Computational Fluid Dynamics

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Dealing with complexity of geometry of the flow domain

Lecture No. # 42

Finite volume method for complicated flow domain

Illustration for the case of flow through a duct of triangular cross-section

In the last lecture, we have seen one particular way of dealing with the flow domain which does not fit into the usual coordinate system that we that we normally adopt, for example, the Cartesian or cylindrical and all that. And if we have a flow domain which is a combination of these, for example, a pipe inside a rectangular duct then we cannot make use of this. And in the last lecture, we have seen an approach whereby we come up with a new coordinate lines, which distort and wrap around the bodies of interest so as to define the flow domain in a structured coordinate frame: psi, eta, zeta instead of x, y, z.

And we solve the equations, we transform the equations from the physical plane into the computational plane and we map the physical plane into the computational plane, and then we do all the computations that are the discretization, solution and all that. In the computational plane and then map, the solution back into the physical plane. So, this body fitted approach is one way of dealing with complicated geometry and it has proved very successful and it retains the characteristics of a structured mesh approach to the solution of the governing equations.

Now in this lecture, we will see in an alternative view point, an alternative approach. It is based on the finite volume method; essentially what we are saying is that we want to solve our governing partial differential equation to get the solution of the flow variable at several points within the flow domain that is a basic CFD approach. And these partial differential equations represent the conservation in terms of the rate of increase of that particular property in a control volume as being balanced by three causes: one is the convective flux, the other is the diffusive flux and the third one is the source or the sync term or the source and the sync term in the case.

For example, of the turbulent kinetic energy, where we saw that there is a production term associated with a large eddies large eddies and then there is a sync term associated with the with the very small eddies and together they determine, the overall level of the particular quantity, which is being conserved; which is the turbulent kinetic energy, in this case at that particular grid point in that particular control volume.

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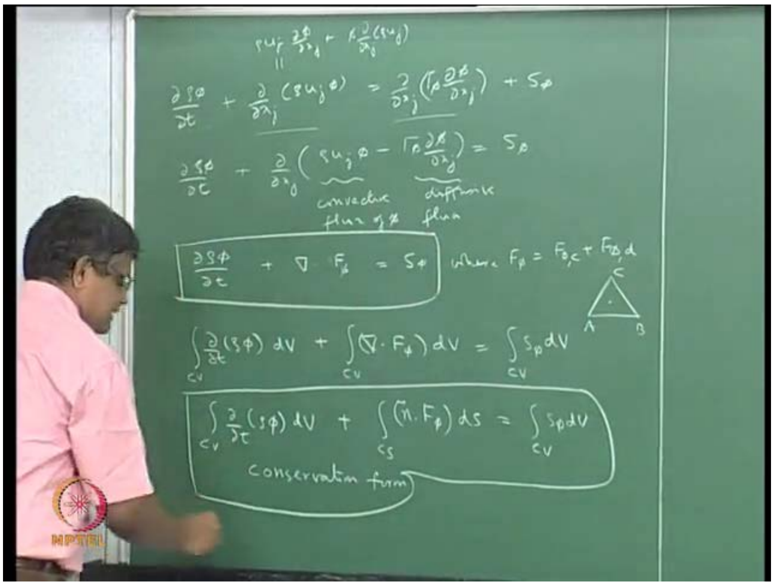

So, we can rewrite our standard conservation equation, which we have written many times as in this way. This is the convective flux is equal to the diffusive flux plus the source term. So, this particular equation can be rewritten in this way by bringing the fluxes together of rho u j phi minus gamma phi d phi by d x j equal to S phi.

So, this is the convective flux (no audio from 04:09 to 04:15) and this is the diffusive flux of the particular quantity phi. And in turbulent flow, we have seen that this particular term will dominate and this diffusibility here is not that of molecular diffusibility, but of turbulent diffusibility. But generally, this is the form here and which can be written realizing that this is a vector divergence operator here. You can write this as plus del dot, for example, F equal to S phi, where F is the total flux. We can put F phi here, where F phi is F phi convective plus F phi diffusive and the convective flux is given by rho u j phi and the diffusive flux is given by rho phi like this.

So, this is athis is a governing equation which is valid at every point. In the finite volume method, we put the equation in this in this form. And we take a control volume and then we integrate this over the control volume dou by dou t of dou phi dv plus integral of del dot F phi dv equal to S phi dv. So, for every control volume that is for we construct a volume around each grid point at which we want to evaluate the variable.

So, we can say that this is integrated over a control volume and a control volume is obviously a closed surface. This is the control volume at the different sides and making up together is the total volume contained in this and it has a closed surface and taking advantage of it. We can using Gausses law, we can convert this volume integral into a surface integral. And we can write this as dou by dou t of rho phi plus integral over the control surface the closed control surface of n ,where n is the normal vector associated with the surface enveloping the control volume equal to s phi d v, where this is the source term here.

So, this is a rewriting of the governing equation in what can be called as a conservation form (no audio from 07:24 to 07:30) and the conservation form refers to the fluxes that are appearing here, the convective flux and the diffusive flux. The conservation form refers to these terms that are appearing here being interpreted as the convective flux and the diffusive flux and therefore associating these with the with the fluxes that are passing through the faces of the control volume, which make up which make up the overall volume of that.

So, giving the meaning the interpretation for these two terms as convective and diffusive fluxes and evaluating these things over the surface enveloping the control volume is what we call as the conservation form. And for example, the alternative view is that you have rho u j phi and even if you take rho to be constant, we know that u is not constant and we can write this equivalently as rho u j dou phi by dou x j plus phi dou by dou x j of rho u j. And so these two are mathematically equivalent and similarly, we can write this as gamma phi dou square phi by dou x j square plus dou phi by dou x j times dou gamma phi by dou x j.

So we can split it up into that and when we do that then we lose the interpretation of this term as the flux, essentially what we are saying is that the quantity phi for which we are writing the conservation equation; it will the value of this within this control volume will change. That is what this term is? This term will be non zero either, because we have a source term which is spread throughout the control volume or because we have some flux, which is coming into the control volume by diffusive action by the fact that the value of the point of the variable in this control volume is less or more than the value of the control volume in the neighboring cells or by the fact there is a flow, because of the flow and because the flow brings in all the fluid properties. And phi is one of the fluid properties like enthalpy or temperature or concentration.

So, the value of the phi within this control volume may change, because it is being brought in and taken out along with the flow or because it is been diffused by gradients that exist between at this particular point, at this particular cell across the… So, these are the mechanism by which the phi can change and that is what is being represented in this interpretation.

Now, when we write in this interpretation, it becomes easy for us to apply this statement of the conservation, of the conservation law here to an arbitrary control volume. Not necessarily, something that has four faces in two dimensions or six faces in two dimensions. We can even take a triangular control volume and then we can say that this this and we can for example, if you say that this is A, B, C here. We can say that dou by dou t of rho phi. Let us say that this is the ith cell with three triangular phase rho rho phi i plus…

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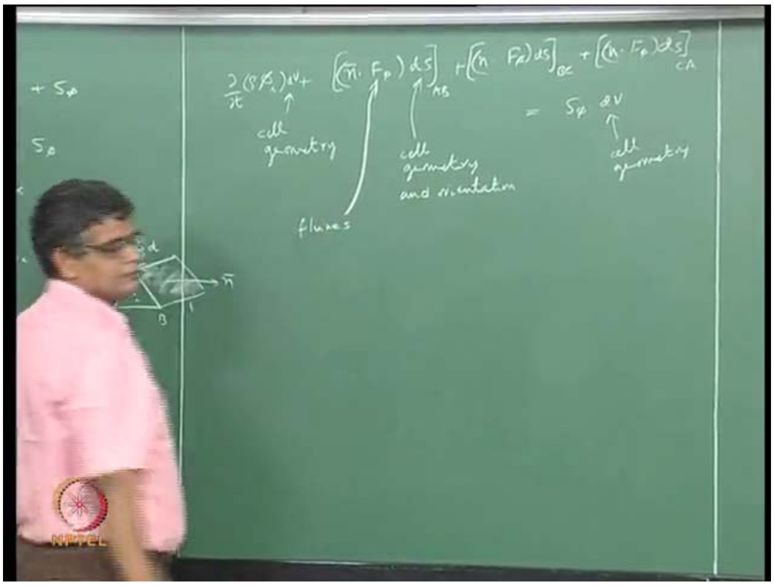

We can associate with this the control surface three phases AB, BC, CA and over each of this we can write. So, we can say that n dot F phi ds over AB plus n dot F phi times ds over side BC plus n dot F phi ds over side CA, that is the integral of all the fluxes that are coming through the phases and there are three phases here is equal to S phi times dv, we have a dv here. So, in this particular case, the volume we are looking at the two-dimensional thing we can make it into three-dimensional by considering unit length in the in the other direction.

So, we need to know what the volume of this element is, so that is we need to the area of this triangle multiplied by the unit distance. So, this is given by the cell geometry and in evaluating this term this is also given by the cell geometry and orientation. And similarly these things and here we have again cell geometry and what we need in order to apply this convert this equation into a mathematical equation is obviously, we have d rho phi by dt times volume.

So, for a given cell we know this delta volume and for a given cell with control phases we know each other areas of each of the side AB, BC and CA. So, when we talk about the area of the side. For example, BC you are talking about the area of this surface with this side as BC and this delta z direction as unity as the length here. So, that is how we get surface and an area and this surface has an outward novel vector n, which is what is coming here.

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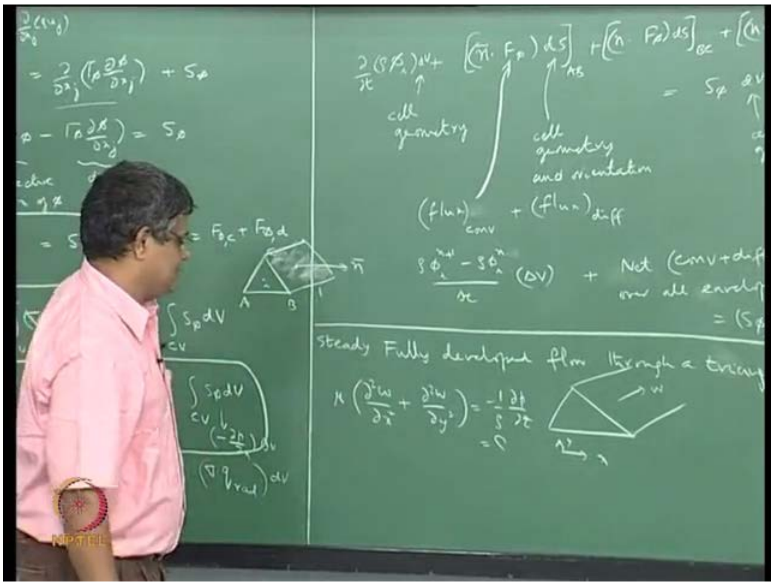

So, in order to apply this equation, we need to evaluate the fluxes. So, the the flux term F p has to be evaluated. So that means that flux coming from the convective and flux coming from diffusive contributions will have to be evaluated at each phase. And that convective flux and diffusive flux has a particular mathematical form rho u j phi and then the diffusive flux and this is where we need information of the values of the phi in the neighboring cells.

So, using those values and using the definition of these fluxes here. We can come up with the evaluation of these terms, in terms of the neighboring cell values. And once you put this here, we will have an overall equation for this particular cell. So, that cell will then be in the form of… At that point, we can write dou phi by dou t as rho phi i n plus 1 minus rho phi i n minus 1 by delta t times delta volume is this and the fluxes will have contribution associated velocity and the value of the phi here and that is value of the phi at… For example, at this point that is the cell centroid here, the plane the surface centroid and that has to be evaluated from in by some means.

So, we can say that this plus net convective plus diffusive flux over all surfaces enveloping surfaces is equal to the s phi at i times delta volume. So, this is the discretized equation in which one can see that phi i n is coming phi n plus 1 is coming and obviously in the diffusive fluxes, we have to evaluate the gradient dou phi by dou x and that means that phi i plus 1 minus phi i minus 1 by two delta x is one possible form. So that is where we are going to link the value of the current cell value to the neighboring cell values.

So, in this way, we can take a governing equation; we can divide our flow domain into cells, small cells and over each cell we can discretize the conservation equation written in conservation form. That is the variation within the cell as a result of volumetric forces and the fluxes here, and it is in this form, we can apply this to a particular cell and then convert this differential equation into an algebraic equation. Now, there are when we talk about the corresponding momentum equation here. We have to make a distinction between sources here that are truly volumetric and those are in a way coming through the surface.

So, when if when you treat the volumetric source as a volume and then as a volumetric source term and any source term which is coming, which is acting on the control phases as appropriately as the phase related source term then we have a strong conservation form. I would like to say that in the momentum equation, the source term here is for example, minus dou p by dou x is something is one of the source term.

And so, we can say that minus dou p by dou x times the volume delta v is the momentum source that is coming from the pressure, but we know that pressure is the surface force. So, this should be actually evaluated as the as something like equivalent to a pressure acting on the surface of the control volume and then that would make it, what is known as strong conservation form.

Again we have… If we have radiation as a source, radiation source term is typically associated with radioactive flux. We can so this is also a flux term, this should be coming through the surfaces not as a volumetric source term, but if you were to evaluate this as a volumetric source term then it is not in the strong conservation form.

So in the evaluation of source terms, we should try to make a distinction between those sources that are coming through acting through the phases like the pressure and the radioactive flux and those that are truly volumetric source terms like a gravitational force term is a volumetric or is a source term which is acting throughout the material which is there in the in the volume. If you have, for example, for the energy equation; if you have a heat generation term, which is paid throughout the control volume then it has to be treated as a volumetric source term, but energy flux coming through the phases in the form of radioactive flux must be treated in in this.

So, we have to treat each term appropriately as either acting through the phases of the control volume or having or being spread out through the throughout the domain and therefore constituting a volumetric source term. So, once we do that then we can claim to have a strong conservation form and the conservation equation that is being put out here, that is being solved exactly at each control volume and that is one of the strong point of of the finite volume method.

The conservation that is incorporated in this equation is being enforced in each control volume by interpreting this as a conservation equation in this conservation form. And so, if the flux terms are evaluated properly, then there is no possibility of generating spurious mass sources or flux sources or sources of this and that is one of the advantages of this. And when you have discontinuities in the within the flow domain as may be arising from shocks in those kind of things.

Evaluating in this conservation form is supposed to give superior results than putting it in the general form like this. So that, enforcement of the conservation in each each discrete cell is a characteristic future of this and written in this way this can be applied to an arbitrary control volume with defined volume and a defined phases which envelope and completely close the cell. So, this is the basic idea, we will we will try to see this in an action through a simple example, before we go onto the formal evaluation of these fluxes and then look at some more complexities.

So, the case that we are looking at the simple example, that we are looking at is flow through fully developed flow fully developed laminar steady flow through a triangular duct, through a duct of triangular cross section. Why did we take this particular example, because this is one can see that at once straight away; it is difficult to get an analytical solution. If it is a circular pipe, we can do and if it is a rectangular pipe may be with more difficulty we can do, but if it is an arbitrarily shaped triangular duct then to get the velocity field is not trivial thing. Analytically and also to apply this to… For example, to fit a structured grid for this is again not an easy task and definitely we will have large distortions of the cells, around this corner points.

So, if you were applying the body fitted grid approach for this then one can expect more difficulty with the numerical solution, and that is also one reason, why this particular example is a suitable example to illustrate the benefits and the advantages of the finite volume method. So, in when we take this particular cross section, we obviously need to have the governing equation and the governing equation is the one dimensional momentum equation through the z direction.

So, and it is since the flow is steady and fully developed. It takes a form of dou square w, where w is the velocity through this duct. So, this is the duct and velocity w is in this direction, and x and y are within the plane of this plus dou square w by dou y square times mu is equal to minus 1 by rho dou p by dou z. And dou p by dou z is constant for the case of fully developed flow and it is a given constant from the fact that you can specify the boundary condition at z equal to zero and z equal to l ,because the you have pressure gradient as constant, we can evaluate this. So, this is a given constant.

So for this particular case we can look ,we can see ,how we can apply the finite volume method for evaluation of this, and we can see how we are going to tackle this and evaluate each of the fluxes through this example.

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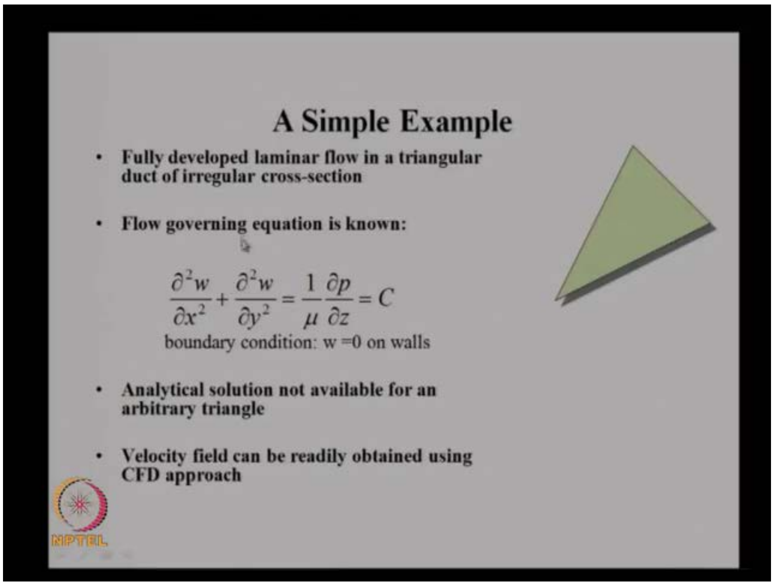

So, here we have the case of fully developed laminar flow in a triangular duct of irregular cross-section. When we say irregular cross-section, the three sides are not the same length; so, it is not an equilateral triangle to make it slightly more complicated and the governing equation for this is obviously dou square w by dou x square plus dou square w by dou y square equal to 1 by nu dou t by dou z. And we are considering constant properties and since it is fully developed and steady this right hand side is a constant, we are calling it as C. And the boundary conditions are, because we have three walls here the velocity w is 0 on all the walls. So, that is the boundary conditions, so the problem statement is very straight forward, but analytical solution is not available for an arbitrary triangle.

We have analytical solutions for equilateral triangles which are given in stand books, but we can use this readily using the finite volume method.

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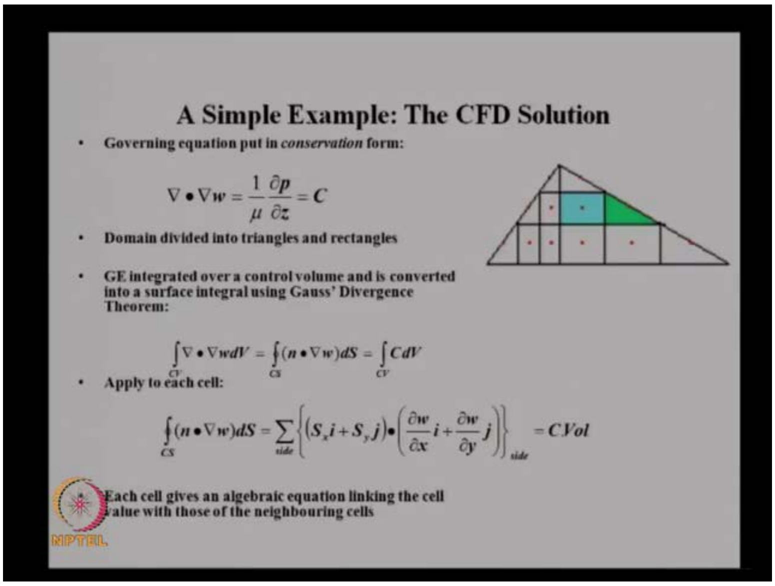

So, as a precursor, we write the governing equation in conservation form. So, what we have here as this term, we recognize that this is the diffusion term and therefore it is a divergence term. So, we put this in the divergence term. So, that part associated with the left hand side here is expressed as a divergence, that is del dot gradient of w is equal to c. And this divergence form when integrated will give us; we can convert this into the area integral, surface integral.

So, we divide the domain this triangular domain into a combination of triangles and rectangles. And that is the advantage of finite volume method, it is not necessary that the domain has only rectangles or only triangles; it can be a combination of all these things. And that makes it an unstructured mesh and we are cheating a bit here. We are taking, for example, we are dividing this into cells like this, a combination of rectangular cells and triangular cells. And we are saying that we want to have the value of the w at these points. These are obviously the centre centroids of these rectangles and we are for the sake of simplicity, we are saying that we want to have the velocity at this point, at these points here. And we know that these points are on the wall, therefore the velocity there is 0.

As we make it more and more, as we introduce more and more number of points that approximation will become less and we can also become more sophisticated and put this point to be at the centroid of this triangle, but for the time being we will say that this is where we want to have the velocities. So, the problem reduces to finding the velocities at these 4 points plus 6 points each of which has a control volume or an area which is unequal here.

And so we… The governing equation is now integrated of the control volume and is converted into a surface integral using Gauss's Divergence theorem. So, that is del dot gradient of w dV integration over the control volume, closed volume is equal to n dot gradient of w this is that diffusive flux of momentum in this particular case, over the control surface. And that is equal to the constant right hand side times, the integrated over the same volume. So, if we can apply this to each cell in this way, so that is integral over the control surface is equal to this n is is an outward normal vector for this case of two-dimensional case. It has two components i direction, S x in the i direction, S y in the j direction.

And the gradient of velocity also has two components dou w by dou x in the i direction and dou w by dou y in the j direction. So the dot product of these two times the constant C times, the volume of that control volume is the discretized form of this equation. So, each cell gives an algebraic equation linking the cell value that is w i, we can see obviously, the w i coming from the gradients here with those of the neighboring cells.

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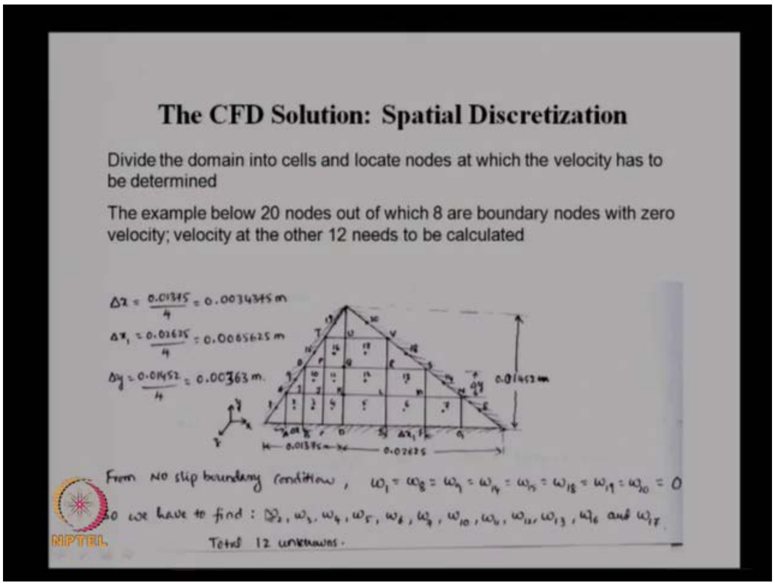

So, we are looking now at the practicalities of the of the of the domain. And here we are looking at 20 nodes; we are breaking up into the left hand side of this triangle into 4 equal parts here, and then 4 equal parts on this thing. And based on that we can do decomposition of this into a combination of triangular and rectangular tiles and again the right hand side of this is also made into 4 divisions in the horizontal and 4 divisions in the vertical.

So, we have a total of 20 points: 1,2,3,4,5,6,7,8,9,10,11,12 all the way like this, out of which some points: 1,9,15 and all those things lie on the boundary. So, out of the 20 points 8 are boundary points with 0 velocities. The velocity is already known and we need to find the velocity at the other 12 points. So that, we can evaluate and having found the velocity at each point, we can multiply for example, the velocity at 16 by the area of the cell, this cell to get the flow rate through this.

We take this velocity times this area will give you the flow rate through this, velocity at 17 time this area will give you the flow rate through this and then we can compute the overall flow rate in that way. And so that, we can get the flow rate for a given pressure gradient, so that solve the problem is post. And in each case we need to find out, the delta the sides of these phases, so we have delta x.

So, we are taking for whatever reason in this example, this length to be 0.01375 and this length to be 0.02625. So, this is broken up into 4. So that, you have a delta x here and this is again broken up into 4 and this whole height is taken as 0.01452. So, that is broken up into 4. So, using these things for any cell here rectangular or triangular; we know the sides and that is the geometric information that we have and once we know the side for each of these. We can find out the length of each side, which constitute the surface and the area of each cell which in three dimensional would be the volume.

So, the geometrical information is obtained from the discretization from the breaking up of the entire flow domain into tiles, rectangular tiles and rectangular and rectangular tiles. So, we need to find out velocity at 2, 3, 4, 5, 6, 7 and then again 10 and 11, 12, 13 and then 16 and 17. So, there are 12 unknowns and over each of these points, we have a control volume which is defined here. And for each cell, we apply this discretized form in order to derive an equation.

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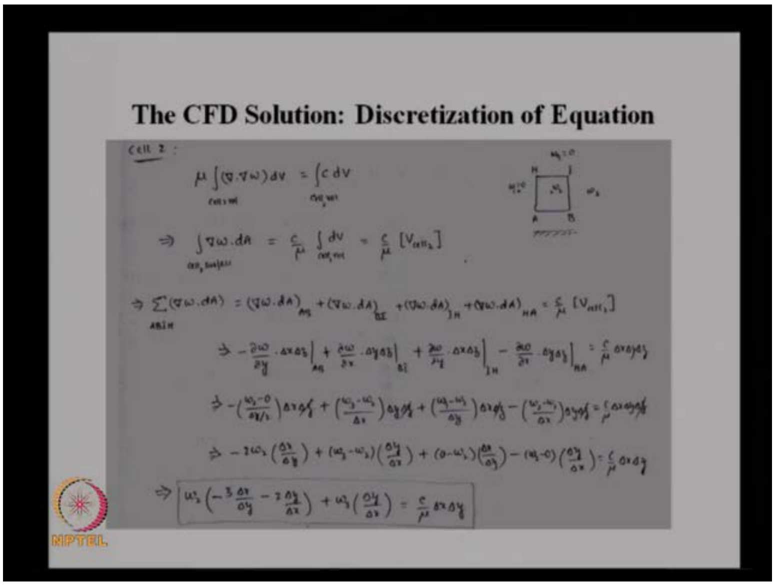

So, that is shown for cell 2 here, so that, this is cell 2 with which is a rectangular phase and here we have w 2. And immediately to the right at the centre of the adjacent cell, we have w 3. And the bottom wall, this bottom phase here is a wall, and left side we have this as part of the wall here; and it has a w 1 of 0 and above this above this again we have 0 here. So, when you look at the 4 neighboring points here. On this side you have 3, w 3 which is to be determined. On the left hand side, we have w 1 at a certain distance which can be evaluated. And that is 0, above this you have w 9 at a certain distance which is 0 and below this, you have a wall. So, you do not have a point here.

So, we can say that the gradient dotted with area of each side is equal to the C by mu times, the total volume of each cell. So, that is what we have volume of… So, we can say that all the cell to surface integral of this over the cell to surface is equal to C by mu times volume of cell 2. So, this particular thing we are putting it as A, B, I, H and so the integral of this over A, B, I, H. So, that is gradient of w times dA over AB plus gradient of w times dotted with dA over BI plus gradient times dA over IH and gradient times this over HA is equal to this.

And we take advantage we take note of the fact that these are oriented in x and y directions the outward norm vectors are located in the x and y directions. And this one on this phase, the outward norm vector for this is located in the positive x direction and here it is located in the negative x direction. Therefore, gradient of w dotted with dA on AB is in the negative x direction. So, we have minus dou w by dou y dotted with area of this cell. So, this is delta x and in the z direction we have delta z. And over BI this is the gradient at this point, so that is dou w by dou x component here, and the area of this cell is delta y times delta z which is in the other direction. And here it is the gradient in the y direction at this point times the area of this cell which is delta x, delta z, and here we have a minus, because outward normal vector is in the negative x direction.

So, we have dou dou w by dou x which is the gradient to the x direction times the area, which is delta y, delta z and this equal to this. So, now in this equation, we know delta x and delta z for this particular cell. And we need to evaluate the gradients at each of these points, now when come to this particular point minus dou w by dou y at AB. So, we take the centre of here we know that this value is 0 from the boundary condition and this value is w 2. So, we can say that is w 2 minus 0 divided by this distance which is delta y by 2.

So, that is the estimate of the gradient that is coming here, when you come to this point here, the gradient dw by dx can be evaluated as w 3 minus w 2 divided by this distance. So, w 3 minus w 2 divided by delta x is the total thing, and here again we have this point and this point. So, w 9 minus w 2 divided by this height which is delta y w 9 is 0. So, we can make use of this and here it is w 2 minus w 1 divided by delta x. So, we are substituting, we are making estimates for the fluxes at each of the phases. In this case, we are looking at the diffusive flux and we can substitute like this. And where ever we know the value 0’s here, we can substitute and then we can rewrite this and then we can finally get an expression an algebraic equation involving w 2 and w 3, because this side and this side, this side the velocities are known.

So, we have an algebraic equation, by applying the conservation equation for this particular cell and we can do the same thing for each of the other cells. For example, if you consider this cell here, you would have the gradient to be evaluated here, here and here and here. So, this gradient will be in terms of w 10 minus w 3, this gradient will w 4 minus w 2 3, this gradient will be w 3 minus w 2 and this gradient is w 3 minus 0 divided by this half distance. So, in that way we can evaluate for each of these.

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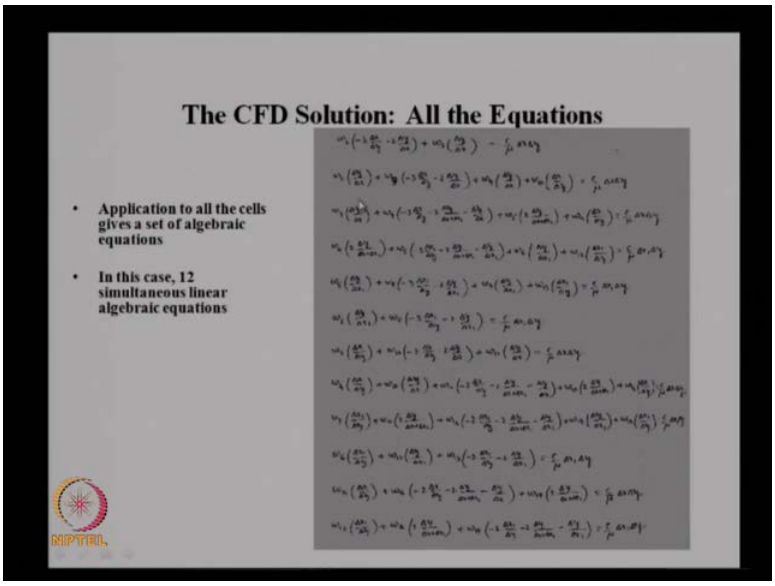

And then, we can come up with with set of 12 algebraic equations for the 12 phases, for the 12 control volumes or tiles. And there in expressed as linear algebraic equations involving the grid information which is coming in form of delta x, delta y, delta z that is here like this and so you have 12 simultaneous linear algebraic equations.

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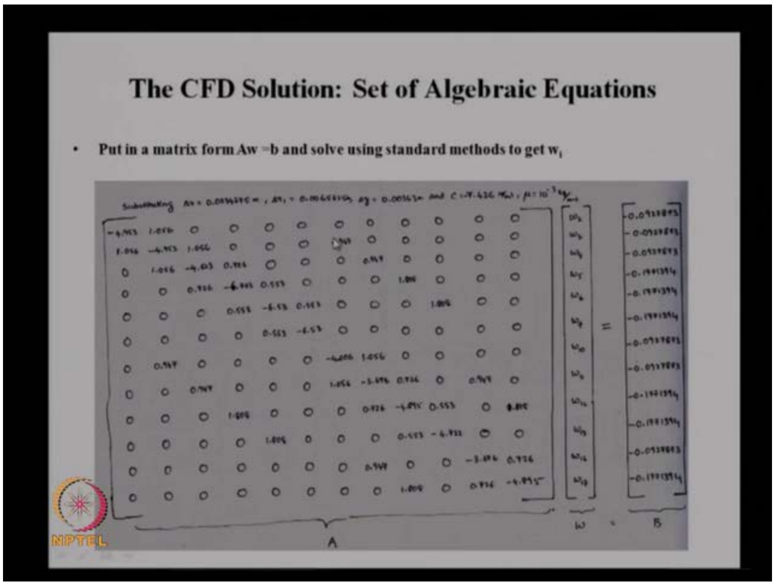

And you can put them in the matrix form and we can see that you have this central diagonal and some adjacent diagonal, there are some things that are coming here, but there are also some other things that are coming here.

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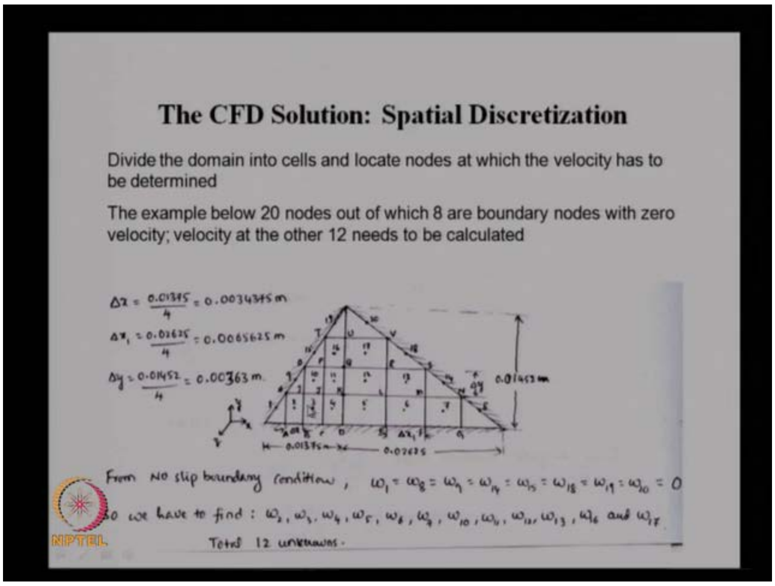

So, it is not necessary that, because of the of the way that the numbering of the cells is made. It is no longer possible to have the kind of structured diagonals that we have in a case of structured grid, because the neighbor if you look at 11 for this point here is 10 and 12 in this direction is ok, but here is 16 and 4. It is not i j plus 1and j minus 1. So, that kind of nomenclature is not possible.

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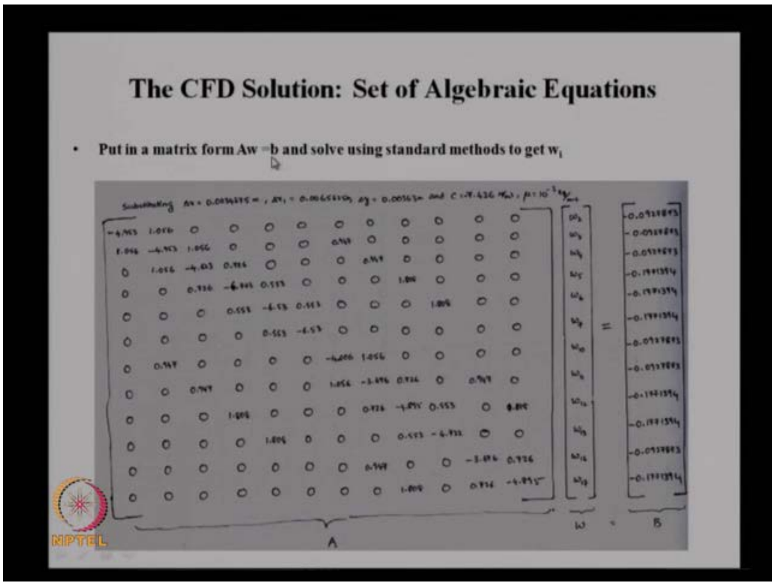

So, that is why we do not exactly have a structured matrix, but it is a still a sparse matrix and still we have a linear algebraic equation of the form Aw equal to b. And this we can solve using Gauss-Seidel method provided that this satisfies the Scarborough criterion, diagonal dominance condition. And it does satisfy the diagonal dominance condition, even in the case of this unstructured grid.

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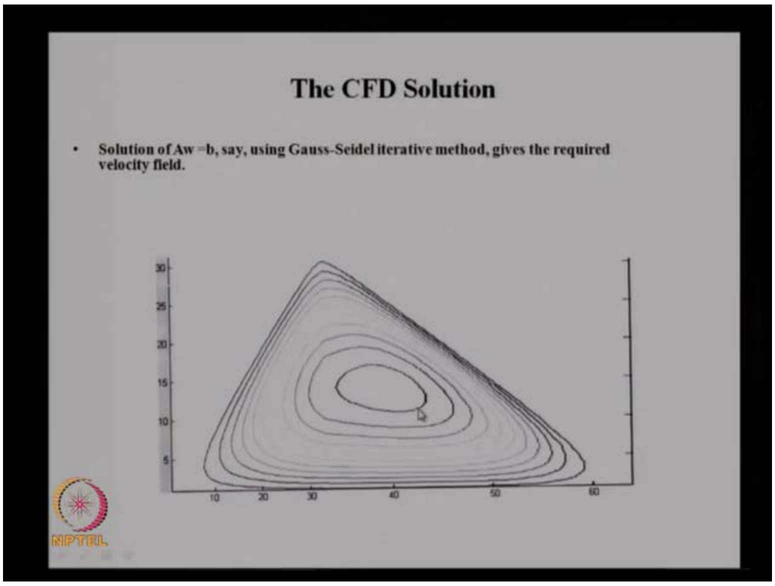

And then you can get a solution at all these intermediate points from which you can draw the contours. So, after solving this using Gauss-Seidel method we can get a velocity field on this.

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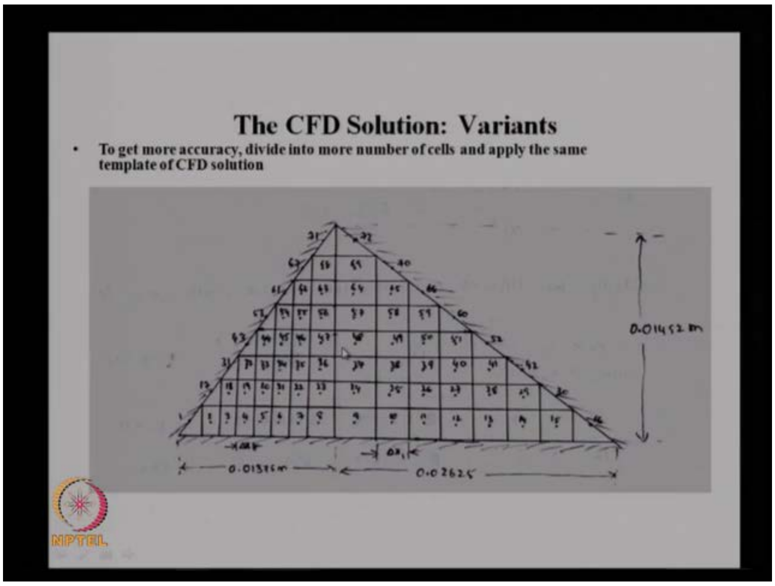

If you want more accuracy, we can divide this into more number of points, and here it is divided into something like 72 points here. And we have each of them is smaller and you get more points and then we can get an equation.

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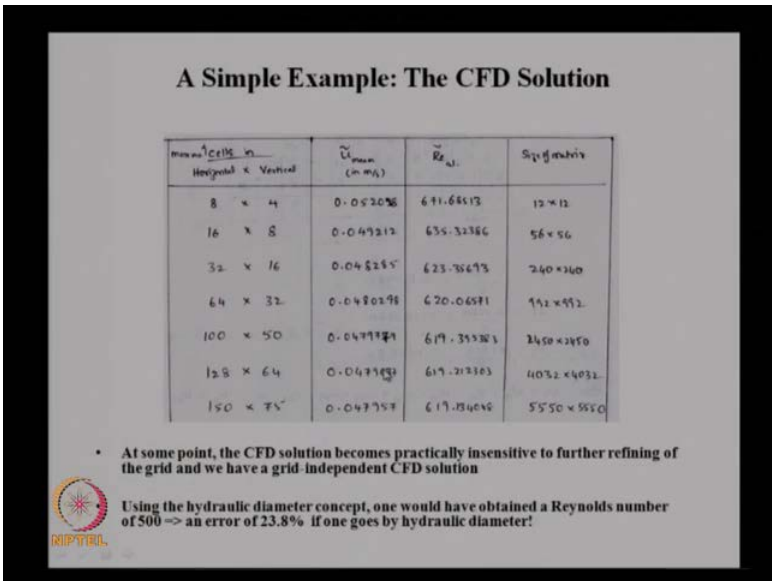

And here, this is an evaluation with each horizontal division into 8 by 4, more number of cells 32, 64 like this. And the mean velocity, that is computed here and the Reynolds number that is computed here and the size of the matrix. As you go to larger and larger matrixes larger larger sizes, we can see that the computed velocity is becoming constant, and the Reynolds symbols is also becoming constant. So, we are looking here at a solution for the mean velocity for a given pressure gradient which which seems to be ok. And which is obtained using a combination of rectangular and triangle cells, not using a structured mesh, but using the control volume approach.

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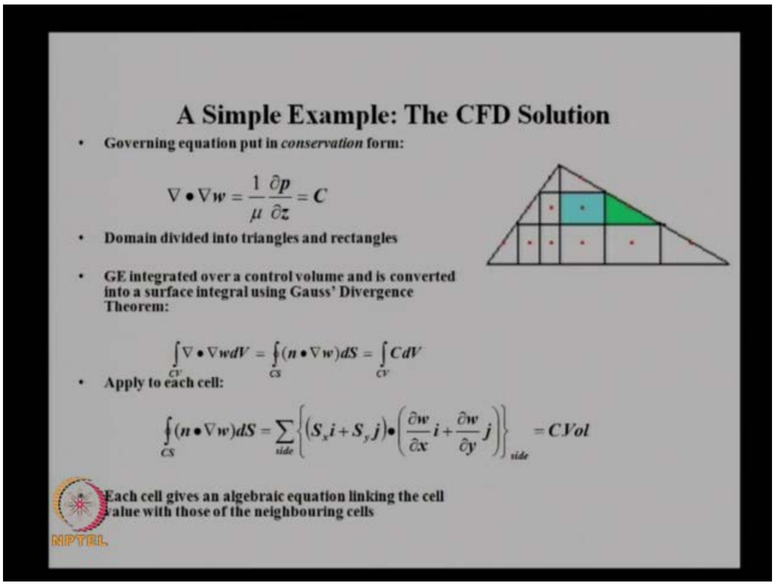

In which, we take the discretized that governing equation conservation form and then converted into evaluation of surface fluxes and volume source terms. So, what we have selected here is a very simple example, because we have seen the case where the flow is a fully developed. And that means that some of the significant components of the fluxes do not appear. If you look at our governing equation here, we only have the diffusive flux coming on the left hand side and here is the source term.

So, we do not have the convective fluxes and that that makes grows simplification or trivialization of the problem, as we have seen here that is appropriate for an example, but in in reality we have to consider the convective fluxes. The fluxes that flux of a particular component coming associated with the flow, and that usually means that that is a difficult more difficult thing to do, because the value of that depends on the upstream value it does not depends on the local values. A diffusive flux depends only on local values, and a convective flux depends on the velocity direction. And that provides a bit of challenge, when we deal with that in the context of finite volume method.

So, what we will do next is to take the general case the take the general case of conservation equation, which not only has the diffusion and source term, as we have considered in these example, but also the temporal and the convective terms into that. And we can see how we can put the whole thing together in the form of a solution procedure, which will be applicable to the general case. The that is it It is important to able to see, how we can put together the whole process of evaluating the fluxes and the areas and the volumes.

And once we have a good understanding, we can see that the overall method is a very flexible method. It leaves us with the choice of defining the cells or the tiles which make up the control volume. So, it gives, it offers us a great deal of flexibility in terms of refinement of grid and refinement of a local zone for improved accuracy. And those kinds of things can be more readily done in a finite volume method, there in the case of structured method, but we will also see that the evaluation of the convective fluxes puts makes it more difficult to incorporate higher order schemes for the first order derivatives.

And that is one of the difficulties associated with a finite volume method. So, there are deftly advantages and there are also disadvantages with the finite volume method. Just as we have them with the structured mesh in the body fitted coordinate system, but comparing the two, I think one would say that finite volume method is more user friendly, more easily adaptable method, more adaptable than the body fitted grid approach. But, body fitted grid approach is more powerful in terms of a systematic exploitation and the full scale implementation of a high degree schemes, compact schemes, high compact higher order schemes is not so trivial in the case of finite volume method.

So, if you are looking at highly accurate solutions probably the structured grid approach is better. But, if you are looking at user friendliness and in terms of being able to tackle complex geometry without too much of difficulty, then I think the finite volume method is preferable. We still have to do lot of work, before we can say that we know how to use these methods for the general case of a three-dimensional flow. So, that will be the subject of the next couple of lectures.