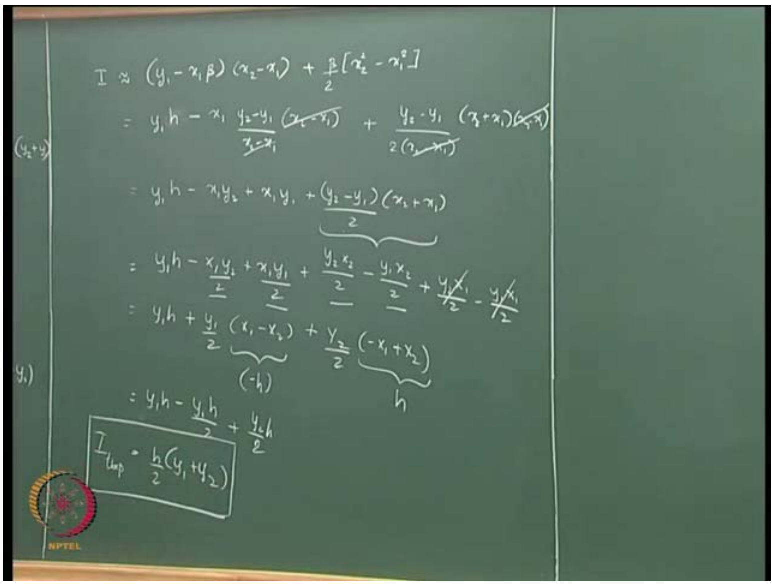

So, let substitute those values in there. So, i is going to be approximately equal to y 1 minus x 1 beta multiplied by x 2 minus x 1 plus I will have beta by 2 x 2 squared minus x 1 squared. I will substitute the value of beta over here in a movement, I will just write it down. We have y 1 multiplied by x 2 minus x 1; I will call that as, h x 2 minus x 1 is this distance h, so i'll just substitute that over there. So, I will have y 1 multiplied by h minus x 1 y 2 minus y 1 divided by x 2 minus x 1, that was the value beta multiplied by x 2 minus x 1, this gets canceled plus y 2 minus y 1 divided by 2 times x 2 minus x 1 multiplied by… So, x 2 squared minus x 1 squared, I will write that x 2 plus x 1 x 2 minus x 1 and the x 2 minus x 1 will get canceled.

So, we have y 1 multiplied by h minus x 1 y 2 plus x 1 y 1 plus y 2 minus y 1 divided by 2 multiplied by x 2 plus x 1. So, this we will be able to expand this as y 2 x 2 by 2 minus y 1 x 2 by 2 plus y 2 x 1 by 2 minus y 1 x 1 by 2 plus x 1 y 1.

And we have x 1 y 1 by 2 and x 1 y 1 over here. So, this will gets canceled and we will just divide this particular guy by 2, we have x 1 y 2 and we have this x 1 y 2 minus So, this will get canceled and we will just be able to divide divide this divide this by 2.

So, we will write this as y 1 h minus y 1 multiplied by x, so we will plus y 1 by 2 we look at these two terms and that is x 1 minus x 2 plus y 2, and then we will look at these two terms and that is going to be y 2 divided by 2 multiplied by minus x 1 plus x 2; minus x 1 plus x 2 is nothing but h, and x 1 minus x 2 is nothing but minus h.

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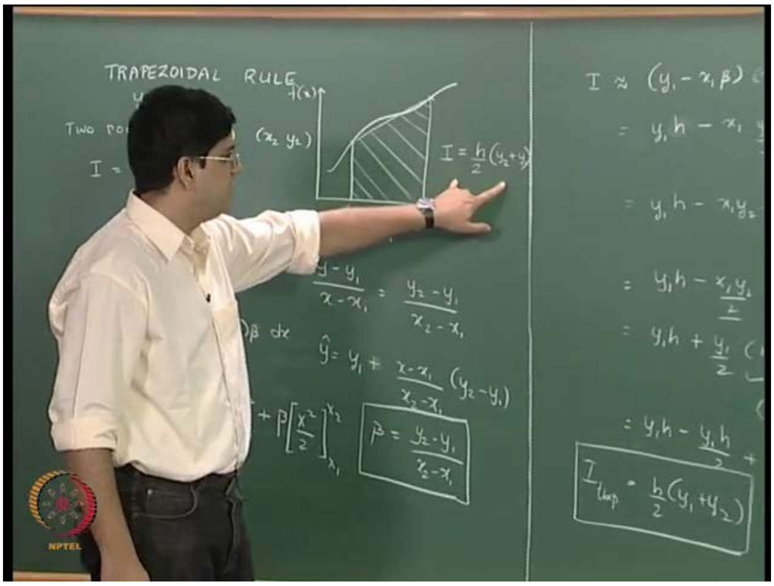

So, we have y 1 h minus y 1 h by 2 plus y 2 by 2 multiplied by h and that we will be able to write, this will get cancel and this becomes y 1 h by 2 will be able to write this as h by 2 multiplied by y 1 plus y 2. So, the integral I using trapezoidal method is written as, I using the trapezoidal method is h by 2 multiplied by y 1 plus y 2. If we go back and look at the geometric derivation, that is exactly what we had obtained as the integral I using the trapezoidal method.

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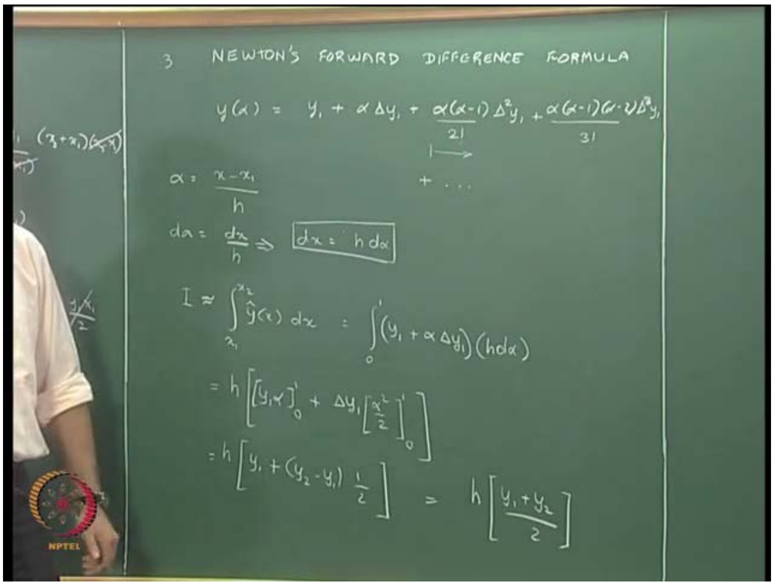

So, the trapezoidal method - the essence of it is you take two data points, connect those two data points by a straight line and find the integral under that straight line. The third method we will use is based on Newton's forward difference formula and if you recall the Newton's forward difference formula for two data points was nothing but y zero sorry y 1 plus alpha multiplied by delta y 1 plus so on and so forth.

So, if you were to write a polynomial y of alpha, then we would be able to write this as y 1 plus alpha times delta y 1 plus alpha multiplied by alpha minus 1 times delta squared y 1 divided by 2 factorial plus alpha alpha minus 1 alpha minus 2 delta cubed y 1 divided by 3 factorial plus dot dot dot.

So, depending on how many data points that we have, will be able to write it as a longer series. Now, when we have just two data points, we are going to retain only the first two terms. For the Newton's forward difference formula, we had alpha was equal to x minus x 1 divided by the h, which is which is del delta x is nothing but h in this particular case.

So, it was x minus x 1 divided by h. Therefore, when we differentiate this, we will get d alpha equal to d x by h or d x equal to h times d alpha. So, this is what we will get when we substitute the value of d x in terms of d alpha.

So, let us recall what integral I was. Integral I was going from x 1 to x 2 integral of y hat y hat x d x that was the approximate value of the integral; when it comes to the trapezoidal rule, x 1 is corresponds to alpha equal to 0; you substitute x 1 over here, we get alpha equal to 0; you substitute x 2, we get alpha equal to 1.

So, based on the Newton's polynomial, we will have the integral going from 0 to 1, y hat is y 1 plus alpha delta y 1, so that is going to be our polynomial multiplied by d x d x is h times d alpha. And now, we do the integration and we will we will get this as h multiplied by y 1 alpha and alpha going from 0 to 1 plus delta y 1 multiplied by integral alpha d x and alpha d x is going to be alpha squared by 2, again going from 0 to 1 and this is going to be nothing but h multiplied by alpha is going from 0 to 1 is going to be 1 minus 0.

So, it is just going to be y 1 plus delta y 1 was nothing but y 2 minus y 1 multiplied by alpha squared by 2; so that is 1 minus 0 divided by 2; so that is going to be 1 by 2. So, therefore, we have y 1 plus y 2 by 2 minus y 1 by 2; y 1 minus y 1 by 2 is going to be plus of y 1 by 2.

So, therefore, we will have this as equal to h multiplied by y 1 plus y 2 divided by 2. When we do this derivation, we find that the final value of the integral was the same that we obtained using our geometric or our algebraic methods for joining the two points with a straight line.

Now the question is why did I go through an additional method in order to figure out what the same integral is? The reason I went through this additional method is because the Newton's forward difference formula, using the Newton's forward difference formula we can figure out what the leading error term is. So, when we are just using two data points, if we had an additional third data point, our polynomial would be even more accurate, but we are not using this additional third data point, and that additional point that we are we are actually throwing away is going to give us an idea of what the error of this particular method is.

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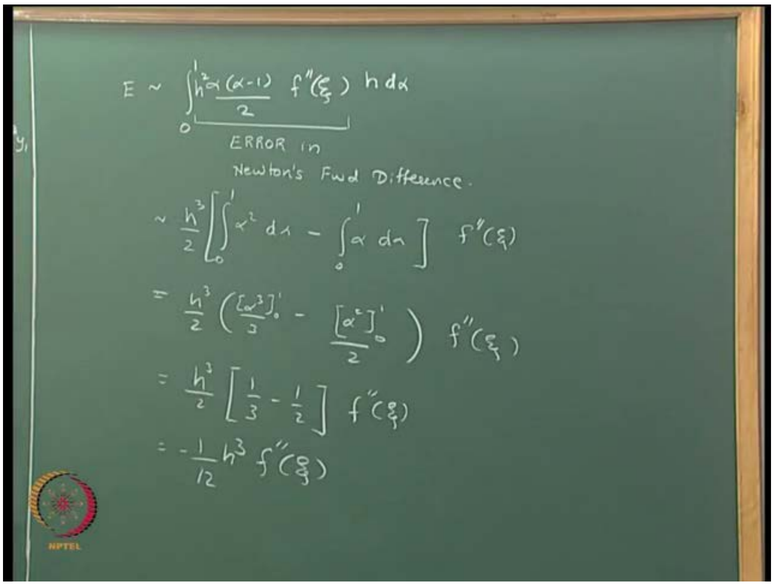

So, the error of this method and this was something that I did not cover in the previous lecture. Using mean value theorem, actually it can be proved that the terms that we are discarding away from this point onwards, the error E is of the order of alpha multiplied by alpha minus 1 divided by 2 factorial f double dash of x that is what it can be proved. So, error in the trapezoidal method is of the order of integral from x 1 to x 2 or integral from 0 to 1 alpha multiplied by alpha minus 1 divided by 2 F double dash of zeta and d x will be replaced by h d alpha. And in addition to that, each of this is associated with h for this particular guy and h for this this particular guy.

So, we will have an x square term over here. This is the standard error in Newton's forward difference polynomial. So, the error for trapezoidal method is of the order of h cube by 2 multiplied by integral from 0 to 1 alpha square d alpha minus integral from 0 to 1 alpha d alpha, which is equal to h cube by 2 multiplied by alpha's cubed by 3 from 0 to 1 minus alpha squared by 2 going from 0 to 1, which is h cube divided by 2 multiplied by 1 by 3 minus 1 by 2.

So that is 2 minus that is going to be 2 minus 3 divided by 6; so that is minus 1 by 6. So, the error is going to be minus 1 by 12 h cube and I have been forgetting f double dash zeta throughout.

So, I will just write that over here multiplied by f double dash zeta and f double dash zeta. So that is really the error when computing the integral using the trapezoidal rule. So, this is essentially the overall derivation using the Newton's forward difference formula.

So what we started off with was, this is the way we will represent using the Newton's forward difference formula, we have because we have essentially only two data points x 1 y 1 and x 2 y 2; we are discarding the rest of the terms in the Newton's forward difference formula. The error associated with discarding this rest of the term essentially is this particular guy. And then what we said is we substitute d x equal to h d alpha in that equation, when we substitute d h equal to h d alpha, we get this error term as well as get this integral term.

Now, next question is can we do better than the trapezoidal method and the answer to that of course, you would expect the answer to that is going to be yes. And we have said that other methods that can possibly be used are the Newton's one third rule, and newton’s sorry the Simpson’s one third rule and Simpson’s three 8h rule and we are going to discuss Simpson’s one third and three 8. So, let us now go on to the Simpson’s one third rule and Simpson’s three 8hs rule and do the derivations for them.

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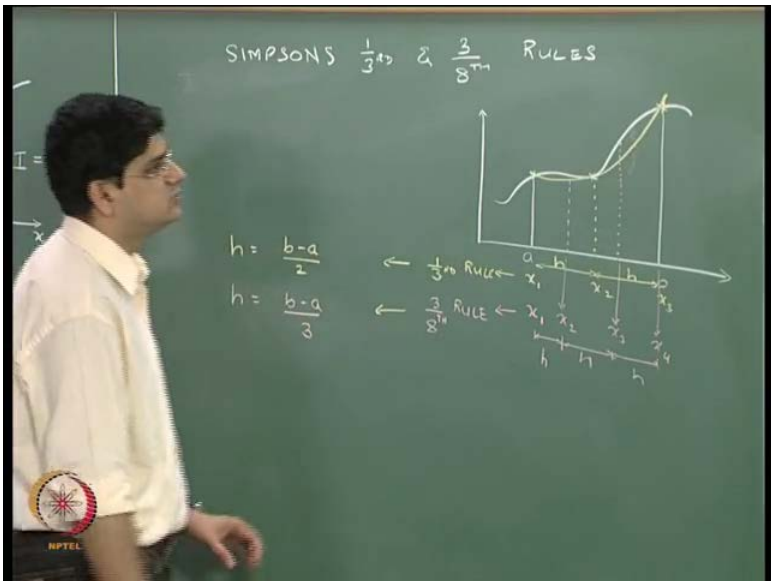

So, the idea is as follows: let us say we have the any arbitrary curve and we want to find integral from a to b, f x d x is to look at another point, which is a mid-point for this particular guy. Now, we have three data points. So, we can fit second order Newton's polynomial to these three data points and the area under that second order Newton's polynomial perhaps, the second order Newton's polynomial would look like this ok

So, area under this particular yellow curve between a to b will be the area of the approximate integral that we get. So, if you are going to use these three data points, we will call them as x 1, x 2 and x 3; this will lead us to the one third rule.

And what about the three 8hs rule? In the three 8hs rule, instead of fitting out second order polynomial, we will fit a third order polynomial. In order to fit the third order polynomial, we need 4 data points. So, instead of bifurcating this particular region, we will trifurcate that region; we will break down that region into three intervals and those three intervals I will mark them sorry I will mark them using this red dotted line and I will write them using the red chalk over here; they will become x 1, this guy is going to be x 2, this guy is x 3 and this guy is x 4 and this will give us three 8h rule.

Please keep in mind; please do not get confused x 1 x 2 and x 3 for the one third rule derivations are different from the three 8h rule derivation. So, please do not get confuse over there. The h for the three 8h rule is going to be there are going to be three h, all of them are going to be equal, although this one does not look like its equal, but believe me it supposed to be equal.

So, we have the overall area from a to b is going to be equal to 3 h in case of the Newton's three 8h rule, whereas in case of the Newton's one third rule, it is going to be 2 h. I am using again the same notations just for notational simplicity; please do not get confused between them, I just repeat that.

So, for Newton's one third rule, our h is going to be equal to b minus a divided by 2 and for Newton's three 8hs rule, our h is going to be equal to b minus a divided by three. So that is what we have so far. Now, what we will do is we will use a Newton’s forward difference formula as we did before and derive the new the one third rule and then, again we will derive the three 8h rule. So, most of it remains the same, until this point remains the same, the change happens this point onwards only.

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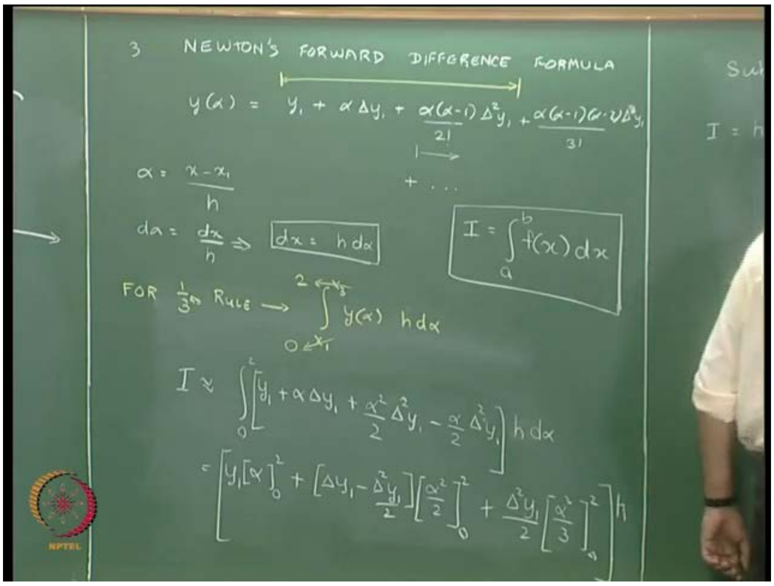

And we are interested in finding I equal to integral from a to b f of x d x, this is our interest. We do not know our function f of x, but instead of that we know the value at several distinct points inside the integral. In case of the newton's sorry in case of Simpson's one third rule, for the one third rule we have integral from x 1 to x 3 y of alpha d x and d x we will write it as h d alpha and what y of alpha are we going to use? We are going to use y of alpha over here, from this point to this point and x 1 is going to be, in terms of alpha, x 1 will be 0 and x 3 in terms of alpha will be 2.

So, we will have 0 to 2 integral y 1 plus alpha delta y 1 plus alpha multiplied by alpha minus 1 divided by 2 delta square y 1. So, I by one third rule is going to be approximately equal to integral from 0 to 2 y 1 plus alpha delta y 1 plus alpha squared by 2 delta squared y 1 minus alpha by 2 delta squared y 1 d alpha .

So, this is going to be equal to y 1 multiplied by alpha, where alpha goes from 0 to 2 plus delta y 1 minus delta squared y 1 i am sorry i just forgot h over here should be h d alpha delta y 1 minus delta squared y 1 multiplied by sorry delta y 1 minus delta squared y 1 by 2 multiplied by alpha and when we integrate alpha, we will get alpha squared divided by 2.

So, we will have alpha squared by 2 from 0 to 2 and the third term is going to be delta squared y 1 by 2 multiplied by integral 0 to 2 alpha squared. So, it will be delta squared y

1 by 2 integral of alpha squared and integral of alpha squared is alpha cube by 3 going from 0 to 2.

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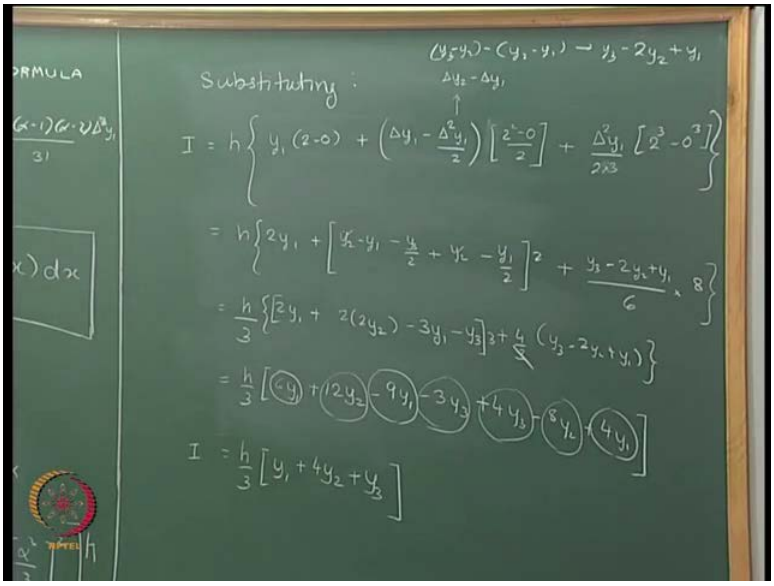

I is an this whole thing will be multiplied by h, I keep forgetting this h all the time; so I will write that write in the beginning. We have h multiplied by y 1 multiplied by alpha; so y 1 multiplied by 2 minus 0 plus delta y 1 minus delta squared y 1 by 2 multiplied by alpha squared by 2, alpha squared by 2 is going to be 2 squared minus 0 by 2, that is what it is going to be; plus delta squared y 1 by 2 multiplied by alpha cubed by 6 by 3. So, this is going to be 2 multiplied by 3 and we will have alpha cubed going from 2 to 0.

So, its 2 cubed minus 0 cubed that is the overall expression. Now, delta squared y 1 is going to be nothing but delta y 2 minus delta y 1. So, this is going to be h multiplied by 2 y 1 plus delta y 2 is going to be y 3 minus y 2 minus delta y 1 is y 2 minus y 1; so this is y 3 minus 2 y 2 plus y 1.

So, we will have delta y 1 as y 2 minus y 1 minus delta squared y 1 by 2 is going to be minus y three by 2 plus 2 y 2 by 2, so that is going to be y 2 minus y 1 divided by 2 this multiplied by 4 by 2 that is 2 and plus delta squared y 1, that is, y 3 minus 2 y 2 plus y 1 divided by 6 multiplied by 8. So that will be h multiplied by 2 y 1 plus, we will take 2 inside over here, so that is going to be 2 y 2.

So, 2 multiplied by 2 y 2; so these two have gone, we have minus y 1 minus y 1 by 2, that is, minus 3 y 1 by 2 multiplied by 2 that is going to be minus 3 y 1 and we have minus y 3 by 2 multiplied by 2, that is, minus y 3 plus 4 by 3 multiplied by y 3 minus 2 y 2 plus y 1, and we take 3 outside and we will get this; we will remove this 3, we will multiply all of these by 3.

And we should It is just a matter of simple numerical jugglery; so what we will have is 6 y 1 plus 4 y 2 multiplied by 3 that is going to be 12 y 2 minus 9 y 1, so that is going to be minus 3 y 3 plus 4 y 3 minus a y 2 plus 4 y 1.

So, the terms in y 1 are 6 y 1 minus 9 y 1, that is, minus 3 y 1 plus 4 y 1. So, 4 minus 3 y 1 plus 4 y 1 is going to be equal to y 1; so its h by 3 multiplied by y 1. Now, we have plus 12 y 2 minus 8 y 2, that is plus 4 y 2 plus 4 y 2 and we have minus 3 y 3 plus 4 y 3 that is plus y 3. So, the Simpson's one third rule is going to be h by 3 multiplied by y 1 plus 4 y 2 plus y 3.

So, that is our Simpson's one third rule; that is how that is what we will get when we use the Simpson's one third rule, in order to find out the integral. So, to recap what we actually did was we in the Newton's forward difference formula, we retained the first three terms and then integrated it for integrated the equation from 0 to 2, and when we did the integration, we were essentially left with this particular formula and then, we substituted the values of alpha and so on into this particular equation and this is what we ended up getting.

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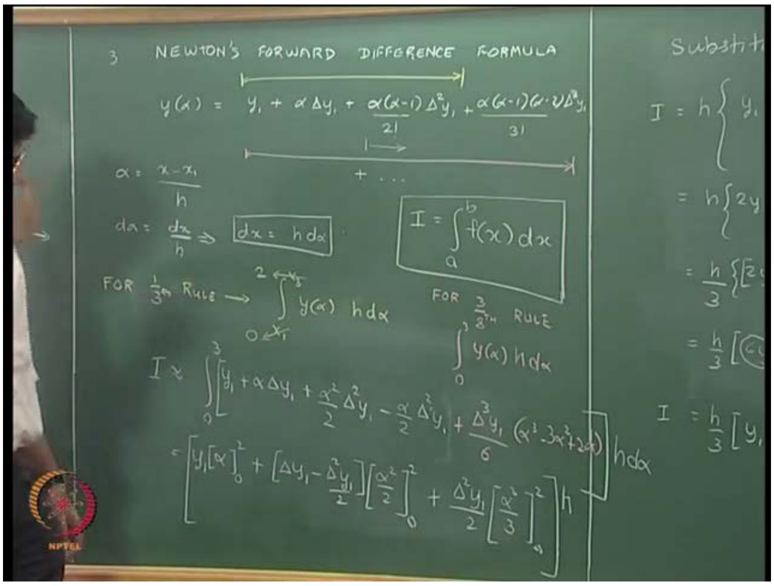

Now, if you want to derive on another higher order formula, that is going to be the Newton's three-eighths rule and let me remind you what we have to do in Newton's three-eighth rule is to go from integral from x 1 to x 4 integral f of x d x. We will use pretty much the same idea, but instead of stopping at third term we will include the fourth term as well and for the three-eighths rule we will have integral. So, keep in mind, we had integral from x 1 to x 3 y alpha h d alpha, in this case we will have integral from x 1 to x 4 y alpha h d alpha and integral from x 1 to x 4 is from alpha equal to 0 to alpha equal to 3, instead of alpha from 0 to 2 now we have alpha going from 0 to 3 y of alpha h d alpha and what do we need to do in order to modify this? In order to modify this, we just take the eraser in order to modify this instead of going from 0 to 2, we need to go from 0 to 3; we have y 1 plus alpha delta y 1 plus alpha squared delta squared y 1 minus alpha by 2 the this guy and i have additional term over here and that is going to be plus delta cubed of y 1 by 3 factorial, which is 6 multiplied by alpha multiplied by alpha minus 1 multiplied by alpha minus 2.

So, that is going to be alpha multiplied by alpha squared minus 3 alpha plus 2; so its alpha cubed minus 3 alpha squared plus 2 alpha multiplied by h d alpha. So, we will have this additional term over here and all the limits of integral are going to be not from 0 to 2, but instead they are going to be from 0 to 3.

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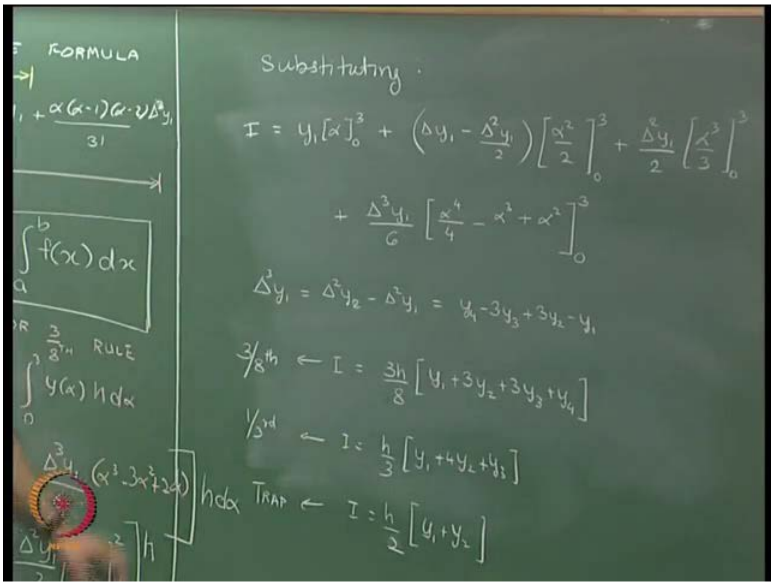

So, I as before, we will use all of these terms as before only thing is that integral is going from 0 to 3, instead of 0 to 2. So, we will have y 1 alpha going from 0 to 3 plus delta y 1 minus delta squared y 1 by 2, this multiplied by alpha squared by 2 going from 0 to 3 plus delta y 1 squared sorry delta squared y 1 divided by 2 multiplied by alpha cubed by 3 going from 0 to 3 and then, we have this additional term in red; this particular additional term in red, we need to get an integral for that.

So, we will have plus delta cubed y 1 divided by 6 multiplied by alpha to the power 4 by 4 minus 3 alpha squared integral of that is going to be minus alpha cubed, and plus 2 alpha it is the integral of that is going to be alpha squared going from 0 to 3, and delta cubed y 1 is going to be equal to delta squared y 3 minus delta squared sorry delta squared y 2 minus delta squared y 1, and if we substitute this particular expression into that, I think we should be getting y 4 minus 3 y 3 plus 3 y 2 minus y 1. So, this is what I believe we will get when we substitute delta cubed y 1 into this particular expression.

So, now, what we have is just left with a bunch of algebraic manipulations and once you do the algebraic manipulations, we expect to get I is going to be equal to 3 h by 8 multiplied by y 1 plus 3 y 2 plus 3 y 3 plus y 4. This is what we expect essentially to get using the Newton's 3 8hs rule and the sorry Simpson's three-eighths rule and the name three-eighths basically suggest that we have 3 by 8 as a constant over here.

So, just to recap the the the Newton's sorry the Simpson's three-eighths rule, is this guy the new is Simpson's one-third rule is I equal to h by 3 multiplied by y 1 plus 4 y 2 plus y 3. The trapezoidal rule, I equal to h by 3 sorry I equal to h by 2 multiplied by y 1 plus y 2.

So, this is the Simpson's three-eighths rules, Simpson's one-third rule and the trapezoidal rule. So, to recap what we have done so far is, we started off with the trapezoidal rule, we recap the geometric interpretation of the trapezoidal rule, then we used Newton's interpolating polynomials, specifically the Newton's forward difference formula in order to derive the trapezoidal rule, the one third rule and the three-eighths rule. With respect to the trapezoidal rule, we also saw how to derive the error conditions, I have not derived the error conditions for Newton's one third as the Simpson's sorry one-third or three-eighths rule, but those can be derived in the same way as we have done the previous derivation. And finally, we have shown how to get the how to use the trapezoidal rule, one- third rule and the three-eighths rule. In this particular case, h is going to be equal to b minus a by 3, in one-third rule h is b minus a by 2, and in trapezoidal rule h is equal to b minus a.

So, now what we will quickly do is go to Microsoft excel and use the trapezoidal rule, one- third rule and the three-eighths rule.

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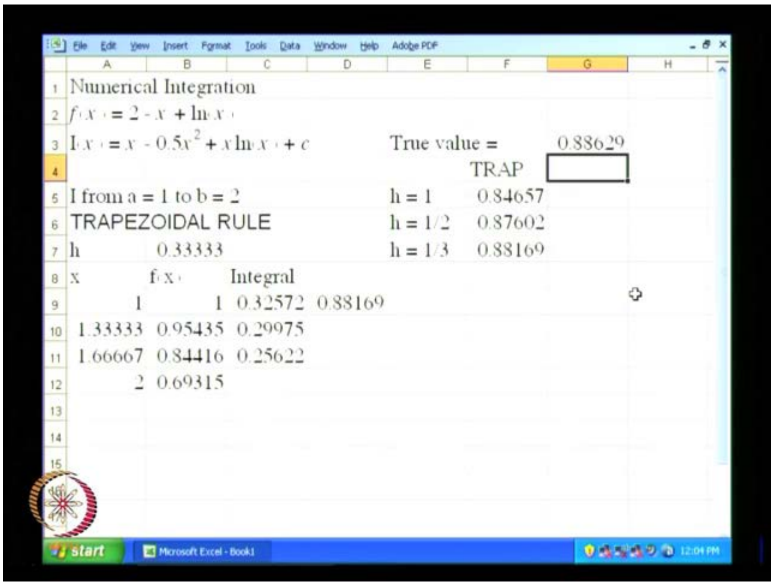

So, now let us use Microsoft excel in order to get the numerical integrals of the equation. This is the equation that we have been using for a fair number of lectures now, f of x equal 2 minus x plus l n x; if we integrate this, we will get 2 x minus x square divided by 2 plus x l n x minus x plus 2 x and minus x will result in just one single x over here and plus there will a constant of integration c.

So, let say we want to get integral I from a equal to 1 to b equal to let us say two and that is what we want to do using say the trapezoidal method. So, the trapezoidal method we want to write out the x, the first value is going to be 1; the second value is going to be h multiplied by 1. So, one straight forward thing that we can do is, we can split the entire interval into one single interval as we have done with the derivation, and our next value is going to be equal to just b. So, let us write down h over here and h is going to be just equal to 1.

So, our next value is going to be previous value plus the value of h and then, we will put dollar signs over there and then, let us compute f of x; f of x is going to be nothing but equal to 2 minus x plus l n of x, that is our f of x. I will call this trapezoidal rule and I will just increase the font size, so that it is clearly visible.

So, this is what we have using the trapezoidal rule f of x is going to be this. So, our integral I, is going to be nothing but h by 2 multiplied by f of x. So, that is going to be equal to h by 2 1 and I will put the dollar signs over there divided by 2 multiplied by y 2 plus y 1.

So, that is y 2 plus y 1 oops yeah y 2 plus y 1 and I press enter; so this is going to be our integral I. The true value of integral is going to be equal to x; so that is going to be b minus a.

So, its 2 minus 4 divided by 2; so that is x minus x square by 2 plus x l n x, that is, 2 multiplied by l n of 2 that is the value at b minus the value of at a is going to be 1 minus 1 square divided by 2 plus 1 multiplied by l n of 1. So, 0.8 8 6 2 9 is the integral. And integral using the trapezoidal rule, if we were to use 1 single one single interval, is going to be as shown over here 0.48657.

So, instead of that, what if we choose two intervals? So, we will choose our h as equal to 0.5 and then, we will just drag this, we will just drag this and the actual value of integral is going to be equal to sum of these two values and that is 0. 87602.

So, we have now improved the value of integral by going, instead of just one single interval, by going to two intervals we have improve that value. What if we go to three intervals? So, it is going to be 1 divided by 3 as the value of h and then, we just drag this, we just drag this and this is going to be sum for this entire. And as we have now increased the number of intervals, the value of the approximate integral has improved.

So, when h you using the trapezoidal method, when h was equal to 1, I will write down over h equal to 1 by 2 and when h equal to 1 by 3 using trapezoidal, using h equal to 1 by 3, the value I will just paste it over here. Paste special values, when h equal to 1 by 2, the value was 0. 8 7 6 0 2, I will paste it over here and when the value of h was 1, that means, we had one single interval, this was the value and I will do the paste special and the values over here.

So, this is the trapezoidal rule. So, what do we find in trapezoidal rule? As we increase the number of intervals or as we decrease the delta x or h value, we improve the integral value and come closer and closer to the actual true value of 0. 88629.

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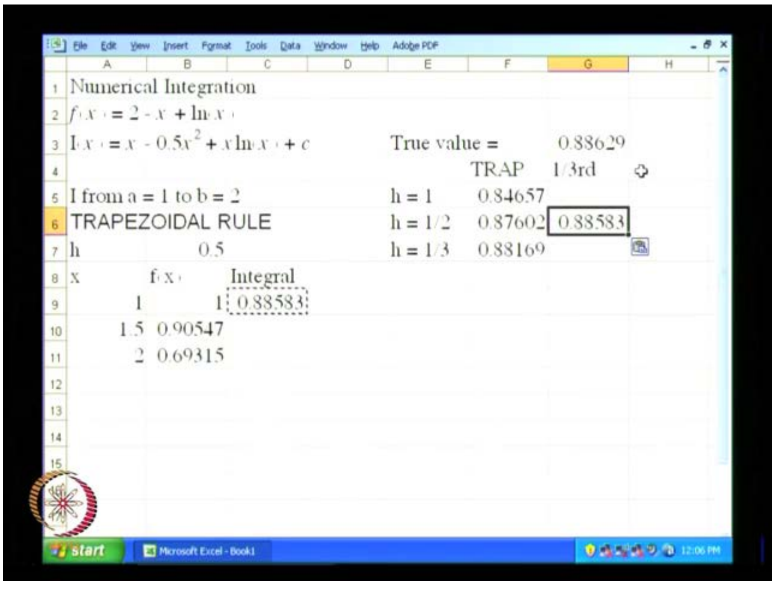

Now, we will use one-third rule. In one third rule, we will have to use h equal to 1 by 2; so we will get the value of x value of f of x at x equal to 1, x equal to 1. 5 and x equal to 2, and the integral is going to be equal to h by 3. So, that is h divided by 3 and I will put dollar signs multiplied by f 1 plus 4 f 2 plus f 3. So, this is the value of the integral obtained using the one- third rule.

So, if you compare the value using h equal to 1 by 2 using the trapezoidal method versus using the one-third rule, we will see that the value of the integral using the one-third rule is closer to the true value than it is for the trapezoidal rule .

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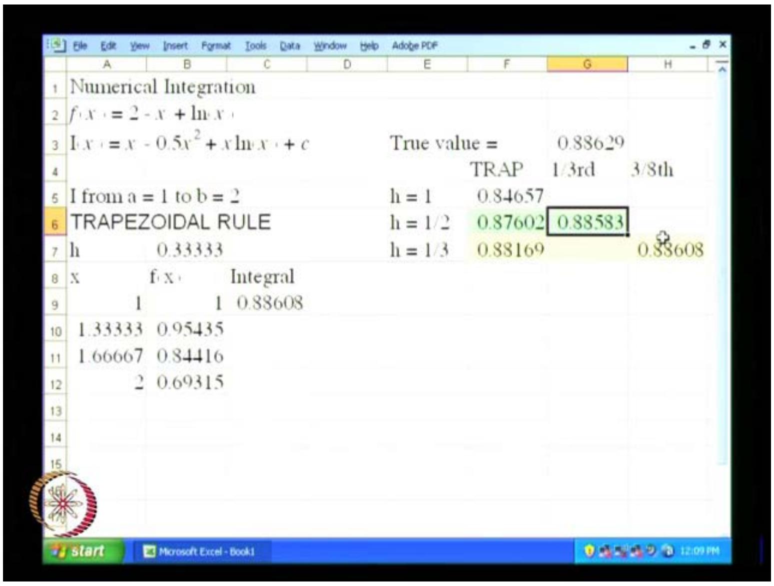

And finally, we will use the three-eighth rule and for three-eighth, h is 1 by 3. And integral using the three-eighth rule is going to be equal to 3 multiplied by h and I will put the dollar signs over here, divided by 8 multiplied by f 1 plus 3 multiplied by f 2 plus 3 multiplied by f 3 plus 1 multiplied by f 4 and that is the value we get for the integral using the three-eighth rule.

And this is what we are going to compare. if we If we intend to, so I will shade them and this is to be compared and so I will shade them with a different color. So, to summarize with h, what we have done so far is use the trapezoidal rule. We wanted to find integral from 1 to 2 f of x, where f is given this way f of x d x the integral 1 to 2 by using just single interval in trapezoidal rule had fair amount of errors, when we go to h equal to 1 by 2 and repeat trapezoidal rule application twice, the error becomes significantly smaller and when we apply the trapezoidal rule 3 times, the error decreases; if we decrease h further and further, the error will keep decreasing further.

Then we considered the Simpson's one- third rule and Simpson's single application of single Simpson's one-third rule allows us to compare the result we get using two applications of the trapezoidal rule. And we find that the Simpson's one-third rule is more accurate than the trapezoidal rule and then, we can get three applications of trapezoidal rule and compare it with the three-eighths rule, and one application is three-eighths rule is more accurate than three applications of this of the trapezoidal rule, and one application of three-eighths rule is also more accurate than the one- third rule.

So, essentially that is what I wanted to cover with respect to the numerical integration using what is known as the Newton cotes integration formulae.