Computational Fluid Dynamics
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Module No. # 07
Dealing with complexity of geometry of the flow domain
Lecture No. # 47
Co-located grid approach for irregular geometries
Pressure correction equation for a coordinate-located structured grid

We have seen how to discretize complex domain which may be simply connected or multiply connected and break it up into tiles in an unstructured way. And we have also seen a structured way of generating a body fitted grid so that; we can now say that if given a complicated flow geometry, at least for two-dimensions, we can come up with the interior locations of all the grid nodes in terms of the x i, y j of all the points such that when you can make control volumes out of it, we can make tiles out of it, and when all the tiles support together, we have computational domain. And the algorithms that we have discussed are for any arbitrary shape geometries.

So, this enables us to deal with… we are now at a stage where we can look at complexity of flow in terms of turbulent reacting flow, and we are also able to tackle part of the geometrical complexity in terms of generation of the control volumes, and the location of this, but there is still one element of the overall solution process that we need to worry about when we come to complex geometries. And this is, with regard to the evaluation of the pressure.

We have seen that in the case of simple geometry; where we expressed our equations in terms of Cartesian coordinates, and where the overall flow domain fitted into something like cube type of situation, then we derived an equation of pressure or pressure correction on a staggered grid, where the evaluation of the velocities were staggered in that particular compound extend by half a grid point. Therefore, we were doing the discretization of the four equations; that is a continuity and the three momentum equations on four separate grids.

Something like that is quite ok when we have a simple grid because we can easily translate the plain by half a grid point in each direction in the x direction for the x momentum, in the y direction by the y momentum equation and so on. But when we have a complicated domain, we have seen how we generate the grid for a body fitted grid or for an unstructured grid, and these methods are not amenable for shifting these things by half a grid point like that. So, to generate a staggered grid is a not so easy, and one could potentially say that we could generate a body fitted in a structured coordinate system.

But we also see that the control volumes are not necessarily rectangular. So; that means, that all the areas and the surface orientations will change from one grid point to the next half grid point. So; that means, that we have to store quiet lot of information about each grid point. For example, when we talk about a three dimensions, we have to know the overall volume and we have six faces. We have to know the face areas, and we also have to know the direction coordinates of that surface normal vector.

So, they have there are six times 3, that is 18. So, directions cosines; that we have to remember for that particular control volume, in addition to the area of the six faces and then the volume of the overall face. So, that is 18 plus 6 plus 1; that is about 25 quantities we have to remember for a grid point for a single point, for a single node in dealing with a structured grid. And if you now have four such grids, so that we can employ a staggered grid system, then the amount of storage associated with the grid itself will become very large. In addition to that, we have the difficulty of generating these things.

So, the staggered grid approach is not very amenable for computation for complicated flow geometry. So, we have to go from staggered to collocated arrangement. Collocated arrangement means where all the velocities and all the pressure and temperature, turbulent kinetic energy all these things are evaluated at the same point within the control volume as supposed to the staggered or non-collocated approach in which some quantities like pressure and temperature and turbulent kinetic energy; these are evaluated at the center, and the fluxes and the velocities are evaluated on the faces. So, we have to come up with a way of using adopting approach for a collocated mesh.

It may seem to be very obvious; the major difference comes when we are looking at the evaluation of the pressure.

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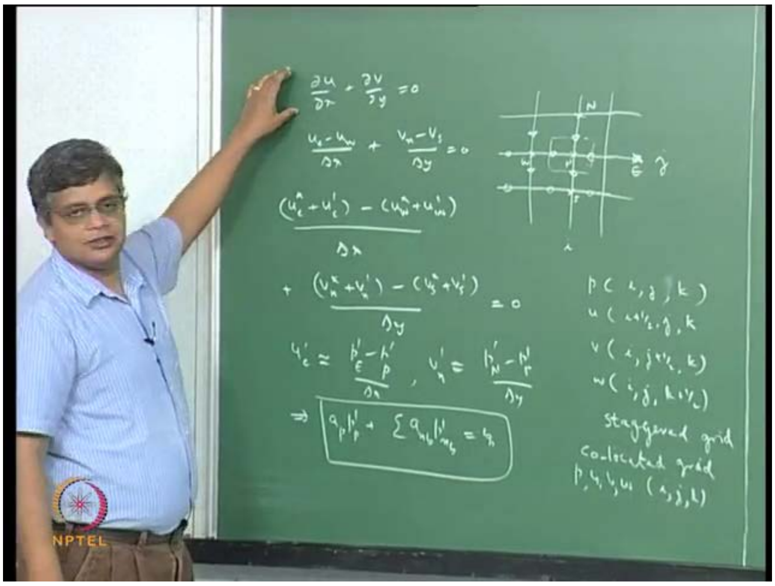

In a simple kind of grid where we have, if say these are the… this is i, here and j here. So, i plus 1 i minus 1 j plus 1 j minus 1 type, and here we are evaluating the pressure and typically in a staggered grid, we evaluate U here and V here. U here and V here. U here and then V like this. This is how we evaluate, and when we solve continuity equation, it is dau u by dau x plus dau v by dau y equal to 0.

We take a control volume centered around the point where we evaluate the pressure, and if this is P, by notation we have this point as capital E, this point as capital E here, and capital west, capital north, and capital south. These are the five points at which pressure is evaluated and when we take this control volume here, and we discretize this, we can write this as U e minus U west divided by delta x plus v north minus v south divided by delta y equal to 0, and this enables us to introduce the velocity correction U e prime and V n prime like that, in terms of the pressure correction, and we therefore, evaluate the pressure correction.

For example, we write this as U e star where star indicates velocity obtained with against pressure field which is P star plus U e prime minus U w star plus U w prime divide by delta x plus V n star plus V n prime minus V s star plus V s prime divided by delta y equal to 0. And we make use of the approximate form of the corresponding momentum equations to say that U e prime is roughly equal to P prime; that is pressure correction at

E minus pressure correction at P divided by some delta x with the appropriate units to some geometrical parameters to make this dimensionally consistent, and similarly V n prime is approximately equal to P prime north minus P prime P divided by delta y and so on.

We substitute these things here and get an equation for pressure correction. Sum of a n b p prime n b equal to b. So, this how we evaluate the pressure correction, and once we solve this, then we substitute it here to get the velocity correction, and then we can go from a guest pressure field into a guest velocity field into new velocities here and here. So, that is a standard way that we discretize the momentum equation on a staggered grid and thereby enable the determination of pressure which is not linked density in the case of incompressible flows. So, this creates a means of using the pressure as a continuity enforcing method, continuity enforcing condition.

So, we can obtain a pressure correction such that the new velocities correction the new velocities of tends to velocity correction that are coming here will satisfy the continuity equation. So, that is how we are we have approached the determination of the velocity field and pressure field in incompressible flows. We have made use of a staggered grid approach.

So, this meant that you had a grid like this for the pressure, and you had a separate grid for the U velocities and even a separate grid for the V velocities and so on. So, and in the case of three dimensional flow, you have four grids; one for pressure at i j, velocities at i plus half j k, the V velocities at i j plus half k, and W velocities at i j k plus half. So, you have the main grid and then three staggered grids like this.

Then you have regular grid like this, then it is very easy to see that the areas are roughly the same or they go in a certain way for example, when you go to a cylindrical coordinate system, it is very easy to compute the areas and the distances that come about here, but when you have a complicated geometry, then it is not readily clear as to how we can implement something like this.

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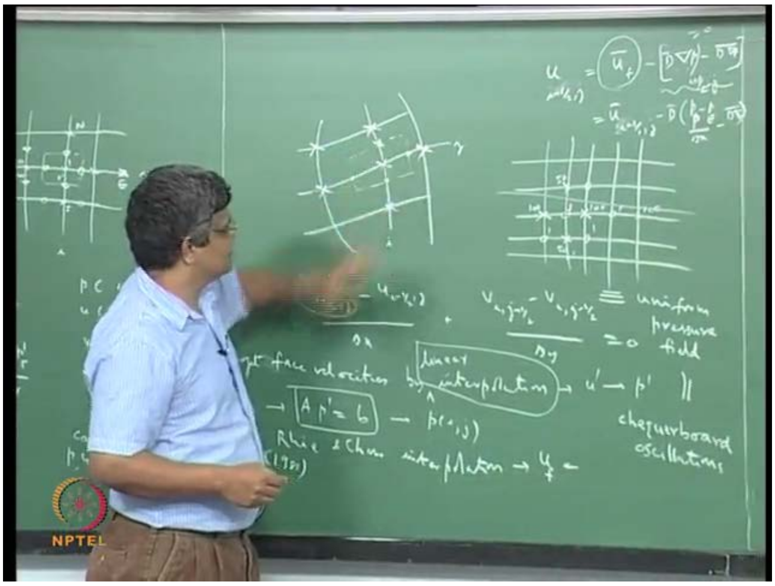

So, the logic that we can use for a structured complicated grid is that one can say that we can do the same thing. We can take similar control volume. It may be like this, and we can construct a control volume like this, but here we what we would like to do is to instead of using four different grids, this is the staggered grid, we would like to use collocated grid in which all P, U, V, W are evaluated at the same i comma j comma k.

So, if you have this kind of single grid like this in the case of and if you use this to evaluate P; all the quantities at the center of this, then and if you want to apply, if you want to discretize the continuity equation in order to get the correspondent pressure in this, then we need to have velocities here and U velocities here and here.

So, we need to we can write it like this that U if you say that this is i here and j here. We need u at i plus half j minus U at i minus half j divided by delta x. We just called this as delta x. It can be slightly more complicated; plus V at i j plus half minus V at i j minus half divided by delta y equal to 0.

This is the discretized form of the continuity equation, and in order to apply this, we need to have values or the velocities at these face points, and these are readily available from the momentum equation on a staggered grid whereas, in the case of collocated grid, these values the velocity values are not available here, and that is the great advantage of the staggered grid. So, we know the velocities only at these points here.

So, we have to estimate the value of the velocities at the faces from… for example, from these two points, and we can estimate one can say that we can estimate this from this two points and this from these two points and like this. If we do that, then we can do interpolation. So, we can get face velocities ((No audio from 15:10 to 15:18)) by interpolation, and then we can substitute this in terms of fluctuating quantities, and then convert these into pressure correction as in this particular way, and then we can try to finally, get an equation for pressure correction like A P equal to b type of thing in this way, but if we do this, this interpolation linear interpolation for example, when we say that what is this value given this value and this value at these distances, we can do a linear interpolation and the linear interpolation is such that these two distances are the same, then this will be average of these two and again this will be the average of these two.

So, if you do that linear interpolation, which will give as a second order estimate for the phase velocity at this point which is which is good, then what we find is that this matrix here will not have the value of P i j. So the value of P i j here is being computed only from these points here without any contribution coming from this. So; that means, that if you have a grid like this, then this pressure and this pressure will determine this. These four pressures will determine this and again these four pressures will determine this. So, that the result that since this is not coming into picture, if you have for example, a strange case of 100,100 and 50, 50 pressure here, then the pressure gradient which is coming into here which into the momentum equation will be such that the pressure gradient in this direction is 0, and the pressure gradient in this direction is 0.

And let us say that 1 here and 1 here. So, in that sense, one can come up with pressure variation like this which is strongly varying between 1 100 1 100 1 100 1; like this, which is seen by the flow as if it is a uniform pressure because the pressure gradient is being determined by two points without by skipping this particular point, and this is 100 and this is 100. So, it is uniform so that the flow will not react to the facts that have these very rapid variations of pressure, so that a pressure variation such as this; 100 1 100 1 100 1, similarly 50 1 50 1; like this is seen as a uniform pressure field.

This is seen as uniform pressure field and therefore, there is no corrective action to smoothen this pressure variation to get rid of this very high frequency pressure variations and this gives rise to what are known as checker board oscillations

((No audio from 18:59 to 19:10)) of the velocity field and the pressure field like that which will be difficult to get rid of because this is seen an as an equivalent uniform pressure field, and this can play have up with this solution, and under one can see that this; obviously, something wrong with this. One cannot do anything about this flow will converge the solution will converge towards this, and these checker board oscillations can be shown to be associated with this linear interpolation.

But when we use the staggered grip approach, we can see here that the pressure at the central point here is strongly influencing the pressure variation at this particular point. The discretize momentum equation at point p describe quadratic equation at point p which determines the value of pressure has a component of four here, weightage of four and this is weightage of one from all these points here.

So, from that point of view, there is a strong say of the pressure at this point, and if you have something like 1 100 like this, then immediately it will see that there is something wrong, and that kind of linkage, that kind of mechanism to eliminate these non-physical, high frequency, small wave length pressure variations; that corrective action is not there. Then we go for the staggered approach with a linear interpolation to get the face velocities which are required in the discretization of momentum equations.

So, it is, the problem is that the discretization continuity equation. So, when we want to discretize the continuity equation at the center of the cell, we need the phase velocities, and these phase velocities are obtained from the discretization momentum equation at about the same point. Therefore effectively, the pressure evaluation at a particular point gets decoupled from the velocity equation at the particular point, and this gives rise to the checkerboard oscillation.

And these are this is a well-known problem with a solution at a collocated grid, and this can be readily avoided by using a staggered grid approach. That while the staggered grid approach is readily implementable for a structured simple grid like this, when you are looking at a complicated floor domain where you have the grid lines curbed around bodies of arbitrary shape. In such a case, staggered grid approach is not possible. We must use a collocated grid.

So, this is where rhie and chow; they have been… the general attempt to deal with this checker board oscillations is to introduce fourth order term in the pressure equation to suppress this things, and the that particular reproach has been formalized by rhie and chow who suggested that the phase velocity here is obtained by the interpolated phase velocity. So, that is what…

Right now here if you want to say that you want to get the U velocity here, then this is this plus this divided by 2, assuming that these are equidistant. So, that this velocity plus this velocity divide by 2 is the interpolated phase velocity minus some contribution of the D which is coming in this year. D is essentially multiplication of this times the gradient pressure which is coming in the momentum equations. And they suggested that this gradient pressure is also interpolatable, and this is broken up into two parts. This is gradient of pressure when completely interpolated.

And this is a contribution with interpolated value of the coefficient, and this is the gradient evaluated at that particular i plus half j. It is this is i plus half j. So, this is the interpolated pressures gradient source term which is coming in the evaluation of the momentum equations, evaluation momentum at this point and this point, and this is the pressure gradient evaluated at the this half point here. So, which we can write as… So, this is the interpolated velocity, phase velocity i plus half j minus some D here, interpolated thing this pressure gradient here. At this point is this velocity this pressure minus this pressure divided by 2.

So, that we can say is Pp minus P e divided by delta x minus this interpolated term. And what this means is that this can be shown to the equivalent of fourth order correction in terms of the pressure gradient here, and what we note is that, in cases where you have a smooth variation of pressure, instead of thing like this, it is varying smoothly like this, then this additional correction is almost 0, where you have this strong variation, then this is bringing in the value of pressure at this point in the evaluation of this.

So, with this additional correction to this, we can introduce correction to the treatment of this sort of rapidly varying pressure gradient as a uniform pressure gradient. So, essentially what are the rhie chow have proposed is that the determination of the phase velocity at the points where the velocity is not being determined, velocity is being determined only at these two points, but we need the phase velocity here, in order to substitute into the equation here. So, they made this phase velocity determination sensitive to the pressure gradient between two success points, and that sensitivity is introduced into this term here.

And the correction that is coming out here will be almost equal to 0 when we have a smooth variance of pressure, but this will be the dominant correction, dominant contributing factor to making this uniform in the case of a strongly varying pressure. So, with this, Rhie Chow interpolation method to determine the phase velocities where the phase velocities partly determined by the interpolated component of the velocity here and partly by the difference between the interpolated pressure gradient and the actual pressure gradient at that point is what this one is. So, that difference will be 0, if you have a linear variation of pressure. If you have a smooth variation of pressure, that difference will be a second order, but if you have a strong variation of pressure as in the checkerboard oscillation that pressure will dominate.

So, this approach has made it possible to use a collocated grid for a structured grid of propagation. And so, this as really enabled the extension of the simple type of calculation procedures to a body fitted grid system because in these systems, it is possible only to use a collocated approach and not the staggered approach. So, this proposed in 19; early 1980s 81 and this was immediately adopted by the large number of people, and this has proved to be a very successful approach to the cure of the checker board oscillations that are found in a collocated grid system.

One has to see this in the general context of trying to make the pressure evaluation sensitive to the smallest variation that is possible. That is crossed to neighboring points and that is what this is bringing. And this sensitivity here, is putting such a way that that sensitivity will that additional term will be almost 0 when you had a smooth variation of pressure, then you do not have this checker board oscillations.

So, the overall accuracy of the computation is not affected, but it is a specific pure, directed at strong variation, strong and local variation of pressure. With this approach, when we when we add this to the overall determination of pressure in the context of determining Ap prime equal to b type of solution with interpolated plus pressure corrected phase velocities, then it lead us to an equation a prime Ap prime equal to b, which can be solved along with the discretized momentum equation to give us the pressure and velocities at the same point. They are collocated. And this type of with this of approach, one can in a fairly straight forward way, one can use a body fitted grid for the calculation of the velocities. Without this kind of pure, it will be very difficult to get to avoid a checker board oscillation. So, this is a modification that is required when we are planning to use a collocated grid approach which is the norm for complex flows.

Now the question is we understand how this can be done, atleast we can guess it how this can be done for the case of structured grids. Now what do we do for the case of unstructured grids. Because in an unstructured grids, one can face the same problem of having to determine the phase velocities and one has to also understand how we can extend this. Actually the extension of this reach out interpolation algorithm for the determination of phase velocity is relatively straight forward in the case of unstructured meshes, and the very similar approach to this can be used and let us just look at one specific case in essentially symbolic terms.

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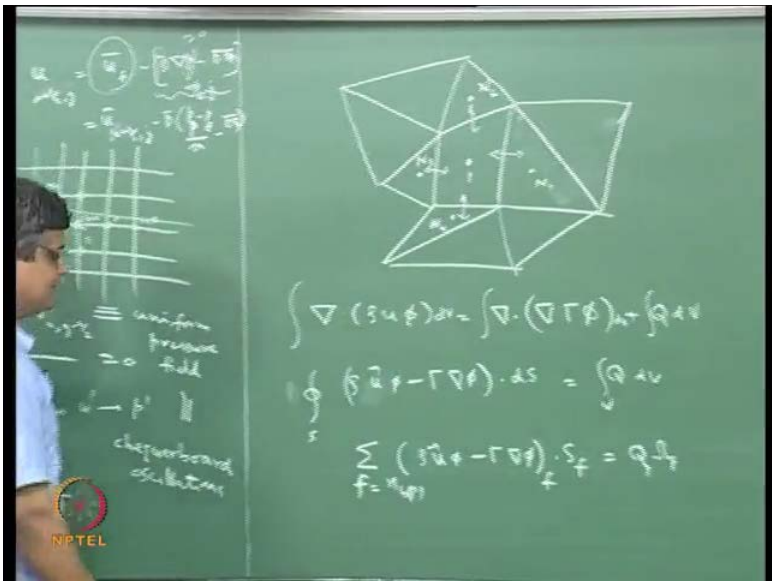

So, let us consider a ((No audio from 31:50 t0 31:54)) unstructured thing like this. ((No audio from 31:57 to 32:19)) We have been too rash like this, and we are concentrating on this particular cell given by these four nodes, and we have the neighboring nodes n 1, n

2, n 3 and n4; these are the nodes at which correspond to the centroid of the neighboring nodes which are sharing a particular phase with this particular thing. So, we have flux.

Across this phase, flux across this face, across this face, and contribution of flux which is coming into this, and this represents this net flux which is coming into the control volume from the neighboring cells based on the values of the variables at these nodes will determine the rate of change of that particular variable in that particular cell.

So, we can write, considering steady equation, we have del dot rho U phi equal to del dot plus Q as the standard governing equation which is valid at every point, and then we integrate this over the volume of this particular cell in order to derive consolation equation applicable for this particular cell in the unstructured finite volume method. And making use of… So, we have diffusive flux here and conductive flux here, and we can bring this two together, and then we can make of the Gauss's divergence theorem to express this as a surface integral dotted with s over the closed surface. ((No audio 34:39 to 34:56))

S is equal to Q dv over the volume here, and this will be discretized, this will be evaluated on each of these faces and which we can represent in this form, where f is neighboring let us say that small f is the neighboring face of Q times omega. ((No audio from 35:36 to 35:55)) So, we are making a discrete evaluation of the fluxes over the face of the neighboring faces of the point p here.

So, this is a neighboring face, this is a neighboring face, this is a neighboring face; like this and we have the evaluation of the convective flux and the diffusive flux, for example, using the upwind scheme and then the central scheme for these things, multiplied, dot producted with the surface area of each face like this. So, this is the finite volume approximation here, and by using the appropriate approximations and all these things, this can be converted into an overall equation like this Ap phi p where phi is the value of the variable at point P here. This can be converted into an algebraic equation like this.

Af phi f, where f is the neighboring point to P; so that is n 1, n 2, n 3, n 4. So, the value of the variable at the neighboring point at the neighboring node which is denoted by capital F. Here the face is denoted neighboring face is denoted by the small f. This is a capital F here equal to some bp.

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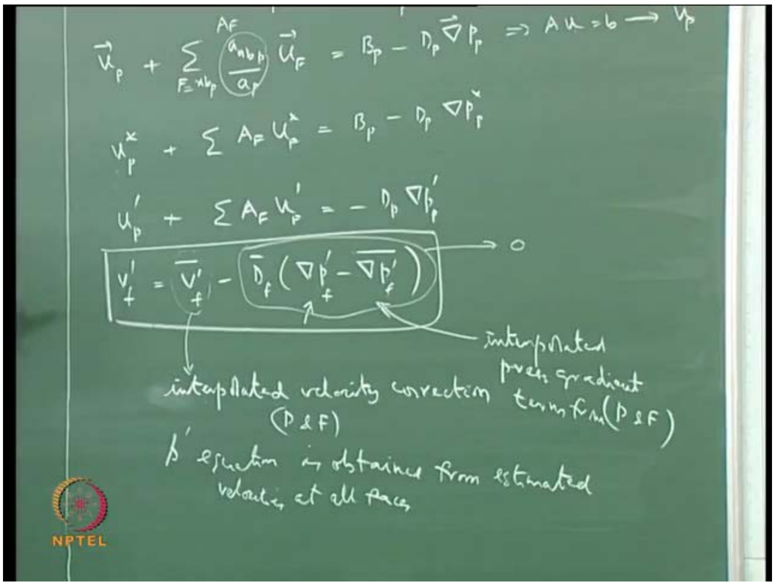

So, this is a overall equation that we can come up with and we can also put it in for the use of the our convenience here as sigma neighboring points over; a neighboring point divide by a p phi f equal to B p divided by A p. So, this is the equation which determines the value of the variable at a particular point p in terms of the neighboring points and also in terms of B p which represents source term and other kind of things. So, this also determines the values of the various velocity components. So, when we apply this method for the various velocities for the momentum equations, we generally treat the pressure term as an additional source term.

So, in such a case, you can write U p here plus A n b p by a p U f sum over all f which are the neighboring points of p equal to some b p corresponding to this minus D p gradient of p at point p, where D p is related to the face volume divided by face fluxes. No, this is face volume, this omega p here divided by this a p term here. So, essentially you have the contribution of terms other than the pressure gradient coming into B p here and the velocity at the neighboring points here and the pressure gradient evaluated like this.

So, this is treatment of the discretization of the momentum equation which in ultimately we converted into that a v p a v equal to b from which we get V, u at every point. So, this is this is the approach that we can use to discretize the momentum equations and convert them into an algebraic type equation for a given point. Now the question is how do we solve the continent equation because we still need. In order to solve this, we need to solve, we need to know the pressure. So, this is where we follow the simple type of approach where we have p star as guest pressure field. And once we have the guest pressure field, this can be convert into an equation which give us the stared velocity field.

So, we can say that we can follow the same procedure, and we can write U p star; the sum of, we can call this whole thing as sum A f, a f u f star equal to some B p minus D p times gradient of p star at P. So, this equation with guest pressure field here will give us the velocity at the node here. Now when we want to discretize the continent equation here, we need the phase velocities.

So, it is the obtaining the phase velocities by simple linear interpolation. If you want the phase velocity here, we take this these two points which is what we get in terms of U p star, and based on that one can get a phase velocity here, but that linear interpolation will give us to give us to checker board oscillations. So, we would like to implement a similar kind of Rhie Chow correction on to this unstructured mesh so as to come up with the interpolation scheme for the phase velocities. So, what we would like to do is that like in the previous case, we know that this is correct expression. That is the final velocity field with the correct pressure field will satisfy this discretized equation.

It is only the approximate pressure field will give us give raise to the approximate velocity field as per this, and we can subtract this from this, and derive the expression for the velocity correction as… So, this is the pressure velocity correction and in the simple scheme for example, we neglect this. So, this equation is also applicable at the faces. So, we can say that we would like to have, what we want to have is an estimate for the velocity corrections here.

So, we need to have a proper interpolation interpolated velocity of the velocity at the phase value. So, we could write this as Vf prime; the interpolated velocity correction corresponding to a pressure correction as the interpolated velocity correction based on the velocity corrections at p and n 1, n 2, n 3, n 4 minus d face times gradient of p prime at the face minus the interpolated gradient at the face. So, this way of writing is similar to writing it like this.

So, we have the interpolated velocity here; that is the interpolated velocity. We are now directly writing in the expression for the velocity correction. So, we have interpolated velocity correction. For example, this can be if you are considering this face here, this will be v p times if you say that this is x 1, and this is x 2, or l 1 and l 2 here. So, for phi f corresponding to this here, interpolated will be given as pi p times l1 plus pi n 3 times l 2 divided by l plus l 2.

So, in this way, we can get linearly interpolated value of phi at this particular face depending on the distance here and the distance here and the value here and the value here. So, this value here is the velocity correction obtained from this type of interpolation. This pressure gradient interpolated thing at the face value of the corresponding face is also the interpolated thing based on the pressure gradient term coming into the momentum equation. And this particular one is the pressure gradient which is coming at the face value. So, this will be evaluated as this minus this divided by this distance. So, this is the interpolated velocity. ((No audio from 46:48 to 47:00))

And this is the interpolated pressure gradient term from P and F. So, that is the variable values corresponding variable values at P here, and capital F which represents the node locations, and this is also from capital P and capital F values; the interpolation, and this is the value which is actually based on this immediate things like Pp minus Pe. So, for example, if you are inter if you are looking at this face here, this gradient of pressure correction at this particular point here will be given by pressure correction at P minus pressure correction at n3 divided by that distance delta x; some the appropriate distance corresponding to this.

So, in this way, in the evaluation of the velocity correction, we are bringing in not only in the contribution from the interpolated thing, but also the pressure gradient associated with the pressure corrections at that same point here, corresponding to the pressure correction at that node. So, the pressure correction at that node is coming into the evaluation of the velocity corrections, and that is precisely what you are doing here in the reach of (( )).

So, by evaluating this for each of the faces, we can bring in the contribution coming from the pressure fluctuation at this node, in addition to the contribution coming from the interpolated values. And again as we said here, in the case where the pressure variation is smooth, this term will be almost equal to 0. And what we are left with is the velocity correction obtained from the interpolated velocities only. So, that is how it is. So, when you do not have any threat of checker board oscillations, then the face, the correction to the velocity at the particular face from the interpolated one is as good as the correct velocity, but where you have severe variation of the pressure correction, this will tend to suppress those oscillations.

So, in this way the extension of Rhie chow method for the interpolation for the phase velocities can be readily done in the case of structured meshes in almost identical identical way in this particular fashion, and this can be incorporated into the, these interpolated velocities can be incorporated to determine to discretize the continent equation and from which we can obtain the pressure correction equation. So, pressure correction equation is obtained from (vocalized noise) estimated velocity at all faces.

So, this is the velocity this is at done at each face; if you have four faces, we will do it for four faces. For this particular volume, we will be getting one here, one here and one here for the three faces. So, in that sense, this can be applied, this discritization is applicable for any number of faces, and same way the estimation of the phase velocity is also applicable for any number of faces, and together we can estimate all the velocity fluxes through each of the faces, and determine the pressure correction contribution coming from that velocity contribution and thereby we can extend this.

So, this method is a… in this way we can make use of the collocated mesh approach for the determination of the pressures and this approach has been very widely used for unstructured formulations, and once we do this, the simultaneous solution of the momentum equations along with continent equation does not pose any additional problems and we can avoid checker board oscillations. And once we do that we can deal with the single grid and you can get an overall solution.

So, in this way, one can see that when we are dealing with complicated flows with respect to geometry, we can represent the geometry either in a structured body fitted grid or in unstructured meshes, and we can adopt more or less the same procedure to convert a given partial differential equation into a discretized algebraic equation like this, and then convert this into that a phi equal to b and go through the overall solution process, and we can solve any number of equations including for troubling kinetic energy and all those things. And in the evaluation of pressure, we have to use the collocated approach, correction proposed by Rhie and chow. Rhie and chow is just one of the popular ways of dealing with the checker board oscillations, and many other corrections are also possible. All of them will are some extensions some equivalences of introducing a damping term with related to pressure variation in the immediate thing with respect to the interpolative pressure variation.

So, with this one can claim that we have an overall solution procedure for dealing with an arbitrarily complicated physical flow as long as we have seen Newtonian fluid and arbitrarily complicated geometrical features. Although we can claim this, there are lots more details that one has to work out before one can go into the full evaluation. A lot of care is needed and a lot of diligence and meticulousness and book keeping and smart programming; all this things are needed for a successful implementation of the principles that we have discussed into an algorithm and a computer program which will enable us to calculate the flow field. We have discussed the principles and I hope you have benefited with this.