Computational Fluid Dynamics

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Module No. # 07

Dealing with complexity of geometry of the flow domain

Lecture No. # 44

Generation of structured grid for irregular flow domain

Algebraic methods

Elliptical grid generation method

So far, we have not paid attention to the aspect of grid generation, and we have seen that in the case of simple domains, where it fits into the cartesian coordinate system or even cylindrical coordinate system; grid generation is not a difficult task, and it is very easy to get structured mesh for such kind of geometries. But when we come to complicated geometries, where bodies are placed inside the bodies, and then the flow domain becomes what one can say as disconnected or multiple connected; in such cases the generation of grid is not so straight forward.

And we have also seen that, when we are looking to fit a body fitted grid, then we have to be very careful about the quality of the grid, because in a body fitted grid, we do not solve the equations in the physical plane, but we solve them in the computational plane; and in the process, we a simple equation like the Laplace equation gets transformed into a general elliptic equation with first derivatives, and also cross derivatives come into picture; and in this cross derivative terms and the first derivative terms are multiplied by the metrics of the transformation from the physical plane to the computational plane. These are the derivatives of the coordinate lines in the two systems with respect to the coordinate lines.

So, these metrics of the transformation have to be small for the cross derivative term for example, not to blow up, and not to be dominated. So, if if you have a smooth grid, then the then as a result of this transformation, the primary nature of the equations is is maintained; and when that is not the case, then the character changes from the dependence on the normal derivatives for example, in the Laplace equation we have only the normal derivatives, we do not have the cross derivatives, we do not have a first order terms; but in the transformed equation, we have these things also. And so, in a that corresponding descritized equation, and contribution of the normal derivatives as opposed to the contribution to the other terms. These are all questions that arise in the case of the general transformed equation.

And if you want to preserve the same nature or the nature to the extent possible as what is contained in the physical plane, then we have to make sure that the additional terms which come, which arise of to the transformation from an x, y, z coordinate system to psi, eta, zeta type coordinate system do not have too much of importance. So, the the this is one of the considerations that is in, that one must take into account in making a body fitted coordinate system. Similarly, when we consider an un-structured mesh, we again that is not a trivial task, because as one can always put several points, but the connection should be such that there is no inter connection of of several tiles, of two different tiles, so and at the same time, all the tiles when put together must add up to the overall control, overall flow domain.

So, this is not a trivial task; and we would like these things to be done in a systematic programmable way; if I if I have a flow domain like this, and with some constants, I can go physically and put points, and then say that these are the grid points, and then I can make up a geometry. But the idea is that it has to be done not manually, but automatically through some kind of programmable algorithm, and generation of that kind of programmable algorithm, which does not make mistakes in in joining the wrong kind of problems, wrong kind of nodes in order to give you that tiles is is a non-trivial task. So, we have to consider grid generation carefully, when we are dealing with complicated geometries. So, we we will look in the next this lecture, and the next lectures, the basic ideas in terms of grid generation; and what we mean by grid generation is in the context of a body fitted grid how to determine points in the physical plane, given the points in the computational plane.

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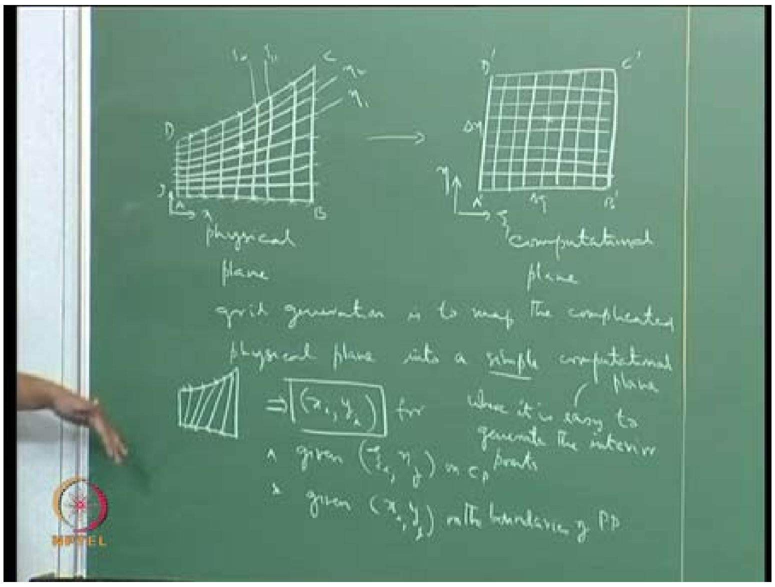

grid generation is to map the complicated physical plane into a simple computational plane

$\Rightarrow \boxed{(x_i, y_j)}$ for

where it is easy to generate the interior points

* given (ξ_i, η_j) in C.P.
* given (x_i, y_j) on the boundaries of P.P.

So, let us this try to understand what we are looking at, we have let us say a fairly simple physical plane like this; and we are mapping it into a two-dimensional computational plane; this is the physical plane, and computational plane. Here we have for example, x and y denoting the coordinates, and these are transformed into psi and eta. If we denote the boundaries by points A, B, C, D, then we have a notion of mapping point A with it corresponding point A prime, B prime, and C prime, and D prime in the computational plane, there is a one to one mapping between a point here, and a point here.

And we would like a similar kind of a mapping between a point here, and a point here; and for a in the from the point of view of grid generation, for a cfd type of computation, we would like to have grid points distributed throughout the domain, we do not want have one point here, another point here, another point here. As we have pointed out right in the beginning, in the case of cfd, we generate the solution that is the we compute the value of the variable, at many, many points a spread throughout the flow domain, and we would like to make sure that we have many points within this, and we do not select a few points.

And so, we need to have every point here, mapped with every point here; and so that is that is what we mean by creating a grid here, in which we have to specify the coordinates of the points within this, and the points at which we want to evaluate the variable, variable values. And for a cfd solution, where the derivatives are being approximated by finite difference approximations, we would like these points to be spread throughout in a systematic way, so that the derivation of the of the the evaluation of the derivatives is also quite systematic.

So, if if we had a a simple domain like this, then we know how to spread the points uniformly within this in a fairly coherent way; for example, we can choose to break up this domain into so many number of eta equal to psi equal to constant lines like this, and we can have so many number of eta equal to constant lines here. And we say that the intersection of psi equal to constant and eta equal to constant constitute the grid point. So, we can decide very easily, we can generate a grid very easily in the computational plane, because in the computational plane, we have a simple flow domain, but the physical plane here is more complicated.

And so the idea of grid generation is to map the complicated physical plane into a simple computational plane. The idea is to have it in a simple computational plane, where it is easy to generate the interior points.

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Because in a typically complicated flow domain that we may have in a computational in the physical plane here. We may not be able to put the points systematically spread throughout this; and systematically means in an in an automatic way, in an algorithmic way whereas, if you have a simple computational domain, then it is possible to put points in a systematic way by choosing so many number of points here, and so many number of points here, and then fixing constant psi and eta lines at the intersection of which we can locate nodes. So, this kind of simplicity is what we desire in the computational plane; and idea of the grid generation is to map these points, which are now identified in the computational as the points, where we want to evaluate the variables into the corresponding points in the physical plane.

So, and the way that we do is that if this is divided into n number of planes here, this also is divided into n number of planes. So, let us let us do it for the simple case, we can do in a simple way that we map side A, B to side A prime, B prime; A, D to A prime, D prime and C prime, D prime like this in this way. And then we put for example, we have put here1, 2, 3, 4, 5, 6, 7, 8, 8 planes, 8 points here; and we put the same number of points on 1, 2, 3, 4, 5, 6, 7, 8. And similarly on the opposite plane this is A prime, B prime; and this C prime, D prime; on the opposite planar side, we put equal number of points. So, this point here, and this point here are joined together; and this point I joined together like this. And similarly this is made up into the same number of planes as on this side, same number of divisions here, and the same is done on this side, and we can join them here.

So, by dividing the mapped surfaces that is A, B and A prime, B prime; similarly A, D and A prime, D prime to have equal number of points in the computational plane and the physical plane; and by making sure that the opposite sides that is A prime, D prime and B prime, C prime have the same number of point, it is possible to ensure that there are points in the interior, the intersection of these points here, corresponds to same number of intersection of these points here. And these points are lines of constant eta, this is eta 1, eta 2, like this; and this is, this line here is psi 1, psi 2 like this.

So, these lines, which are straight lines separated by constant delta eta and delta psi here; are lies which are curved in this, and the intersection of these curved lines, this psi line, and this eta line constitute the node at which we want to evaluate this. So, the idea of so in this way we can map, and interior point here, with a corresponding interior point here, this is 1, 2, 3, 4, 5; and from there 1, 2, 3, 4, 5, 6; so this is 1, 2, 3, 4, 5; 1, 2, 3, 4, 5, 6, so this point, so this point is mapped with this point; and then this point is therefore, mapped with this point.

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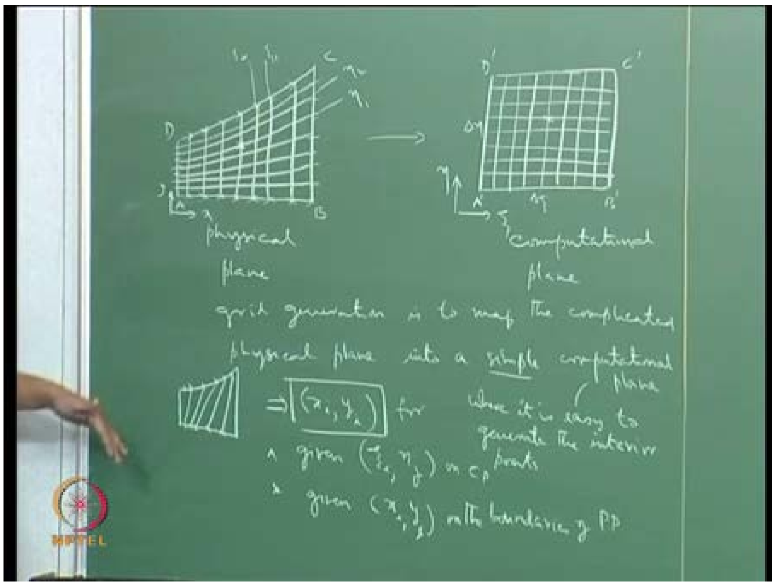

grid generation is to map the complicated physical plane into a simple computational plane where it is easy to generate the interior points

$\Rightarrow \boxed{(x_i, y_j)}$ for

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So, the idea of the grid generation is to fix, to map the physical plane boundaries and the computational plane boundaries, and divide the computational plane, which is rectangular in the case of two dimensions, and like a cube in the in the case of three dimensions. And divide this into lines of constant delta psi and delta eta not necessarily same, we need not have the same number of divisions on this side, and this side, but we maintain the equal number of divisions on opposing faces here, and similarly on these two faces; and we maintain the same number of divisions on this side and this side. And thereby we can fix the total number of points that are going to be on the… That we are going to be forming the nodes within this physical domain.

Now the location of these interior points in the computational domain is very straight forward to evaluate, because we have them as a horizontal and vertical lines. And here it is not so easy; it is not it is not straight forward to evaluate what is this how this particular this varies as a function of x and y; this obviously, a curve. So, that is why the evaluation of the corresponding interior points for a given number of interior points on the computational domain and the physical domain is not straight forward, and it has to be done systematically; and it cannot be done in a simple way.

So, it is the objective of the grid generation is to evaluate the location of x and y on the interior of the physical domain for a given number of points, and for a given location of the points on the physical boundary. The important variables here are the number of points that you want to put on on this face and also on this face, and also where you want to put them. Here I have put them as a almost equal, but instead of that, I can make them if I feel that in this particular thing, there is more variation here, I can put more number of points here, and then have it like this. And I can put if I wanted to I could put like this, and have a grid, which is like this. Although this is not something that one would recommend, but the specification of the problem is not only in terms of the number of points that we want in the computational plane here in the two directions, but also on where we want to put these number of points on each side here; and they can be put like this or they can be put like this, preferably we would like to put them in in such a way that we get good lines here, but that is this part of the domain.

So now, once we specify the number of points and the location of points on the boundary. The next task is to evaluate them on the interior, so once we evaluate the location of x i y j within this interior throughout, then our task on grid generation is almost completed, in the case of structured grid. So, this mapping of the physical plane into the computational plane is should result in the computation of x i y j for given psi i eta j on the computational plane, and given x i y j on the boundaries of the physical plane. So, we have to find the entire x i y j of all the interior points of this.

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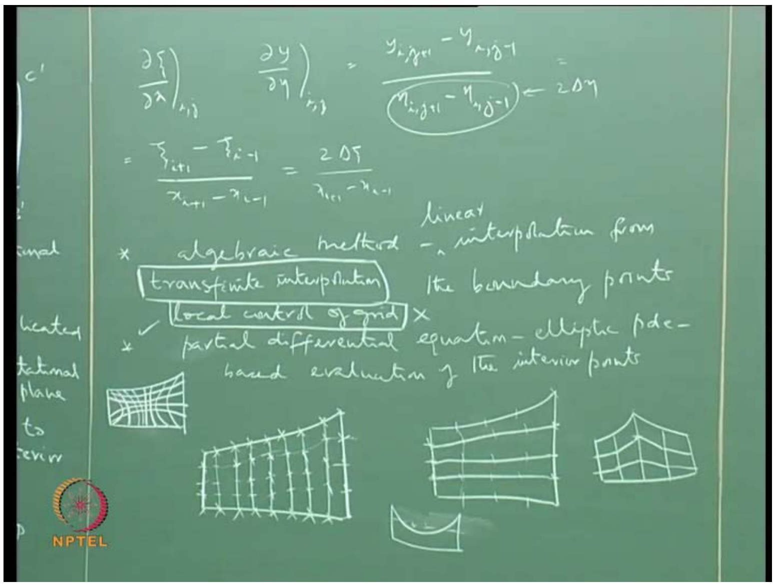

$\left.\frac{\partial \xi}{\partial x}\right)_{i,j}$ $\left.\frac{\partial y}{\partial \eta}\right)_{i,j} = \frac{y_{i,j+1} - y_{i,j-1}}{\eta_{i,j+1} - \eta_{i,j-1}} \leftarrow 2\Delta\eta$
$= \frac{\xi_{i+1} - \xi_{i-1}}{x_{i+1} - x_{i-1}} = \frac{2\Delta\xi}{x_{i+1} - x_{i-1}}$
× algebraic method - linear interpolation from the boundary points
transfinite interpolation
local control of grid ×
✓ × partial differential equation - elliptic pde - based evaluation of the interior points
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Once we have this, then the metrics can be evaluated, because metrics typically are like dou psi by dou x or if you look at the inverse dou y by dou eta. So, we can write this at a particular point i j, we can say that this is equal to this is variation in the x direction, so we can write this as i plus 1 minus psi i minus 1 by x i plus 1 minus x i minus 1; and this is for this particular point, and these are the neighboring points, and we know these things, because these are along constant psi lines in this. And this is what we can do when i j is in interior point, this we know is second order accurate approximation for this; and for boundary points, we have to come up with similar evaluation of the of the metrics of the transformation using the one sided formulas.

And this, also can be written at i j as y i j plus 1 minus y i j minus 1 by eta i j plus 1 minus eta i j minus 1. Now, here as a result of the mapping, we have for each point i j, we have (x i,i jy j) and also (psi i,eta j). So, the evaluation of the metrics is straight forward, once we compute, once we do this internal mapping, mapping of every point in the computational domain including the boundary points with the corresponding points on the physical plane. And once we have the metrics done, then we have the whole equations specified, because we have a transformed equation, which has the metrics has a coefficient; now the metrics are evaluated numerically; and our equation becomes a partial derivative equation with specified coefficients plus variables that are coming as a part of the Navier’s Stokes equation or the governing equation.

So, the grid generation, the task of the grid generation that one can see is to, evaluate these things. And one would note that epsilon i plus 1 minus epsilon i minus 1 is nothing but 2 times delta psi, this is psi here, because in the computational plane, you have uniform spacing; and this is x i plus 1 minus x i minus 1, this may be more not so straight forward, because you have we have curved lines here, and similarly this one is the same as this this whole thing here is 2 times delta eta.

So, the fact that we have uniform spacing means that this is the, this evaluation is straight forward. Now that question that we have is, how to get the interior points x i y y j? Given epsilon i eta j from a simple discritization of the simple computational plane domain; on what basis do we get this interior point? One can imagine several ways of doing it, but we will discuss two possible two possible and readily acceptable ways; one is by algebraic method; and this algebraic method is based on interpolation - interpolation from the boundary points; and the other approach we know that interpolation is essentially linear interpolation; and this is a fairly easy way of doing it; and it has we will see how it can be done; and it has its advantages, because of the ease with which it can be done. But the fact that it is linear interpolation creates some problems.

The other possibility is to use for example, a partial differential equation approach; essentially, for this particular will be using an elliptic pde, which is use to do the evaluation. So, elliptic pde based evaluation of the interior points, in which we can impose a condition that the constant eta lines and constant psi lines that we have on the physical plane are smooth. They do not go, they do not go like these, but if they have to go like this, the will be going smoothly. So, the additional condition of smoothness is incorporated in the pde based approach; and the smoothness important, because we are using the metrics, in evaluating this; and the metrics means, if the derivative of for example, dou psi by dou x; and so this would be sensitive to any kind of non-smoothness; for example, this particular thing means if the derivative here, when seen from this side is one thing, and seen from this side is another. So, there is a discontinuity in the derivative and the metric.

And that is a problem that we do not want to have, because in transformed equations, these derivatives are squared in some of the cases; and we also have the second derivatives epsilon x x psi x x and so on. And this value will will take a beating, when you look at a discontinuity in this. So, the algebraic methods do not have any condition on how these how these constant psi lines are to be evaluated, because each point is evaluated on its own without worrying about the neighboring points; whereas, in the case of partial differential equation approach, it is possible to lay additional conditions on how these grid lines should behave in the physical plane for the interior points.

So, this is obviously more beneficial, when we are dealing with body fitted grid system. So, this is to be preferred, but we will see that the generation of these interior points based on this approach is not so trivial, and that itself requires a solution of additional partial differential equations. So, that makes it more time consuming, so let us now see, how we can use the algebraic method; when we talk about interpolation, the simplest way of doing this is that something as simple as this. We fix the number of points, anyway this is coming from the computational plane, and we choose to fix them wherever we want.

So, the first thing is to do the boundary discritization into equal number of points on opposite sides, and similarly here; and we can fix a line here, we can join these two by a line here, and then within this line, we divide this into as many segments as we have on this boundary here, we have 1, 2, 3, 4 segments. So, we break it up into 4 segments here; you take the next line here, join them by a straight line; so you break it up into 4 segments. So, each line element here is broken up into equal number of segments, and these are the interior points.

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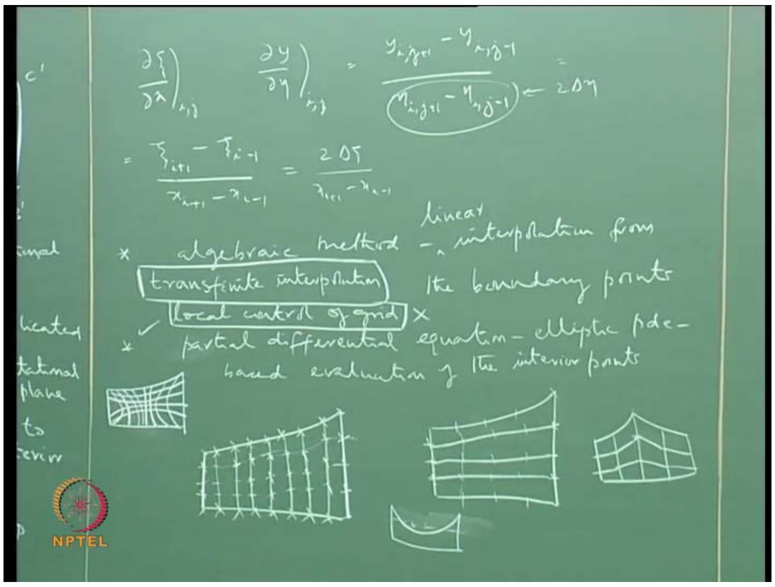

$$\left.\frac{\partial \xi}{\partial x}\right)_{i,j} \qquad \left.\frac{\partial y}{\partial \eta}\right)_{i,j} = \frac{y_{i,j+1} - y_{i,j-1}}{\boxed{\eta_{i,j+1} - \eta_{i,j-1}}} \leftarrow 2\Delta\eta =$$

$$= \frac{\xi_{i+1} - \xi_{i-1}}{x_{i+1} - x_{i-1}} = \frac{2\Delta\xi}{x_{i+1} - x_{i-1}}$$

* algebraic method – linear interpolation from the boundary points

transfinite interpolation

local control of grid ×

✓ * partial differential equation – elliptic pde – based evaluation of the interior points

So, you are not essentially reconstructing any constant psi line and constant eta line like that. It is understood that if you want to join them by a line that is your constant eta line correspond to this. But we are not at any point creating a line, which is a curve, which is joining, which is going through this, because for the evaluation of the metrics, it is sufficient to know what the coordinates x i y i of the interior points are for given boundary points. So, in this way we can do an interpolation in this way, that is with join the the two exchange boundary points by a line here, and then we divide each line into equal number of segments, corresponding to the number of segments on the other side we can do this.

We can also do the same thing by taking the other we have, and then we have the boundary points here, let us say we have this much, we can join these two here, and each line segment here can also be broken up into equal number of segments. So, in that sense, we can do interpolation either in in these parallels, in in this direction or in this direction. And ultimately, we get the internal points it is not necessary that both of them will give us the same same points here; for example, in this particular thing here, we have this, this is a straight line, which which need not be exactly corresponding to this line here, and the other thing about the…

So, from that point of view, generation of the interior point by interpolation is very straight forward; but in situations, it can give raise to problems; for example, if you have a domain like this, then if you want to do some interpolation based on, if you want to do an interpolation like this.

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One can immediately see the disadvantage from this interpolation; and the disadvantage is that the discontinuity of the surface at the boundary like, what we have here is propagating into the interior. So, this kind of discontinuity slope that is present on the boundary, because of the constraints of the physical domain is also propagating into the interior, because we have we have done a linear interpolation. And in some sort of curved boundaries, this kind of interpolation here can also give raise to points, which are lying outside for example, one were to take domain like this, a highly curved domain, one can conceivably get a point like this, and we can get points on on lying outside the domain here.

So, in such a case, we must have an algorithm, which takes account of not only interpolation in this direction, but also in this direction, and that kind of algorithm is what is known as transfinite interpolation. Essentially, it is an interpolation, which which is done in this way, and then which also takes corrections based on the interpolation from this side, so that we do not ever get to have points, which lie outside the domain. So, with transfinite interpolation, it is possible to make sure that all the interior points are on the inside. But this is still a linear interpolation, and there is no condition on this that their grid lines are smooth; and even with this, the disadvantage of boundary discontinuities, slope discontinuities propagating into interior that still remains.

So, that is one of the disadvantages of the algebraic interpolation method. One disadvantage is that boundary discontinuities will propagate into the interior, and we would like to avoid those kind of discontinuity, because the computational, the transformed equation in computational plane depends on the evaluation of the metrics of the transformation, and the metrics will will have different values, when you have slope discontinuity here; so this is one disadvantage.

One advantage is that not work much not work not much work is needed in order to find all the interior points. And also the extension from two dimensions to three dimensions is very straight forward in this. So, when you go from two-dimensional plane to three-dimensional plane discretization, it does not give raise to additional difficulties, not too much. One other disadvantage with the algebraic method is to have local control, local control of the grid; this is not there in the case of algebraic method; in the sense that let us say that you would like to have in in this domain here, you expect strong gradients here, and you would like the grid lines to be closer here.

So, you would in this domain, instead of having a a grid line like this, you would actually like to have closer grid here. If I have this kind of possibility, I can make sure that the grid spacing is much less. So, if I had this, I can see that the spacing here is less, and here the core spacing; but I am not really bother about the core spacing here, because this is where my derivatives are, and this is where I want to evaluate with accuracy. So, if I want to have this kind of possibility, where I would like to have a control over the grid density within the interior, then it is not possible with linear interpolation. Using linear interpolation, I cannot make my curves my interior point lines; curve towards one particular point or away from a particular point. So, that sort of control over the local grid density grid size is not available in the case of algebraic method.

Both the disadvantages of the algebraic method namely, local control over the grid size and smoothness of the interior coordinate lines are possible, if you use a partial differential equation approach. And so, in in this approach what we are looking at is that we are trying to have the lines of constant psi, and constant psi, and constant eta here to be smooth. And how can we ensure that, we can say that if if one were to say that a smooth variation, then that the second derivatives are are small.

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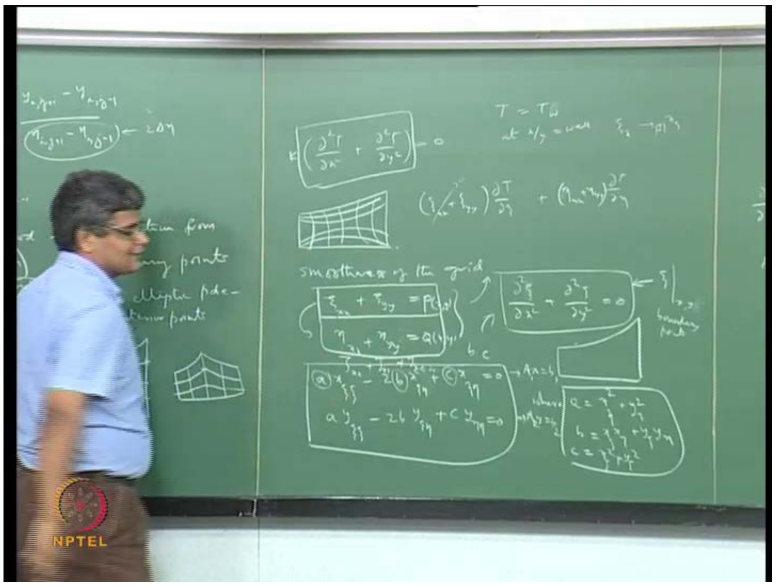

And for example, if you have temperature contours in the case of pure conduction, in which case, they they they satisfy the equation like this. Then we know that for whatever be the temperature boundary conditions here, then the isotherms here, isothermal lines, constant temperature contours will always be smooth; they will not be in any way. So, these are constant temperature lines, they show smooth variation. So, if you want to look at this and this, one could say if I want to have my constant temperature line to be smooth, and I would like my eta variation here to also exhibit to also satisfy this equation. So, and similarly, if you compare these lines here, with these lines here, so, which are the constant psi lines; so the constant psi line should also satisfy the Laplace equation; so, that is a basis for the partial differential equation.

So, in order to maintain smoothness we would like psi to satisfy the Laplace equation that is psi xx plus psi yy should be equal to 0; and similarly my eta line should also be smooth in the xy plane. So, we also would like eta xx plus eta yy to be smooth. So, the constant eta, and constant psi lines in the physical plane satisfy these conditions, then I can expect smoothness of the grid; and one would note also that if we were to transform this Laplace equation for temperature into the corresponding psi and eta coordinates, then we would have dou t by dou psi term, which has psi xx plus psi yy here, and dou t by dou eta term will have eta xx plus eta yy terms, which appear here. So, that is the first derivative terms will appear with these kind of things.

If you have these equal to 0 as a condition for for the interior points, then this terms will be 0, the first derivatives will not appear in the transformed equation, it is only the second derivative that will appear. So, there is also the simplification of the transformed equation possible, if we make this as a condition. Now, how can we apply this condition? So, if we want to solve this, this is a a partial differential equation of this form, and this has to be obeyed on this boundary here, and that means that and we can see that this is in elliptic equation, we need to have boundary conditions; and what kind of boundary conditions we want? We want the boundary conditions on the enclosed surface; so, that is on all these lines.

And I would like you to compare this with this equation, which is a heat conduction equation; and here we are specifying temperature thermal boundary conditions at the walls, that is on on all these surfaces here. So, essentially we can specify the Dirichlet boundary condition that T equal to some T of T wall at x or y corresponding to wall. So, essentially, what we are doing is, we are specifying the temperature here, and here, and here, and here, and subject to those things, we we can solve this. So, similarly we need to have the psi value on these boundaries; and similarly here, we need to have eta values on the boundaries here.

So, based on those considerations, so we solve these with boundary conditions for psi at the x and y boundaries at boundary points. So, by doing this, we can we can ensure the we can get this constant psi lines all these things; but what we would like to say is that in this form, these equations are not very useful, because we know the interior points on on this here, and we know the boundary points are here, it would be easier to work in in the complimentary way. So, instead of saying psi of x x and psi of y y here, if we can write it in terms of x of psi and all that, then it becomes simpler.

And using the equivalence between the metrics of transformation and the psi x in terms of xx psi involving the Jacobian of the transformation and all that; it is possible to rewrite this equation in this form; and a y psi psi minus 2 b y psi eta plus c y eta eta equal to 0, where a is equal to x eta square plus y eta square b equal to x psi x eta plus y psi y eta and c equal to x psi square plus y psi square. So, these two equations, these two equations here are transformed using equivalence between psi i and x eta in terms of the mutual transformation possibilities into these equation here, where the coefficients a, b, c are not actually constants, but they themselves are function of this.

So, these equation are now the variables are x and y and the independent variables are psi and eta. So, using this, it is possible to say since we know the since we know the eta and psi in the computational plane, because it is an because it is a simple grid, we will be able to get corresponding values here by solving this. And the x and i, x and y will satisfy these equations, which will mean that the inverse relation psi and eta will satisfy these equations. So, this is actually derived from this.

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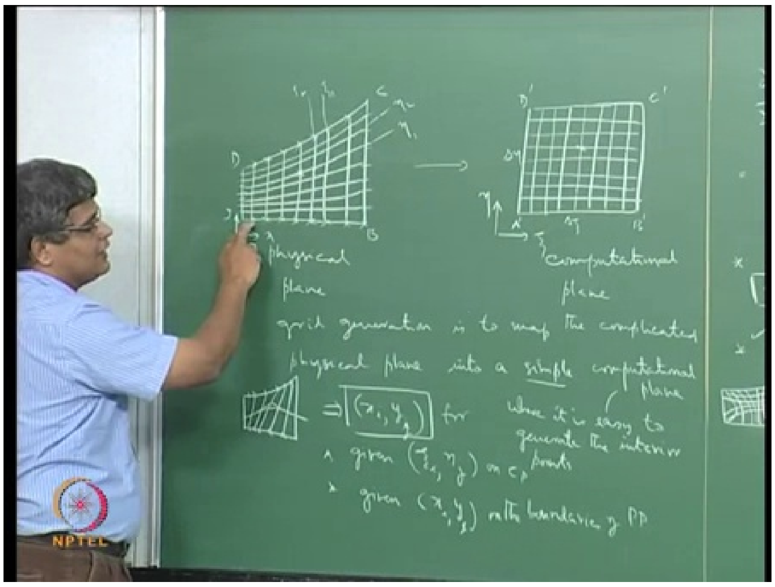
physical plane
computational plane
grid generation is to map the complicated physical plane into a simple computational plane
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And for this as a boundary condition for each psi equal to constant line here, there is a corresponding physical point here, and we say that at psi equal to psi 1, x equal to x correspond to this; and similarly we need to have at psi equal to psi 1, what is y? So, the boundary points here that we select based on our understanding of the problem here, I used as the boundary conditions for the solution of these two equations; and with these things we will be able calculate the internal points.

So, that is the that is how we can generate the internal points; in such a way that the constant psi and eta lines are very smoothly within the domain here. So, they show a smooth variation like what we have what we have shown here or here. But this, requires a solution of two Laplace equations with coefficients which themselves are part of the solution; for example, at the beginning of the solution, we cannot evaluate this at a point *i* j.

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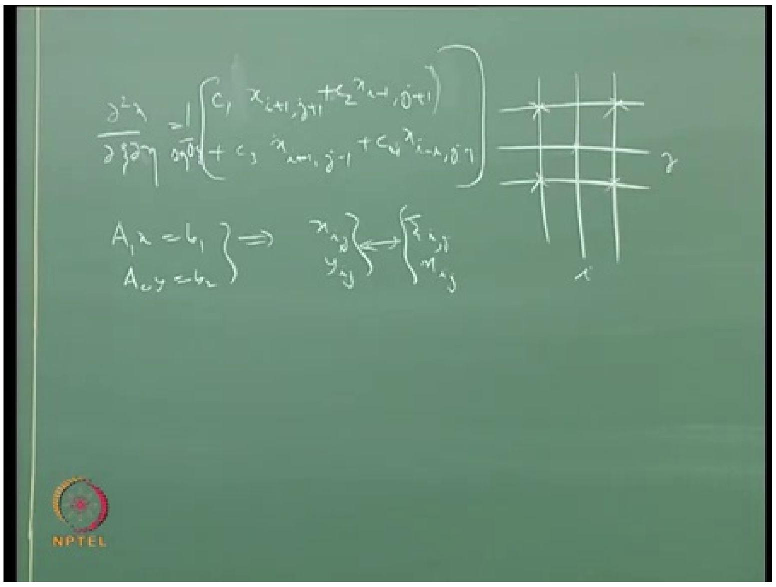

$$\frac{\partial^2 x}{\partial\xi\partial\eta} = \frac{1}{\Delta\eta\Delta\xi}\left\{\begin{array}{l} c_1 x_{i+1,j+1} + c_2 x_{i-1,j+1} \\ + c_3 x_{i+1,j-1} + c_4 x_{i-1,j-1} \end{array}\right\}$$

$$\left.\begin{array}{l} A_1 x = b_1 \\ A_2 y = b_2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_{ij} \\ y_{ij} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} \xi_{ij} \\ \eta_{ij} \end{array}\right.$$

So, we can descritize these things as in terms, for example, this this term here is dou square x by dou eta dou psi dou eta, and on a two-dimensional grid, we can write this; it is a cross derivative, we have unfortunately choose a wrong thing; but we can write this, if this is I, and this is j, we can write this as these four coming into picture. So, that will be x i plus 1 j plus 1 minus x i minus 1 j plus 1, we have we have an a expression for this, let us just put it in this way some c 1 times this plus c 2 times this plus c 3 times x i plus 1 j minus 1 plus c 4 times x i plus 1 j plus 1 i minus 1 j plus 1, we have i plus 1 j minus 1, so we have I minus 1 j minus 1.

So this, these are the constants divided by 1 by delta eta delta psi. So, we can find out these constants from their descritization, based on the spacing's here. And so, we can write like this, and the and we can convert this into an equation of a x equal to b, a 1 x equal to b 1, and this as a 1 y equal to a 2 y equal to b 2; and the solution of these two will give us x i j and also y i j. So, these correspond to the to the x i j y i j correspond to the psi *i* j and eta *i* j point. So, there is the a correspondence between the this and this, which is, which comes as a part of the solution of these two equations for known interior points in the computational plane.

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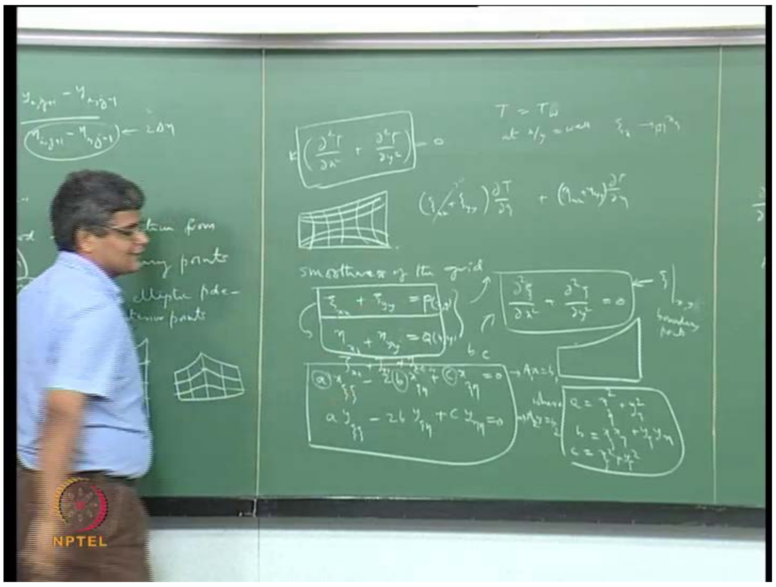

With the boundary points, with x and y are the boundary points here, forming the boundary conditions for these two elliptic equations. So, in order to do this transformation of these partial differential equations into a 1 x equal to b 1 like this, we obviously, need to know the coefficients a, b, c, they themselves depend on this, so, these unknown, not known x prior a.

So, we can start with a linear interpolation or transfinite interpolation to get some rough known values for a rough values for the interior points; and use those interior point information to calculate the metrics here. And then we can put them into this, now we have partial differential equation with known coefficients; this we conduct convert into a 1 x equal to b 1, and also this into a 2 y equal to b 2, solve this, and get all the interior points; and using those interior points, we re-estimate the values of the a and a b and c here, and then we put them again in this, we now have a different set of these equations and then we can solve them.

Now we get different kind of interior points; we can go through a few iterations to evaluate a, b, c here, which go into this; convert them into matrix equations for x and x i j and y i j, and then use those solve them using Gauss-Seidal method or any other method of our choice, and then get them back here do a few iterations to get the converge solution. So, the converge solution of this iteration, and this iteration together will give us the interior locations for each interior points on the computational plane and the physical plane, the corresponding psi eta and x i j x and y. So, that kind of mapping is possible with this partial differential equation approach.

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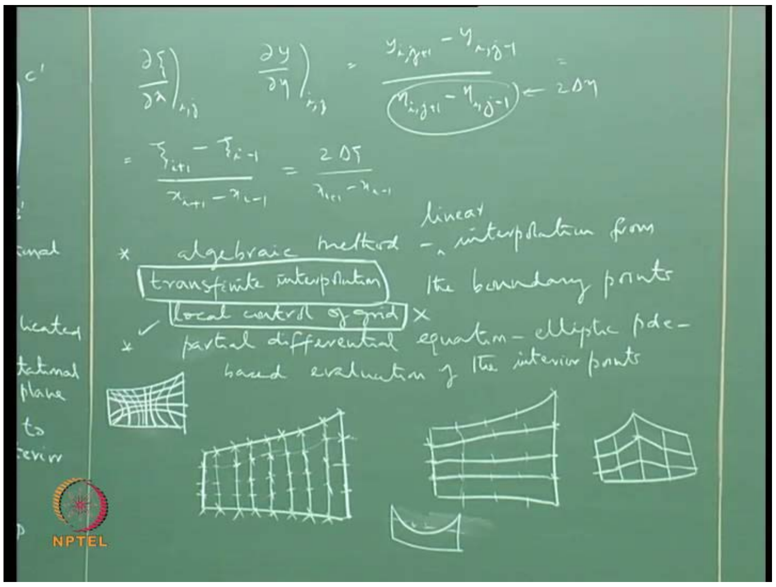

$$\left.\frac{\partial \xi}{\partial x}\right|_{i,j} \quad \left.\frac{\partial y}{\partial \eta}\right|_{i,j} = \frac{y_{i,j+1} - y_{i,j-1}}{\eta_{i,j+1} - \eta_{i,j-1}} \leftarrow 2\Delta\eta$$

$$= \frac{\xi_{i+1} - \xi_{i-1}}{x_{i+1} - x_{i-1}} = \frac{2\Delta\xi}{x_{i+1} - x_{i-1}}$$

* algebraic method – linear interpolation from the boundary points

transfinite interpolation

local control of grid ×

✓ * partial differential equation – elliptic pde-based evaluation of the interior points

And the resulting solution will definitely has smooth variation of constant psi and constant eta lines, because that is the condition for the for this particular method. Now if you want to have local grid control, we can do that by making this things, this points attract; for example, if one were to imagine these to be constant temperature lines, and if the contour lines are space together that means, that the gradient is high; for example, there can be a source of heat here or a sink of heat. So, if you have a local source of heat, then these things, these contour lines will get attract towards this; and local sink means that they can go away from that. So, they can we can make them attracted or not attracted by putting these kind of quantities.

And so, in such a case we would not be solving just this Laplace equation, but we can solve these with with a passa equation, which will be some a source points which are put it different locations the strength of which, and the location of which is a point for our consideration; and using making use of strength and location, we would like to make these grid lines constant eta and constant psi lines get attracted towards this in order to make the grid requirement possible.

So, by using judiciously some source point here of a different strength, one would be able to generate this kind of distorted mesh, which makes sure that you have lesser grid size in area of desired interest, but it is not something which is, which immediately jumps to the i as to walk these values should be, one has to do it in iterative way; but the possibility of having a grid, which is not only smooth, but which goes into which which can have smaller grid size in areas of desired location is possible.

Further refinement of this method would also mean that you can have orthogonality; and orthogonality of grids is especially important at the boundaries; in the general case, we cannot maintain orthogonality, but we can half as a post processing of this we can readjust these lines by imposing the orthogonality boundary conditions or we can use orthogonality as a boundary conditions in the specifications of the problem itself. So, one can have localized orthogonality of the grid lines as part of the grid generation process using this partial differential equation.

So, the pde approach is a more robust approach which will give us a grid which is, which has smoothness of the grid as a guaranty and that is always desirable; and even the pde approach is readily extendable to three-dimensions; here we have put only two things, we can also add a third dimension zeta x x a zeta y y plus zeta z z equal to 0; and similarly we can put the eta z z and psi z z things here. So, by solving three partial differential equations together, we can generate a three-dimensional. So, in that sense this algebraic methods and pde methods are extendable to to general three-dimensions; and these methods can enable us to generate the interior points for known boundary points, and for known discritization on the computational plane.

And at the end of all these, we will be able to get the x i, y j, z k for every point on the computational plane and the corresponding physical plane. Using this we can evaluate the metrics of the transformation, and we can evaluate the coefficients that go in to the transformed equations, which we will then be disc1ritized in the computational plane and the we can solve them the values of the phi’s for the computational plane; and then map the solution on to the physical plane. So this completes the way that we can go for a solution for complicated geometries, and the generation of the boundary fitted approach for this. In the next lecture, we will see how we can do something similar not for a structured grid that we have discussed here, but for an unstructured grid.