Computational Fluid Dynamics

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Module No. # 07

Dealing with complexity of geometry of the flow domain

Lecture No. # 40

Need for special Methods for dealing with irregular flow geometry

Outline of the Body-fitted grid approach

Coordinate transformation to a general, 3-D curvilinear system

We now are at a stage where we can say that we know what equation to solve for a general three-dimensionalterminate chemically reactant non-isothermal flow. So, using the time averaged approach,we cansolve forfairly complicated flow problem, but we still are saddle with difficulty that our flow domain has to be very simple.Because we have so far considered onlyrectangular dimensions. And it is fairly straight forward to extend this tosay polar coordinates, where we haveflow through a pipe or flow in the annulus space between twopipes;two circular pipes or even for the case of flow overperfectly spherical ball.

So, in all these cases one canuse appropriate cartesian coordinate system or a polar coordinate system or a spherical coordinate system to represent the flow domain. And as long as we do that we are quitewith whatever techniques that we have leant, but if you are looking at a more complicated case. For example, if you are looking at a cricket ball where you have specific stitch pattern that you would like to take care off.Then you have a difficulty in rist in describing the stitch pattern in a spherical coordinate system. If you are looking at a conicalsurface,if you are looking at a flow through a converging diverging duct.Then you cannot represent the whole flow domain in exactlypolar coordinate system.

We are looking at a flow domain which can be described in terms of constant.In which the boundaries of the flow domain lie along constant values of the coordinatecoordinates,for exampleconstant x, constant y, constant z, constant r, constant theta, constant phi. So, these are all the kind oforthogonal coordinateframes of constant coordinatedirections, which can be easily representedusing the techniques of c f t that we have learnt.

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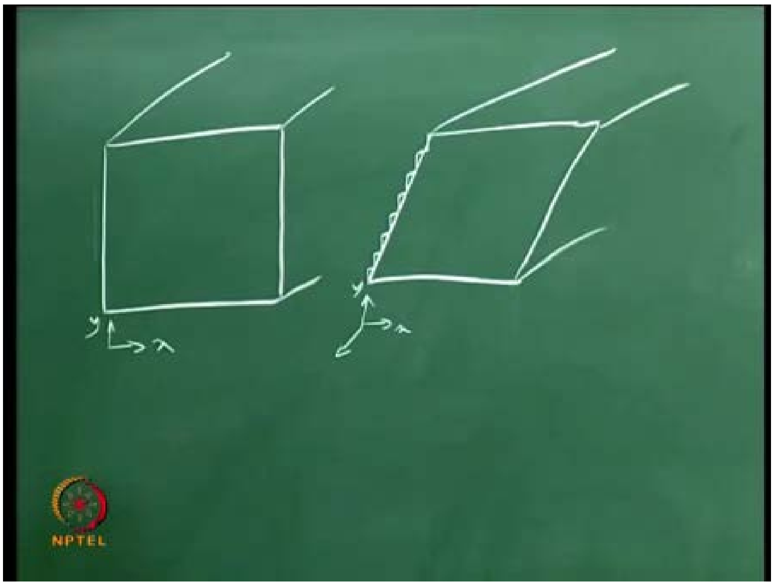

But if you have for examplea simple case ofa flow domain, which is not really like this, which we already considered that a duct like this.Here, this particular wall if you want to describe this in terms ofx and y coordinate system. This wall is alongconstant y line and this wall is along constant x lineand this surface is along constantz and so on. But if you consider, if you want to describe the same thing in terms of x y z coordinate thing one which is like thisthen; obviously, this wall is not along constant y line. So, it is this does notfallen to thegeneralkind of technique.That we have dealt with and you can break it up into something like constant x, constant y like that, but that is not a true representation of the surface.

You have a surface, which iscoming up like that and if you if you are very conscious of the nature of the surface then you; obviously, cannot havethis stair case kind of pattern to represent an inclined wall.For example, if you looking attheeffect of the seam of of theon thethe seam on the cricket ball. And the effect on that on the aerodynamics thing then; obviously, you cannot treat that seam to be in this stair case pattern.

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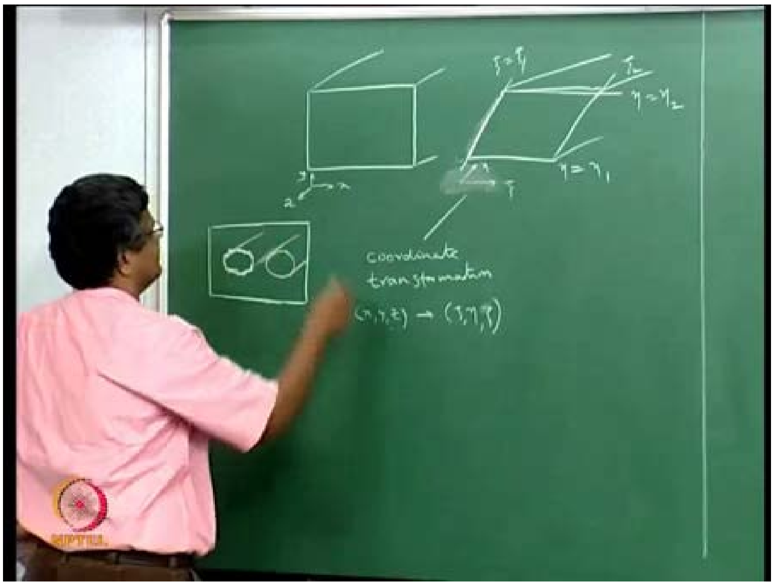

You would like to be a smooth surface and if you consider anothercase, where you havea rectangular duct withtwo tubeswhich are placed. And then you have flow going over these tubes and so, in such a case. You cannot represent this wall here using a cartesian coordinate system. Again you would have to use something like this stair case kind of approach with this.So, this in these kind of cases, where it is difficult to represent the bounding surfaceson constant x, constant y, constant z or constant r thetaplanes like that. Then we have a difficulty inusing the conventional c f t that we have so, far considered.

We have to go from here, we have to tackle these kind of geometries in a different way. We have to treat them in a such a way that either we have a system thereby. We cantreat the bounding surfaces as the constant coordinate lines not necessarily x y z that is one approach. And the other approach is to treat them using finite volume of finite element kind of methods.So, when we talk about geometry like this. We have the possibility of using coordinate transformation (no audio from 05:54 to 06:04)fromx y z, which is what we are using heretopsi psitau something like this.Where this is our regular coordinate system and this is a different coordinate system and constant lines of psi psi and zeta here willrepresent the values here thebounding domains here.

For example, we know that, we cannot represent this is a constant x or constant y line in this, but if you had if you had these coordinate lines is that.This is psi and this islet me put it psi eta psi like this. So, if this is let us worked in two dimension. This is our psi here and this is our eta. So, that this corresponds to eta equal toeta 1.And this line this surface corresponds to eta equal to eta 2.And this coordinate line corresponds to psi equal to psi 1.And this corresponds to psi equal to psi 2.Then these surfaces the bounding surfaces can be describedin as being a coordinateconstant planelike that. So, in in this case we can then do the discritization.

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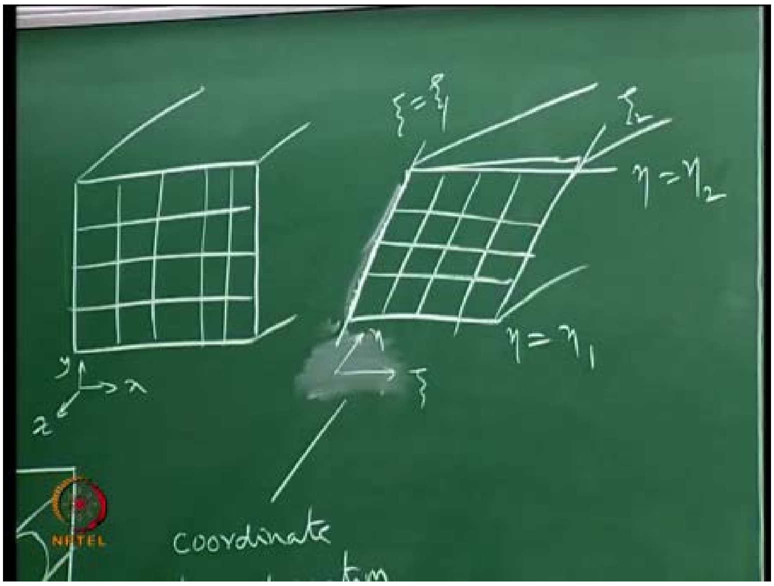

So, we have a grid lines here,which are parallel to the coordinate lines. (no audio from 08:06 to 08:10) In this case, we can have grid lines which are parallel to these coordinate lines.And these are not along this direction both y and x are changing. So, in that sense this is not either constant x or constant psi line constant y line, but this is a constant psi line and this is a constant eta line. So, by going by doing a coordinate transformation from the x y z into a different coordinateframein which the bounding surfaces are along constant values of these coordinate directions. We can tackle a complex geometry and the simplest example of this is going from x y z tor theta z.

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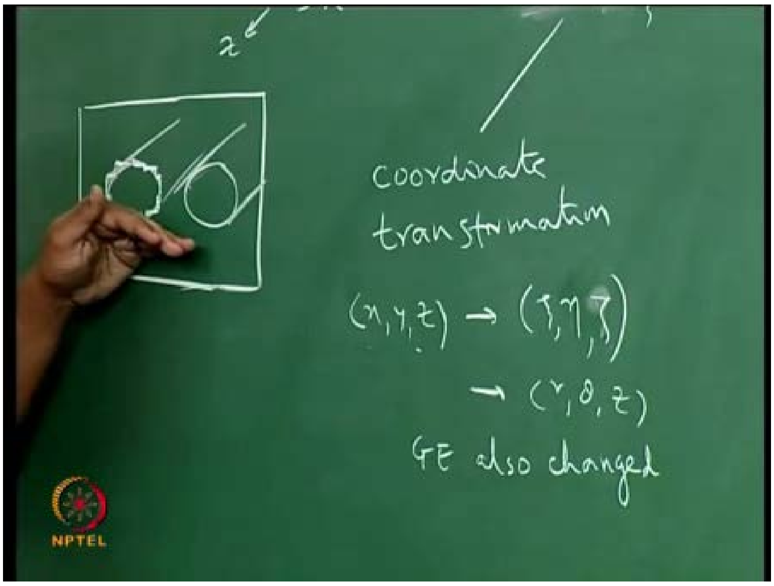

For example when you want to look atflow through a pipe. You do not represent the correspondingsituation in x y z. You would use an r theta z coordinate system. And so, you are going from x to r and y to theta. So, you have changed we have done a coordinate transformation. And in the process what we also know is that the governing equations also are changed. (no audio from 09:31 to 09:37) So, in this approach although we can describe thethe domain. Although we are consume in x y z coordinate system the typical cartesian coordinate system. We actually do not write down the equations in cartesian coordinate system.

Because it is easier to write them down in a cylindrical polar coordinate system. So, we go away from x y z coordinate system to r theta z coordinate system. And we deal with a transformed equations representing the same consideration.

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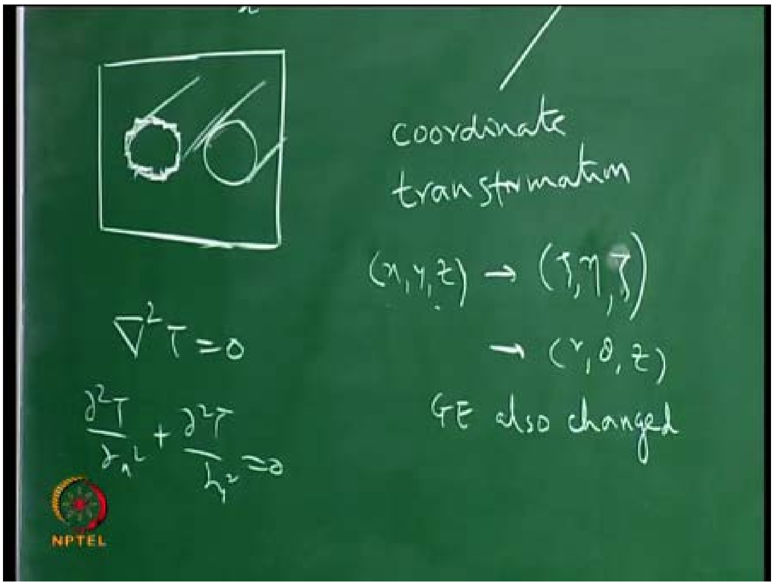

Exampledel square T equal to 0 is a heat conduction problem.It is represent a heat conductionand this this is applicable in any geometry, when we are dealing with a rectangular geometry. We write it as dou square T by dou x square plus dou square T by dou y square equal to 0 in x y coordinate but if the same thing where ininina polar coordinates in in a circular pipe. We willwriting theradialcoordinateequation for this depending on whether it is r theta r z theta z. There it can in many different two dimensional combinations of that. So, one approach is to move away from the restriction ofx y z or r theta z or r theta phi type of transform coordinate frames.

That we are very familiar with into an arbit arbitrarilydefined coordinate system psi etazeta.Such that the domains of the bounding surfaces on, which we want to apply the boundary condition will lie along parts of constant psi eta and zeta lines. And in this way we can take proper account to the shape of the of the flow domain. And it is restriction boundaries andandtherefore, we can take proper account to the boundary conditions and so, this is one approach.

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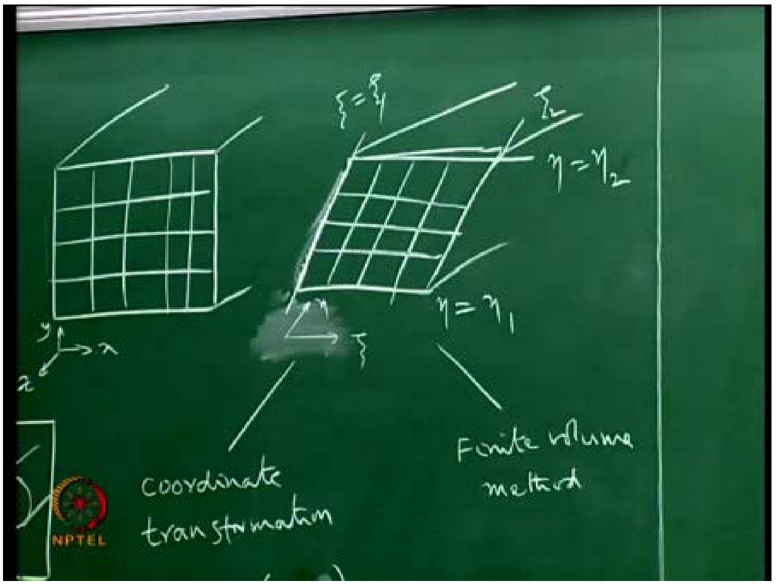

The other approach is to use for example the finite volume method, which can be considered as a variation of the finite element method, where inyou do not (( ))deal with aa structure grid. But we try to write down the consideration equations in each of these in a specific form. And thereby dividedefinederive equations for for thevariables and how they change from point to point and all that.So, the finite volume method is much more flexible.

It does not enforce on you that, you must have a structure grid.For example, with fourfaces in a two dimensional coordinate system. The moment you say it is constantconstant eta lies and constant psi lies here, every control volume has four faces. And that is a structured grid inin two dimensions in a finite volume method. You do not need to have only four faces you can have more number of faces.

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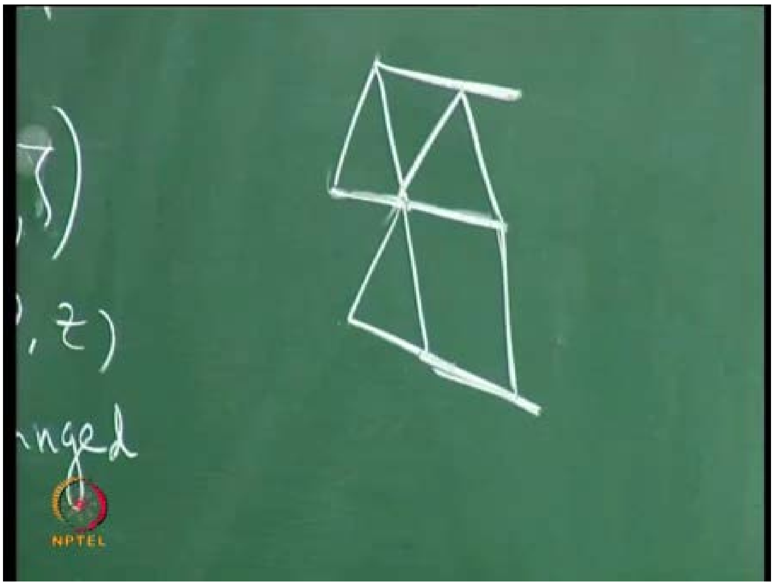

So, if you haveadomain like this. You can make up a control volume; you can make up your using you can have a control volume. In which you start your equations either having foursided thing or a threesided thing andany sided. You can even make a this whole thing to be one control volume. And thereby you can tackle any complicated geometry, where you readilyusing this using the finite volume method. So, there are twodifferent ways ofdoing this two different approaches both are tactics and both have their advantages anddisadvantages. What we will do is we will try to take a look at theprinciples of each of these.And try to understand what is involved in in doing this once; we understand this approach and this approach.

Then we can choose one of these twoapproaches and tackle any complicated geometry. And one would say that being able deal with the coordinate transformation and being able to deal withwith non-orthogonal (no audio from 14:25 to 14:31) coordinate system. Satisfactorily has led to theexample the explosion of the useof c f d further range of practical problems. So, this is this can considered as this has happened inearly eighties in a way. And that is when people have startedusing thiseven in processindustries in real earnest because one is not restricted to tackling these simple geometries.

One could consider the fullcomplexity of the flow domain. And then the corresponding flow changes modifications associated with that. And finite volume method has also led to a similar kind of explosionand it has led to lot moreease of competition. And some of thea lot more user friendliness into the use of computer course has been brought in by finite volume and finite elemental methods. And this has also led toa great development in the usage of c f d. And now,a days both have practiced to such an extent that one can say that any complicated flow in any complicated geometry can beattacked using thec f d techniques.

Of course there are lots and lots of areas in problems, where c f d cannot be readily applied and used but the range where one cansay with confident.yesThis is a problem that I can attempt and I can doresolve. Using c f d that range of problems has tremendously increased withthese twomethods, which will enable us to tackle realistic geometries that are often found in practice. So, let us try to understand first this and then this approach to tackling aa non-standard geometry, which involved it into either ofthe cartesian or cylindrical or spherical coordinate system. So, what we are looking at is that the approach that is followed in thisis that as given in this example.

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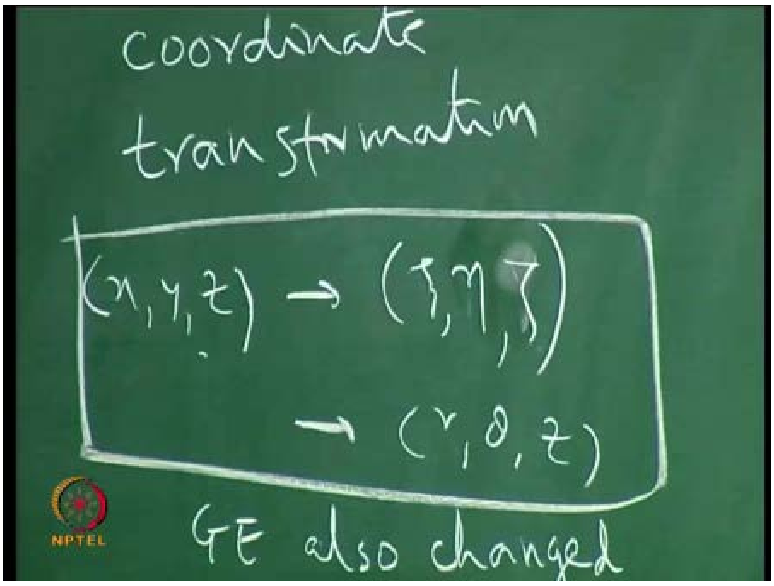

We abandon or attempt to describe the flow domain in x y z coordinate system. And we move on to a more convenient coordinate system. In which the bounding surfaces or along constant coordinate lines. And along with the process, where the moment we abandon x y z. And use this we have to represent the variation of a parameter of interest not in x and y.

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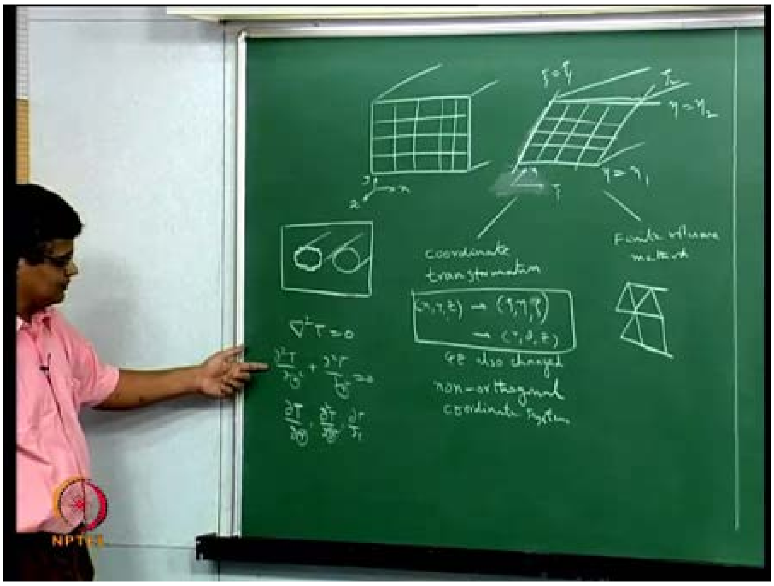

But in terms of dou T bydou r for example, if it is r thetadou square T bydou theta square anddou Tby dou zor let so, in the sense we move away from x and y. And we write it in terms of r and theta. So, we have to change transform or coordinate system from here to here.

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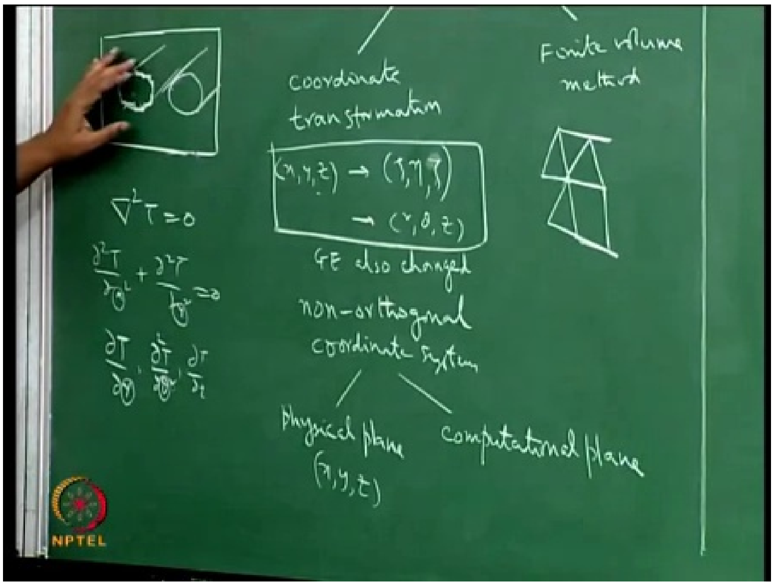

So, this is where the idea oftwo grid approach comes into picture it is not a multi grid. We are looking at a physical plane and a computational plane (no audio from 18:18 to 18:26) physical plane is the plane in which we describe our system.For example, this is our domain and if this cylinder is placed here; and in this cylinder is placed here; with a center of of this and this radios is this and so, on.So, this is an x y z plane, but we do not solve the equations in this plane. We transform from this physical plane into psi, eta,zeta here. And then we transform our equations also from here into this.

We compute the solution so, in that computational plane here. And then transform the solution back into the physical plane. So, we have an identification of where we want to have the variables of interest here. And wewe export them into the computational plane, which is in described in terms of psi, eta, zeta. And we write the governing equations in this plane we discreties this.

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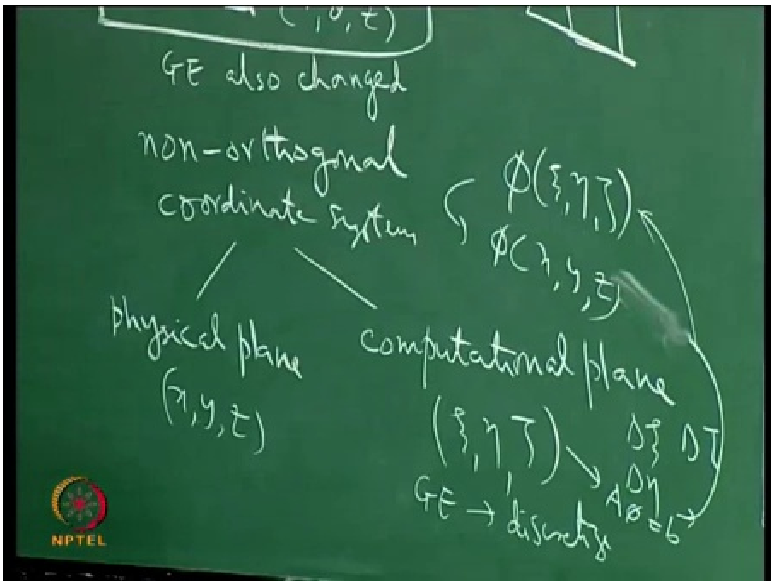

So, we are talking about delta psi, delta eta and deltazetabeing use towrite express these things. So, from these we convert the governing equation into A phi equal to b, where a consist ofthese instead of delta x, delta y, delta z. We have now delta psi, delta eta and all these things and then we finally, get by solving this. We get phi at every psi eta zeta that we wanted to do. Now, we have an understanding of which point of x y z corresponds to which point of psi eta zeta. So, once we have thisphi at many many points on the computation domain.

We say that since this point there is a oneto onemapping between the computation plane and the physical plane. Then we say that this point, which is computational plane of psi naught and eta naught and zeta naught here has a corresponding physical space location, which when we say that x y z. So, there is a oneto onemapping between a point in the physical plane and point in the computational plane. We do all the computations that is starting of the governing equation isdiscritization analysis. Andconverting into a phi equal to b solution of the simultaneous equation.

And all that is done and finally, we get wegetthe solution of phi in the computational plane.And since will have what point this corresponds to in the physical plane. We say that at this point in the computational plane the phi is this and therefore, at the corresponding point the value of phi is here. So, only the solution is isgiven back is mapped from the computational plane and physical plane. So, in the process we have a mapping of the physical plane. The geometry in the physical plane into the computational plane and using that mapping.

We transform the governing equations, which are described in physical space in terms of x y z into computational plane. And the transformed governing equations are descritized and solved in the computational plane to get the solution in the computational plane. And using that mapping between the physical space and the computational space. We we map the solution back from computational plane into physical plane. So, we havesomebody else doing all the computations and giving us the solution. And that somebody else does not; obviously, work in x y z he works in a coordinate frame of his own interest.

So, associated with this approach is the idea of deriving a mapping between x, y, z between the physical plane and the computational plane. And transforming the computational planethe governing equationsinto equations, which are applicable in the computational plane. Once it is done, it becomes same as our mathematical problem. We have certain derivatives, which we try to approximate using a Taylors series expansion and so, on. We put them together we do analysis for consistency and stability.And then we choose a descritization scheme, which was good and then we that becomesmatrix equation.

We have several such matrix equations and we solve them using either gauss siedel ormulti grid or whichever, which we think isthe important thing is the most appropriate and then you get the solution here. So, in this approach what we need to understand in addition is to this transformation of physical and computational plane. And transformation with boundary conditions and we will see what is involved in in this process.

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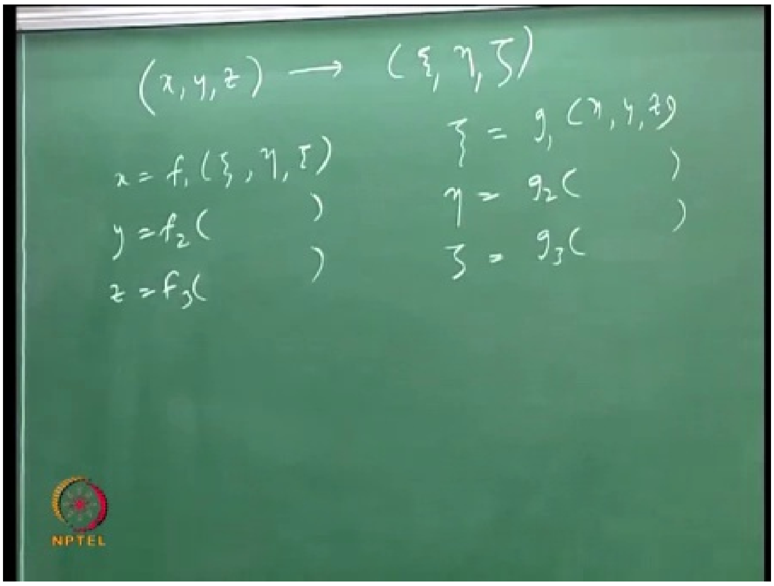

First we will look at transformation from x y z into psi, eta,zeta here. Andwe notice that because of this x is a function ofpsi, eta,zeta. And y is also function of this z is also function of this. And similarly, psi is a function of x y z; eta is a function of x y z and zeta is also a function of z y z in the general case.

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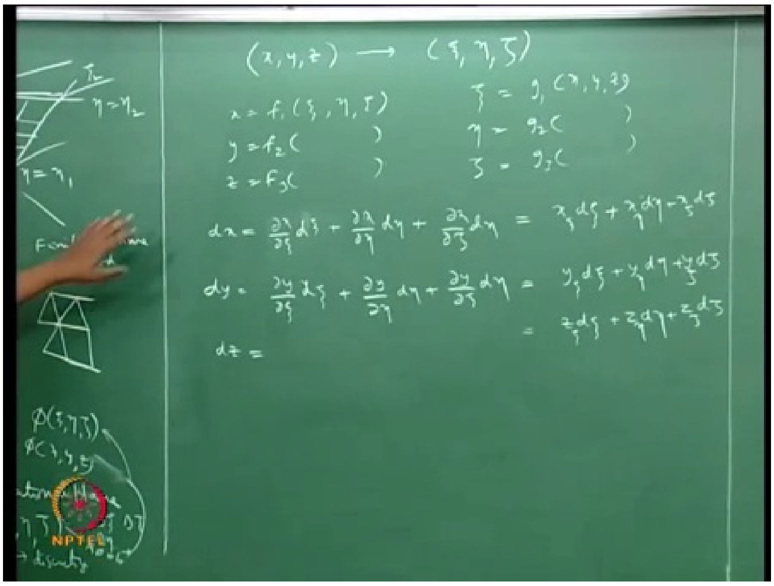

So, we can write x is a function of these things. So, we can write d x equal to (no audio from 25:06 to25:39)thisis a function of (no audio from 25:42to 25:49) dou x by dou times d eta. And similarly, we can write d y, which is a function of those three variables can be written as the specific derivatives here. Depend on what kind oftransformation we have (no audio from 26:13 to 26:24). And we can wecan introduce a small simplification of notation here this dou x by dou psi here.

We write as x subscript psi this indicatesa differentiation of this with respect to psi here. So, we can write this x sub eta (no audio from 26:51 to 26:59) this is a change of notation otherwise it is nothing.Plus y eta d eta y psi y zeta d zeta and similarly I can write d z to be by the same equal to z psi d psi plus z eta d eta plus z zeta d zeta.

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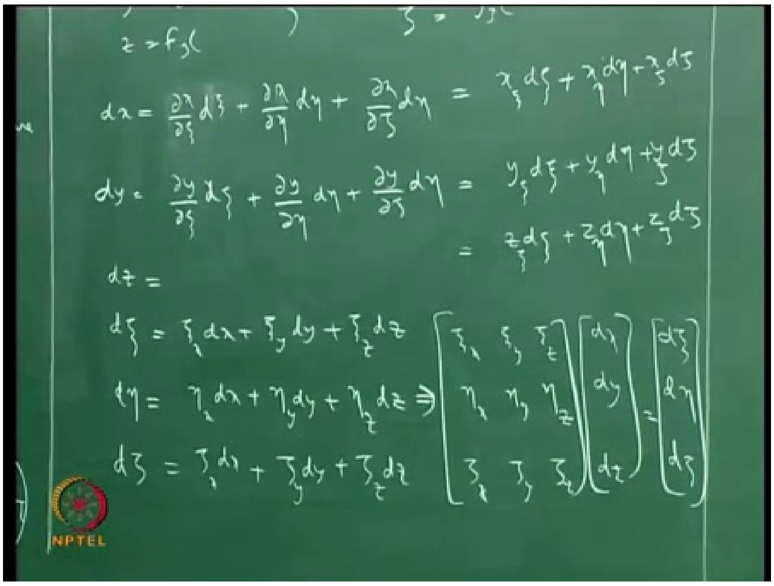

And so, we can write therefore, we can also write from here d psi, which is function ofx y z can be written. Similarly, as psi xdx plus psi y dy plus psi z dz d eta can be written as eta x dx eta y dy plus eta z dz at finally, d zetacan be written as zeta x dx this; obviously, means dou psi by dou x as per our notation and psi y dy plus psi z dz,which we can write in matrix notation as psi x psi y psi z andeta x eta y eta z zeta x zeta y zeta z. So, we can combine all these things here and say that this is equal to d psi d eta and d zeta.

What it means is that thethe totaldifferential of psi is now; a sum of differentialswith respect to differentcoordinates that of, which this is a function here. And this transformation the differentials the total differentials dx dy dz can also be express like this. We can also write(no audio from 30:08 to 30:19) equal to here. We come x psi x eta x zeta y psi y eta y zeta z psi z eta z psi times d psi d eta and d zeta.

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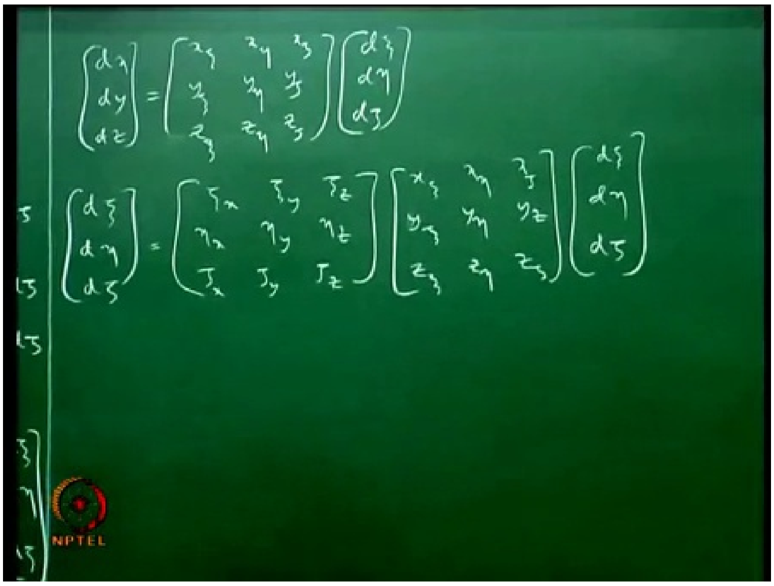

So, we can substitute this for dx dy dz here in this. And therefore, writed psi d eta d zeta equal to psi x psi y psi z eta x eta y eta z zeta x zeta y zeta z times. We substitute for this this thing here x psi x eta x zeta y psi y eta y zeta z psi z eta z zeta times d psi d eta d zeta. So, from this we have this here and this here. So, obviously, these twoshould give us the identity matrix the multiplication of these twoidentity matrix.

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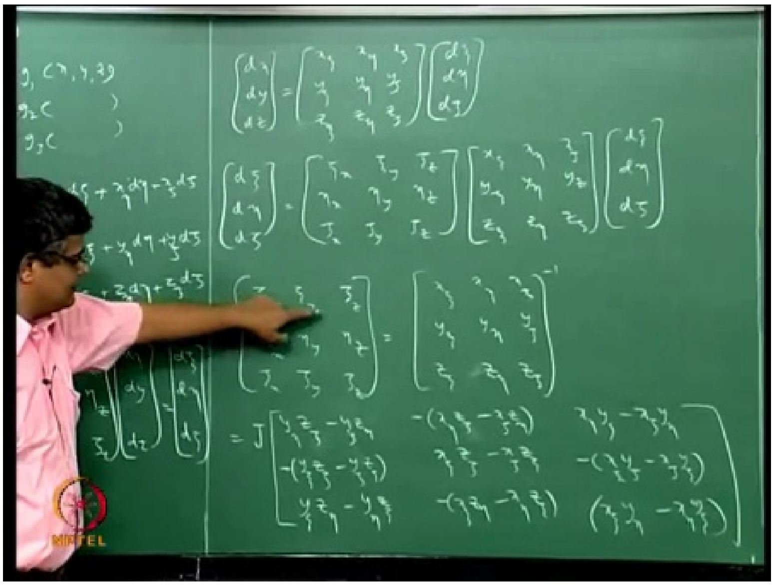

And therefore, we can write a very useful result psi x psi y psi z eta x eta z zeta x zeta y zeta z equal to inverse of psi x x psi x eta x zeta y zeta z psi z eta z zeta inverse and this can be written asJ which is a Jacobean. We can y eta z psi minus z zeta. So, these two minus these two.So, that is. So, that is the first term and thenhere we have minus y psi z zeta minus y zeta z psi and y psi z eta minus y eta z psi. So, these are the three elements here and here we have three more x eta z psi x zeta z eta x psi z zeta minus x zeta z psi minus x psi z zeta minus x eta z psi.And x eta y zeta minus x zeta y eta minus x psi y zeta minus x zeta y psi and finally, x eta x psi y eta minus x eta y psi.

So, the matrix of the transformation. So, that is dou psi by doux dou psi by douy dou eta by doudou y all these things are given in this. Where J is known asJacobean of the transformation (no audio from 36:03to 36:19) is this the determinant of zeta x zeta y zeta z eta x eta y eta z and zeta x zeta y zeta z. And this can also be written as 1 by J inverse inverse of the transformation.

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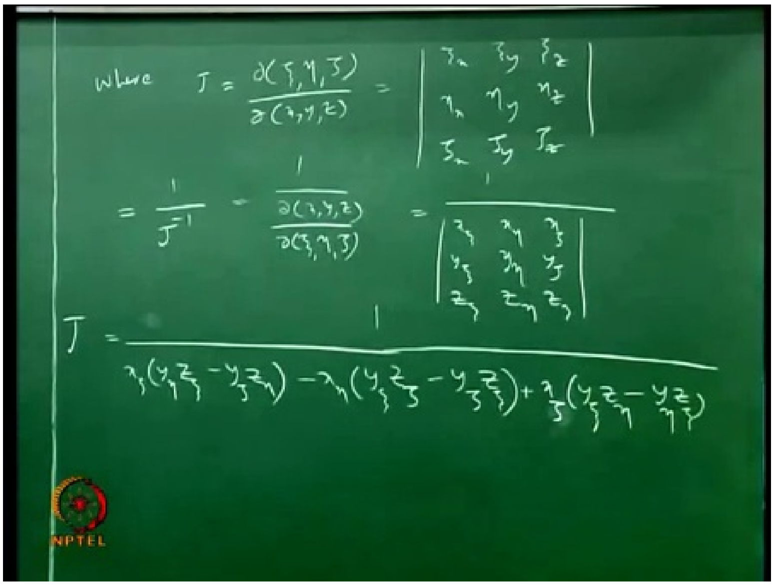

So, this is 1 by dou x y z by dou psi eta zeta. So, this is 1 by x psi x eta x zeta y psi y eta y zeta z psi z eta z zetaand finally, this is equal to 1 by (no audio from 37:28 to37:44) minus x eta y psi z zeta minus y zeta z psizx zeta times y psi z eta minus y eta. So, this is what J is and what this actually tells us is that this matrix each of thispsi x is given by J times this here. And similarly, this is given J times this and this given by J times thisand so, on. And together these relations define thehow we can evaluate the matrix of the transformation these matrix are important because these matrixes appear inthe, when we transform our governing equations from x y z tofrom x y z to psi eta zeta like this.

So, we need to be able to evaluate this and these relations and the corresponding relation for x and all these things tell us that if iffor example, in doing this. If we know the relation of how x y z are related to this. If we know this then we can immediatelydifferentiate this with respect to psi eta and zeta. And evaluate the xthese matrix and from this we can get the Jacobean. And these are alreadyderivable from the known transformation here. So, we can therefore, get the corresponding matrix of psix psi y and psi z like this.

So, either we should know this or we should know this, if you know this. Then we can getx x psi and all these things all these things from similar relation or if you know this. Then we can get the corresponding matrix in this now, the why we want to do this is at when we are doing computations in the computational plane. We are going to write derivatives with respect to derivatives of the parameters of in case with respect towith respect topsi eta and zeta. So, we want express things in terms of doudou phi by doueta dou psi like that and so, that what is going toappear. And that is where this this information comes. And here what we have is this is the mapping that we are looking at in terms of x in terms of psi and all these things.

So, how for example, if we can express this in terms of x and y then, we get a transformation like this.And if you express the corresponding this line in terms ofthese two things then we get a transformation like this. So, when we start about doing the problem like this.We want to have a coordinate system in which these lines here correspond toconstant x and constantpsi lines. So, from finding the transformation one is for one way finding with transformation come out with some analytical thing for example, we can say that.

This line if this is theta here then we can say that this is equal to this divided byby tan theta or something like cosine theta is what this this will give us. So, we can have this kind of algebraic trigonometric expression betweenthe x and the corresponding x y z and the corresponding psi eta zeta lines. Otherwise in the general case, we have to come up with a numerical description ofthis kind of transformation. So, that is where these transformation relations will actually will be helpfulthese will be (( ))using this.

One canone can come up witha coordinate frame,coordinate transformation numerically for an arbitrarycoordinatefor an arbitrarily complicated geometry like this. So a geometry, which is described.In in this physical plane like this can be transformed into a correspondingcomputational plane description.

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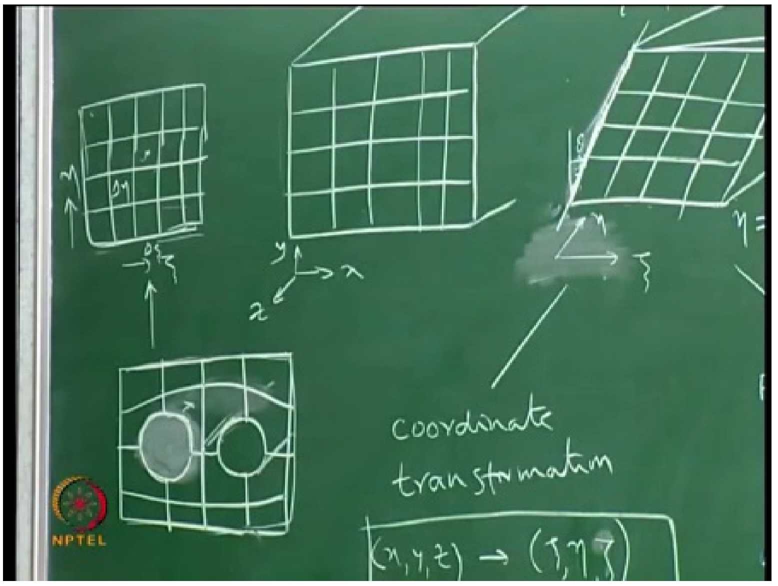

In which we have for example,psi here and eta here and lines in in this computational plane. We have deadlines, which are straight and easy anddescribed in terms of constant delta psi and constant delta eta like this, but this line here.In reality that is in the x y plane may correspond to something like this. And this line may correspond to something like this. And thereby we can expect one more here and this line here corresponds to this this line. Corresponds to this and this line corresponds to this. So, you have this line here corresponding to this and these lines may correspond to something like this.

So, what is a rectangle here, with delta psi and delta eta as a two dimensions in physical plane.It is actually likelike this this represents that particular volume. So, this transformation from a physical plane. So, the shape in computational plane is nice and easy, but the same thing is may to correspond to the real complicated shape in the physical domain. So, as a result of this, we may get something likethe velocity or something at this point. And that point corresponds to in the physical plane to some here. So, we can get this value by doing the computations in thisand getting the valuehere.

And then same that this control volume corresponds to this. So, this centroid corresponds to this and this where I have the points. So, we are there is a certain relation between what thiscoordinate lines are in computational plane.And what the coordinate lines are in the physical plane. So, in the physical planethe psi eta zeta lines are curved. And they are allow to curve in such a way they go throughthey also go through at least some of them will go through thebounding surfaces here, but in the computational plane for ease of discretization.

These are lines you constantpsi and constant eta. So, that you have justa square matrix a rectangular matrix with rectangulardomain with delta psi and delta eta. So, in this is thekind of relation, we are looking at by this is the advantage also. That we are getting by doing this transformation a complicated shape here with curved boundaries is made to represent in the computational planea rectangular boundary. And the treatment of these rectangular boundaries is very easy in our computationaldomain, but the treatment in the physical domain is more complicated.

So, instead of doing all the derivatives and all these things in the physical plane along curved boundaries. We do it in the computational plane alongsimple grid kind of approach. And then we come back into into the physical plane and in the process. If we say that this represents this corresponding variation ofdou Tdou x here corresponds to something else in terms ofdou T bydou psi anddou T bydou eta, because now x is a function of both psi and eta here.

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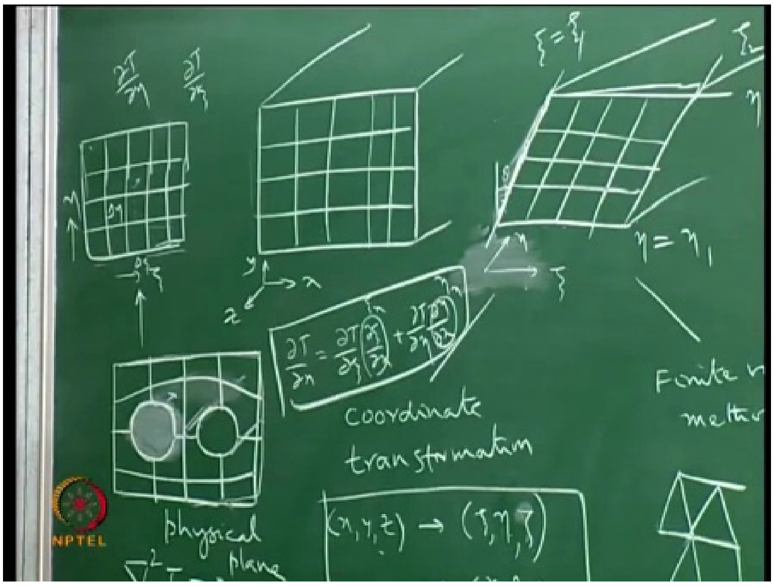

So,dou T bydou x now becomes a function ofdou T bydou eta and alsodou T bydou psi. So, one can say that this is equal todou T bydou psi timesdou psi bydou x plusdou T bydou eta timesdou eta bydou x and one can see that this is our psi x and this is our eta x. So, these are the metrics of the transformation that we are expressing here. So, andthat is how we we make the relation between we do the whole computation for this complicated shape. We get a relation we get a transformation between x y z space, andpsi eta zeta space and using this this kind of chain rule of differentiation.

We convertdou T bydou xdou T bydou ydou square T bydou x square all those derivatives into derivativesin terms of psi eta zeta. And in the process we have to make use of we have to come up with these metrics of the geometric transformation. And these will compute from the corresponding variation of y with respect toeta, and z with respect to zeta like this. And then using this transformation rules we get the metrics of the transformation.

And that will give us over all discretized equation in terms of the derivatives of the variable with respect to psi eta zeta and the metrics. So, the metrics are known from the physical grid generation. So, that we have now a discretized equation having only the derivatives as the unknown variables. So, thosederivatives are approximated withdifferent approximations. And then, the result in partial differential equation is (())in a phi equal to b and then you solve finally, for phi of psi eta zeta here. And then from this centroid you go back to here. And say that this is the value of phi in the physical plane.

So, this is the approach that we solve that we use here. We have done only part of this we have seen how we canget the metrics of this kind of transformation from a knowledge ofvariation of x with respect to this this physical mapping. We will also look athow to do the transformation and what this transformation enters in terms ofc f d solution. We will see that in the process of this transformation additional complexities are a raise. We will look at this (( ))complexities and then take a complete view of this approach for dealing with complicated geometries that is the approach where we have to do a coordinate transformation and then carry out the solution.