Computational Fluid Dynamics

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Module No. # 02

Lecture No. # 06

Topics

Equations governing flow of incompressible flow

Initial and boundary conditions

Well posedness of a fluid flow problem

Let us take stock of the situation. We have derived already the continuity equation which is nothing but the mass conservation equation. We have also derived the momentum conservation equation on a control volume. Let us write this down and see what form we finally have.

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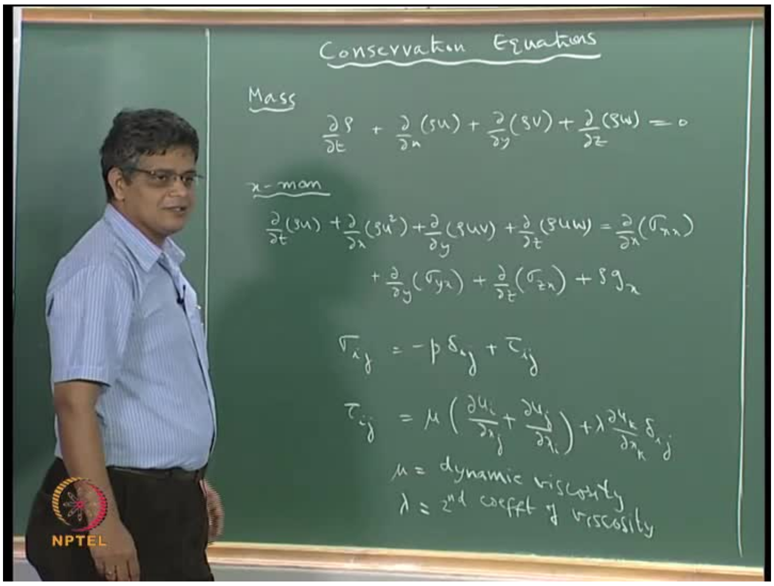

So, we can start with the conservation equation. We have derived the mass conservation equation as partial with respect to t of density plus partial derivative with respect to x of rho u plus partial derivative with respect to y of rho v plus partial derivative with respect to z of rho w is equal to 0.

For the momentum, we considered three separate directions in the x y z directions, and we could write x momentum conservation equation as d by dt of rho u plus d by dx of rho u square plus d by dy of rho u v plus d by dz of rho u w, and this, we said was equal to the stresses acting on the different surfaces to the rate of change of momentum rising out of the stresses in the x direction plus stresses in the x direction from phases in the x direction plus stresses in the x direction resulting from faces in the z direction plus the body force, which is the gravitational force, and here, we could write down similar equations for the y momentum and the z momentum, and we said that these equations are not sufficient in themselves and that we need it to find extra relations for the stresses, and here, we made a certain assumptions.

We first of all divided the stress into a hydrostatic component plus a stress component - a viscous stress component - which arises only from relative motion of fluid, and we said that the stress arising out of relative motion arises only from sheer deformation and extension deformation but not from rotation strain rate, and we saw that we could write this as mu du i by dx j plus du j by dx i plus du k by dx k delta i j with a lambda there, where mu is the conventional dynamic viscosity that we have, and lambda is the second coefficient of viscosity; mu is the dynamic viscosity, and lambda is the second coefficient viscosity; it is also known as the bulk viscosity.

While the value of u is easily measurable, for example, a rheometer, the second coefficient of viscosity - the lambda value is not easily measured, and in many case is neglected, and in the specific case of incompressible flow, this whole term will become 0. So, the value of lambda does not really matter. Whatever it is, it does not come into the picture at all. So, we do not have to worry too much about the value of lambda. We just would like to say that it is usually not a big problem when we consider this.

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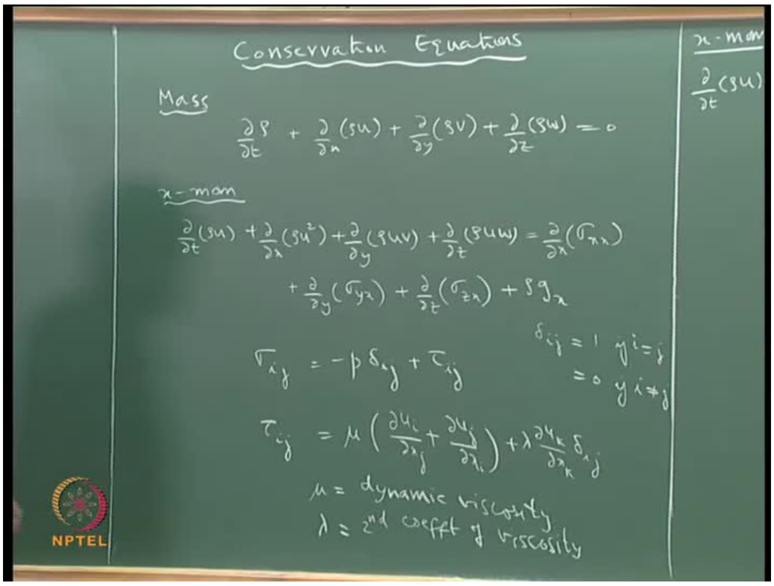

Now, we can substitute these two expressions into this and derive the x momentum equation. We can finally get an x momentum equation like this. The p coming from minus p here. (Refer Slide Time: 05:54) We know that delta i j is the conique delta - where delta i j is equal to 1 if i is equal to j and is equal to 0 if i is not equal to j.

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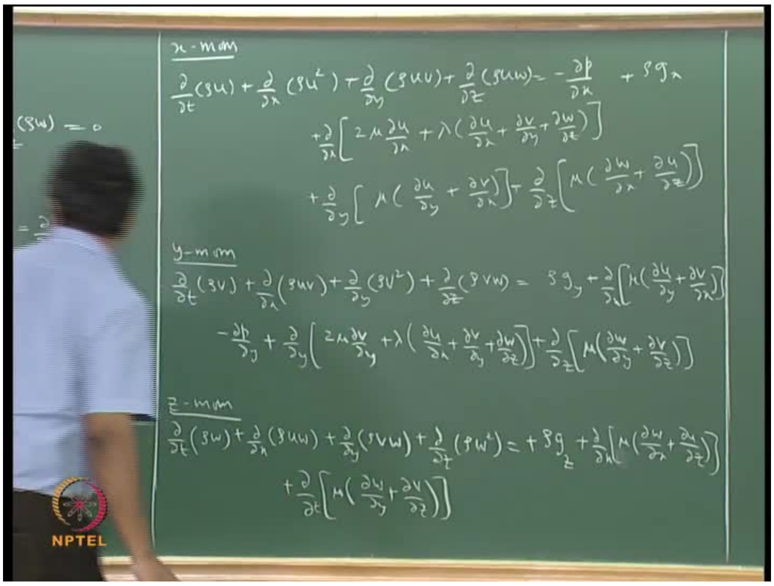

So, with that notation, then we have dou by dou x of we can add the rho g x term. We have in this, we have already accounted in this for the pressure term. We have tau x x, and tau x x can be written from this as mu times dou u x by dou x plus dou u x by dou x plus lambda times dou u k by dou x k.

Because when we say x x i is equal to j equal to 1, so this thing becomes non-zero. So, we can write it as two mu dou u by dou x plus dou u k by dou x k is a term with repeated index k and it implies summation over the 3 values of k, so, that is k equal to 1, in which case, this becomes dou u by dou x k equal to 2; in which case, this becomes dou v by dou y and k equal to 3; in which case, this term becomes dou w by dou z.

So, we can write this as plus lambda times dou u by dou x plus dou v by dou y plus dou w by dou z. This is what we have for this term. We have minus dou p by dou x plus this. Then we come to sigma y x. This p is here i is equal to 2 and j is equal to 1. So, this term here becomes 0 because i is not equal to j, and this term becomes 0 here, and so, the pressure term does not appear in this term here, and the viscous term here becomes mu times i is one. So, this is dou u by dou v plus dou v by dou y or dou x by dou x so plus dou by dou y of mu times dou u by dou y plus dou v by dou x.

Now, we have z x; z x means that i is equal to three and j equal to 1 in this expression. So, again, this becomes delta 3 1, so that is equal to 0, and here, tau 3 1 is the tau z x, and so, this becomes mu times dou w by dou x plus dou u by dou z, and this term becomes 0 because this is 0. So, we can write this as plus dou by dou z of mu dou w by dou x plus dou u by dou z.

So, this is the x momentum equation that we have, and we can similarly write the y momentum equation. In a similar way, taking the u converting one of this use into v's here. This dou p by dou x becomes dou p by dou y, and this becomes rho g y the y component, and these things will also change; we will just look at that. So, we can say that rho v plus partial with respect to x of rho u v plus partial with respect to y of rho v square plus partial with respect to z of rho v w is equal to…

Now, we come to, we have this rho g y the gravitational component, and here, we will have sigma y x, and sigma y x means that this will be 0 and, tau y x, tau y x we have seen already that this thing will be 0; this will be dou u by dou y plus dou v by dou x. So, we can write this as plus dou by dou x of mu times dou u by dou y plus dou v by dou x, and here, we have sigma y y will be coming.

So, sigma y y is i equal to j equal to 2. So, this has a value of 1 here; this has a value of 1 here. So, you get minus dou p by dou y plus tau y y is given by mu times d v by d y plus d v by d y, so that is 2 mu d v by d y plus lambda times dou u by dou x plus dou v by dou y plus dou w by dou z, and then, we will finally have sigma z y; so, sigma z y means that i equal to 3 and j equal to 2. So, this becomes 0 and this is equal to 0, and tau z y becomes mu times dou w by dou y plus dou v by dou z, and similarly, we can write the z momentum equation in a very similar fashion on the left hand side, this v here, which is appearing in these 4 terms differential w.

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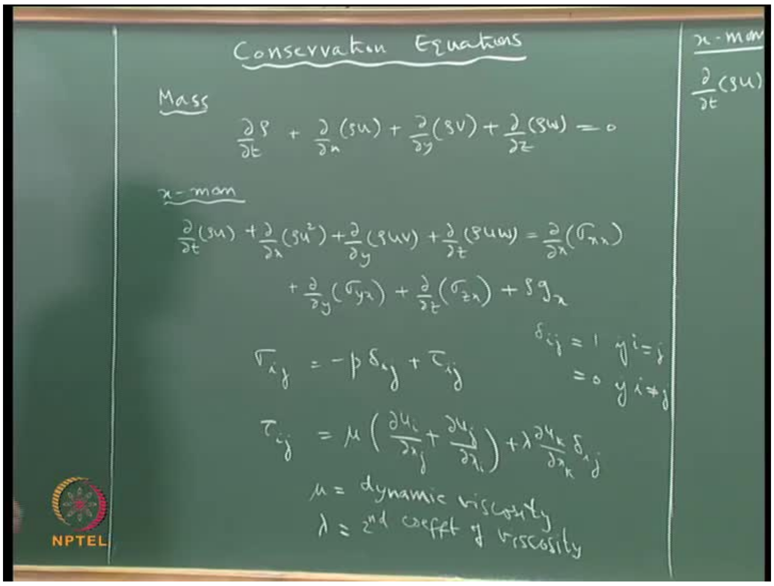

And here, you will have sigma z x sigma z y and sigma z z, and we will have the gravitation term as rho g z. So, sigma z x is dou by dou x of sigma z x will be mu times dou w by dou x plus dou u by duo z, and sigma y z will be dou by dou z of mu times dou w by dou y plus dou v by dou z, and sigma z z means that z equal to i equal to j equal to 3, so this is 1 here and this is 1 here, and this becomes dou w by dou z dou w by dou z.

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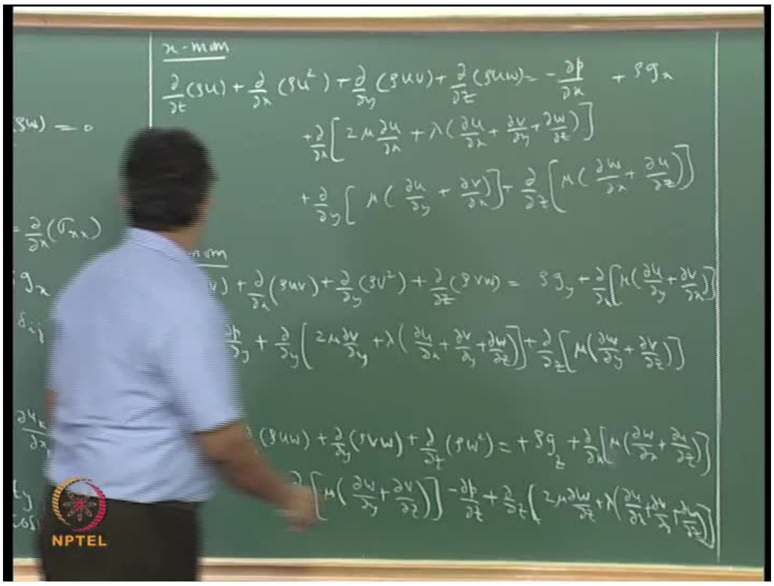

So, this term here becomes, so, we have minus dou p by dou z plus dou by dou z of 2 mu dou w by dou z plus lambda times dou u by dou x plus dou v by dou y plus dou w by dou z. So, we can now look at the overall equations that we have.

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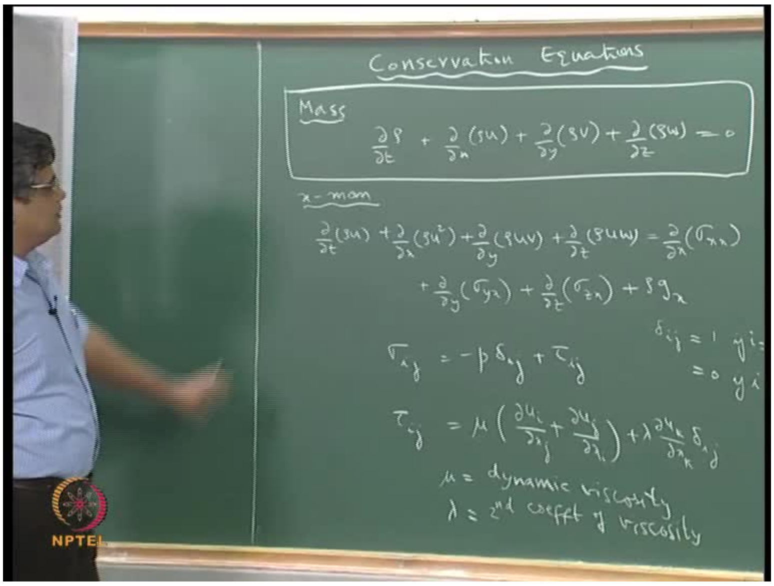

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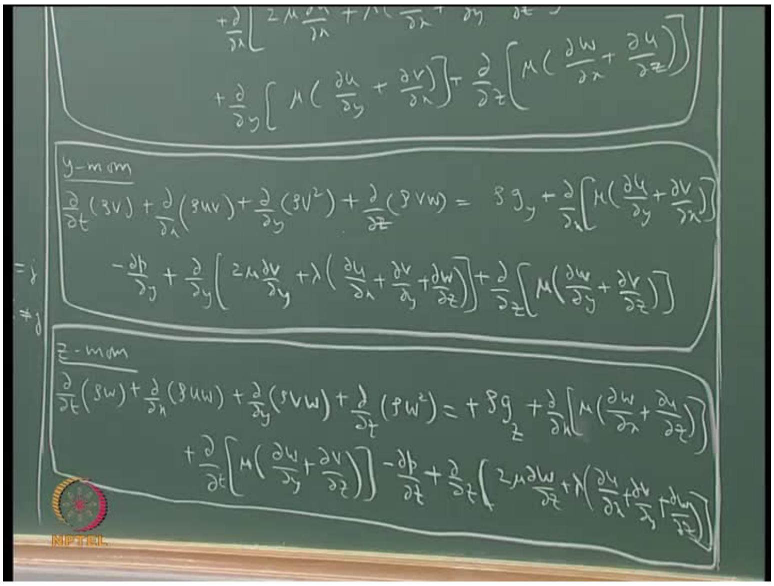

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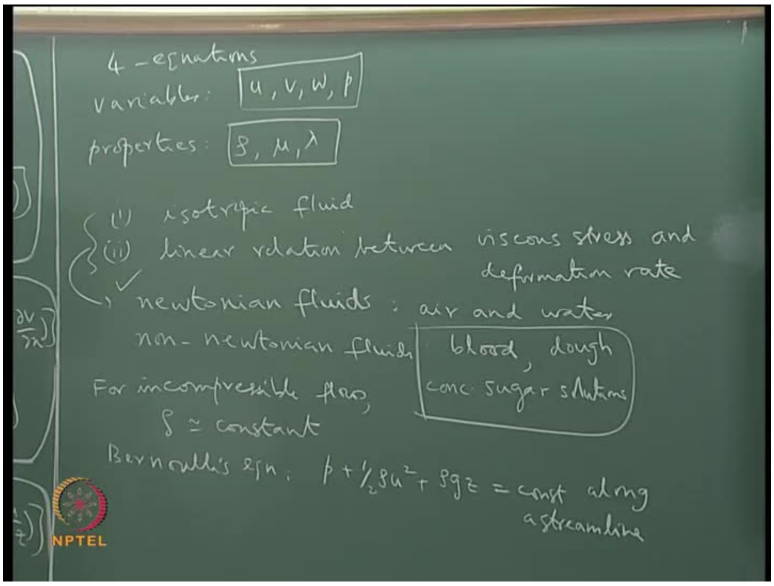

We have the mass conservation equation otherwise known as the continuity equation. We have an x momentum equation and we have a y momentum equation and we have a z momentum equation. So, we have four equations, and the variables that are appearing in this apart from the properties of the fluids are here. Rho is obviously a property of the fluid which is the density here. In the continuity equation, we have u v w. Then we come to the x momentum equation, and let us also put the properties. We have rho which is the density, and in the x momentum equation, you already have rho u w here, and pressure is coming which is a flow variable, and of course, g is the gravitational vector; we expect it to be specified in terms of orientation of the vector volume.

We have mu and lambda. Otherwise, everything else is given here. In the y momentum equation, no additional variables; in the z momentum equations, no additional variables. So, we have now 4 equations for the 4 variables for a given properties of the fluid.

So, the properties that are needed are the density, the dynamic viscosity and the second coefficient of viscosity. So, in that sense, we can now claim that we have as many numbers of equations as there are the numbers of variables.

We have derived these equations subject to some conditions. We have derived them, subject to the condition of an isotropic fluid, and linear relation between viscous stress and strain rate and deformation, right, and in the process, we have also made use of the condition of material, frame invariants of the linear relation between viscous stress, and stress in the strain rate. So, with these things and, as a, as a result of this frame in various conditions, the rotational, strain, strain rate which is expressed in terms of for example, dou u by dou y minus dou v by dou x.

So, in terms like this, when you have a pure solid body rotation, then there is a corresponding non-zero strain rate, but we say that, so there is a corresponding relative motion, but we say that that is not causing any stress. So, that comes as an inbuilt package of the frame invariants of the relation.

So, with under these conditions, so those fluids which satisfy these conditions are called Newtonian fluids, and for these equations, for these kinds of fluids, these equations are valid. We have said earlier that fluids like air and water are newtonian fluids, and there are number of non-newtonian fluids which obey either, which do not obey either this relation or this relation or both. For example, polymeric fluids which preferred orientation of the chains may not be isotropic, and typically the relation between stress and deformation rate is also not linear in such cases. So, such fluids are called non-newtonian fluids, and these kind of fluids include very common fluid like blood. The dough that is used to make in India like the dosa and idli and chappati and bread and all those kind of things and also concentrated sugar solution.

So, many other polymer related fluids are not Newtonian fluids, and for these kind of equations, these kinds of fluids, these equations are not valid. So, only for these Newtonian fluids, we have these equations valid.

We have not made any assumption here on compressibility of the fluid. So, these equations are equally valid for a compressive fluid as for an incompressible fluid. But in many cases of chemical engineering interest, the fluid, the flow is usually incompressible. So, for example, the usual condition of incompressibility of a flow not of a fluid, we have an ordinary gas is typically incompressible, but if the gas is flowing at not too high velocities, for example, if the gas is flowing at a velocity which is less than a third of the speed of sound in that particular gas is medium. Then we can assume that the flow is incompressible and that the density changes, arising out of the pressure changes, arising out of velocity changes within the flow are not significant enough for us to consider seriously the effects of compressibility.

So, from that point of view, we can say that when the velocity of the flow is not very high compared to the speed of sound. Typically, if it is less than a third of the speed sound or the velocity is less than point 3 3 of a mass number. Then we can say that the fluid is incompressible or the flow is incompressible. For incompressible flows, density is can be taken to be constant; it does not vary from place to place, and here, we are talking about density being constant in an isothermal flow where we have no temperature changes, and whatever density changes that we are talking about are density versus pressure versus density relation, for example, if you consider the bernoulli's equation, we have p plus half rho u square plus rho g z equal to constant along the stream line. What this means is that when u changes, then p changes.

When we say that the flow is incompressible, we are saying that the velocity changes arising out of all the forces that are acting on it. As the fluid goes along the stream line, does not cause sufficient changes in the pressure that the density is greatly affected. So, that is the condition of incompressible flow that we are bringing here. So, that is why incompressible flow is a flow property; it is related to the velocities and the pressure, and then, the corresponding density. It does not talk about the fluid itself. So, you can have an incompressible flow even of a gas. So, for such cases, you can say that density is constant.

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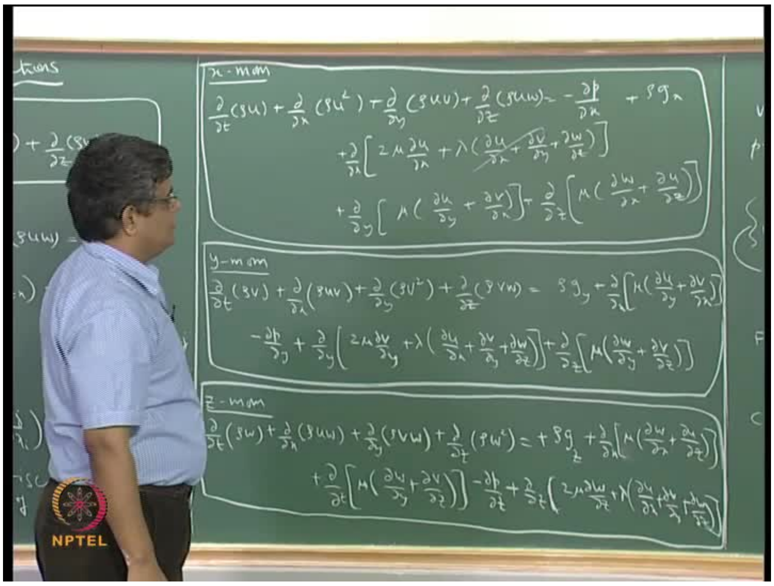

So, when you come to the continuity equation, this term becomes 0, rho is constant, so it can be taken out of the domain and we can write the continuity equation reduces to dou u by dou x plus dou v by dou y plus dou w by dou z equal to 0, and in the x momentum equation also here dou can be taken out, and it can be put here as one by rho dou p by dou x. This rho here gets cancels out, and this whole term becomes 0, because from the continuity equation, and again, here, it becomes 0, and here, it becomes 0. Therefore, the number of the fluid properties in here, lambda is not necessary.

So, we need only two fluid properties - which is rho and the viscosity; each of which are very easily measurable, and we can show that under the condition of incompressible flow with the continuity equation becoming like this. These x momentum and y momentum z momentum equations are considerably simplified.

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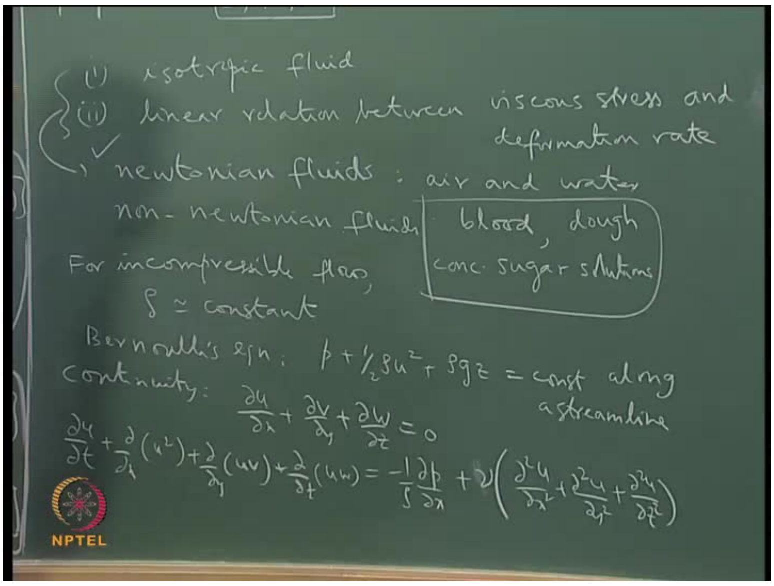

So that we can write the x momentum equation as, we will write here - dou u by dou t plus dou by dou x of u square plus dou by dou y of u v plus dou by dou z of u w equal to minus 1 by rho dou p by dou x plus mu times dou square u by dou x square plus dou square u by dou y square plus dou square u by dou z square. So, this, where u here is a kinematic viscosity, that is, the dynamic viscosity divided by the density.

Similarly, the y momentum equation reduces to dou v by dou t plus dou by dou x of u v plus dou by dou y of v square plus dou by dou z of v w equal to minus 1 by rho dou p by dou y plus mu times dou square v by dou x square dou square v by dou y square plus dou square v by dou z square.

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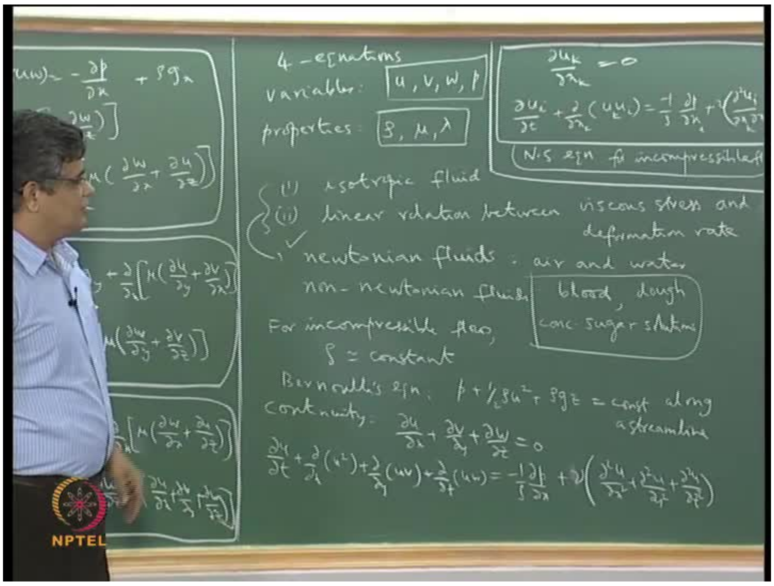

So, we can, in that sense the final equations here are much simpler, and we can take advantage of the index notation that we are familiar with, and write the continuity equation as dou u k by dou x k equal to 0, and here, we are following the Einstein's convention that in a term, this is a term here, in which, if we have a repeated index like k, then that implies summation over all the 3 values of k here k equal to 1 means dou u 1 by dou x 1 k equal to 2 is dou u 2 by dou x 2 plus dou u 3 by duo x 3 equal to 0, and we know that u 1 is u and x 1 is x u 2 is v and x 2 is y and u 3 is w and x 3 is z.

So, from that, we get back this equation, and we can also write the i th momentum balance equation as dou u i by dou t plus dou by dou x k of u k u i equal to minus 1 by rho dou p by dou x i plus nu dou square u i by dou x k into dou x k. So, this are the Navier Strokes equations for incompressible flow, and we can see that for the Navier Strokes equation, we need only the density here and the kinematic viscosity, which is reducible from the dynamic viscosity.

So, there are only two properties that are required. There are four variables - u v w p, and there are four equations; that is the continuity equation, and the three momentum equations for the three directions.

So, in computational fluid dynamics, the objective is to solve these four equations for the four variables for a given flow domain and so on. So, that is the objective.

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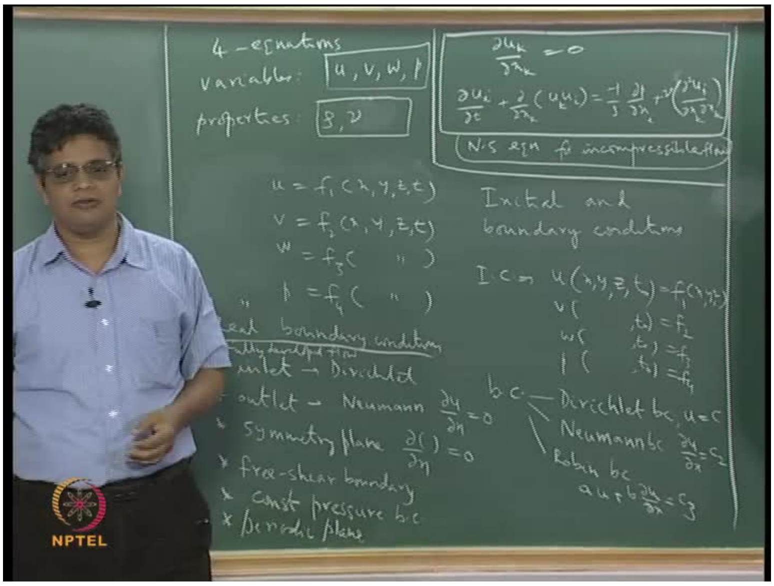

We now have the equations which describe the motion, and these equations are here. We have the continuity equation and the three momentum equations written in the index notation for a cartesian coordinate system, and these equations are valid at every point within the flow domain.

So, if you consider this whole room to be the control volume, the fluid domain of interest, and if you are interested in the circulation pattern of air, then we can apply these equations with the properties of air in terms of density and kinematic viscosity, and we expect these equations to be valid at every point, at this point, this point, that point, and anywhere else, and the equations also, such that, if we solve these equations, then we would be getting u as a function of x y z and t v has a function of x y z and t, and w also has a function of x y z, and p also as a function of x y z, and in that sense, the information of how the flow variables, that is, the three velocity component pressure vary within the flow domain is contained in these equations, and if we solve these equations, then we should be able to predict the flow variables at any point and every point within the flow domain of interest.

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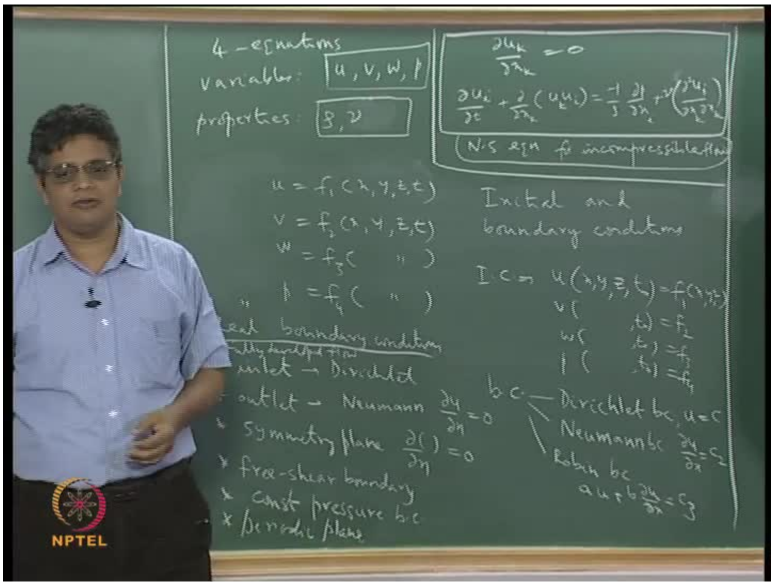

So, in that sense, the equations contain all the information that is necessary but not quite all, because what we see here are these are partial differential equations and these are not algebraic equations, and that means that it is necessary to give the boundary conditions and initial conditions in order to get a proper solution.

So, let us consider these boundary conditions and initial conditions. So, we need to have, in addition to the equations ,we need to have initial and boundary conditions, and these initial and boundary conditions are applied in the flow domain, in which, we want to compute this.

So, the initial condition is specified throughout the volume. So, the initial condition is of the form that u at x y z t 0 is equal to f of f 1 of x y z, and similarly, v of is equal to f 2 and w is f 3 and p is this f 4. So, all this at time equal to t 0 are functions, given functions of x y z.

So, this is how we specify the initial conditions. What about boundary conditions? Boundary conditions are typically of three types - one is where you specify the variable of interest, where you are talking about a boundary condition of u you say that u equal to something some value. So, that is known as Dirichlet boundary condition - where you specify u equal to a constant for example.

We can also specify the derivative, for example, dou u by dou x is equal to something. So, that is we specify the gradient of that particular quantity. Then that is known as Neumann boundary condition - where you say that dou u by dou x is c 2, and you can also have sometimes a mixed boundary condition, where you say that a value plus it is derivative is equal to something, and that is known as Robin backward boundary condition - where you say that a times u plus b times dou u by dou x equal to c 3, where a b c are specified constants.

So, these are the three types of boundary conditions, and in the general case, you can even have more complicated boundary conditions. For example, you have a gas liquid interface; you can have the curvature interfaces coming into the boundary conditions to satisfy the kinematic boundary condition and so on.

So, in such a case, we can have even more complicated things, but generally speaking, these are the boundary conditions, the types of boundary conditions we have, and when we talk about a fluid flow situation, we talk about more realistic boundary conditions. Realistic in the sense down to earth more physical boundary conditions as we have a flow domain, and the flow domain implies that there is some flow coming in and some flow going out.

So, in such a case, we can define something as an inlet; a particular zone of the surface as an inlet and another zone as the outlet conditions, and we can also have conditions of constant pressure or constant symmetric plane and so on. So, let us call this as real boundaries in terms of popular parallels.

We have an inlet to the flow domain where we normally apply a Dirichlet boundary condition; that means that we have to say that on the inlet surface, all the flow variables are specified - u v w p; p is a special thing and we will right now leave it at that point, and we have right an outlet boundary condition. Again, this is where the flow is leaving the boundary, the domain of interest, and typically the values of these things are extrapolated from interior values and that can be done in a number of ways. So, this can be considered as a Neumann type of boundary, and essentially we may say that dou u by dou n is equal to 0, where the variable normal to the outlet boundary condition, outlet surface is equal to 0 is something and we can have a symmetry plane.

So, symmetry plane is where the variation across the plane of symmetry is equal to 0. We can also have free shear boundaries, for example, on the top surface, we can have a sheer free boundary, that again implies something like a Neumann boundary condition, and we can have also constant pressure boundary condition. This is useful when you are looking at a periodically varying flow. For example, we can have a big heat exchanger and it has baffle plates, when coming like this, another going like this. So, the flow is made to go through like this. So, it is going through sections in this wavy pattern and you can take one section of this and say that you have a periodicity, that is, so, we can write that also as a periodic boundary condition, and in a periodic boundary condition, what we mean is that the variables at the two planes have the same profiles. So, that is what we mean by a variable.

So, a particular velocity profile is repeated after a certain distance, and usually the pressure drop between the two planes which is driving the flow is specified here, and we also have a fully developed flow condition. This in a way is similar to outlet, whereas an outlet implies, fully developed plain place dou u by dou n is equal to 0. At an outlet, you can also get some use some extrapolation to get the outlet, the flow variable from the outlet plain from the interior.

So, you can have many kinds of these. When we come to practical problems, we can discuss more of this in detail. Now, before we before we leave this particular section, we have to understand the implication of boundary conditions and initial conditions especially in the context of well posedness of a problem, well posedness of a mathematical problem.

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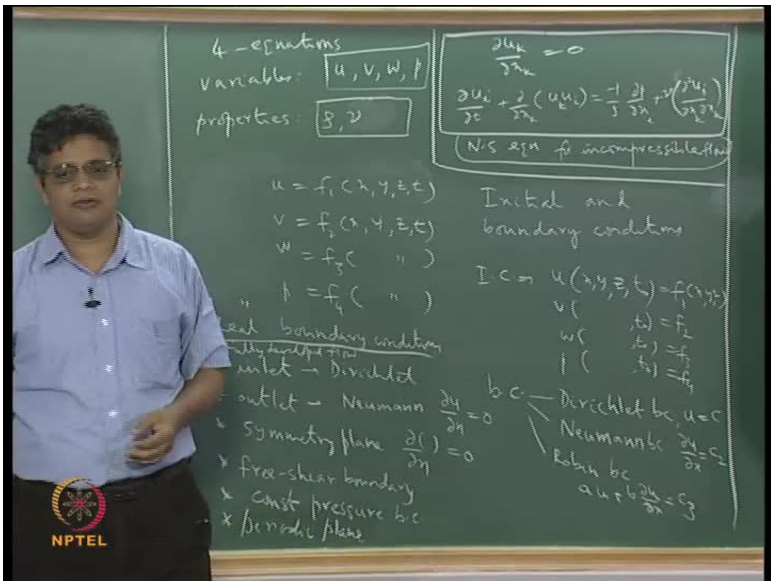

What we are trying to say is that we have an equation and we have a flow domain, and we know that we have to specify the initial and boundary conditions, but the point that we have to consider especially from the point of well posedness is that any type of boundary condition for any kind of problem is not permissible if you are looking for the well posedness of the mathematical problem, and what we mean by well posedness is that if the problem is well posed, then it has a solution and it has a unique solution and the solution depends continuously on the boundary conditions and initial conditions. So, that is, if you change the initial condition or a boundary condition slightly, then the corresponding flow solution should also change.

So, that it depends continuously, and it depends, so, when we change the boundary condition, the solution changes, and not only that, it should change continuously in the sense that it should not suddenly go off into a discontinuous solution. So, that means that small changes in the boundary condition should give rise to correspondingly small changes in the flow variables.

So, this kind of sensitivity to the boundary conditions and initial conditions must be exhibited by the solution, and in, only in such case that you have a unique solution, and the solution exhibits to the boundary conditions and initial conditions which a part is specification. Can you claim that your problem is well posed? So, what kind of conditions may arise in which this well posedness requirement is broken.

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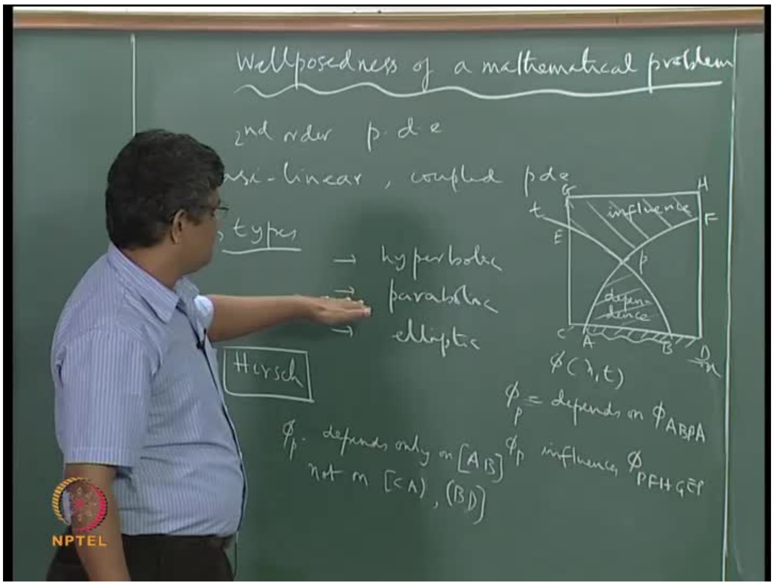

It depends very much on what physics the equations that you are trying to solve contain, and what we see here is that we have a partial differential equation, and we have a partial differential equation which has a second derivative as the highest. We have the second derivative which is coming here, and we have first derivatives coming here and here, and not only that, we see that what we are dealing with are second order partial differential equations, and these are also quasilinear and coupled partial differential equations.

Quasilinear in the sense that the highest derivative, that the second derivative that is occurring here is appearing as a linear term at least in these equations, because u here is a property of the fluid, and we are, for the time being, we are assuming it to be a constant, and so, this is quasilinear coupled partial differential equations, because there are four equations and you cannot solve any of the four equations in isolation. You have to solve them together, in the sense that. Therefore, these equations are coupled together, but for the time being, let us forget this coupled equations. We can say that the equations that we are trying to solve share some features that are common with a second order partial differential equations, and we know that second order partial differential equations are typically of three types - these are hyperbolic, parabolic and elliptic.

We will not go into the details of, what, when it is hyperbolic, parabolic and elliptic. Those are well known for a standard; this classification is well known, but we would like to look at the physical implication, the physical interpretation of what is hyperbolic and how these things may affect the boundary conditions.

When we say hyperbolic solution, we mean that it is like a wave like a solution. A wave like solution has the property of a wave; which means that it has a certain sense of progression; it has a direction of progression, and it also crucially has a velocity of progression.

So, this is, so, when we say that solution, a problem is hyperbolic, it implies that it the corresponding equations like the Navier Strokes equations that we have admit a certain a wave like solution with a certain sense of propagation and a velocity of propagation, and when we talk about a second order partial differential equation, then there are two wave like solutions, and along the lines associated with this propagation direction which are called the characteristic lines.

We have two wave like solutions which are propagating along those line at a characteristic speed, and in a parabolic type of flow, we have only one way, one direction of propagation, and in elliptic problem, does not admit a wave like solution. Therefore, it has no specific direction of progression. It progresses if it can be said we were progressing; it is progresses in all directions.

So, it is with respect to the wave like or the nature of propagation and the characteristic speed at which the propagation happens. That distinguishes the flow from being hyperbolic or parabolic or elliptic. This aspect is discussed in detail in the book by Hirsch. We will give the reference later.

So, it is, in this context, we have to be vary of the boundaries conditions and initial conditions. So, depending on the existence of a wave like solution or not, we can see that, we can come across conditions, in which, the well posedness of a particular mathematical problem is threatened by an arbitrary specification of the initial and boundary conditions.

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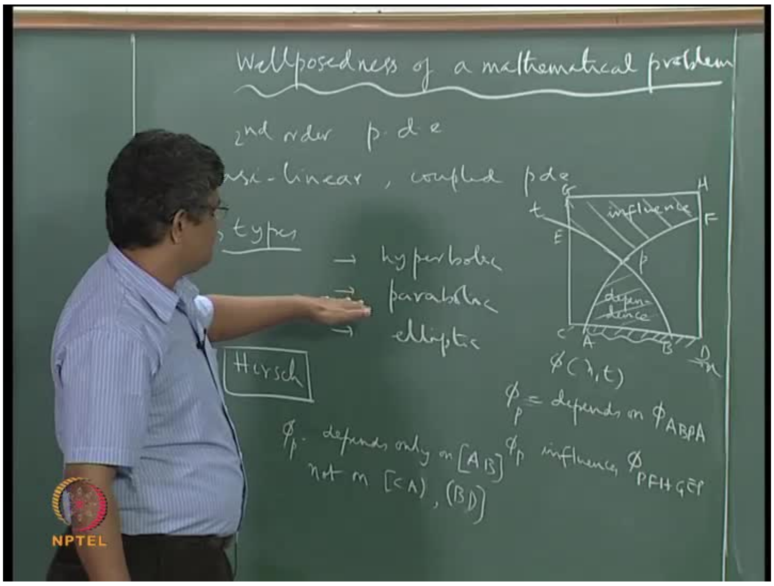

Let us consider a 1 dimensional domain. So, this is the boundary, and this is, let us say that this is x and this is the time. We are looking at variation of particular variable phi as the function of x and t, and this particular thing is given by a standard second order partial differential equation.

So, in which case, if you are looking at a particular point p here, if it admits a wave like solution, we have two characteristic lines, which are, in general, curved lines. So, these are the characteristic lines which are passing through the point here, and which when extrapolated go backwards like this and forwards in this direction. So, associated with this particular point p here which is in this solution domain, we can identify some part of this solution domain which influences the value of phi at p.

So, the value of phi at p depends if you call this as p here and let us call this is A B and C D E F G H, let us say that. So, phi p depends on phi within the domain A B p A. So, this is the domain of dependence of p, the value of phi on this, and this is the value of phi here will influence the values of the solution contained within this domain.

So, this is the domain of influence, and this is the domain of dependence. For this particular point, within the solution domain which is in C D H G. It also means, so, we can say that phi p depends on this and phi p influences the value of phi within P F H G E p. So, for a hyperbolic problem, we can define clearly the zone of dependence and the zone of influence within the solution domain, and what it also means this that the value of phi here does not influence the value of phi in this domain or in this domain, nor does it depend on either of this.

Now, when you look at it from the point of specification of the boundary conditions and the well posedness requirement, the value of p phi at the point p here in this solution domain depends only on the boundary condition between point A and B. It depends only on this part of the boundary condition and not on the entire boundary condition field.

So, if we have a solution scheme which says that phi of p depends on the entire boundary condition C A B D, then that is not correct. So, phi p depends only on A B; it does not depend on not on C A or B D, where I am indicating these small brackets to say that this point A does not belong to this. So, that means that the true solution of phi at this particular point p will be continuously varying with any changes in boundary condition of point A and B but not with any changes in on the boundary condition between point C and A or between B and D.

So, this means that the solution of phi at particular point depends only on part of the boundary condition and not on the whole boundary condition. So, this, and if you are trying to evaluate phi as a function of the entire boundary condition, then it is going to be wrong, because it should ideally depend only on part of the boundary; on that part of the boundary which is contained within the two character lines along, which the wave like solution propagates.

So, that means that if you had a solution scheme which specifies, which evaluates phi at point p in terms of the boundary point C A and B D as well as the A B point, then it is wrong because the solution is exhibiting a dependence on C A and B D which should not be there; it should be dependent only boundary between A and B. So, that kind of boundary condition the dependence sometimes of the boundary condition of the solution only on part of the boundary is characteristic feature of a hyperbolic type of solution.

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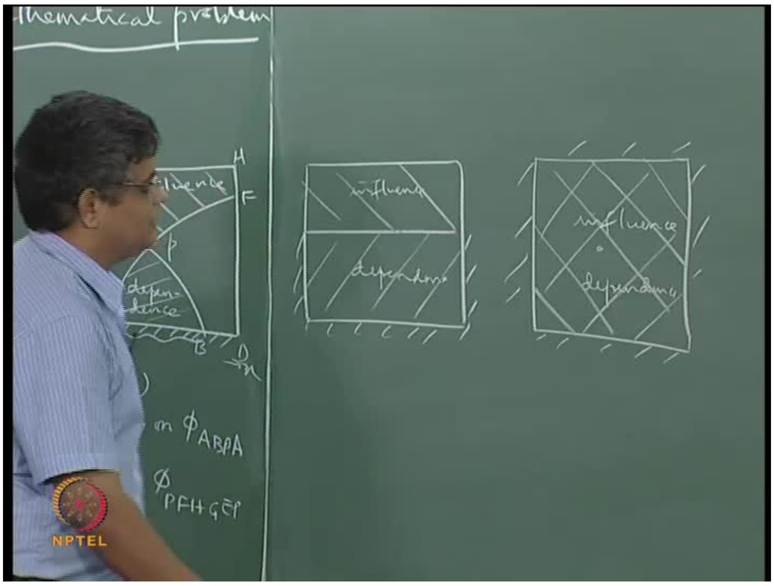

In a parabolic solution, typically you have in a parabolic, this is the zone of dependence and this is zone of influence; so, that means that this whole boundary at initial conditions will be influencing this, and the, here we have the initial condition and this is where the boundary condition comes into picture. So, and this part will not be influencing this. So, that is the specification of the boundary condition at this particular point x equal to 0 at this particular time will not be influencing the solution here.

In the case of elliptic for the same boundary, we have the whole boundary is the zone of dependence, and the whole boundary is also the zone of influence. So, that means that the conditions on the entire boundary of this solution domain will be affecting the solution at any point within the domain. Whereas here, the solution at a particular point will be dependent only on this part of the boundary, and here, it depends only on this part of the boundary.

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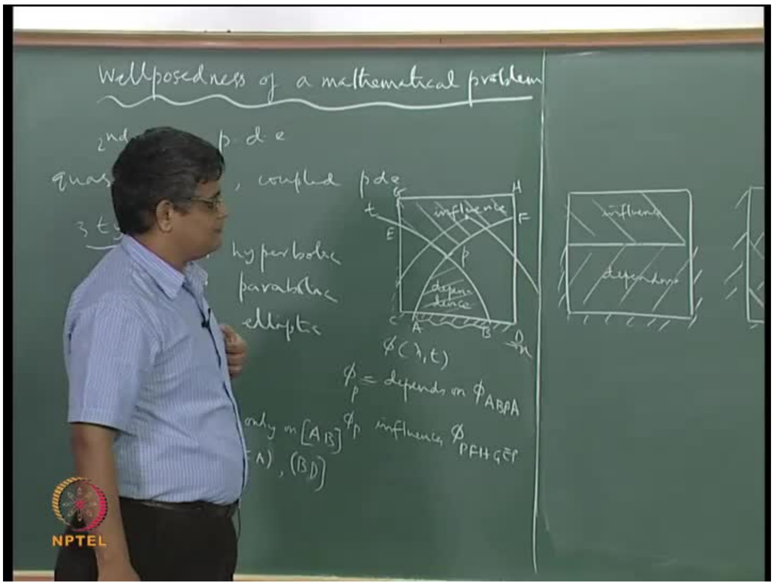

So, it is only on some part of the initial condition may be influencing this. If you go to some other point here and then if the two characteristics are like this, then the solution value here depends on this part of the initial boundary condition; this part of the boundary condition and this entire part of the initial condition.

So, in that sense, that depending on where you are within the solution domain, you may have only part of the initial condition or part of the boundary condition influencing the solution value if the problem is hyperbolic or parabolic. Whereas in the case of an elliptic problem, the entire boundary condition is will be influencing and also depending on the value.

Now, this has an implication on the specification of the initial and boundary condition for a well posedness problem, because if you are solving a hyperbolic problem, then it is necessary to take into account only that part of the initial slash boundary condition which will be influencing the value, and if you are solving an elliptic problem, you should not attempt to solve the problem without specifying all the boundary conditions.

So, in a hyperbolic problem to include influence of all the boundary condition on the solution at every point will be incorrect; it will lead to ill posedness of the solution, and in the case of elliptic not to include some part of the boundary in the calculation procedure will lead to ill posedness.

So, it is in that sense, we have to consider the well posedness of mathematical problem, and specify the initial and boundary conditions appropriately. So, this is something that we have to keep in mind when we consider the solution of this mathematical problem, and the mathematical problem consists of the equations like the partial differential equations that we have as well as the boundary conditions and initial conditions as appropriate.

We must keep in mind that this is what we have for a quasilinear second order partial differential equation. What we are dealing with when we are in fluid flow are quasilinear coupled partial differential equations. So, it is much more complicated, but in a case where you have highly hyperbolic nature or highly elliptic nature of a problem, then we have to do, we do have to consider these kind of effects in making a solution.

So, it is not exactly valid because we are dealing with it is much more complicated, and the solution, the equations that we have are mixed hyperbolic parabolic or parabolic elliptic or purely elliptical type equations. They are not as pure hyperbolic and pure parabolic and type of equations. So, this is just a guideline for us to see that we are imposing the right kind of boundary conditions and initial conditions for a particular problem.

So, with this, we have completed the basic equations; the derivations of the very basic equation for a fluid flow. We have not considered the equations for problems, in which, heat transfer takes place or reactions take place or even for turbulent flow. So, those will be doing towards the end after we have looked at how to solve these equations using the computation fluid dynamics.

So, we will, in the next lecture, we will start looking at how to solve these equations using the computational fluid dynamics approach.