Computational Techniques

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Module No. # 04

Lecture No. # 03

Nonlinear Algebraic Equations

Non-linear algebraic equations: these equations, these methods were classified into two main classes - one was bracketing method and the other was open method. Bracketing methods basically bracketed the solution of our interest; whereas the open methods worked either with one or two solutions and - the two - the two solutions may not necessarily bracket the true…, the two guesses may not necessarily bracket the true solution.

So, that was our bracketing and open methods. Then we looked at - you - you…, basically implementation of these methods using Microsoft excel for one particular example; what we are going to do in this lecture is essentially to look at some of the three specific methods, out of the methods, that we have done in the previous lecture and try to do more in depth analysis of the convergence of these methods.

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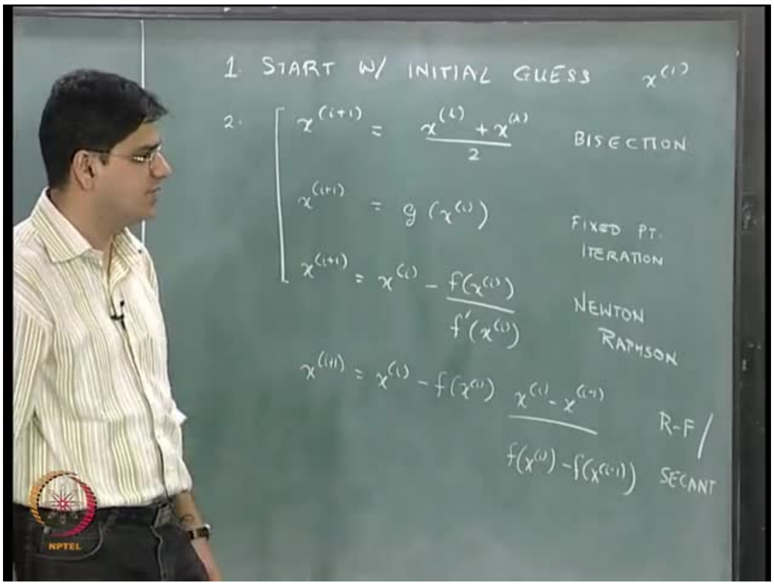

So, what do I mean really by convergence is this, is what we said is, each method is going to start with an initial guess, let us call that as x 1; and then we are going to use some method to get x i plus 1 equal to something in order to get from basically either from x i or from x i and x i minus 1.

For example, in case of bisection method, what we had was, x i plus 1 is equal to x l plus x r divided by 2 was in case of bisection. In case of a fixed point iteration method, we used x i plus 1 as nothing but g of x i; and if we look at the Newton Raphson method, we have x i plus 1 equal to x i minus f of x i divided by f dash of x i, that was newton Raphson; and likewise, we had secant method; and secant method we had x i plus 1 equal to x i minus f of x i multiplied by x i minus x i minus 1 divided by f of x i minus f of x i minus 1.

This was regula-falsi or secant method; of course, the difference between regula-falsi and the secant method was, in regula-falsi we h ave this as x l and x r instead of x i and x i minus 1; so, these where the methods that we looked at in our previous lecture; what we are going to do is, take these three specific methods and we are going to analyze how these particular methods converge; what we mean by that is, the next step beyond this what we looked at was, how, when to decide whether or not to stop this particular procedure of iteratively finding out x i plus 1 starting with x i using any of these - these - methods mentioned over here.

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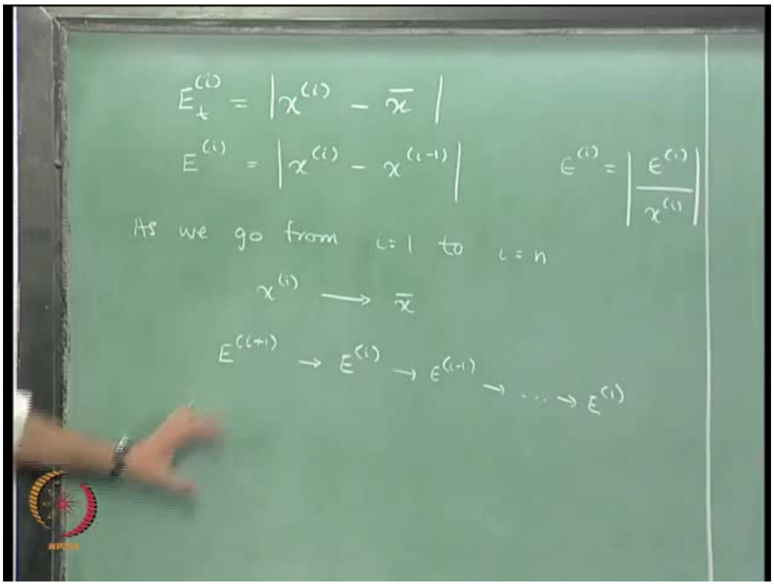

So, the idea over there is that, as the - i - index i keeps increasing, we need our x to somehow go to the - the - true solution x bar; so, let us again go back and think about the definition of errors that we had - come up - come up with earlier; the definition of true error E t at, let us say, the ith iteration is going to be x i minus x bar, where x bar is the true solution; and our approximation error E is going to be equal to x i minus x i minus 1 and of course, we do not bias ourselves whether this error is positive or negative.

So, we are going to put…, we are just going to look at the absolute values of these particular errors; so, this is the error at iteration i, whether we are looking at the true error or the approximation error.

So, as we go from i equal to 1 to say i equal to n, where n is a big - big - enough number, we want our x i to start converging towards x bar; and in the limit, that n is equal to infinity, we really one our x i to actually represent our - our - x bar, that is what our aim in a numerical technique is going to be; so, the convergence criterion that we are going to look at is, how basically our E i plus 1, how E i plus 1 depends on E i, or for that and then E i then is going to depend on E i minus 1 and so on up to E 1, that is the error in the initial condition that we started out with.

So, we start out with a fair amount of error in the initial condition and as the number of iterations increase, we have this particular x i or - x i - x i tending towards some solution x bar; and since, in general, x bar is not really known, the stopping criteria we actually use is not that base on the true error, but it is either based on the approximation error E i or the stopping criteria is based on the approximation error epsilon i, which is nothing but E i divided by x i.

Again these error concepts were considered earlier in module 2 and this is what we are interested in; so, for all these methods, we are going to look at how E i plus 1, how that depends on E i, so that is essentially what - we are - we are interested in doing.

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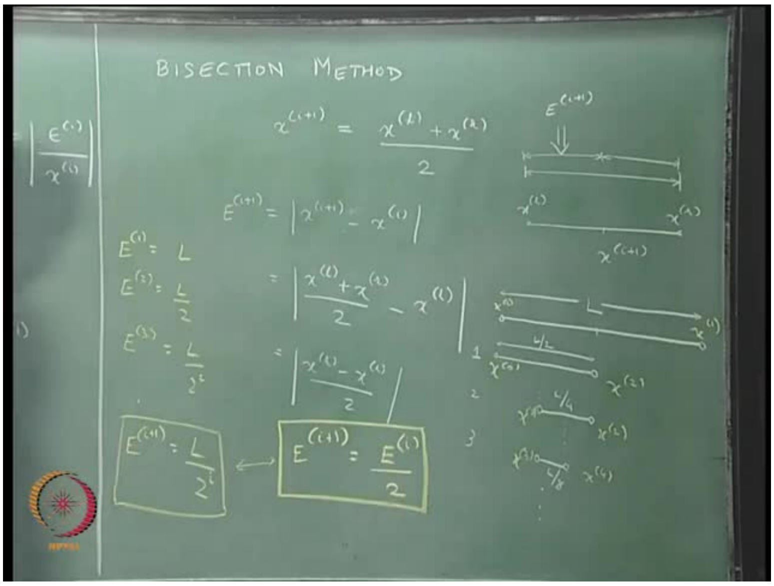

So, now, let us take - an - the bisection method example; and I am taking the bisection method the first simply, because it is the easiest - of the - of the methods for us to analyze, that is the reason why we are taking the bisection method; and at any time, at any iteration i, we get x i plus 1 as nothing but x l plus x r divided by 2, an average of our left assume guess value and of the right guess value; these two values again to recall or going to bracket the true solution.

So, this is how we are going to get our x i plus 1; so, geometrically - what - what that means is, basically this is going to be our x l, this is going to be our x r, and this is going to be our x i plus 1. And our error E i plus 1 is going to be nothing but the absolute value of x i plus 1 minus x i; keep in mind, x i is either going to be x l or it is going to be x r; so, x i is either going to be equal to x l, or it is going to be equal to x r, it really does not matter if we substitute this particular expression x l - minus sorry - plus x r divided by two minus, let - let - us for now, just substitute x l over here, what we will get this as this is going to be nothing but x r minus x l divided by 2. If we substitute x r over here, we are going to get x l minus x r divided by 2, since we are taking the absolute value, this is going to be the actual error, is going to be exactly the same; it is going to be the difference between the two solutions and half of that.

So, what thus x r minus x l mean, x r minus x l is nothing but this particular length, that is what our x r minus x l is going to be; and x r minus x l by 2 is going to be essentially this length, or it is going to be this length; so, this, for example, is nothing but our E i plus 1; so, to summarize at any given iteration - iteration - count i, our the next error E i plus 1 is going to be just the half the length of the current segment form x l to x r.

So, that is what - this - this is essentially going to be. Now, let us look at where we - we - have started this particular problem, we start this particular problem by giving two initial guesses, two initial guesses which lie on either side of the true solution; so, let us say, this is the entire line segment where we started out with - with - the two initial guesses; at the first iteration, so we started with x 0 and x 1, at the first iteration, this is where we got our x 2 at, so at the first iteration perhaps the length segment - where - when we started out with now has been haft, half after the iteration number 1.

So, let us just consider that, initially the difference x l minus x r that we started with was that particular length was l, after the first iteration this length became l by 2. Now, in the second iteration, we get our x 3 as again the midpoint over here; now, let us say in this particular, let us argue that, in this particular occasion, it is the x l that is drop x i x 3 becomes equal to x l; in that particular case, we will get this particular segment.

Irrespective of whether we retain the left half of the segment or the right half of the segment, we have essentially half the length that we are interested in; so, this essentially has become l by 4; at the end of the third iteration, the length of the segment has become l by 8 and so on and so forth.

So, we started out with x 0 and x 1; let us, let us write that down and I will use different colored chalk here, we started out with x 0 and x 1; this remains our x 0, this becomes our x 2; now, we have x 3 and x 2; and now, we have x 3 and x 4.

So, the initial difference that we really started out with was x 1 minus x 0, that is nothing but our E 1, or so E 1 was equal to l, so that is what i am going to write down over here; our E 2 is then, this particular difference our E 2 was nothing but l by 2, E 3 was l by 2 squared and so on our E i plus 1 was nothing but l divided by 2 to the power i.

So, this is the error in the current iteration of the bisection method; and as you see that, as the number of iterations of the bisection method increase every time we are going to half the particular error that we - we - have.

So, this is essentially what we are going to get in bisection method; the final error E i plus 1 is going to be just the initial length l divided by 2 to the power i; also we are interested in seeing how the error E i plus 1 is related to the error E i; and that is very simple to know because E i is going to be nothing but l divided by 2 to the power i minus 1; so, we will be able to write E i plus 1 is nothing but E i divided by 2; so, these are the two important expressions and they essentially mean one and the same thing with respect to convergence of a bisection method.

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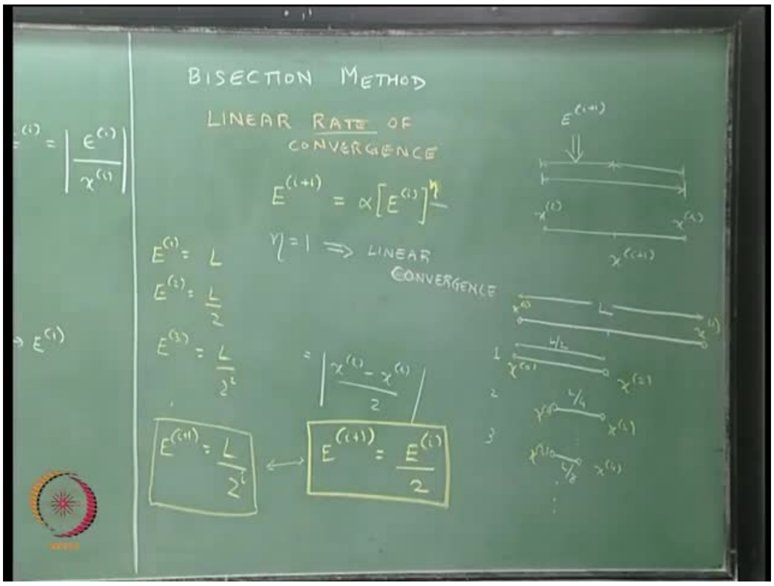

So, these are essentially true two important expressions to keep in mind; this particular way of convergence is what is known as linear convergence; so, the bisection method has a linear rate of convergence; what we mean by linear rate of convergence? Or in general, what we mean by rate of convergence, is how our E i plus 1 is related to E i; so, E i plus

1 is some constant alpha multiplied by E i to the - power - power n or let us call it eta, just to distinguish from the total number of iterations.

So, this particular eta determines the order of convergence of a particular numerical technique, whereas this particular expression gives the rate of convergence of this technique; so, what that essentially means is, how the error in the i plus 1th iteration is related to the error in ith iteration.

If eta equal to 1, we call this as a linear rate of convergence and this is what we really get for the bisection method; E i plus 1 is nothing but half with that means alpha equal to half multiplied by E i to the power 1; and the so in this particular case, eta is equal to 1 and we get linear rate of convergence.

In a few - few - minutes from now what - what - we will look at is also look at fixed point iteration which also has a linear rate of convergence; and the newton Raphson method which has a quadratic rate of convergence, what that means, quadratic, by quadratic rate of convergence is that eta is equal to 2; the two methods that we are not going to cover, we are not going to cover the derivations of are the - fix sorry - the regula-falsi method and the secant method; and in both regula-falsi method and secant method the eta lies essentially between 1 and 2, so that we call as a super linear rate of convergence.

But coming back to where we were at, for the bisection method, we have eta equal to 1 with linear rate of convergence; the at, what is another advantage of bisection method is, that given a particular tolerance value, we can determine a priori - what - how many iterations we are going to require for - this particular the - this particular method to converge.

For example, let us say, epsilon tolerance is going to be the desired accuracy that we are interested in, or we will write that as E tolerance as the desired accuracy that we are interested in.

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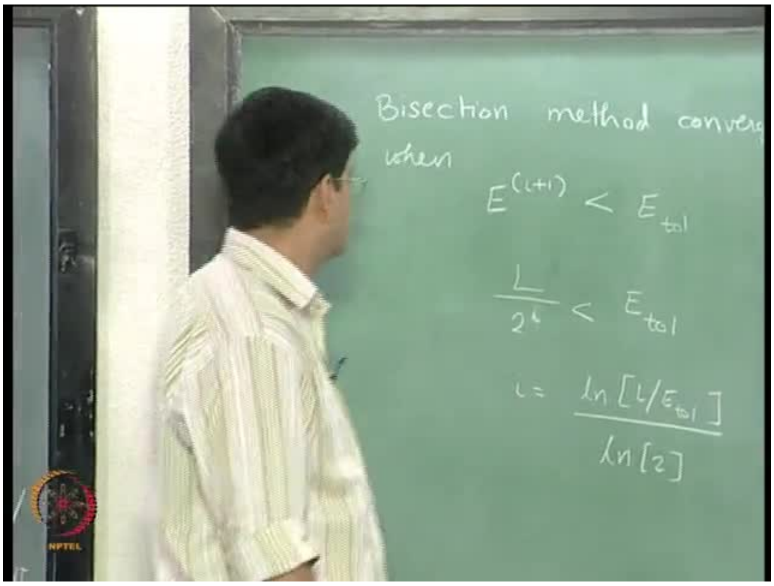

So, the bisection method is set to have converged when - E tolerance is sorry - E i plus 1 is less than E tolerance; and we are going to start with the length l so our E tolerance E i plus 1 is nothing but l divided by 2 i so that is going to be less than E tolerance; and we take our 2 to the power i - on the - on the other side, we will, we will essentially get i and - divide it - divide by E tolerance, and take algorithm, we will get i equal to log of l n divided by E tol divided by log of 2.

So, i has to be at least equal to this particular number or - greater than - greater than this, this of course is a fraction; so, for example, if we get this particular number as ten point five then the minimum number of iterations of the bisection method we will require are going to be equal to eleven; and we can predetermine how many iterations of bisection method we are going to require in order to solve this particular problem.

So, that is essentially all we have to do about the bisection method. The next method that we are going to consider and we will do the same kind of stability analysis on…, and that is going to be fixed point iteration. What i am going to do with fixed-point iteration is, again go through a derivation of a similar type, we will come, we will derive the expression, how E i plus 1 depends on E i and there we will again see E i plus 1 is some alpha multiplied by E i and then what that particular value of alpha means with respect to this stability of that solution we will talk about that; and then we will talk about the graphical interpretation of what that means.

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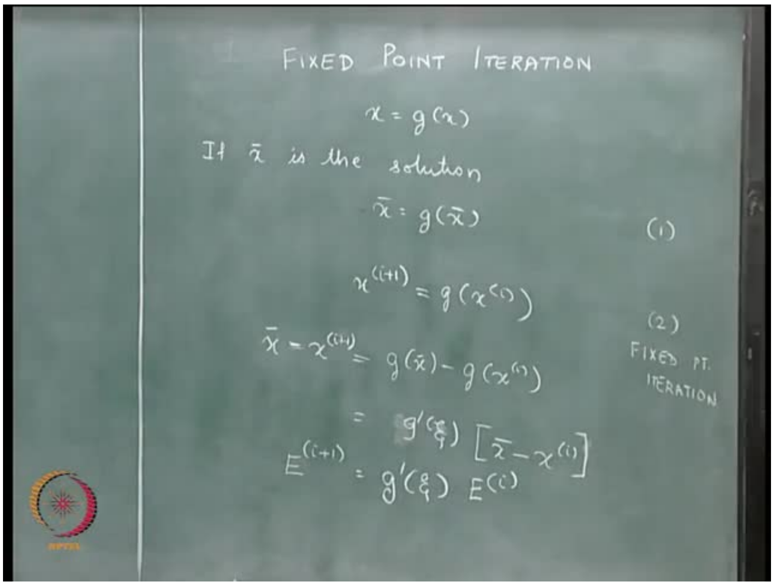

So, that is, that is coming up right now, is going to essentially fixed point iteration. So, the objective in solving the fixed point iteration; in fixed point iteration was to get the solution of the equation x equal to g of x; and if x bar is the solution, then x bar is going to satisfy this particular equation. So, x bar is going to satisfy the equation x bar equal to g of x; x bar is going to be exactly equal to g of x bar, that is what we really mean by the true solution of that particular problem.

So, this let us call this as our equation number one. Now, the fixed point iteration, the way that the fixed-point iteration works is that, we basically calculate our g of x at the current guess x i; and based on the current guess, we are going to calculate the g of x and that we are going to assign it 2 x i plus 1; so, what we have again was x i plus 1 is going to be equal to g of x i; that is, that is our equation two; that is our fixed point iteration.

So, this is, this is essentially - what what - what - we are - we are going to get; what we can just do is, subtract equation one from equation number two, so or or rather we will subtract equation two from equation number one; and the result of that subtraction is going to be x bar minus x i, is going to be equal to g of x bar minus g of x i.

Now, by essentially, the mean value theorem what this particular expression really means - what - what we can write this particular expression down as, we can write this as, g dash of some value zeta, where zeta is a value that lies between x i and x bar; so, g dash of this value zeta multiplied by the delta x, which is essentially going to be equal to x bar minus x i.

So, this is going to be - our - our expression, sorry, this should have x i plus 1, we have, when we subtract these two equation, we will have x bar minus x i plus 1; so, this essentially is nothing but our E i plus 1; and this guy is nothing but our expression E i; and - we can - we can write - those - those - things - things down, so we have E i plus 1 is going to be equal to nothing but g dash of zeta multiplied by our error E i; so, the error in i plus 1th iteration is going to be some particular constant g dash of zeta multiplied by error at the ith iteration, where zeta is a point which lies somewhere between x i and x bar.

And as the error becomes smaller and smaller, our g dash zeta tends to go towards - x x i - x i or x bar; so, it is essentially what - we need - we really need to do in order to figure out whether this particular method is convergent, sorry, this particular method, what this particular value is going to be is, calculate the first derivative of the function g of x at the value x i; what we are seeing over here is x i is close enough to the value zeta and that will essentially give you how this particular method is converging from E as the number of iterations increase.

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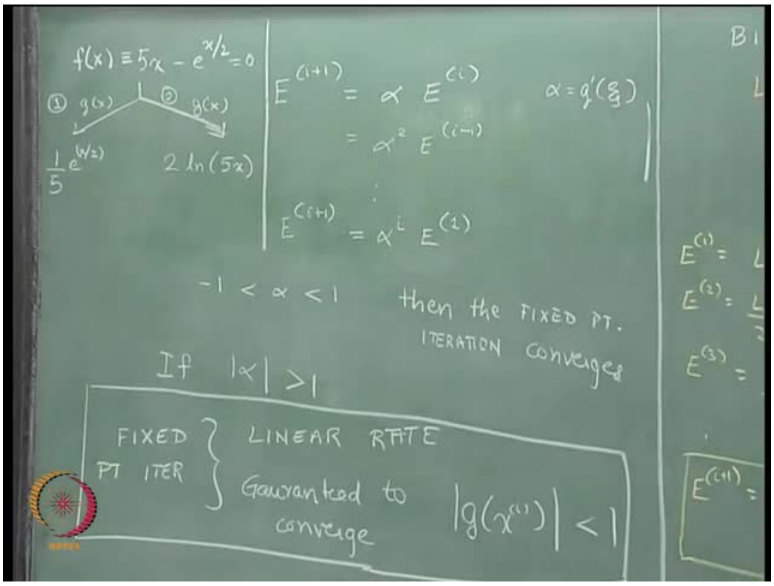

So, what do we mean by that is, again let us - look at - look at our expression E i plus 1 equal to alpha times E i, where alpha in this particular case is - g of zeta and sorry - g dash of zeta, and rather i should be writing g dash of zeta i, because zeta i is a number that rise between x i and and x bar.

Likewise, we will have the expression E i is going to be the same constant alpha - multiplied - multiplied by E i minus 1, so on and so forth. So, that we can write as alpha squared multiplied by E i minus 1 right, because E i is nothing but alpha multiplied by E i minus 1 and alpha multiplied by alpha would be essentially alpha square into E i minus 1; and if we keep writing that over and over again we will get this as alpha to the power i multiplied by E 1.

So, this is going to be our E i plus 1; now, if this particular value alpha is lies between minus 1 to plus 1, alpha to the power i decreases continuously; and as i tends to infinity, alpha to the power i is going to become equal to 0. So, if minus 1 is going to be less than alpha less than 1, then the fixed-point iteration converges; and if alpha lies is greater than 1 or if alpha is less than minus 1, in other words if the - alpha - absolute value of alpha is greater than 1, then the fixed-point iteration is not guaranteed to converge.

So, just summarize what alpha - is is - is nothing but g dash of zeta and we are going to approximate zeta as our x I; so, to summarize our fixed point iteration has linear rate of convergent and it is guaranteed to converge if our absolute value of g of x i is less than 1. So, this is the important result that we get about convergence of the fixed point iteration method; so, what we are going to do is, we will take up an example in Microsoft excel and try to use basically the fixed-point iteration in order to get the solution to that particular expression.

The expression that we are going to use is, f of x equal to 5 x minus e to the power x by 2; and f of x - in this particular written - in this particular form, we can, we can write this this in two different ways; so, f of x is equivalent to this equal to zero; and the two equivalent, two equivalent ways that we can write this as, we this, we can take e to the power x by 2 on to the other side and divide by 5; so, our g of x for the - first - first case is going to be nothing but 1 by 5 e to the power x by two.

And in the second case, we take this - to the left hand side - to the right hand side and take algorithm on both sides and we will get our x is going to be equal to 2 times log of 5 x, that is going to be the second approximation, the second way of getting g of g of x.

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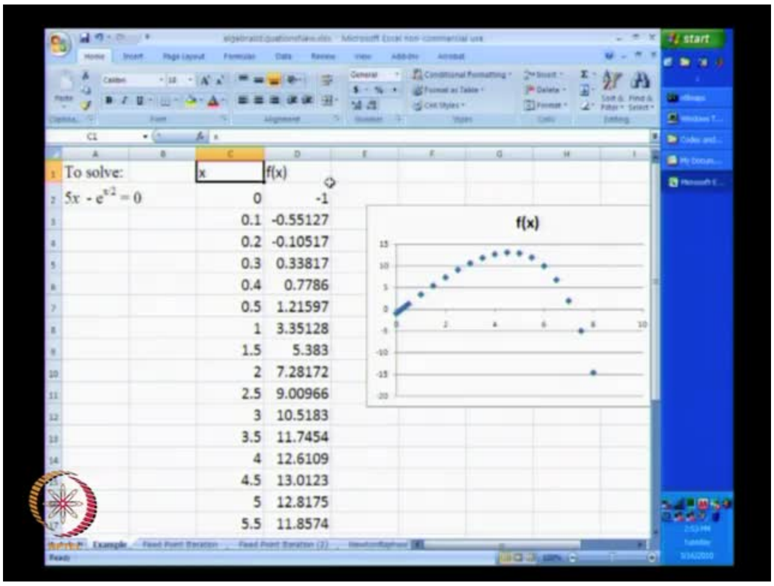

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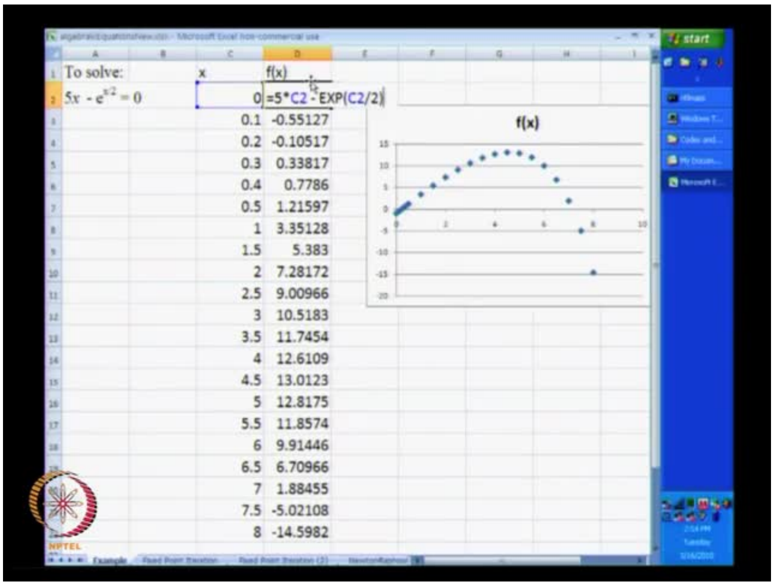

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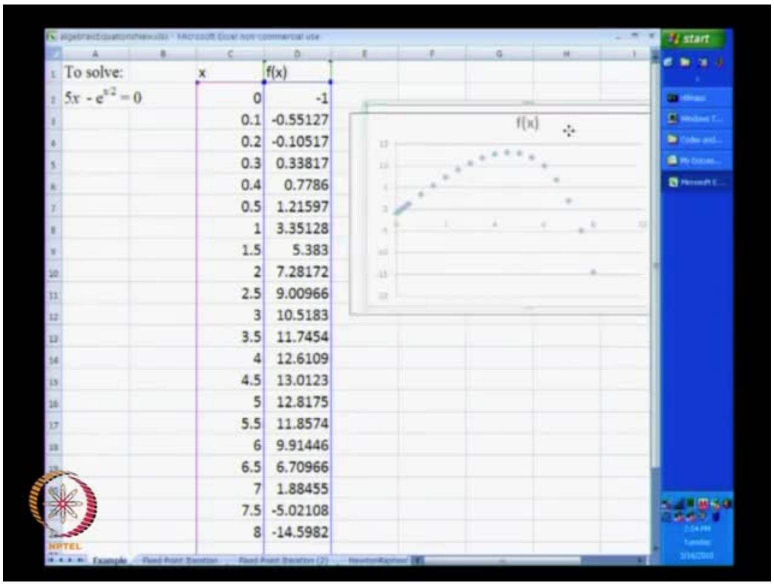

So, what we are going to do essentially is, consider these two examples; so, these two ways of solving this particular problem using the fixed point iteration method. So, we are now going to move on to excel in order two look at the fixed point iteration. So, just to give us an idea of what this particular functions looks like, what i have done over here is, I have taken…, so this is the particular equation, 5 x minus e to the power x by 2 equal to 0, that is what we - we - want to solve; so, the left hand side of this particular expression is nothing but our function f of x; so, for various values of x going from 0 to 8, what I have done is plotted our f of x; and if you click f 2, you will be able to see what f of x means, it is it is basically 5 multiplied c 2; c 2 is nothing but, as you can see over here, c 2 is nothing but our x value minus e to the power x by 2; e x p is the exponential; c two is again x divided by 2; so, that is our function f of x; and what then I had done is just dragged it along the entire column and when i dragged at along the entire column and then plotted this is what we - we - get as a solution f of x as a of x.

So, this particular function intersects the x axis at two points; these two points are the points of solution, so what - we can - we can do is, just format the data series and - add - add a line.

So, if you look at this, this is one solution, and that solution is lies between seven and seven point five perhaps something like seven point one, seven point two, or something like that; and then there is another solution which lies essentially between 0 point 2 and zero point three that is when the function changes the sign; and so this is perhaps going to be something like zero point two - two - or something - something - of that is sort.

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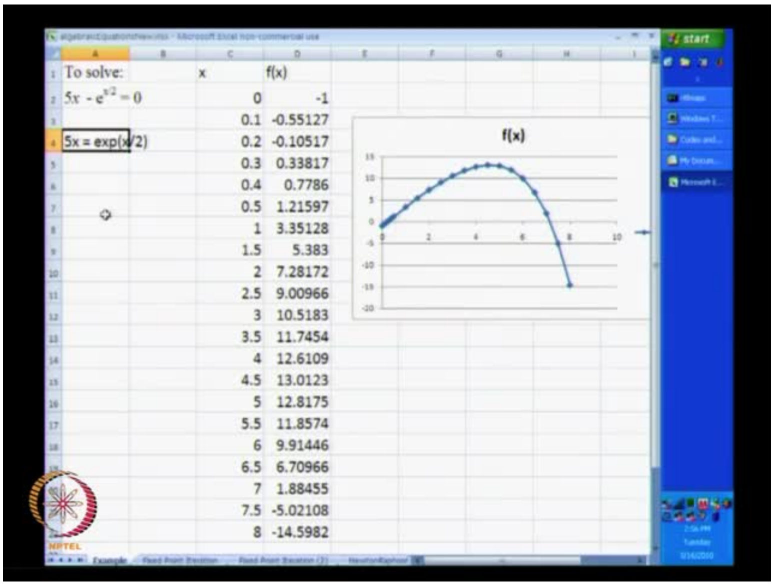

So, as now, we had, we had said before this particular equation can either be written as …, so this we will write as 5 x equal to exponential of x by 2; and I will just increase the font; so, 5 x equal to x 1 and of x by 2, we can either divide this particular expression by 5 to get our first example of g of x; and when we do that, this is what we get g of x, the equation that we want to solve is x equal e to the power x by 2 divided by 5.

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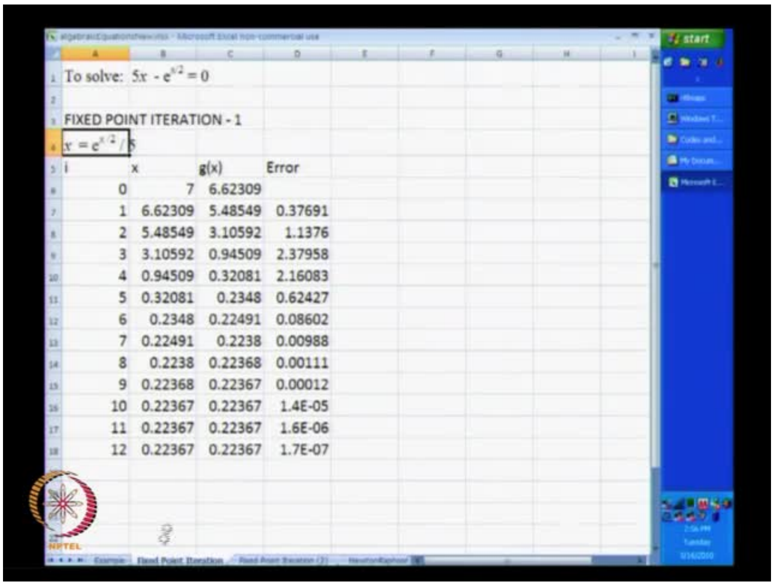

Or the other way to do this is, take algorithm, so you will have log of 5 x equal to x by 2, or x equal to 2 times log of 5 x and this is what we have got - in in - our second expression over here; so, x equal to 2 times log off five x.

So, now, as we can see this particular curve intersects the x axis at two points, what we will do is, we will solve the fixed point iteration method using the first expression that is x equal to one fifth e to the power x by 2; we will solve that particular expression starting with various different initial guesses between 0 and 10.

The areas of real interest for us are - are - the areas where x is going to be less than - the particular solution - this particular solution; the second area of interest is going to be when x lies between this solution and this solution; and the third area of interest is when x lies beyond this particular solution.

So, let us look at the first particular, first example, x equal to e to the power x by 2; and I have used the various iterations over here just to keep it consistent with notations we have used previously, I will start i with i equal to 1.

So, essentially, let us start with in the beginning, let us start very close to zero, let us start with a number equal to say zero point - zero zero - zero zero one; and when we start with a number equal to zero point zero zero one, what we see is that fairly quickly our solution goes and converges to this particular point, so we start at this particular region; so, what I will do over here as I will just zoom this.

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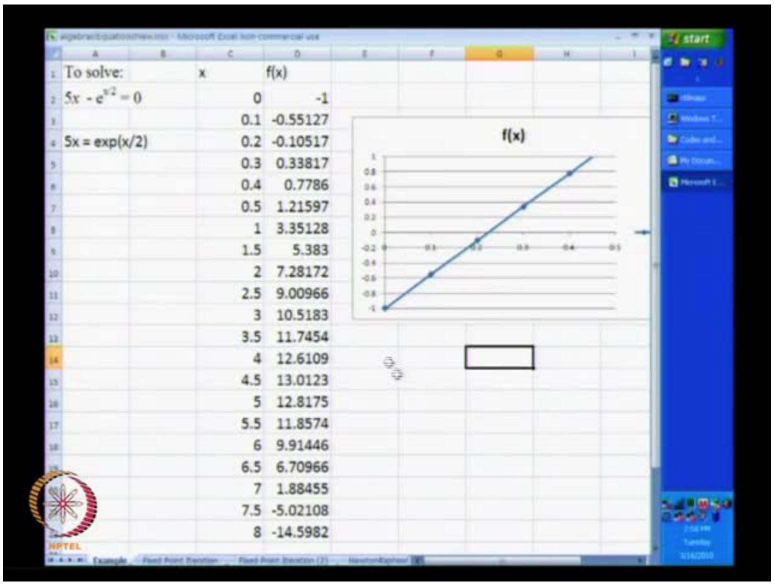

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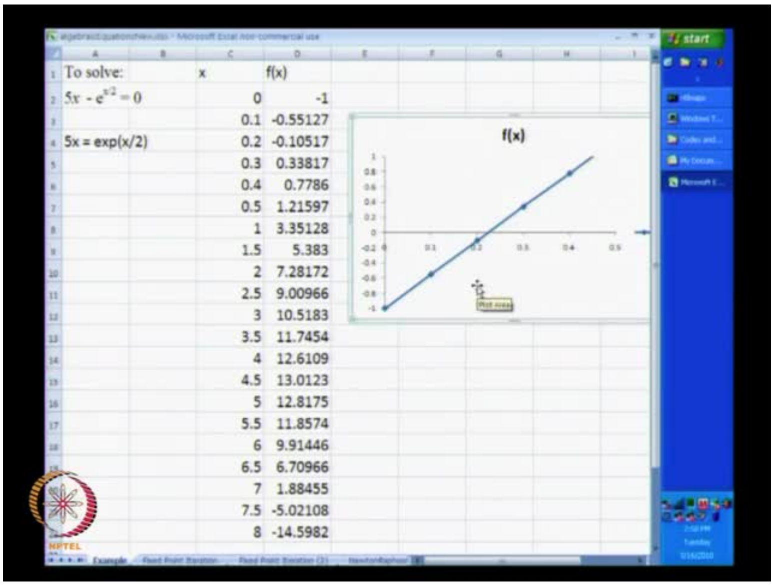

So, this is, this is the same curve I have not done anything to that curve, what I have, the only thing I have done is zoomed in the x and y axis in order to show you what exactly happens in the x and the - y - y axis; and this is our x axis; and this is the point of intersection.

So, when we are started with some - some - value very close to zero over here - what - what happened with the fixed point iteration was that, particular value ended up going to this particular solution, when - when - we use x equal to e to the power x by 2 divided by

5. Now, let us take - take - a value very too close to the solution, but again on this side of the solution, let us take the value as zero point two.

And if - if our - our error criterion was 10 to the power minus 4, it has taken about six iterations in order to converge the solution to the predefined error criteria; let us look at what happens when our initial guess is x equal to zero point two, what we find when the initial guess is x equal to zero point two, again is that the solution converges to the desired solution and this solution converges so fairly quickly, it converges now this time in five iterations.

And again starting from here, we have ended up at this particular solution, that is go a little bit further and start at zero point five, and see what is going to happen; so, again I will change this particular number to zero point five; and so from zero point five we ended about zero point two, five point two - two - and so on.

So, what - what - we did is from this particular point, we had a big jump, and we ended up over here and then fairly slowly we ended up to this particular solution; let us now again zoom out this particular system and over here also.

So, now, we have gone beyond that point and - let us - let us go somewhere close to say 4 and we will start with that instead of point five will change x equal to 4, again what we find is that, the solution has indeed converged; and let us try again x equal to say x equal to 7 x equal to 7 is fairly close to the other solution; and when we substitute x equal to 7, again we find that the number of iterations taken are fairly large, but again what we find is that, our solution has indeed converged from starting at this particular value; although we have started very close to this particular solution, irrespective of that when we started at this value, we ended up going essentially to - this - this particular guy.

So, that is what has happened; for this entire range, what we have done is, we have spanned the entire range from x equal to zero point zero zero one to x equal to 7; and for this entire range, we get this, we the fixed point iteration method converges to this particular solution.

Now, let see what happens if we go to point 8; and what the interesting thing that we see when we get to x equal to 8 is now the solution is not converging in fact the solution is diverging; the first value of x is x equal to 8, the second value of x is x equal to almost eleven, forty seven, 3 in 10 to the power 9, and then there is a floating error, because the numbers have become too large compared to the storage capacity, essentially the number has become greater than 10 to the power 309.

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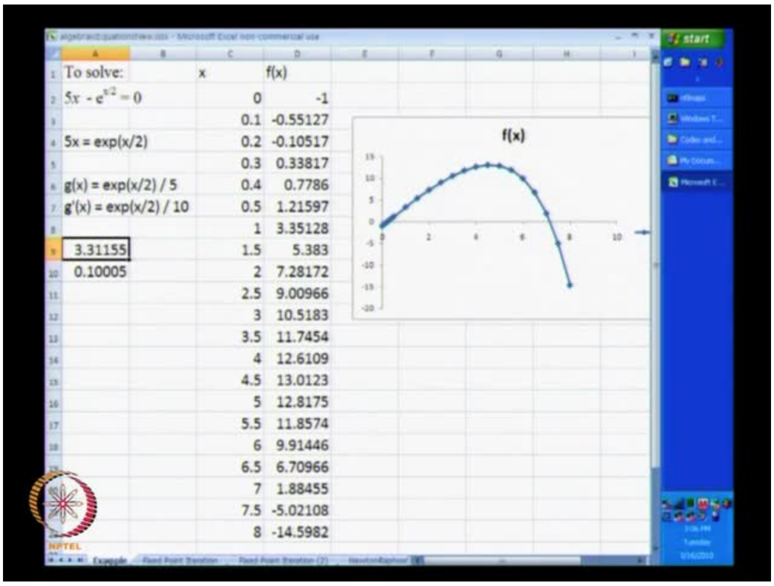

So, that is, that is where essentially this particular methods - stops - stops working; so, in a sense when we started with x equal to 8, rather than converging to this particular solution, we ended up diverging towards infinity, that is what - this - this method shows; now, our g of x was nothing but e to the power x by 2 divided by 5, so g dash of x was nothing but e to the power, so g of x was e x p x by x by 2 divided by 5; and g dash of x was e x p x divided by 2 divided by 10.

So, this were our g of x and g dash of x values. Now, when - when - we started off with the value of 8, we will let us take substitute that particular value and see what we get; so that is equal to e x p of 8 divided by 2, the whole thing divided by 10, we get that value as five point four; so, what we essentially see over here that, the exponential the g dash of x computed at that particular value is essentially greater than 1, which basically means that this particular method - is going to - is perhaps going to diverge.

Let us look at the, case, another case when - when - we started with say zero point zero zero one, so if we do e x p of zero point zero zero one, what we find is that, indeed that particular method value is less than 1.

So, when we start with x equal to say zero point zero zero one, we are guaranteed to converge; when we start with x equal to 8, there is no guaranties of convergence; it is possible that the particular method would converge; it is possible that the particular method with will not converge; and just to take an example, if we were to replace this with 4, so e x p of 4 by 2, that is going to show us convergence.

So, the convergence is…, so if we were to use our - our - particular initial guess in this range between zero to say four point two or four point three, we are definitely going to get the solution to converge; so, we know that for a fact that, for x equal to from x equal to 0 to x equal to say around 5, we are going to get the solution to converge; but what happens when we substitute 7 in this particular expression; when we substitute 7, g dash x evaluated at x equal to 7 is indeed greater than 1.

But as we had seen, when we had seven in this particular expression our out fixed-point iterations converges; so, what does that mean it, what it really means is that, the absolute value of g prime of x lying between minus 1 and plus 1 is a sufficient condition for convergence, but it is not a necessary condition.

So, that means, if g dash of x lies between minus 1 and plus 1, we know for sure that, this particular method is going to converge; but if g dash of x does not lie between minus 1 and plus 1, there is a possibility that the method will converge, but there is also possibility that the method will not converge.

So, we cannot really say anything for sure if g dash of x is going to be greater than 1, but if it is less than 1, or rather if it is between minus 1 plus 1, we can say with hundred percent certainty that, the method is going to converge; the other way of writing 5 - five - x equal to e to the power x by 2 is by taking log on to the both sides and then multiplying by two throughout.

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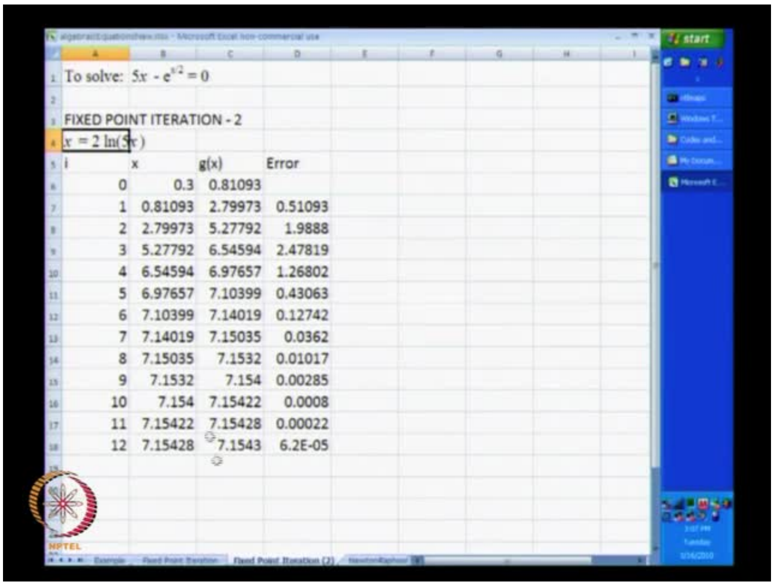

So, we will have l n five x equal to x by 2 and then we can write x equal to 2 times of l n 5 x; and when we do that and then we can run the fixed-point iterations. So, it is the same expression as you see over here; it is exactly the same expression that we have solved previously, only thing is that, we have rewritten x equal to g of x in the form x equal to 2 times log of 5 x; so, again iteration - iteration - number 1, 2, and so on, this is to keep up with the - the - notations that we have been using previously.

This is our x; this is g of x; I will just show you what g of x is, it is 2 multiplied by log of 5 multiplied by b 6, b six is nothing but x; and so, if let us say, we start with a very low value, let us say we start with a value of say zero point two, what is going to happen in this case if we start with the value zero point two, the g of x that we will get is equal to 0 and then log of 0 is not defined and therefore, we - we - cannot continue further.

So, if - if if - the particular initial guess is say at zero point two, we are not getting - getting - the solution to converge; let us, let us see if it was say zero point two - two - again the solution is not converging.

So, for all the values beyond this particular solution our numerical method of fixed-point iteration is unable to converge; if we use the value of zero point three, this value of zero point three is fairly closed to this particular solution, but when we start with the value of zero point three with the second expression for the fixed-point iteration, that particular method converges to this particular solution and not to this solution.

And that is what happens if we take the value equal to 4, again we are converging if we take the value equal to 7, again we converge and if we take the value equal to 10, again we see that we are converging to the solution seven point one five, which essentially means that, from this range onwards right up to infinity no matter where we start off with - with - that particular form of fixed-point iteration, we will end up at this particular - this - this case.

And what you can then try yourself is, for this particular, this g of x 2 log 5 x, you can get your g dash of x and once you really have g prime of x then find out numerically where the fixed point iteration is guaranteed to converge and where the fixed point iteration does not have guarantees to converge.

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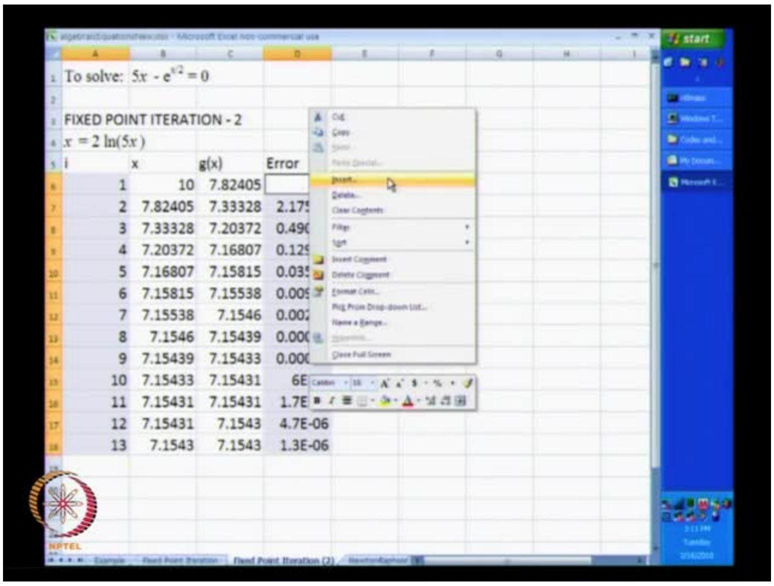

And what you will see essentially from this particular example is that, the point iteration has a guarantee to converge in this entire range starting from approximately four point two and going right up to infinity, whereas in this range, there are there are no guarantees of convergence, but we find that in spite - that - of the fact that, there are no guarantees of convergence, because this is a sufficient condition; as a result of this, we will get.., we could get convergence if g prime exceeds 1 or is less than minus 1.

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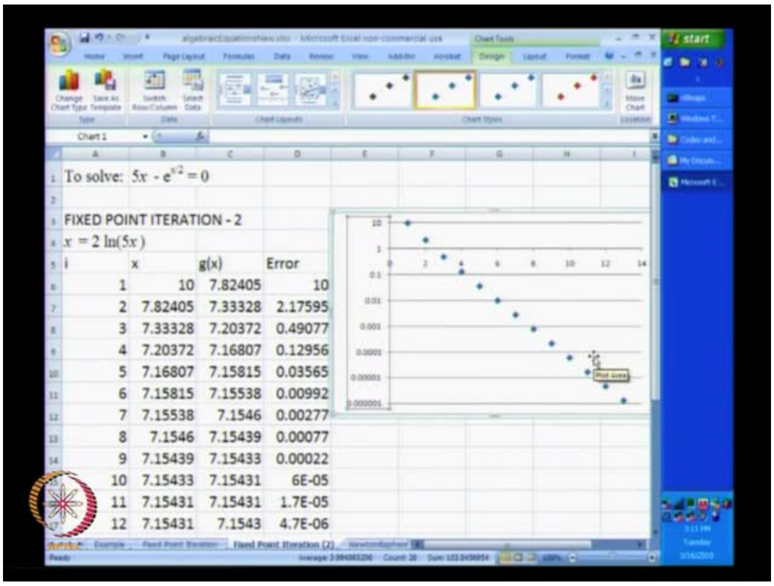

And then the final thing is, what we can do is plot our error against the iteration number I will just do this particular plotting over here and I will essentially try to plot this on - log - log scale and we will have to go away from this insert scatter plot, it will insert, and so what we are going to do is, we are going to make this plot as algorithm plot.

So, this y axis is going to be a algorithmic scale; so, when we essentially make this particular scale as algorithm mix scale, we find that that particular error line is straight line approximately; and so this particular curve is going to be essentially a straight line with a slope of minus 1, what, why we get that is essentially because we have a linear rate of convergence.

By linear rate of convergence, we mean the error E i plus 1 is essentially going to increase linearly with the number of iterations as the number of iterations increase. So E i plus 1 is going to be equal to some constant multiplied by E i to the power 1 and then when we take algorithmic plot of that we have essentially going to get an approximately a straight line in this; in this particular case, an approximate straight line with slope of minus 1; so, that is about the fixed point iteration; similarly if we look at the bisection method, we will also get a similar algorithmic plot for how the error changes - with - with respect to the number of iterations.

What we are going to do next is, next we are going to look at the Newton Raphson's method. We will derive the Newton Raphson's method based on essentially the Taylor series expansion and then we will use the Taylor series expansion in order to - find - figure out how the errors in the Newton Raphson’s method change with the iteration.