Computational Techniques

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Module No. # 05

Lecture No. # 05
Regression and Interpolation

Hello and welcome to lecture 5 of module 5. So, far we have been talking in module 5 about regression and interpolation and in this particular lecture, we are going to cover a few more methods of interpolation. Basically, the main method we are going to cover today is Newton's interpolating polynomials.

So, far what we have done in interpolation is first considered the polynomial regression method, just extended it to an interpolation problem, where we had x 1 y 1 up to x n y n as n data points, and for n data points we could fit an n minus oneth order polynomial function.

So, using a polynomial fit, we were able to get an interpolating polynomial for this particular system. After that, what we looked at is Lagrange interpolating polynomials and in Lagrange interpolating polynomial, the polynomial had a particular form, because of which finding out the coefficients was a very simple task. The polynomial The form for the polynomial, was there where n polynomials p 1 to p n and the form was such that, if you substitute x equal to x i in any ith polynomial p i, then the value is going to be of p i is going to be 1. If you substitute any other value of x, that means x, x 2, x 3 or x 4 or so on p 1 that value of that particular polynomial will be 0.

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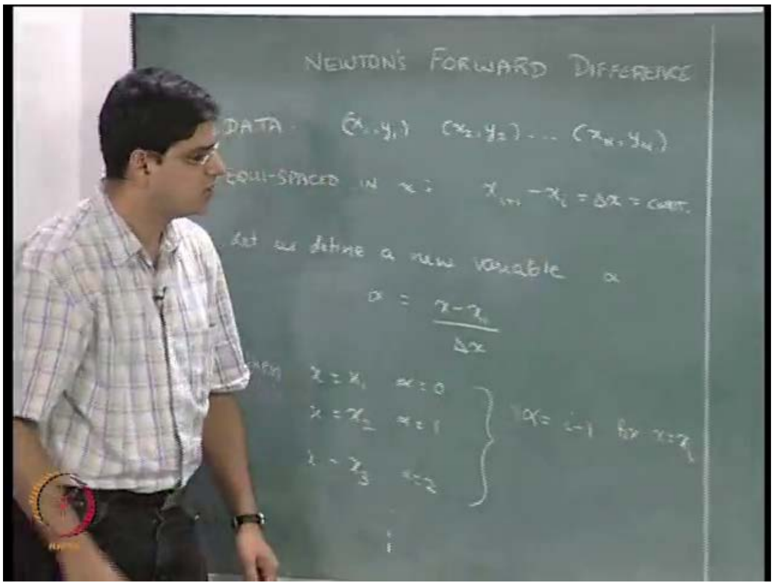

We use that property in order to easily find out what the overall functional form, our Lagrange interpolating method will going to is going to give us. Today, we are going to talk about another method called Newton’s forward difference method and just like Newton's forward difference, there is a Newton's backward difference method as well, as well as there is something called Newton's divided difference. I would not cover backward difference; I will cover forward difference and use that to talk about Newton's divided difference. Compared to the other methods that we have seen so far, the Newton's forward method difference method requires our data to be equally spaced in x in the depended variable independent variable x.

So, the data that we have is (x 1, y 1) (x 2, y 2) and so on up to (x n, y n), in addition to that there is one more requirement - equi-spaced in x, which basically means that the difference x i plus 1 minus x i, which we will write as delta x is constant.

So, this is another requirement; this requirement we did not have in the interpolating polynomial up to this type. So, this particular requirement of x i plus 1 minus x i equal to delta x and that delta x is constant, is a requirement for Newton's forward difference and also Newton's backward difference formula. So, let us define a particular variable alpha, and alpha will be defined as x minus x 1 divided by delta x.

So, what alpha is going to do is, for alpha equal to 0, 1, 2, 3 up to n minus 1 are going to represent the points x equal to x 1, x equal to x 2 up to x equal to x n. For example, when I write x equal to x 1, alpha of course is 0; when x is equal to x 2, alpha is x 2 is x 1 plus delta x. So, alpha equal to 1, when x equal to x 3, at that time x 3 is nothing but x 1 plus 2 delta x, which means alpha equal to 2 and so on. So, based on all this what we see is alpha equal to i minus1 for x equal to x i.

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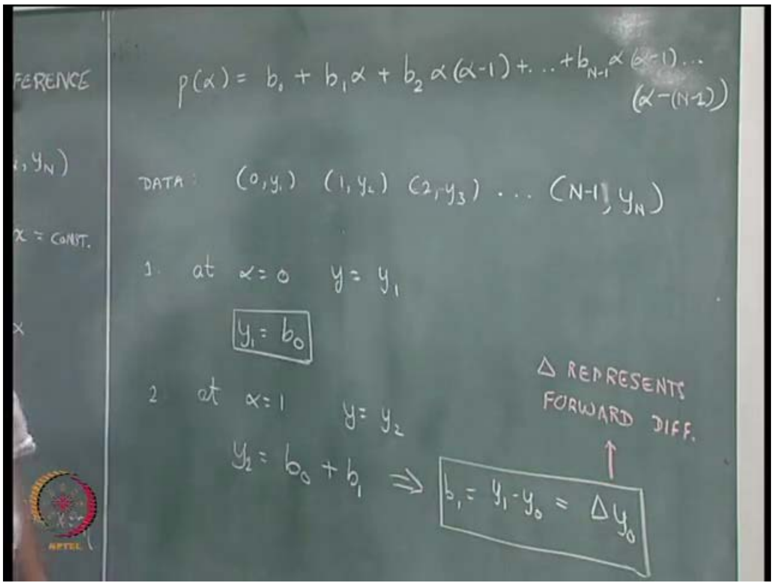

So, the way we will represent these particular polynomial functional forms is going to be very similar to what to we did in the idea is going to be very similar to what we did in the Lagrange interpolation, though with a little bit of a difference. We will describe a polynomial p of alpha equal to some constant, let say b 0, rather than using a plus, I will say b 1alpha plus b 2 alpha multiplied by alpha minus1 plus so on up to b n minus 1 alpha multiplied by alpha minus 1 and so on up to alpha minus n minus 1 or rather sorry I should be saying alpha minus n minus 2.

So, keep in mind what we actually have is the data consist of (0, y 1) (1, y 2) (2, y 3) and so on up to (n minus 1, y n). This is now our data in terms of excuse me this is now our data in terms of alpha and y; so when we substitute alpha equal to 0, all of these terms are going to vanish away, pretty much similar to what we saw in the Lagrange interpolating polynomials. When we substitute alpha equal to 1, these two terms will remain all other terms will vanish; when we write alpha equal to 2, these 3 terms will remain and all other terms will vanish so on and so forth. Now, we are interested in finding out what b 0, b1, b 2 and so on are.

So, our first thing we will do, substitute alpha equal to 0 and when we substitute alpha equal to 0 in this particular equation, what we are going to get is y 1 equal to b 0 plus b 1 multiplied by 0 plus b 2 multiplied by 0 so on and so forth all other things vanish away. So, our b 0 is nothing but y 1. Next, at alpha equal to 1, y equals y 2; when we substitute alpha equal to 1 in this particular equation, what we will get is y 2 equal to b 0 plus b 1 multiplied by 1 plus b 1, b 2 is 0, b 3 is 0, b 4 is 0 all of them are 0s and from this particular equation we will get b 1 is y 2 minus b 0 and b 0 was nothing but y 1. So, b 1 is y 1 minus y 0 and this we denote as delta y 0, where delta represents of forward difference and that is the reason why this is called a forward difference method.

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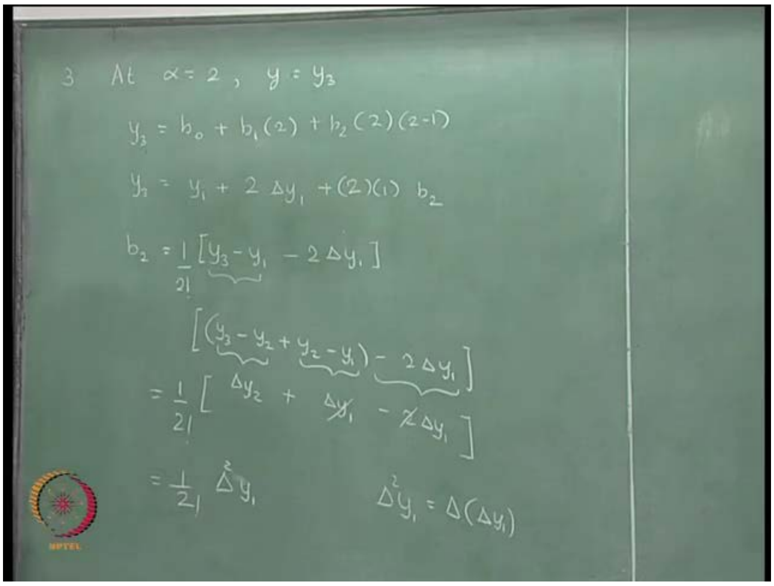

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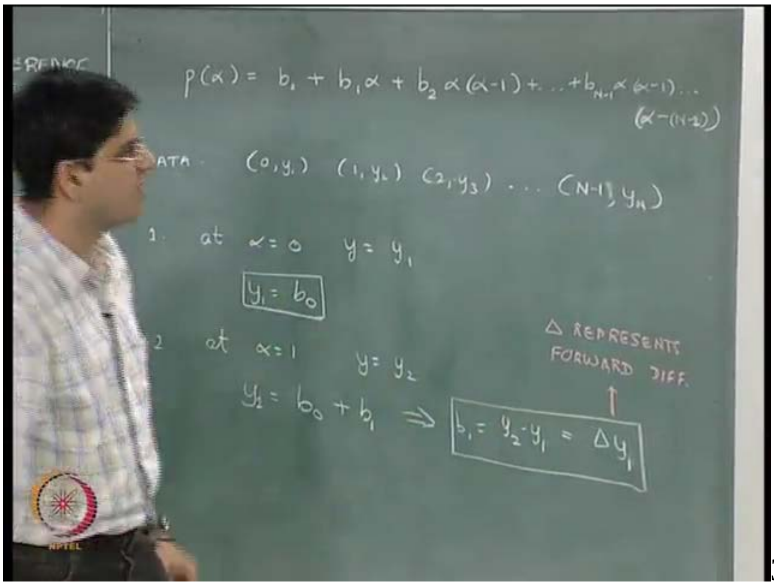

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So, now, we have the value of b 0; we have the value of b 1; now, we need to get the value of b 2. And of course, what we are going to do is follow the same pattern at alpha equal to 2, y equals y 3. So, we will have y 3 equal to b 0 plus b 1 times 2, where alpha is 2 plus b 2 times alpha multiplied by alpha minus 1 plus b 3 is all 0, b 4 is 0, b 5 is 0; so, all the other terms vanished away when we substitute alpha equal to 2 in this particular equation. We will substitute the values of b 0 and b 1; so we will have y 3 equal to b 0. Now, b 0 if we look over here, b 0 was y 1 and b 1 was delta y 0 oh i am sorry i should have been this should have been y 2 minus y 1 y 2 minus b 0 that is delta y 1; so this is delta y 1 and this is y 1 over here.

So, y 3 if we write b 0 is y 1 plus 2 times b 1 multiplied by delta y 1 sorry b 1 is delta y 1 so 2 times delta y 1 we will have So, y 3 equal to y 1 plus twice delta y 1 plus 2 multiplied by 1 multiplied by b 2. We rearrange this and we will be able to write b 2 equal to y 3 minus y 1 minus 2 times delta y 1 divided by 2 multiplied by 1 and I am write this as divided by 2 factorial.

y 3 minus y 1, let us write this particular guy y 3 minus y 1 as y 3 minus y 2 plus y 2 minus y 1 minus twice delta y 1. Now, y 3 minus y 2 is nothing but delta y 2; this is nothing but delta y 1; this is of course, minus 2 delta y 1 multiplied by 1 by 2 factorial, delta y 1 and this one of these delta y 1s will get cancelled.

So, we will have delta y 2 minus delta y 1; delta y 2 minus delta y 1 we will call it as delta squared y 1, which is the second ordered finite forward difference of y 1, which is equal to 1 by 2 factorial delta squared y 1, where delta squared equals delta of delta y 1, which is delta of y 2 minus delta y 1, which is basically y 3 minus y 2 minus y 2 minus y 1 that is what our delta squared essentially is going to meet.

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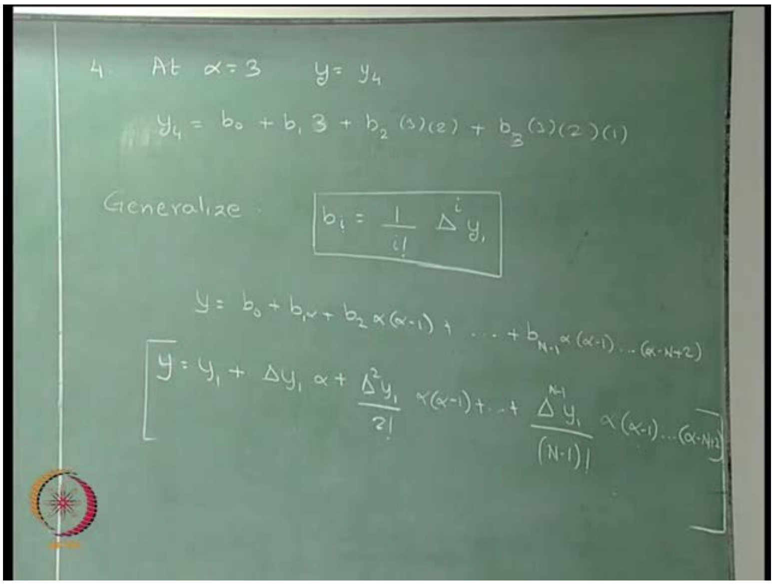

So, our b 2 is going is going to be 1 by 2 factorial delta squared y 1. And we continue doing this again at alpha equal to 3; y is going to be equal to y 4. So, y 4 equal to b 0 plus b 1 alpha beta is 3 plus b 2 multiplied by alpha multiplied by alpha minus 1, so that is 3 multiplied by 2 plus b 2 multiplied b 3 multiplied by 3 multiplied by 2 multiplied by 1 and we substitute this as y 1, this as delta y 1, this as delta square y 1 and then we keep rearranging the terms and so on and we will essentially get our b 3 as delta cubed y 1 divided by 3 factorial.

So, we are actually seeing a pattern when it comes to… So, b 0 is y 1, b 1 is delta y 1, b 2 is 1 by 2 factorial delta squared y 1 so on and so forth. So, to generalize, we will have b i b i is going to be 1 by i factorial multiplied by delta to the power i y 1. This is going to be to be our b i's.

So, we will write our polynomial y equal to b 0 plus b 1 alpha plus b 2 alpha alpha minus 1 plus b n minus 1 alpha alpha minus 1 and so on up to alpha minus n plus 2. And substituting the values of b, we will have y equal to y 0 plus b 1, which is delta y 1 times alpha plus delta square y 1 divided by 2 factorial alpha multiplied by alpha minus 1 and so on up to delta to the power n minus 1 y 1 divided by n minus 1 factorial multiplied by alpha alpha minus 1 and so on up to alpha minus n plus 2.

So, this is the overall equation for Newton’s forward difference polynomial y equal to y 1. i am sorry i keep using the term y 0 So, y equal to y 1 plus delta y 1 alpha plus delta square by 2 factorial y 1 alpha alpha minus 1 plus delta cube y 1 divided by 3 factorial multiplied by alpha alpha minus 1 alpha minus 2 so on and so forth up to n infinite series; that is the overall expression for Newton's forward difference method that we will get.

So, this particular method relies on the assumption that x 1 x 2 x 3 x 4 and so on are equally spaced data point, but as we have seen previously, these data points need not be equally spaced; under those conditions what do you do? Well, there is a different method which is known as Newton's divided difference interpolating polynomials and that is what we will look at next.

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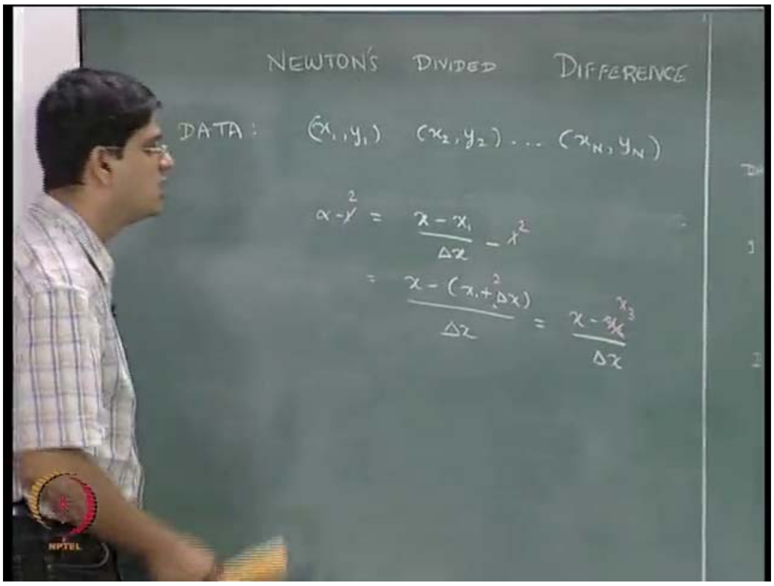

Newton's divided differences, as before we have data (x 1, y 1) (x 2, y 2) up to (x n, y n), but for newton's divided differences, we do not need equi-space data. What we are going to do is, we are going to modify the definitions of the forward differences, back ward differences or whatever the difference might be, appropriately we are going to change them so that we are able to use the same formula that we had previously, but in a slightly different or a slightly changed context.

So, we do not have the data to be equi-space and when we do not have data to be equi-space, there is no question of defining the variable alpha, because that particular definition is not exactly going to work; so all these things are actually going to go away.

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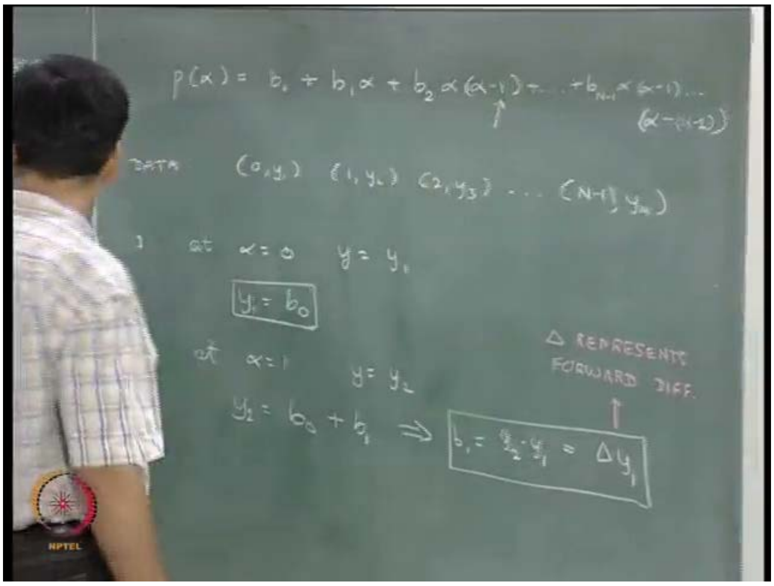

What we will keep an eye on is the final expression that the polynomial actually gets. In case of the Newton's forward difference formula, the polynomial had the form p of alpha equal to b 0 plus b 1 alpha, b 1 alpha was nothing but this alpha was nothing but x minus x 1 divided by delta x; b 2 alpha multiplied by alpha minus 1, that was x minus x 1 divided by delta x, this was x minus x 1 divided by delta x minus 1.

So, let us look at what alpha minus 1 becomes. Alpha minus 1 was x minus x 1 divided by delta x, that is, alpha minus 1; this becomes x minus x 1 plus delta x divided by delta x, which is equal to x minus x 2 divided by delta x.

So that is what this particular guy becomes. This is x minus x 2 divided by delta x, I hope you are seeing the pattern. Now, when we have alpha minus 2, I will use a colored chalk over here; now, when we have alpha minus 2, we have again x minus x 1 divided by delta x minus 2, minus 2 basically would end up appearing over here; this sign will be plus, because the minus sign we have taken outside the brackets, and x 1 plus 2 delta x in Newton’s forward difference formula was nothing but x 3.

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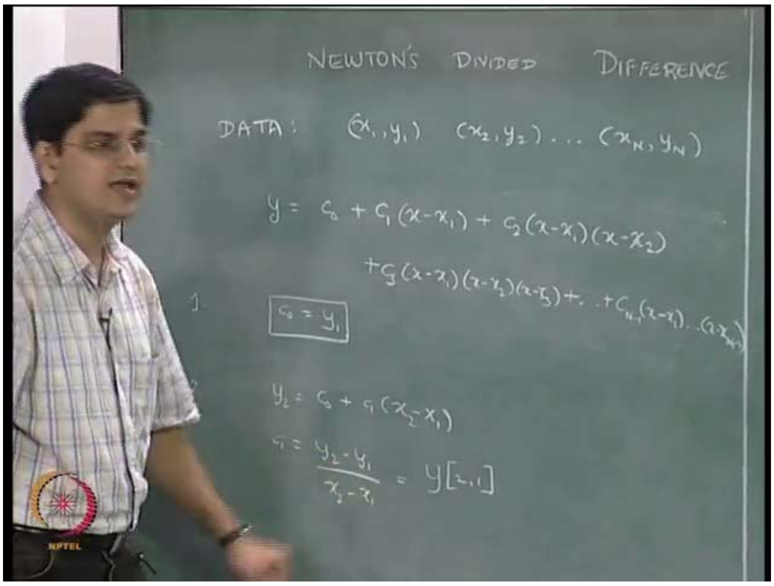

So, now, what we are now doing we are what we are essentially doing is, we are pattern matching based on what we did with Newton’s forward difference formula in order to get a short cut to the Newton's divided difference formula over here. So, the Newton's divided difference formula essentially is going to look like - this y is going to be equal to let us again call it b 0 or in fact we should call it c 0 may be, because the values are going to be different than b's. So, we will have c 0 plus c 1 multiplied by x minus x 1; we would not divide by delta x right now, do not worry about that, we will come to that later, plus we c 2 multiplied by x minus x 1 multiplied by x minus x 2 plus c 3 multiplied by x minus x 1 x minus x 2 x minus x 3 plus so on up to c n minus 1 x minus x 1 and so on up to x minus x n minus 1.

Now, when we substitute x equal to x 1, we will get y equal to y 1. So, c0 as before is going to be nothing but y 1; just as we got b 0 equal to y 1, previously we will get c 0 equal to y 1. When we substitute x equal to x 2, what will be left is y 2 equal to c 0 plus c 1 multiplied by x 2 minus x 1 and all the other terms are going to go away. So, c 1 is going to be equal to y 2 minus y 1 divided by x 2 minus x 1.

This is what this term is going to be. In standard notations, this is written as y square brackets 2, 1.

So, it is a divided difference between y 2 and y 1, then we proceed ahead and again you see what we are getting, instead of delta x, we are getting delta y 1, we are just getting y 2 minus y 1 divided by x 2 minus x 1.

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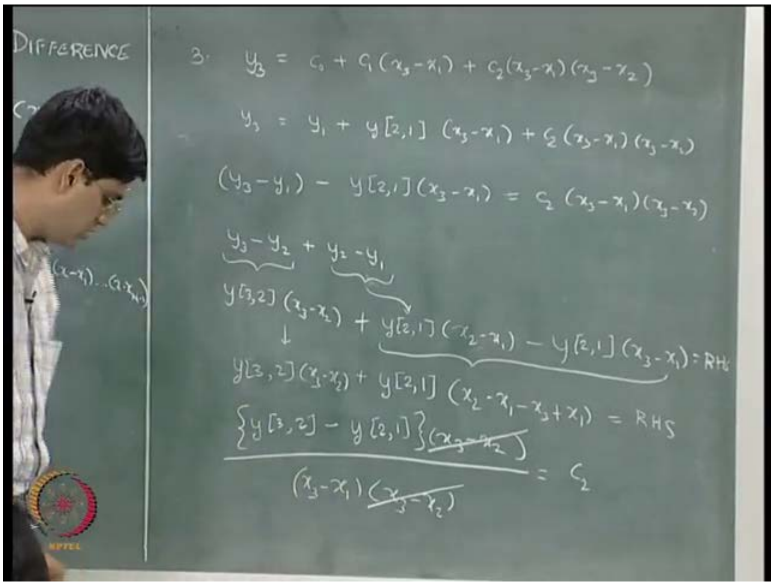

Now, we substitute x 3 in that particular equation; so y 3 equal to c 0 plus c 1 x 3 minus x 1 plus c 2 x 3 minus x 1 x 3 minus x 2 none of the other terms remain, because when we substitute x equal to x 3 all the other terms are going to disappear from this equation.

So, this we are going to write y 3 equal to y 1 plus y of c 1 was y of 2 , 1 multiplied by x 3 minus x 1 plus c 2 x 3 minus x 1 divided by x 3 minus x 2. We take all these equations, all these guys to the right hand side and we will get y 3 minus y 1 minus y 2, 1 multiplied by x 3 minus x 1 plus c 2 times or sorry equal to c 2 times x 3 minus x 1 multiplied by x 3 minus x 2. And this particular term, we will we should be able to write this as y 3 minus plus y 2 y 3 minus y 2 plus y 2 minus y 1, just the way we did that before.

So, we will write this as y 3 minus y 2 plus y 2 minus y 1; (Refer Slide Time: 24:00) this we will write this as y of 3, 2 multiplied by x 3 minus x 2 yeah remember what was y 2, 1 was, y 2 minus y 1 divided by x 2 minus x 1. So, y 3, 1 if (3, 2) is going to be y 3 minus y 2 divided by x 3 minus x 2.

So, y 3 minus y 2 is just going to be x 3 minus x 2 multiplied by y of 3, 2 ; this is what this term is going to be, and this term is going to be nothing but y of 2, 1 multiplied by x 2 minus x 1; so plus this. This term remains as it is; so it is minus y of 2, 1 multiplied by x 3 minus x 1 equal to right hand side; I am not I am just being lazy, I am not writing the entire right hand side.

So, we have y 3, 2 multiplied by x 3 minus x 2 plus y 2, 1 multiplied by this guy minus this guy. So, this particular expression, we will write down again, y 2, 1 x 2 minus x 1 minus x 3 plus x 1 equal to the right hand side and I will just copy this down over here y 3 , 2 multiplied by x 3 minus x 2.

Now, we will divide throughout by this particular term and we will see what we get with respect to c 2. So, we will get y 3 , 2 minus y 2 , 1 whole multiplied by x 3 minus x 2 divided throughout by this particular term x 3 minus x 1 x 3 minus x 2 is going to be equal to c 2.

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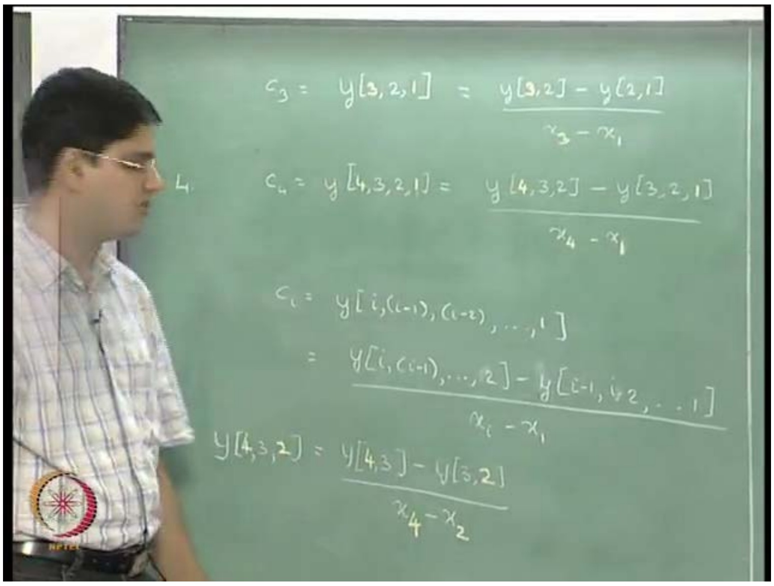

So, c 2 this term remains sorry and this term gets cancelled. So, c 2 is y 3, 2 minus y 2, 1 divided by x 3 minus x 1. So, we will write our c 3, which y of 3, 2, 1, which is defined as y 3, 2 minus y 2, 1 divided by x 3 minus sorry x 1. Proceeding further, we will be able to write c 4 as y of 4, 3, 2, 1, which is going to be equal to y of 4 , 3 , 2 minus y of 3 , 2 , 1 divided by x 4 minus x 1. I will just use different color colored chalks over here to write this down as 4, this as 1, x 4, x 1, and this 4 comes over here; this 1 comes over here. See what is happening is you have two yellow color guys over here, one of them disappears; the last one disappears from this square sign; the first one disappears from this square sign divided by x 4 minus x 1.

This is the pattern that you get. Same pattern is repeated over here also; again I will color this in yellow. In the numerator when you have these divided differences, the first guy the second yellow number- the last yellow number - has disappeared; in the second guy, the first yellow number has disappeared; in the denominator, it is the first yellow number minus the second yellow number. I am talking about the subscripts in this in this section.

And you will have that particular pattern repeating c 5 is going to be y 5, 4, 3, 2, 1, c 6 is going to be y of 6, 5, 4, 3, 2, 1 so on and so forth. So, in general, our c i is going to be y of i, (i minus 1), (i minus 2) , so on up to 1 , which is going to be equal to y of i , (i minus 1) and so on up to 2 minus y of i minus 1 , i minus 2 and so on up to 1 . So, notice that, 1 is missing over here; notice that i is missing over here; divided by x i minus x 1.

Now, how do you find y of this particular term? We find that y again using essentially the same set of equations. So, y i... So, in this particular case, let us look at y 4, 3, 2 what is going to be y 4, 3, 2, is going to be, I will just color this again. So, this will be y of 4, 3 minus y of 3, 2 divided by x 4 minus x 2, and again if we color this with yellow color, this is what we get.

So, this is how the overall divided differences are going to work. The forward differences also work in the similar manner and if you substitute all these equations, all these numbers in those particular equations, you will see that what we were getting previously as in this particular expression for Newton's forward difference formula; we will be able to recover that same expression using the Newton's divided difference formula.

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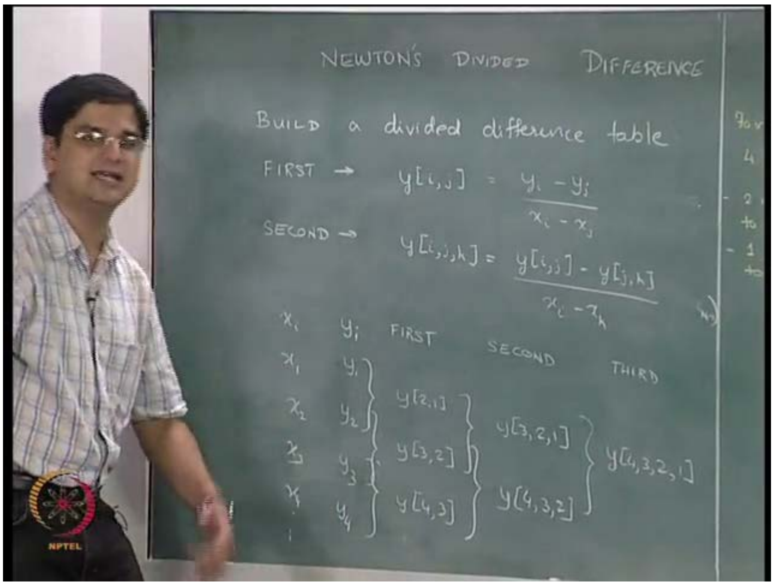

In implementing Newton's method what we will do is, we will get a divided difference table; we will build. The first order differences the first order differences are of the form y i , j , where y i , j is going to be nothing but y i minus y j divided by x i minus x j, example y 2 , 1 y 3 , 2 y 4 , 3 so on and so forth those are the first ordered differences.

We will have our second order differences; second order differences are going to be y i , j , k , which are defined as y i , j minus y j , k divided by x i minus x k, and so on we will have third order differences i, j k l; y i, j, k minus y j, k, l divided by x i minus x l so on and so forth. So, these are the various orders of differences. Let say we have the data x i and y i; so we will have x 1 x 2 x 3 x 4 and so on; likewise, we will have y 1 y 2 y 3 y 4 and so on.

So, our first order differences are going to be the first difference is going to be y 2 minus y 1 divided by x 2 minus x 1. So, this we are going to get y 2, 1. Our second difference is going to be between y 3 and y 2; the second first order difference is going to be y 3 , 2 , which is y 3 minus y 2 divided by x 3 minus x 2. Then the next difference is going to be 4 minus 3; so, this is going to be y 4, 3 and so on and so forth. So, we have now created the first the table of first differences.

Then we will have a table of second differences; table of second differences will consist of these differences over here. So, y 3 , 2 , 1 is y 3 quite y 3 , 2 minus y 2 , 1 divided by x

3 minus x 1; so that is going to be our y 3 , 2 , 1 . Likewise, we will use this to get y 4 , 3 , 2 , which is y 4 , 3 minus y 3 , 2 divided by x 4 minus x 2 .Next, we will have another row for the third ordered differences.

The third ordered differences are going to be y 4 , 3 , 2 minus y 3 , 2 , 1 divided by x 4 by x 4 minus x 1; so that is going to give us y 4 , 3 , 2 , 1 . Likewise, between this and the next guy, we will get y of 5, 4, 3, 2 so on and so forth and we keep building that table, until we do not have we have we are left with only one value in that particular row.

So, the first the first column over here - the y column - has n number of elements; the second column, which represents the first differences has n minus 1 elements, n minus 2 elements, n minus 3 elements up to 0 elements.

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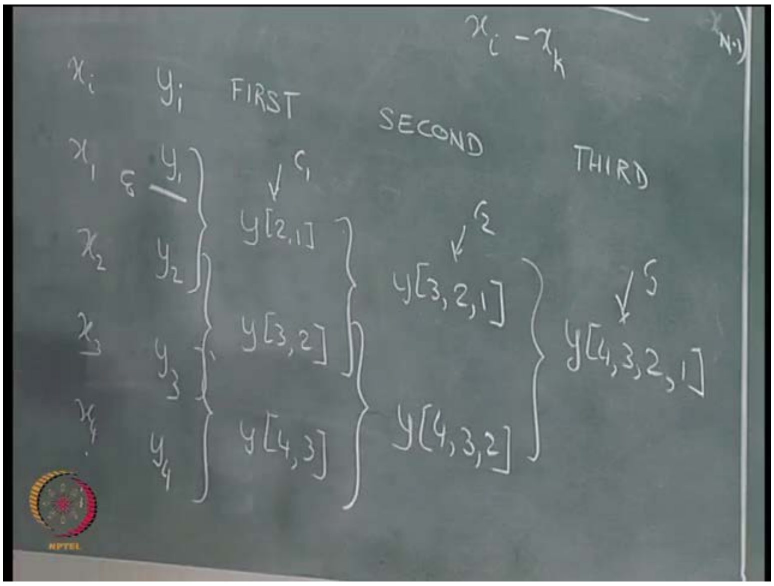

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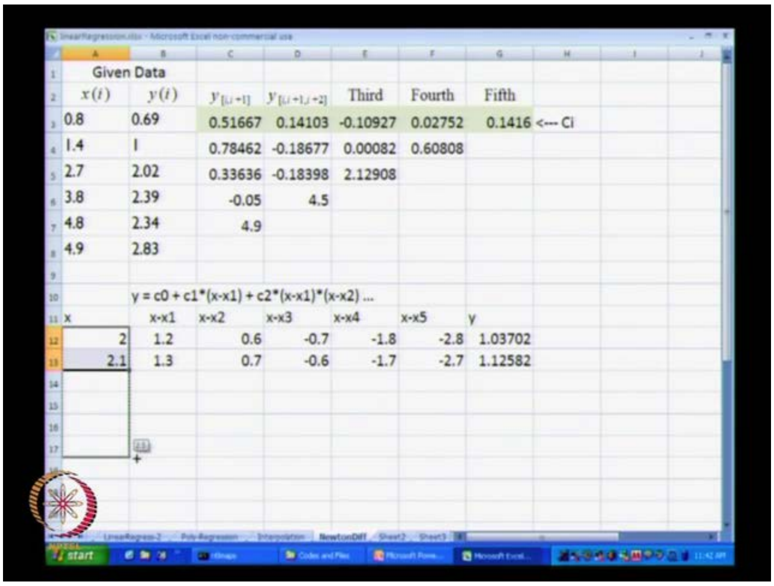

And the top most value in each column starting from y is going to be over c 0, c 1, c 2 and so on. So, this is our c 0; this is our c 1; this is our c 2; this is our c 3 so on and so forth, and that is essentially how you will calculate Newton's divided difference. Let us start off with the example, I have Microsoft excel over here and I will put it in the full screen mode; it is the same data that we have seen previously x versus y data. Now, because this is not equally spaced data, we cannot use Newton's forward difference backward difference formulae; we will actually be using Newton's divided difference formulae. We created divided difference table consisting of x and y and then, the first differences, second differences, third differences and so on. This is going to be the first difference; first difference y 1, 2 y 2, 3 y 4, 4 so on and so forth.

We will then make a few more tables i y i plus 1 i plus 2 that is the second difference; we will have a third difference fourth difference and fifth difference. So, if you remember the difference, the first difference was nothing but y i plus 1 minus y i divided by x i plus 1 minus x i. So that is what we will write it as, this minus this guy divided by again bracket open this minus this; please be careful about the brackets otherwise, the Bodmas rule - brackets exponential division multiplication addition subtraction that rule will kick in and we will get incorrect results.

So, we will have y i plus 1 minus y i divided by x i plus 1 minus x i and what I will then do is, just drag it for the remaining five, remaining four data points and we can click on f

2 as before and check that this indeed represents y 3 minus y 2 divided by x 3 minus x 2 so on and so forth. Now, y i plus 1 i plus 2 is going to be y of i , i plus 1 , i plus 2 minus y of i , i plus 1 divided by x of i plus 2 minus x i.

So, these are the two divided differences that we are going to use in the numerator, and in the denominator we are going to use this and this value. So, that is going to be equal to this guy minus this guy divided by this minus this and I will just drag and draw, and I will just write down below what we are doing, so that it is it is very clear. y I will just change the font over here; first font, make it font size eighteen, in fact I will do that for the entire set of data, make the font size 18.

So, I will just write down what we are trying to do, we are trying to get y of 3, 2, 1. The numerator for y of 3, 2, 1 is y of 3, 2 minus y of 2, 1 that is the numerator. Format cells, align it to the left yes and the numerator is going to be y of 3, 2 minus y of 2, 1 ; the denominator is going to be x 3 minus x 1.

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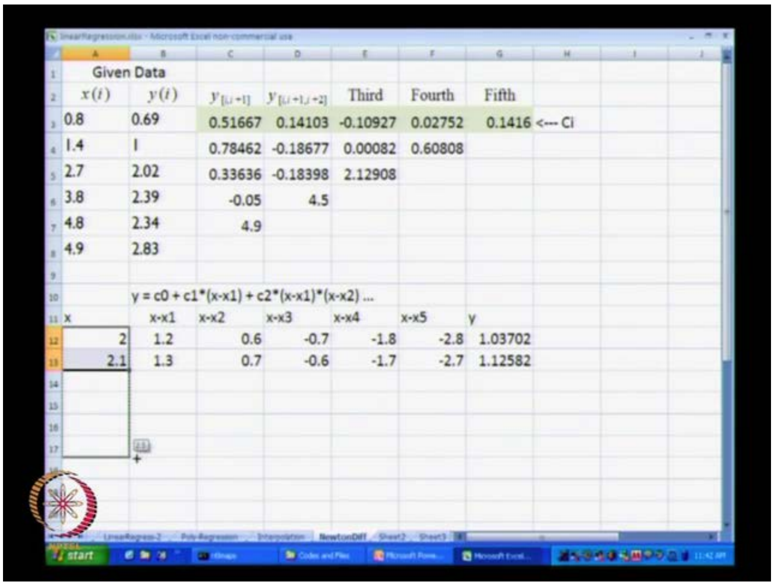

This is what we have been consistently doing. So far, y of 3, 2, 1 will not have 1 in the… first guy will not have 3 in the second guy and it will in the denominator will be just y 3 divided by y 1.

Now, y of 3 , 2 - So, this is y of 2 , 1 this is y of 3 , 2 , y of 4 , 3 , y of 5 , 4 , y of 6 , 5 those are what we have in the third column over here. So, this becomes y of 3, 2 , 1 why because this is y of 3 , 2 minus y of 2 , 1 divided by x 3 minus x 1. Let us look at this guy, this should be y of 4, 3 , 2 y of 4 , 3 , 2 is going to be y of 4, 3 minus y of 3, 2 divided by x 4 minus x 2, that is exactly what we get over here and so on so forth; that is what we will get, if we look at the bottom also.

So, the third differences are going to be this minus this you know, for the first row will have the second row minus the first row divided by…, because it is a third difference divided by we will have x 4 minus x 1. We will compare this; just keep a look on not the the values of the cells; when I show it up, just look at the blue, yellow, green and purple tabs when I click f 2. Let us click f 2, see the green and blue tabs are over here; the brown and purple tabs are over here; when we go on to the next guy, see the green and blue tabs have just moved the column, but not the rows; but the brown tabs remains the same, but the purple tab, instead of this guy, it has now come over to the bottom person over here.

So, if you look at this, it has shifted from 2.7 to 3.8 when we go on to the third differences. So that is what we need to keep in mind when we make this divided difference table, and now I will just have to drag it over here. The fourth divided differences are going to be this minus this divided by this minus this, follows exactly the same pattern. And the fifth divided differences are this minus this divided by x 6 minus x 1, those are the divided differences.

So, now, that we have the divided differences; the first row becomes our c 0, c 1, c 2 and so on. I will just shade this and this row is going to be c i. So, now, the Newton's difference formula was… So, the data I will just write it write this as x i and y i; this was the data. Let us get an arbitrary point x and we need to find out what the value of y is going to be, y that we had said was equal to c 0 plus c 1 multiplied by x minus x 1 plus c 2 multiplied by x x minus x 1 multiplied by x minus x 2 and so on and so forth.

So, this was what our y was going to be. (Refer Slide Time: 45:05) So, in order to construct y what we need is, we will need x minus x 1 x minus x 2 x minus x 3 x minus x 4 and x minus x 5. Remember we do not have x minus x 6 term at all in this expansion. Let us select arbitrarily any x of our choice and we will select our x to be 2 for this case, and the final column is going to be just over y. And x minus x 1 is going to be equal to x minus x 1; x minus x 2 is going to be equal to x minus x 2; x minus x 3 is this minus this; x minus x 4 is this minus this; x minus x 5 is x minus x 5.

So, this is all the data that we that we have and using the data, we have obtain x minus x 1 x minus x 2 and so on and so forth. So, y was equal to… and so let us also do this; let us use the dollar signs over here; so that we do not have to we can just drag and drop all these things below, because x 1 x 2 x 3 x 4 the positions are not going to change.

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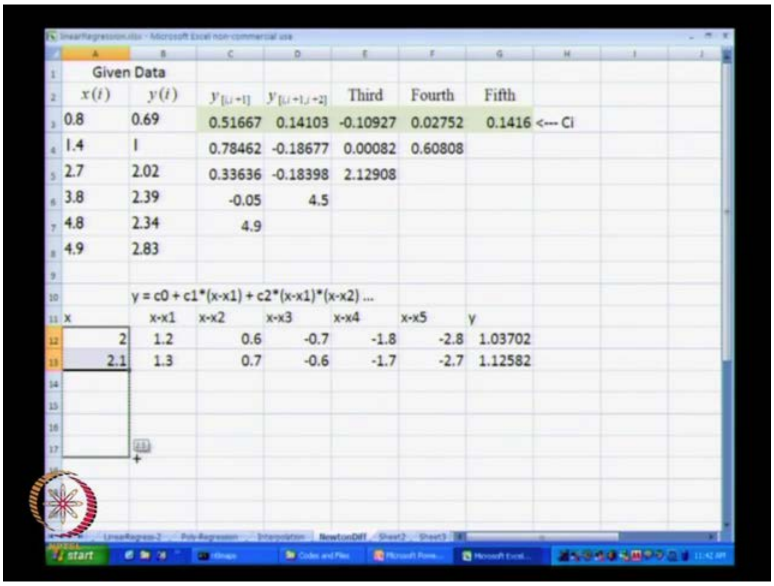

So, we will put the dollars over there; so that when we drag and drop, we are not going to change any of these positions so that is what we have. So, y as we said was equal to c 0; c 0 comes over here and we will put the dollars in their c 0 multiplied by sorry it is just going to be c 0 not c 0 multiplied by anything plus c 1, that is, dollar c dollar 4 multiplied by x minus x 1 plus should be 2 it should it would not be dollar c dollar 4 it is dollar d dollar 3, I apologies for that mistake. So, its c 0 plus c 1 multiplied by x minus x 1 plus c 2, which is this guy over here, c 2 multiplied by x minus x 1 multiplied by x minus x 2.

We will go back and put the dollars over there, plus we will have c 3 multiplied by x minus x 1 multiplied by x minus x 2 multiplied by x minus x 3, dollar f dollar 3 and then, we will have a last term, which is c 0 c 1 and this last term is this multiplied by this multiplied by this guy multiplied by this multiplied by this multiplied by this.

So that is actually So, what we have is y equal to c 0, which this is… there is mistake over here, y c 0 is actually not this but c 0 is this guy. So, I will just move it over there; c 1 is this; c 2 is this; c 3 is c 3 is this; c 4 is… so it is very convenient in Microsoft excel if you do a mistake to just rectify it easily.

So, let us again look at what we have obtained - c 0, c 1, c 2, c 3, c 4, c 5 those are the various c values. So, it is c 0, which is this value b 3 plus c 1, which is c 3 multiplied by x minus x 1 plus d 3, which is our value c 2 multiplied by x minus x 1 multiplied by x minus x 2, then when we look at this; this is c 3 multiplied by x minus x 1 x minus x 2 x minus x 3 plus c 4 multiplied by x minus x 1 x minus 2 x minus x 3 x minus x 4, I will put the dollar signs over there again.

And then, the final c 5 multiplied by this multiplied by this multiplied by this multiplied by this multiplied by this term. So that is the value of the function at x equal to 2; this is the value that the function will get at x equal to 2. Let see at x equal to 2 .1, what we should have? We will have to just drag this and we will we will just we can just confirm that the values are correct or not. So, this is indeed; this minus this, which is x minus x 1 by clicking f 2, x minus x 2 x minus x 3 x minus x 4 x minus x 5.

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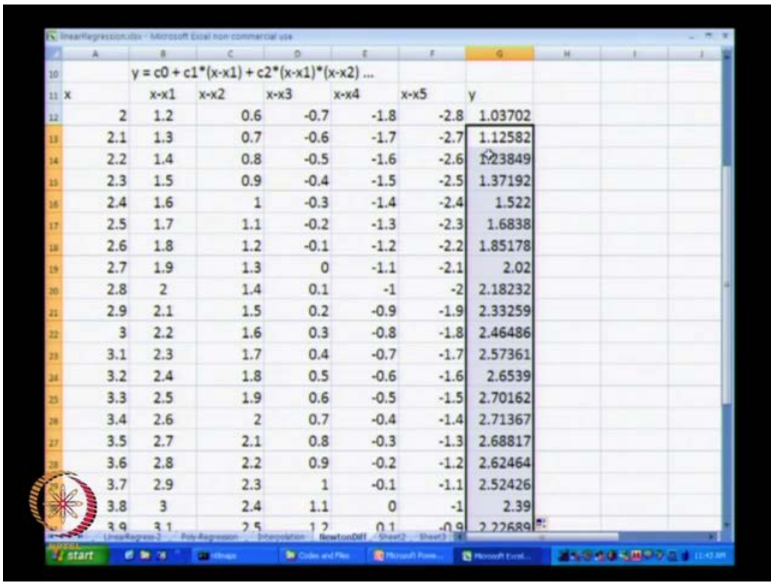

y = c0 + c1*(x-x1) + c2*(x-x1)*(x-x2) ...

x	x-x1	x-x2	x-x3	x-x4	x-x5	y
2	1.2	0.6	-0.7	-1.8	-2.8	1.03702
2.1	1.3	0.7	-0.6	-1.7	-2.7	1.12582
2.2	1.4	0.8	-0.5	-1.6	-2.6	1.23849
2.3	1.5	0.9	-0.4	-1.5	-2.5	1.37192
2.4	1.6	1	-0.3	-1.4	-2.4	1.522
2.5	1.7	1.1	-0.2	-1.3	-2.3	1.6838
2.6	1.8	1.2	-0.1	-1.2	-2.2	1.85178
2.7	1.9	1.3	0	-1.1	-2.1	2.02
2.8	2	1.4	0.1	-1	-2	2.18232
2.9	2.1	1.5	0.2	-0.9	-1.9	2.33259
3	2.2	1.6	0.3	-0.8	-1.8	2.46486
3.1	2.3	1.7	0.4	-0.7	-1.7	2.57361
3.2	2.4	1.8	0.5	-0.6	-1.6	2.6539
3.3	2.5	1.9	0.6	-0.5	-1.5	2.70162
3.4	2.6	2	0.7	-0.4	-1.4	2.71367
3.5	2.7	2.1	0.8	-0.3	-1.3	2.68817
3.6	2.8	2.2	0.9	-0.2	-1.2	2.62464
3.7	2.9	2.3	1	-0.1	-1.1	2.52426
3.8	3	2.4	1.1	0	-1	2.39
3.9	3.1	2.5	1.2	0.1	-0.9	2.22689

And we will just drag this over to the bottom and this is the value that we get clicking f 2. This is an error over here; we will just move this and put dollar signs; we had forgotten those dollars. So, select this and just drag it and just double click this, double click this, same thing over here and the y's are going to be the interpolated values.

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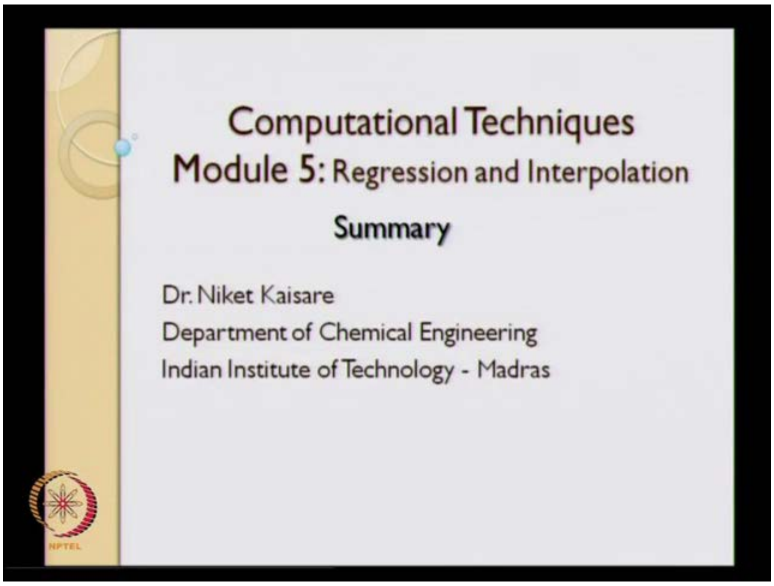

Now, what I will go ahead and do is, summarize what we have done so far in the regression and interpolation, basically the module 5 of this particular course.

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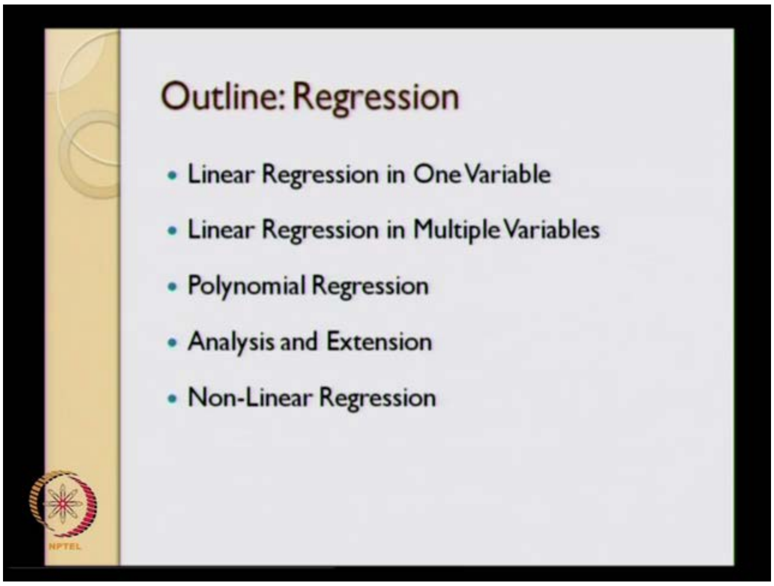

This is the overall over view that we have covered in the regression part; we first looked at the linear regression in one variable, then we looked at multi linear regression; we then talked about polynomial and functional regression. So that is just the same idea as multi linear regression, but where we have the same variable and different polynomial forms of the same variable. We analyze the overall relation methods and we showed that fitting a straight line x versus y is not the same as fitting the straight line y versus x.

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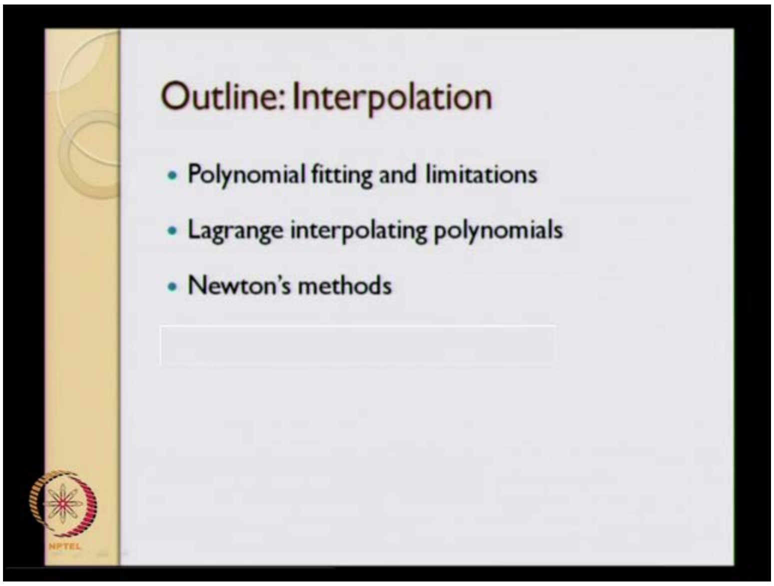

And very briefly we covered non-linear regression; we did not go into any of the details, but just gave the idea behind non-linear regression. After that, we talked about interpolation and interpolation, we covered polynomial fitting and saw some of its limitations or rather I just stated those limitations, rather than discussing them in in any way.

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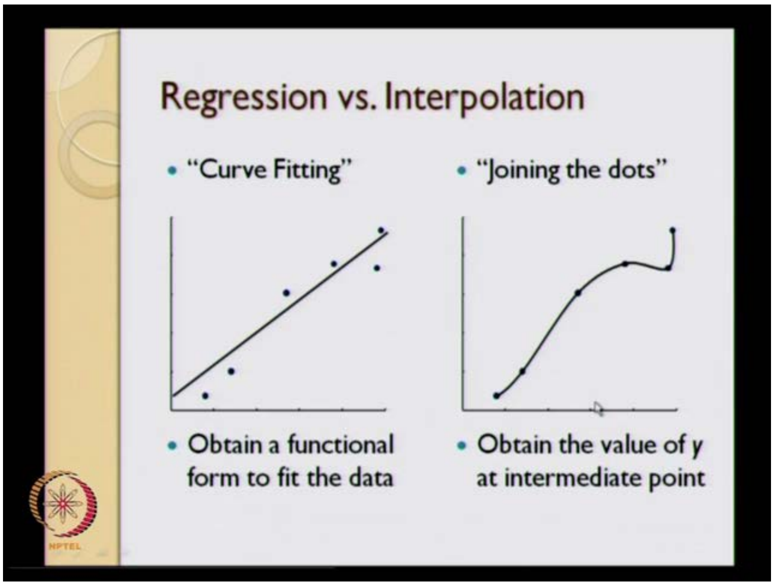

Then we talked about Lagrange interpolating polynomial; we did Newton's forward difference and Newton's divided difference method. Now, to summarize further, the differences between regression and interpolation - regression can be thought of as curve fitting; we have or we decide a certain functional form of the equation and for that functional form, we are going to fit a specific curve to the overall data.

As against that, interpolation is kind of joining the dots. You have some data point and we want to join those dots using a smooth curve of some sort and that is what interpolations is going to do. The use of regression or curve fitting is to fit a functional form to the data, that functional form can be useful somewhere else.

For example, we have this Arrhenius type of relation for rate of reaction that is a functional form that we have fitting to the data. In case of interpolation, we want to find out the value of the y; y is that independent variable at some intermediate data point. Let say at this particular data point, we want to find the value of y; we do not have the data at that particular value.

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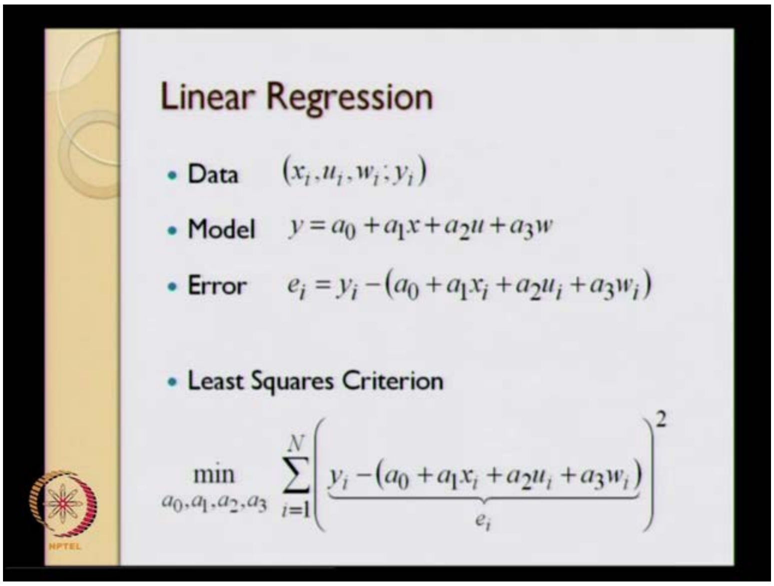

So, we fit some kind of an interpolating polynomial to this data points and then, just substitute x value over here and what we get is going to be the value of dependent variable y for that point. So that is the difference between regression and interpolation. Then we saw multi linear regression; this was the data that was that is given to us. The model that we are going to fit is of this form; this particular model is linear in the various parameters. This is the way we defined our errors and then, we try to minimize some kind of a least squares criterion, which is minimization of sum of square of errors, which can be solved.

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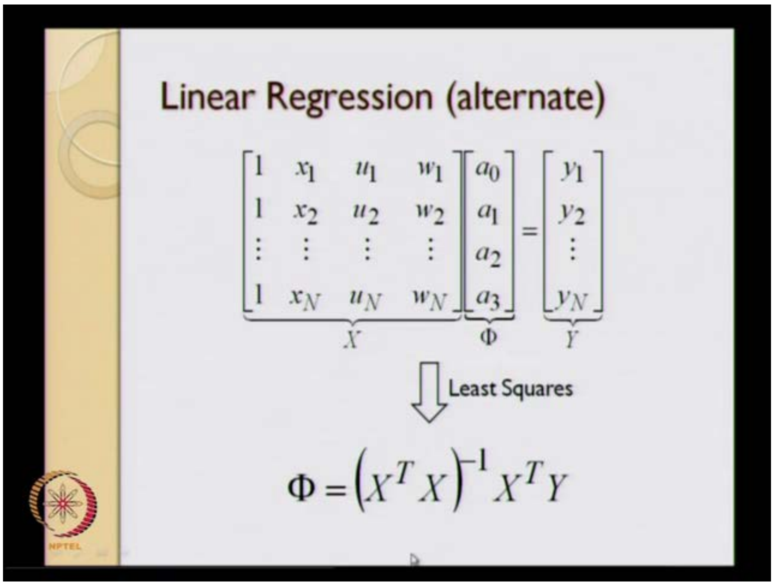

An alternative way of doing that was to write down in this particular form. This is again something we have seen before in a Victorian notations, where the first column is 1's which correspond to the constant term a 0; the second column is x 1 x 2 up to x n, which correspond to the term a 1; the third column is u's, fourth column is w's.

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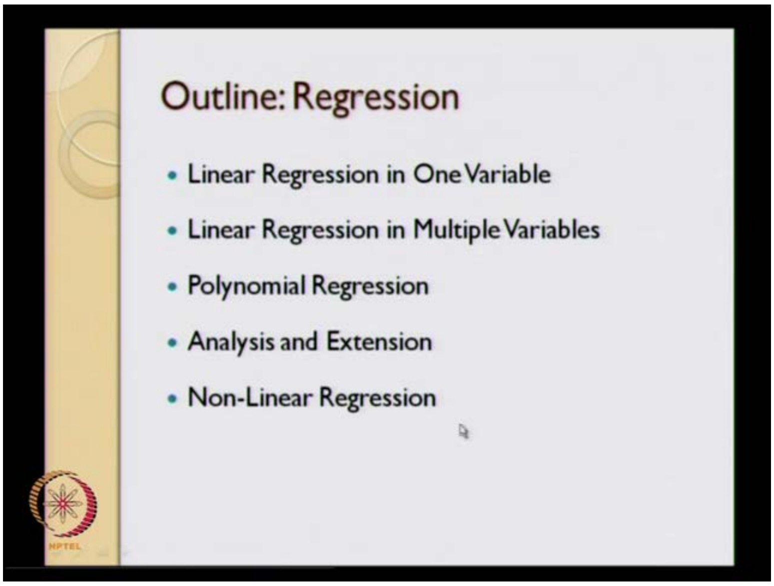

So, we got this particular equation in the form x phi equal to y, and the least squares solution for that is given in this particular form. So, this is what we have seen in multi linear regression, then we looked at non-linear regression; we very briefly covered non-linear regression in about 5 minutes or so on.

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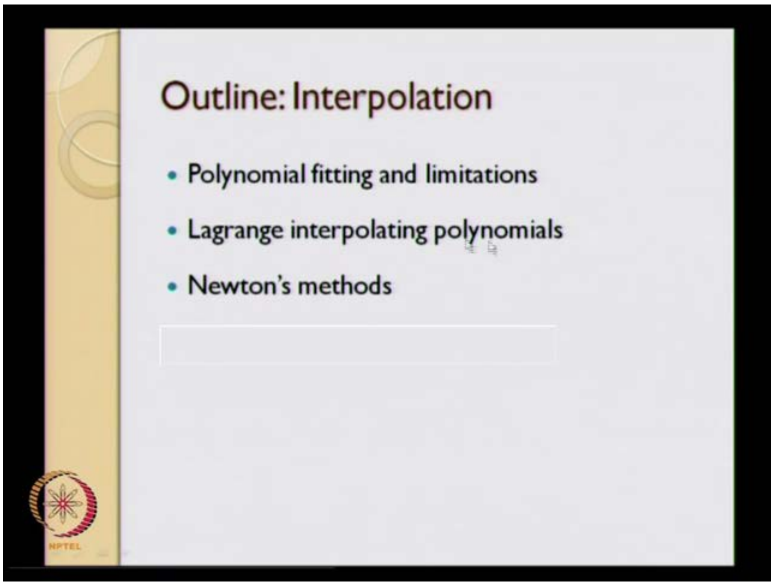

Then we saw polynomial extension of polynomial fitting for this data, then we talked about the linear Lagrange interpolating polynomials; implementing Lagrange interpolating polynomials is straight forward. Finally, we talked about the Newton's method.