Computational Fluid Dynamics

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Module No. # 02

Governing equations

Lecture No. # 4

Topics

Forces acting on a control volume

Stress tensor

Derivation of the momentum conservation equation

Closure Problem

Deformation of fluid element in fluid flow

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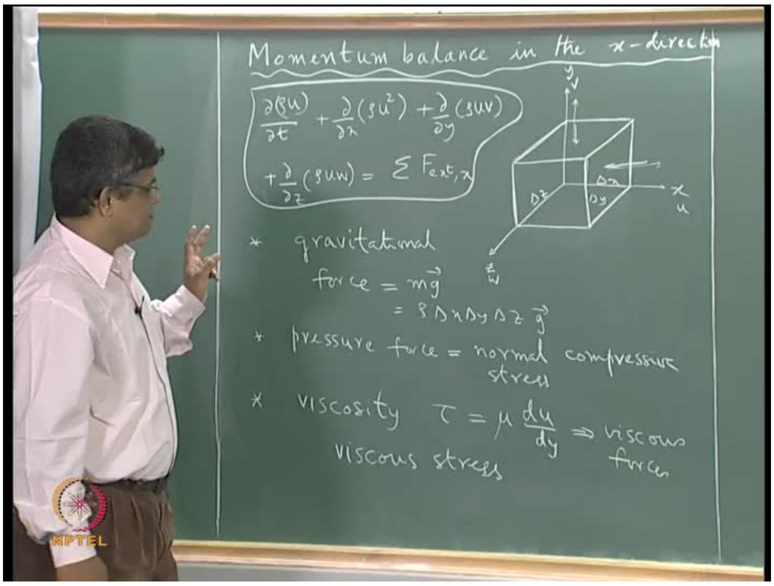

So, in the last lecture, we are considering the momentum balance in the x direction and we are able to write the left hand side of the terms which represent the rate of accumulation and the net outflow of momentum, and we said that the rate of accumulation of momentum in the x direction plus the net outflow of x momentum through the control volume is equal to all the sum of all the external forces acting in the x direction.

So, today we have to specify what those external forces are, so that we can complete the mathematical formulation of the momentum balance equation. So, let us just see where we are for a control volume like this with x length of delta x of y length of delta y and z length of delta z with velocity components u in the x direction, v in the y direction and w in the z direction. We wrote that the statement of momentum balance in the x direction could be expressed as partial derivative with respect to time of rho u which represents the net rate of accumulation per unit volume in this plus partial derivative with respect to x of rho u square plus partial derivative with respect to y of rho u v plus partial derivative with respect to z of rho u w is equal to sum of all external forces acting in the x direction.

So, this was the relation that we have derived and, this statement, this equation is incomplete without specifying what the external forces are. Now, what kind of external forces can be think of acting on this domain on this small domain which has small x delta x delta y delta z and which is one element of the entire fluid domain. So, we are looking at forces which are present everywhere in the fluids, not at the boundaries as coming as boundary conditions and so on.

So, what kind of forces can we imagine? When we consider fluids, we already know that they something called pressure, and so, we can imagine pressure as a force which is acting on this on a fluid element and we know that pressure is a compressive stress. If you imagine a plane like this and it is immersed in the fluid, the fluid above it and around it will be exerting a compressive stress on this which is normal to the surface area vector of this particular plane.

So, in, in that sense, pressure is a compressive stress. We also know another force which is the gravitational force, which is present everywhere. So, we can consider gravitational force which is m g and which we can write as rho times delta x delta y delta z times g. We can consider pressure force; pressure being and inherent property of the fluid. There is a different between a gravitational force which is a volumetric force, which is spread throughout the domain, and a pressure force which is a force exerted on a surface, which is normal compressive stress, and if we imagine, for example, this particular plane which has an outward normal vector in the oriented in the positive x direction coming out like this.

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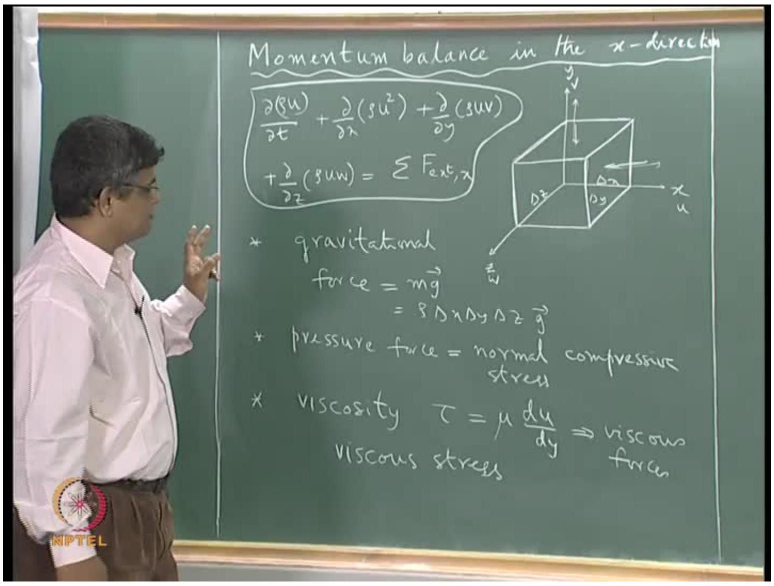

Then the fluid which is external to this control volume will be exerting a compressive stress which is acting in the opposite direction along the same outward normal directional vector but acting in the opposite direction. If you consider this top surface, the outward normal vector is oriented in the positive y direction. The fluid above it, that is, external to the control volume will be exerting a pressure which is acting downwards and, it is, it is a compressive stress acting on this particular plane.

So, the fluid which is external to the body will be exerting a pressure against each other six control planes, the six sides of this control volume and that is what we mean by this pressure force. So, we would like to distinguish between a gravitational force which is a body force, which is acting, which is the essentially the acting on every fluid particle which is contained within this.

And the pressure force which is acting only on the surfaces of the body by the fluid external to this. Now, is there any other force? We know that a fluid has a property called viscosity. Viscosity is a, we call it as a flow resistance offered by the fluid to relative motion and the simplest form is the shear stress is equal to mu du by dy is a what is considered as the Newton's law of viscosity - where this shear stress causes a gradient of velocity or a strain rate which is given by du by dy and it is made propositional to this. What this actually means is that one can interpret it as a relative velocity, velocity gradient here will induce stress which is governed by the viscosity. So, one can say that a fluid has will have viscous forces in addition to the pressure force and gravitational force.

We have as the third element viscous forces just like pressure forces are normal compressive stresses acting on the stress viscous forces are also arising from viscous stress, because this is a shear stress is actually a stress. So, what we can see from this? In addition to this, we may have, for example, for magnetically conducting fluid, we may have ah magnetic forces and electromagnetic forces and other forces which may be coming into picture but we restrict our attention to simple fluids without any extra effects. So, we can consider these as the main forces that are acting on this.

You can, you can say that surface tension is of course that we have to consider and it is an inherent property of the fluid, but surface tension is a force which appears only at something like an air water interface; only at the edge of the boundary, it is not in inside in the interior of the fluid continue, and we are talking about the fluid control volume which is everywhere within the fluid continue. So, unless we have an air water liquid, air liquid interface, at this point with in this control volume, we need not consider surface tension. So, surface tension would come up as a boundary condition rather than as a force, within this, within these considerations.

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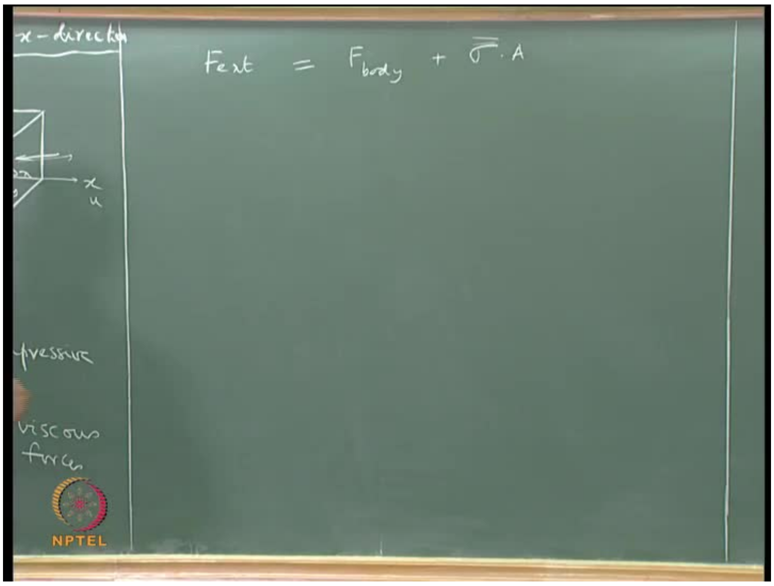

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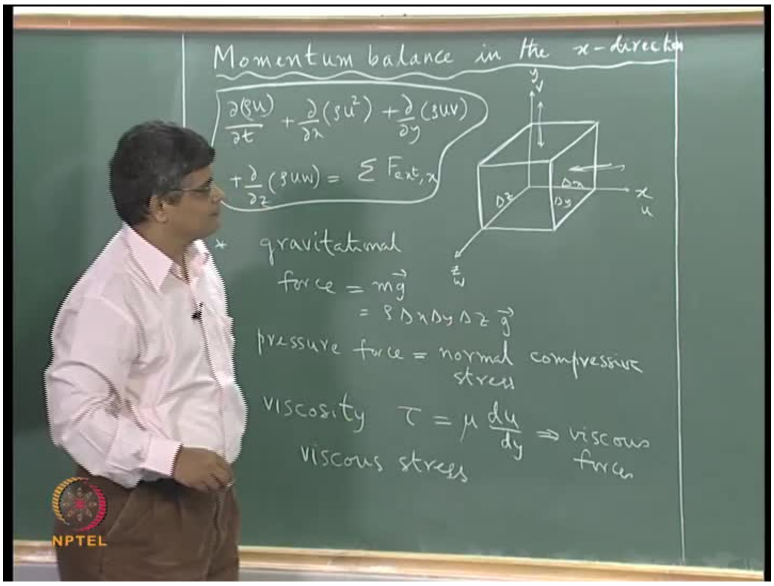

So, from looking at these things, when we talk about a general frame work for external forces, we see that we have to distinguish between two types of forces - those that are body forces and those that are stresses. So, we can say that external force is a type of body force plus a stress. Now, we have to be careful with the stress because stress times particular area is what will give rise to a force here. So, we have a body force and a stress, stresses. We evaluate both of these are separately, and among the body forces, we consider gravitational force which is m g; so, this is rho delta x delta y delta z times g.

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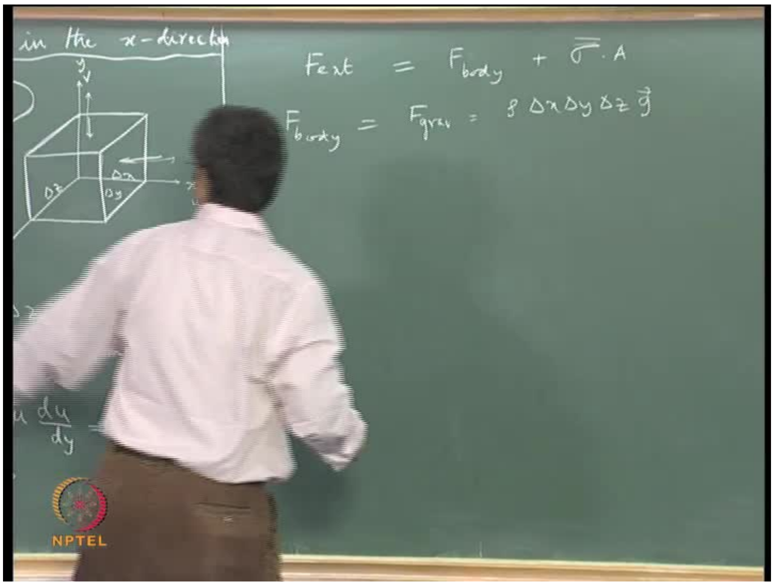

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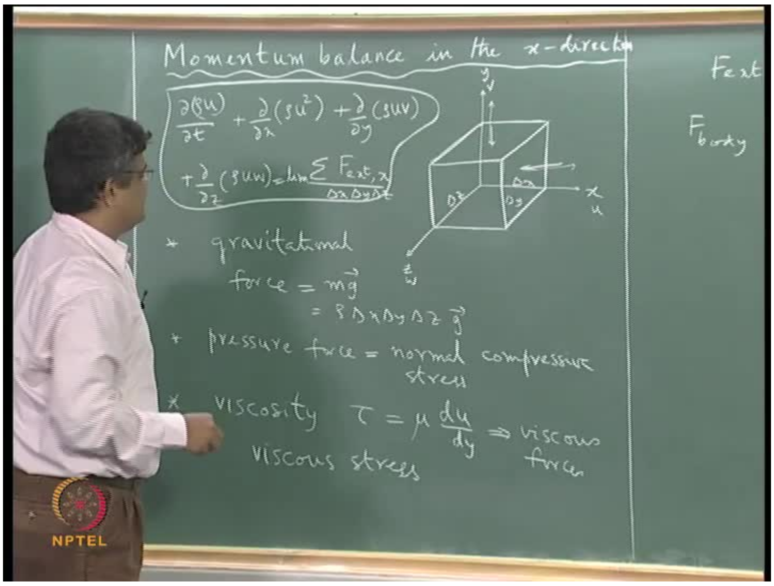

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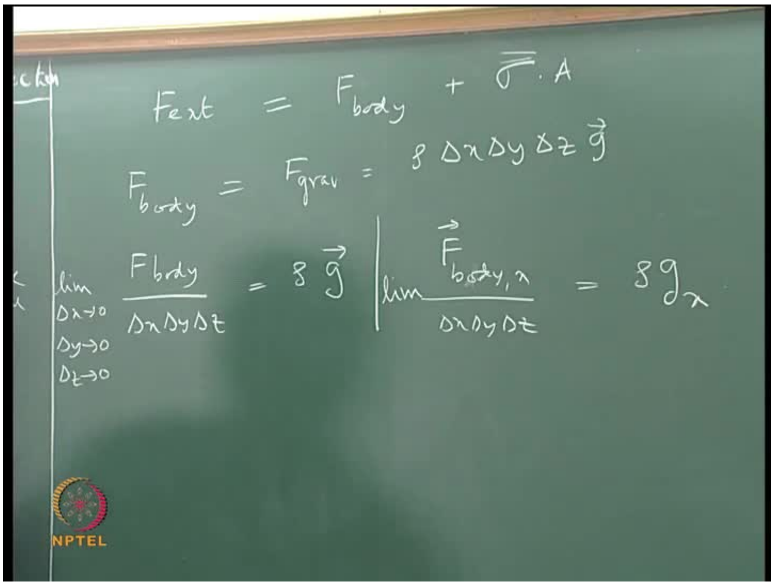

So, when we talk about the body force here, if body force is nothing but a gravitational, so this is rho delta x delta y delta z times g, and when we take the, we have forgotten here, divided by delta x delta y delta z, and in the limit as delta x tends to 0 and all this things. So, now, the equivalent gravitational force in the limit of this divided by delta x delta y delta z are going to 0.

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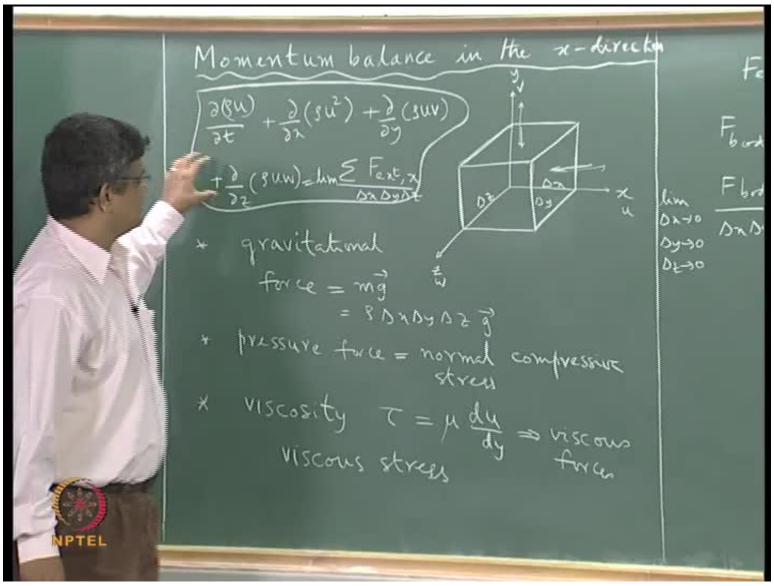

So, we can say that F body divided by volume in the limit as delta x tends to 0 delta y tends to 0 delta z tends to 0 is nothing but rho g, because this cancels out and, we have, we are left with this, and specifically, the x component of the body force by unit volume is rho g x – so, where g x is the x component to the gravitational vector. So, from that point of view, when we consider the momentum balance equation and, when we, when we want to look at the external forces acting per unit volume of the control volume,

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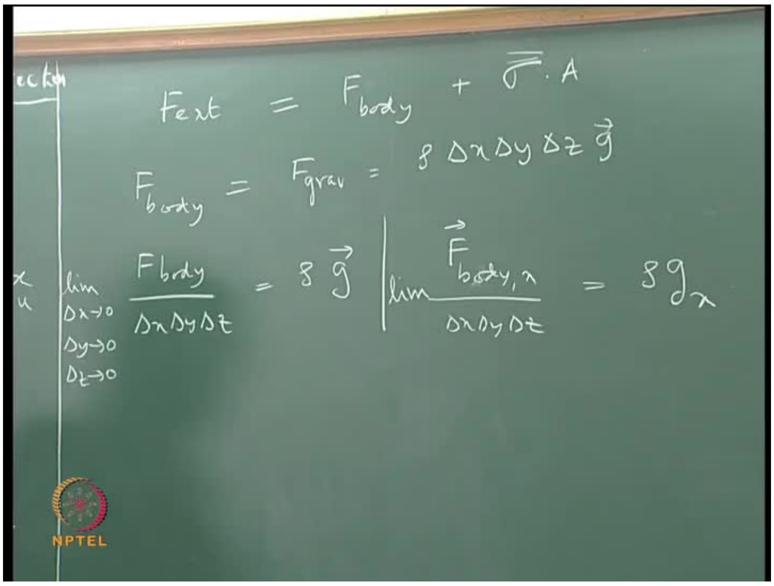

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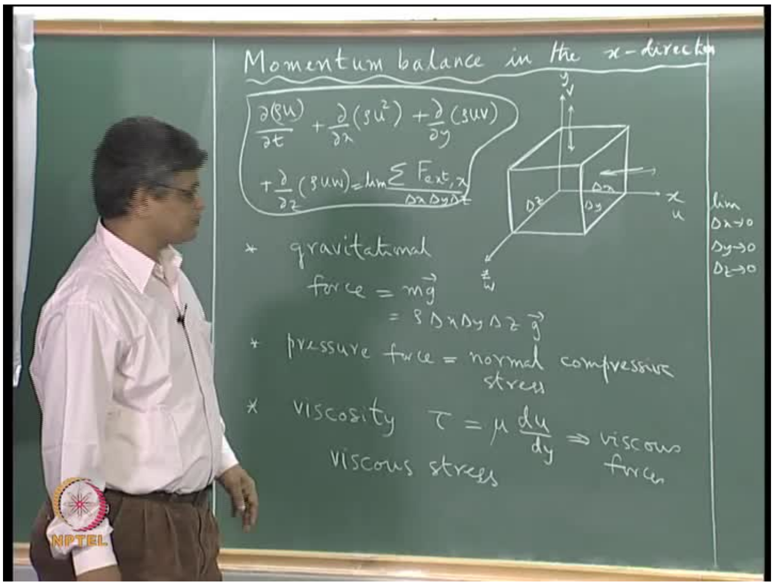

Then this is nothing but rho g x for the body force which is which we are identifying as a gravitational force. Now, what about stresses? Stresses which are coming from the inherent property of the viscosity or from the inherent fluid property of the pressure. As of now we treat them together in a stress tensor sigma because we need to consider that when we when we are looking at a body like this, - three-dimensional body like this - the stress is a tensor which is acting on all the phases and we have to identify this properly and we have to have an establish notion of work stresses.

If we imagine a surface like this, - a plane surface - then we can have a stress which is oriented on to this normal to this stress; normal to the to the surface. We can have a stress in the plane of the stress, for example, in this direction or in this direction. So, we can for every surface, we can have three stress components - one which is normal to the surface. If it is positive, then it will be like this; if it is negative, then it will be going into the surface and a stress which is perpendicular to this direction, for example, if this is x, then this may be y direction, and this stress which is x direction and z direction.

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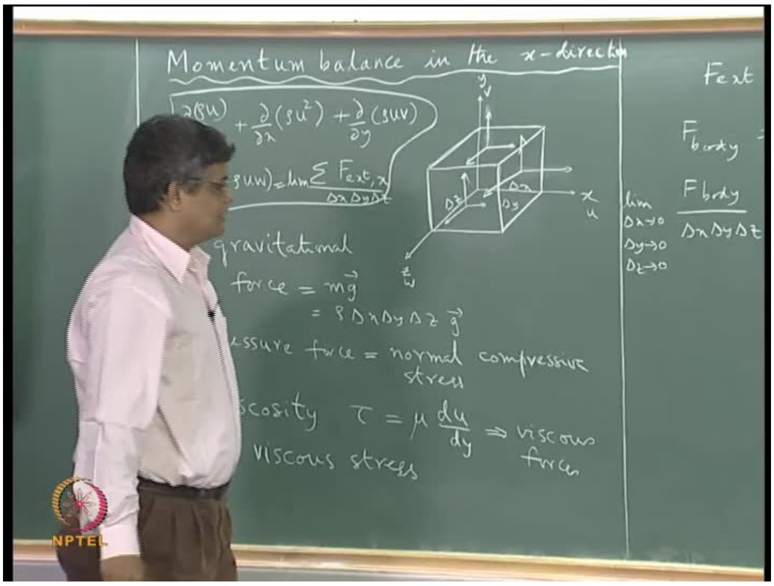

So, we can have stress normal stress and two shear stress components which are trying to deform the plane. So, for every plane, if you come here, we can have a normal stress and we can have a stress acting in the y direction and we can have a stress acting in the z direction. When you come to this surface again, you can have a normal stress acting in this direction and a stress acting in this direction and a stress acting in this direction. (Refer Slide Time: 14:10) So, for every surface, we can imagine three types of stresses - 1 normal stress and 2 shear stress components and we see that the normal stress component for this plane and the one of the shear stress component for this plane are in the same x direction.

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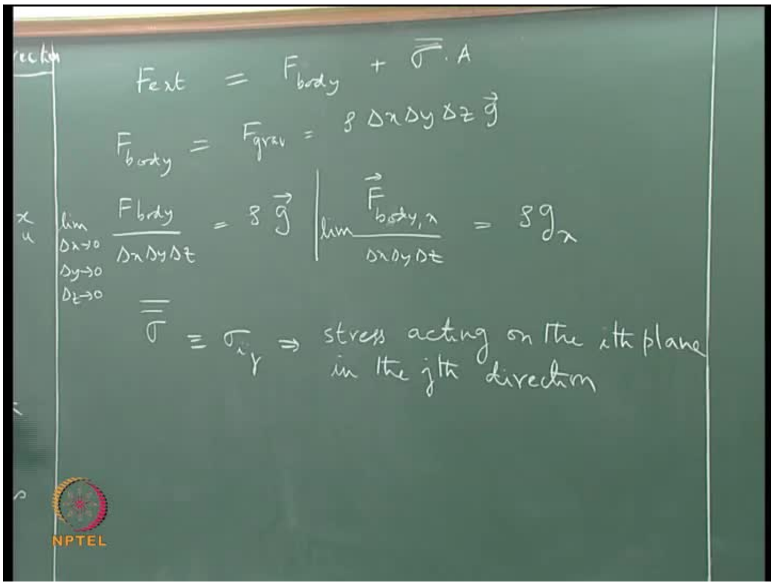

And if you consider, for example, this plane here, this front face, we have a normal stress coming in this and a shear stress coming in this direction and in this direction. So, each phase has three components and we have to when we talk about a stress acting on a surface, we have to identify on what surface it is acting and in which direction it is acting on the particular surface. So, the stress tensor here is identified with two subscripts which we call as i and j and where this implies the stress acting on the ith plane, in the jth direction.

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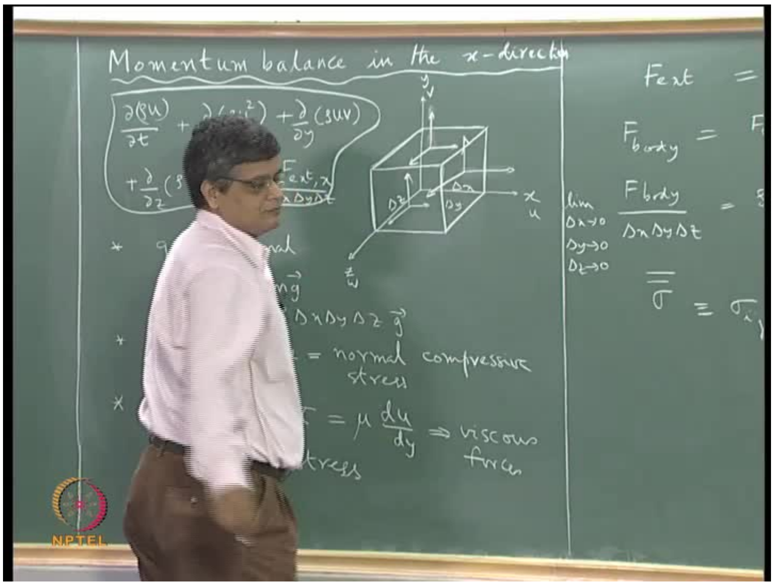

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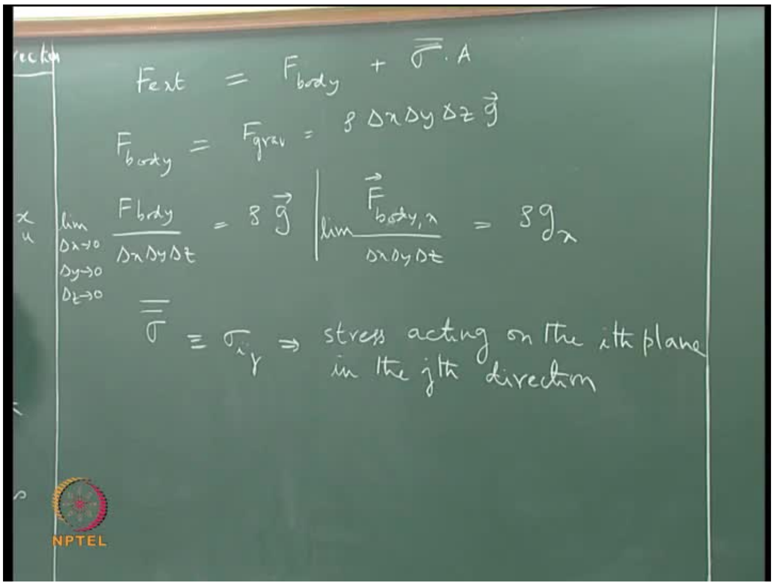

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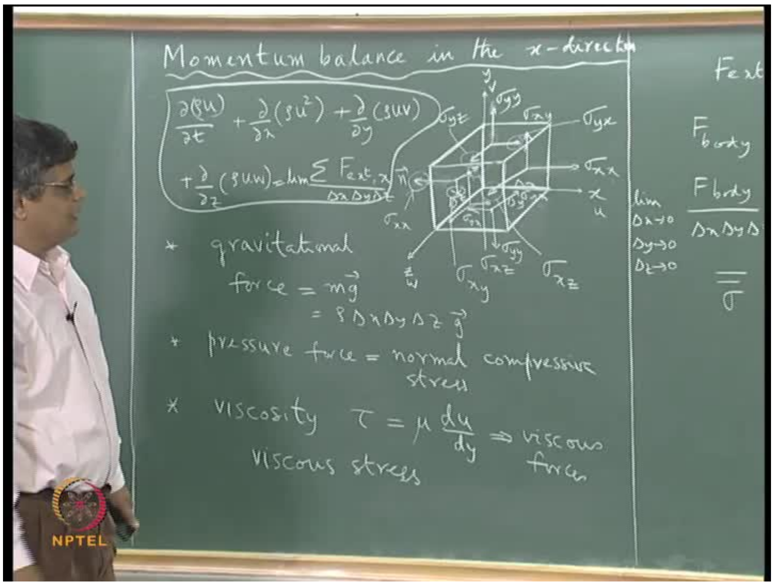

So, this is a convention that we follow, and using this definition, we can now put names to this stress component. When we consider this particular phase, it has an outward normal vector which is aligned in the positive x direction; it is aligned in this. So, we call this as the x phase. So, this ith phase here refers to the outward, the, direction and which the outward normal vector is aligned.

So, this is an ith this is an x phase and because outward normal vector is aligned in the positive x direction, it is a positive x phase. So, this component, this stress here is aligned in the x direction. So, this j here is x. So, when we talk about this sigma here, the i is x and j is also x because this particular component is acting in the x direction.

When we look at this component here, this is acting on the same x phase; so, this is sigma x and it is acting in the y direction; so that is sigma x y, and this component here is acting on the x phase; so, sigma x, and it is acting in the z direction sigma z x z. Now, when you come to this, this is acting on the y phase because of outward normal vector is aligned in the positive y direction. So, this is a positive y phase.

So, this is sigma y and this particular component is acting in the y direction. So, that is sigma y y. This component here is again acting on the positive y phase, so the first subscript is y sigma y and this is acting in the x direction. So, this is sigma y x, and this component here is again acting in the positive y direction positive y phase, so, sigma y, and its acting in the z direction; so, this is sigma y z. So, we can see that each component here specifies what phase it is acting in and in what direction that is acting in, and by convention, we also indicate whether for a control volume like this; we also include in the notation here, in the convention here. The distinction between a positive x phase and a negative x phase.

For example, if you consider this phase here, that is, the left side phase of this left right bottom top back and front. So, on this particular phase, the outward normal vector is aligned in the negative x direction. So, this is the normal vector of this particular phase, and so, this is aligned in the negative x direction because positive x direction is in this direction. So, this is a negative x phase, and on negative x phase, the normal stress will be, so this will be acting, this is again x phase because this is obviously x phase and it is acting in the x direction, but its direction is in a negative x direction. So, instead of being in the positive x direction, it will be considered to be acting in negative x direction, and the y directional stress on this suppose to be down.

So, this is sigma x y and this is acting in the negative y direction because it is a negative x phase, and similarly, the x z component will be acting in this direction. So, this is sigma x z; so, x indicating the phase, on which, it is acting, and z indicating the direction, in which, it is acting, but for this particular control volume, this happens for negative phase. So, it is acting; this acting in the negative z direction and this acting the negative y direction.

So, we can see that outer we have three positive phases and three negative phases. On the positive phase, this three stress components act in the positive directions of the coordinate phases like sigma x x positive x direction; sigma x y acting in the positive y direction; sigma x z acting in the positive z direction, but on the negative phase, example, this one - sigma x x is acting in the negative x direction and sigma x y is acting in the negative y direction downwards here and sigma x z is acting in the negative z direction.

If you consider the bottom phase here, so this component here is sigma y y because this is a negative y phase, and y indicates, the second y indicates the direction, in which, is acting, it should be acting in the y direction. So, ideally, it should be upwards but because it is a negative y phase, you put it down, and correspondingly, sigma this plane, y plane will also have a sigma y x component and a sigma y z component which are the two shear stresses, and because it is a negative y phase, sigma y x will be acting in this direction.

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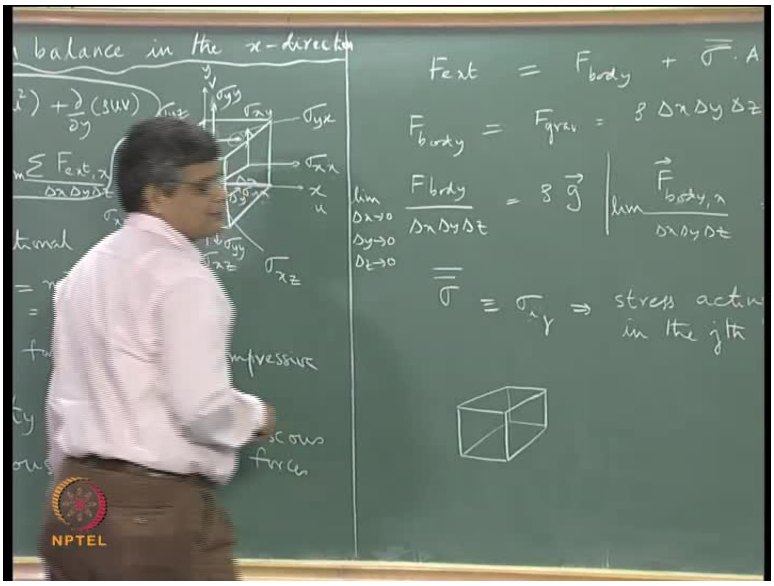

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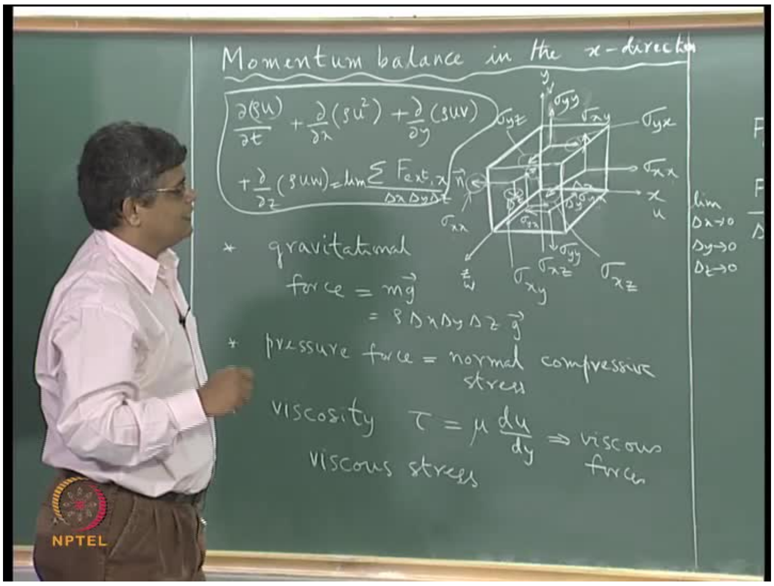

And sigma y z will be acting in this direction. So, when we consider a three-dimensional, a cube like this, then on each of the six phases, there are three stress components and these are acting, and out of the three stress components, 1 is in the, 1 is a normal stress; the other two are shear stresses, and the notation here, sigma i j here has a convention that i indicates the outward normal vector of that particular surface, on which, the stresses acting, and j indicates the direction, in which, that particular stress component suppose to act, and in the general case, when we want to consider a stress acting on a six sided figures like this, there are eighteen components.

When we do the momentum balance in the x direction, we are not interested in the forces acting in the y direction. For example, the stress which is sigma y y is acting in the y direction; sigma y z here is acting in the z direction. We are not interested in the stresses acting in the y direction or the z direction. We are, acted, interested only in those stresses which are acting in the x direction, that is, those stresses, for which, j is equal to x. For example, this sigma y x here, it is acting in the x direction; this sigma x x is acting in the x direction; this sigma x x is acting in the x direction - negative direction - but still it is acting in the x direction.

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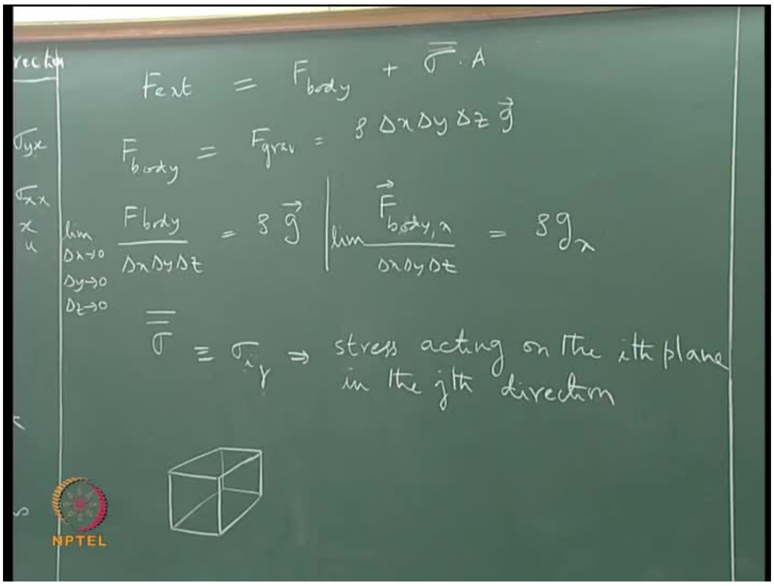

This sigma y x is acting negative direction x direction, and we have not, here we have a sigma z x which is acting in the x direction. So, as far as the momentum balance in the x direction is concerned, apart from the body forces when you consider the stress process here, we have to consider only those stresses which are acting in the x direction.

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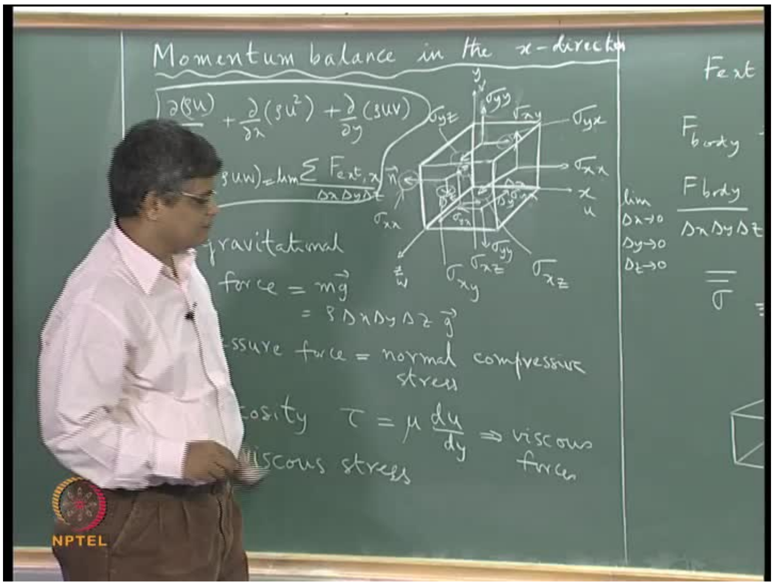

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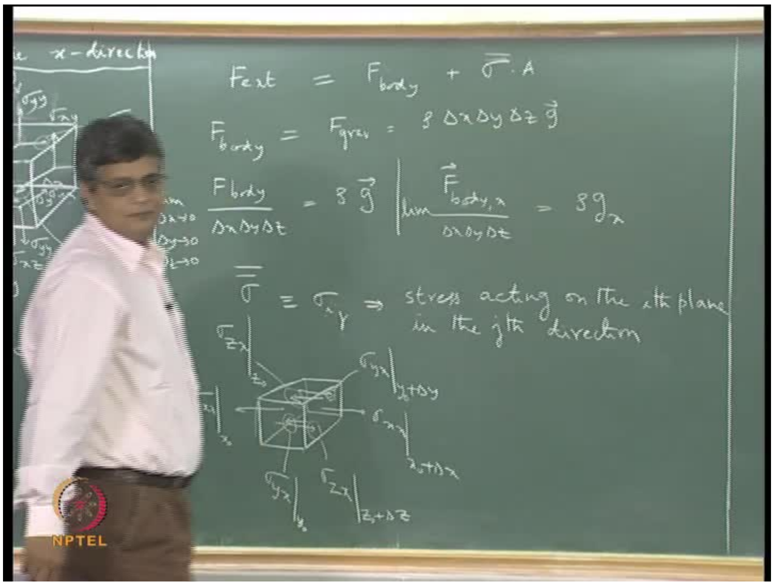

There are eighteen stresses, out of which, six stresses are acting in the x direction, and six in the y direction; six in the z direction. Now, let us identify the stress components acting in the direction. We follow the same notation that this is x; this is y and this is z direction. So, when we consider, this, this phase, that is, the right hand side phase, this stress acting is sigma x x, and we put the origin here and this particular plane is at a distance of x naught plus delta x distance. So, we can write this as x naught plus delta x.

The stress acting on the top surface in the x direction, it is from here, it is acting in this direction, and this is sigma y x and this is acting in the plane which is located at y naught plus delta y and that is acting in the positive x direction. When we consider this front phase here, this again has, this is a z phase and it has a stress acting in the positive x direction, and this is sigma z x, because this is acting in the z plane, and it is acting at on the plane, which is located at z plus z naught plus delta z.

Now, let us consider a negative phases. So, this phase which is the left phase has a stress acting in the x direction, that is, sigma x x at x naught and it is acting in the negative x direction, and the bottom phase here has a stress acting in the negative x direction, and this is sigma y x, because this is acting on the bottom phase, that is, the y phase it is acting at x direction, and this is at a plane - y plane - which is located at y naught, and now, we consider the back phase. This again is a negative phase because outward normal vector is aligned in the negative z direction. So, this has an x component which is acting like this.

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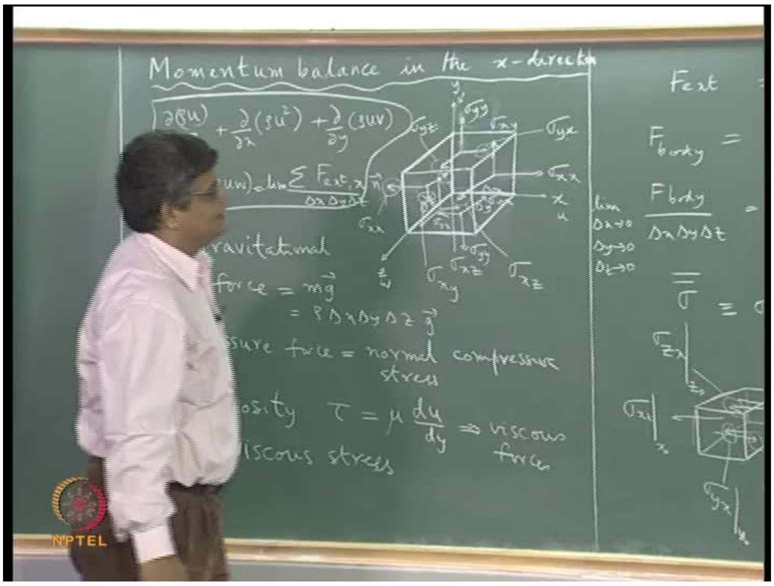

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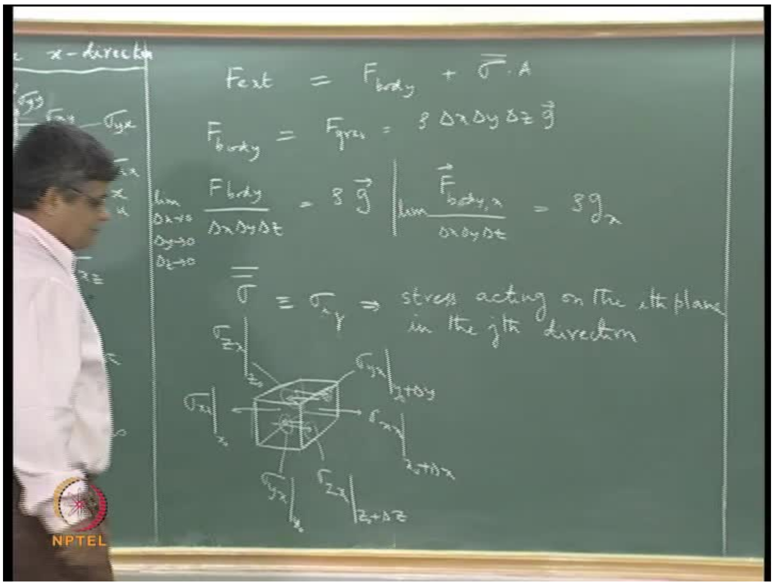

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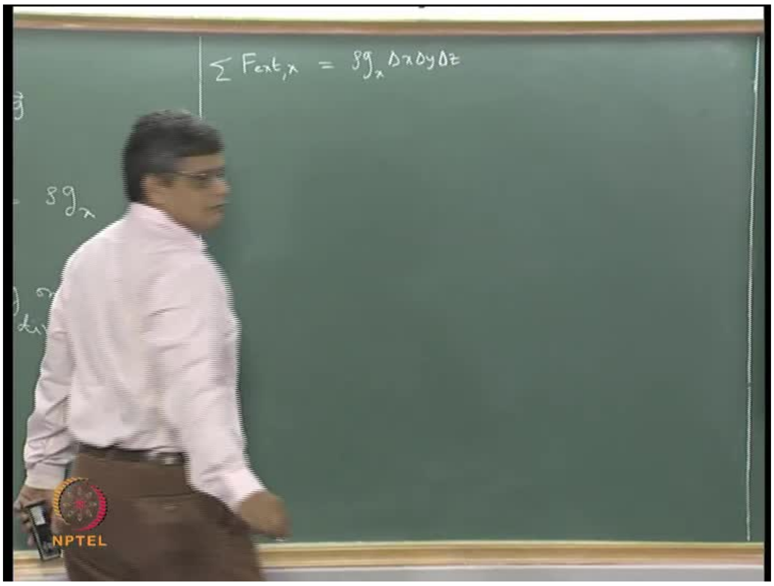

And this is sigma z x on a plane locate on a z plane located at z naught. So, these are the stress components and we know that each surface here is a plane with either delta z delta y delta z as the dimensions. So, using this, you can now compute the external stresses active in the x direction by multiplying each stress component by the corresponding area, and because we have taken these planes to be aligned with the normal directions, then this dot product becomes very easy. So, we multiply corresponding stress with the area, and then, we can, therefore, write F external x sum of all external x as we have already got rho g x times delta x delta y delta z. That is the body force.

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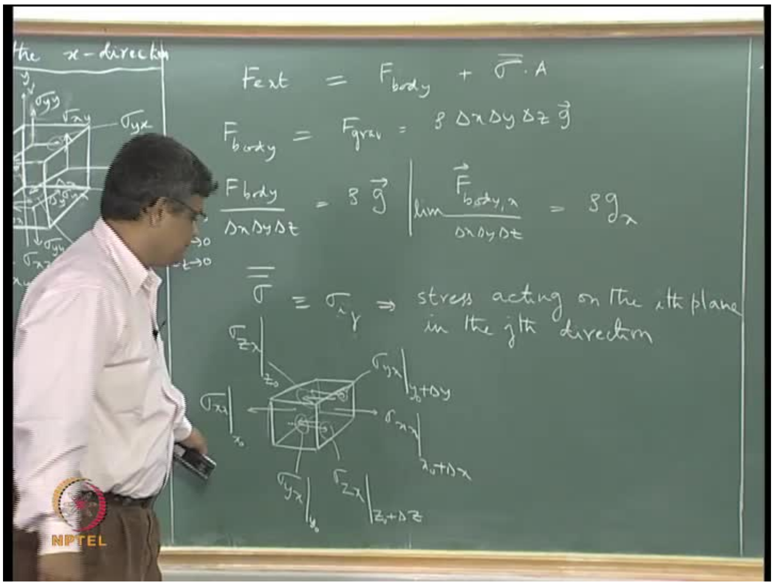

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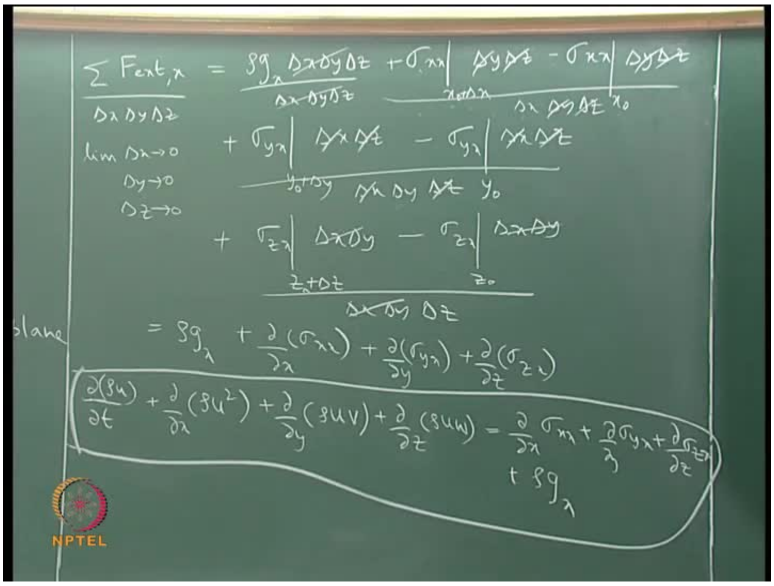

And we now have to evaluate each stress and multiply by the corresponding area. So, let us first consider the sigma x axis. So, we can say, we consider this, then we get sigma x x at x naught plus delta x times the area which is delta y delta z, and on the opposite phase, that is, on the left phase, we have a sigma x x and it is acting in the negative x direction. So, that will be with minus sign minus sigma x x, and now, this is at x naught and an area of delta y delta z.

Then we have plus the top phase which is sigma y x at y naught plus delta y times the corresponding area which is delta x delta z minus sigma y x at y naught, which is on the bottom phase acting in the x direction times delta x delta z plus the stress acting in the front phase in the x direction, so, that is sigma z x at z naught plus delta z times area which is delta x times delta y minus sigma z x at z naught which is the back phase times delta x delta y.

So, we have external forces as the gravitational component and six stress components acting on the six planes in the x direction multiplied by the respective areas. Now, what we want is the sum of all these forces on a unit volumetric base in the limit as delta x delta y delta z tends to 0. So, we divide this whole thing by delta x delta y delta z and take the limit as delta x delta y delta z tends to 0.

Now, let us before we take the limits, let us try to simplify. So, as we have seen earlier, they cancel out here delta y delta z cancel out from this and we left with delta x, and here, delta x will cancel out and delta z will also cancel out. We are left with delta y here and here we are left with delta z.

Now, if we take the limit as delta x tends to 0 delta y tends to 0 delta z tends to 0, so that is we are considering a smaller and smaller and smaller volume of this particular unit volume. This thing here becomes rho g x and this is sigma x x. Let us say it is a variable at x plus delta x naught plus delta x minus the same variable at x naught divided by delta x as x delta x tends to 0 is nothing but the derivative of that particular quantity in the x direction.

So, this is dou by x of sigma x x, and this one here is again sigma y x at y naught plus delta y minus the same quantity sigma y x at y naught divided by delta y as in the limit as delta y tends to 0 will give us variation with respect to y of sigma y x and this gives us plus dou by dou z of sigma z x. So, the left hand side, the right side term in the momentum balance equation. Now, it can be written like this.

So, this is equal to what we have on the left hand side. So, that we can write down the complete momentum balance in the x direction as d by dt of rho u plus d by dx of rho u square plus d by dy of rho u v plus d by dz of rho u w equal to d by dx of sigma x x plus d by dy of sigma y x plus d by dz of sigma z x plus rho g x.

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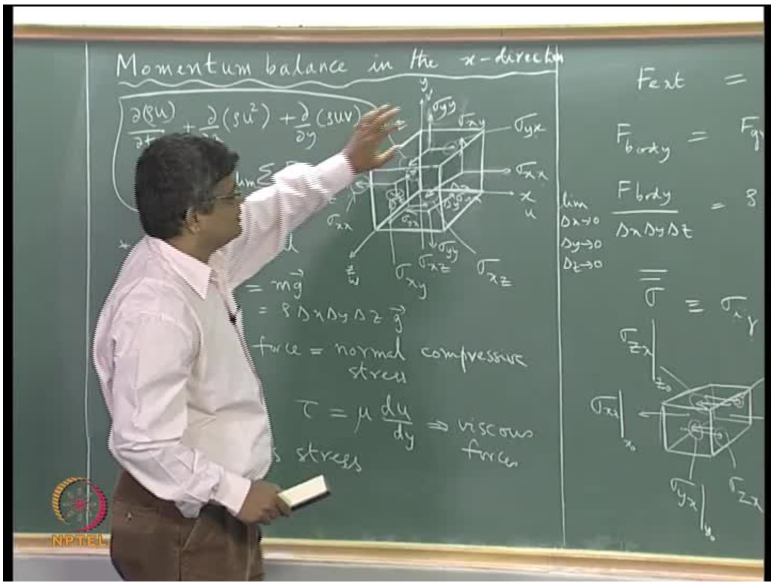

This is a x momentum balance and we can similarly write the y momentum balance by doing exactly the same thing. We look at the rate of accumulation of the y momentum in this and the rate of ah inflow of y momentum through the x phase and y through the left phase and bottom phase and the back phase and we evaluate the rate of outflow of momentum through the right phase, the top phase and the front phase.

Then we evaluate also the forces acting on the body, the, on this control volume, like the body force, the gravitational force as the body force and the various stress component acting in this and we can divide those whole thing by delta x delta y delta z and take the limit as these things tends to 0 so that we are considering almost a point, and at that point, we can get an equation very similar to this.

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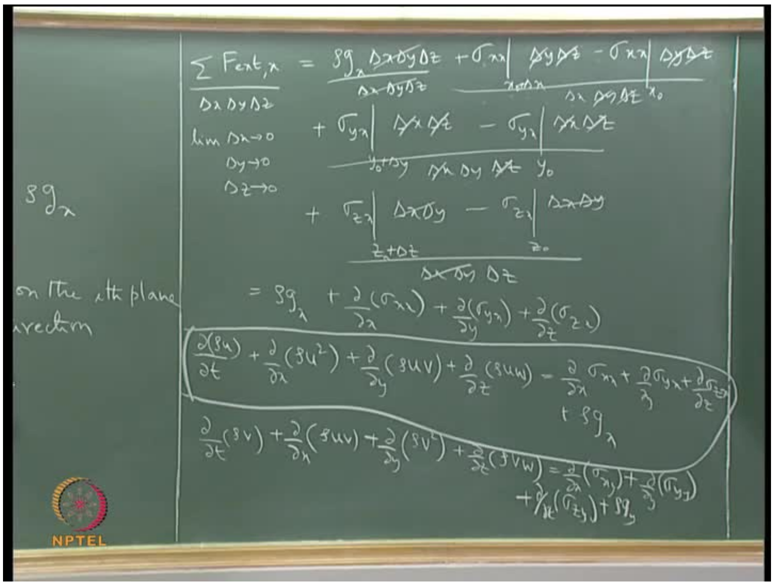

Instead of u here, we have v, and 1 of the u becomes v here dou y of rho v square plus dou by dou z of rho v w equal to dou by dou x of now sigma x y plus dou by dou y of sigma y y plus dou by dou z of sigma z y plus rho g y. So, this is the y momentum balance. When you compare the two, you see that the stress that are appearing in the x momentum balance. All have x as the second index, because that indicates the direction in which that particular stress component is acting.

So, in the x momentum balance equation, that second index must be x, and in the y momentum balance, the second ah index, that is, sigma x y here, sigma y y and sigma z y; that is in the y direction, because we are representing the y momentum balance, and we can write a similar expression for the z momentum balance equation. So, this completes the derivation of the momentum balance in the x y z directions. So, now, what have we got from this? The first balance in the three directions, in the momentum balance has given us three extra equations.

And the three extra equations are the momentum balance in the x direction, momentum balance in the y direction and the similar equation from the momentum balance in the z direction, and we also have one equation for the continuity equation which we have derived in the previous lecture.

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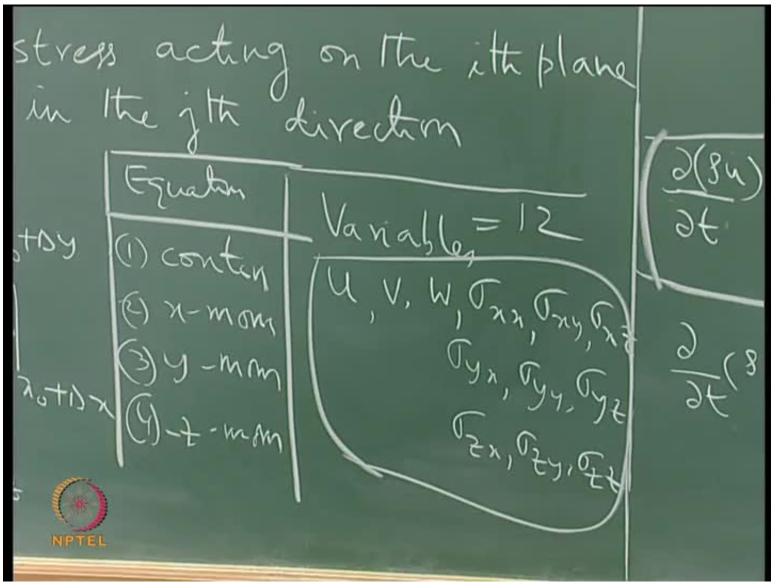

So, now we have four equations and what are the variables that are appearing in these equations? Let us just see. When you consider this, u is obviously one variable; u is a variable; v is another variable; w is another variable, and we still have to specify what these stresses are sigma x x sigma x y sigma x z sigma y x sigma y y sigma y z sigma z x sigma z y sigma z z. So, the variables that are appearing, in, in this four equations are they are nine stresses plus three, that is twelve. Out of which we have, for these twelve variables, we have only four equations.

Equations are - 1 is the continuity equation, x momentum balance equation, y momentum balance equation and z momentum balance equation. So, we are face with the tricky situation that we have far few numbers of equations to describe the variables that are appearing in this. We can say that we have additional equations like the energy equation; we have not impose the energy equation, but energy equation obviously brings in a new variable which is the enthalpy or the temperature. We can bring in one more equation - the entropy balance equation that brings in one more variable. So, which is the entropy?

So, by adding more number of equations, we are not resolving this imbalance between the number of equations available and the number of variables that are appearing in these things. So, this kind of imbalance is a something that cannot be just resolved by adding more number of equations, more number of fundamental equations, and we have to bring in empiricism; we have to bring in some kind of expectation of the fluid behavior to describe what this stresses are.

The major problem is coming from these stress components, there are nine of the stress components. Unless we specify what we mean by the stress components, we cannot hope to get a balanced number of equations and a number of variables. So, we need to instill more information into a model especially in the form of what these stresses are so that we can hope to get a as many equations as the number of variables there are.

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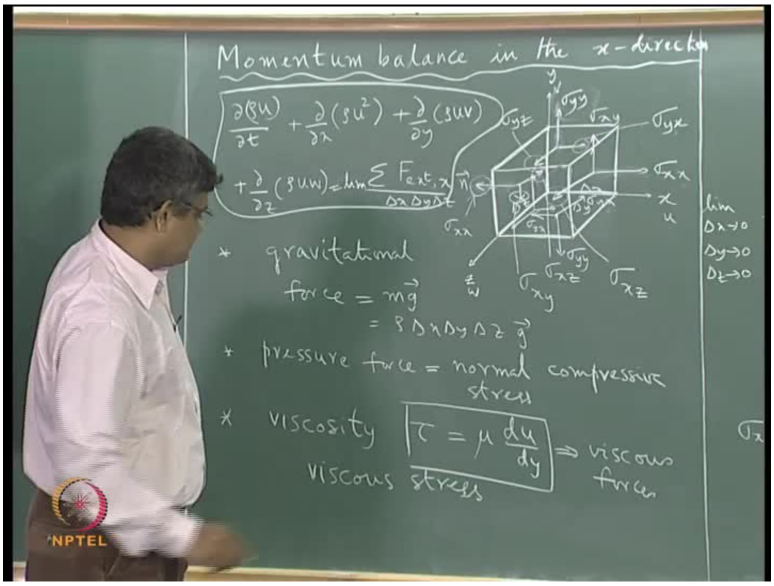

And we need to bring in some sort of conception of this stresses that are appearing with in the fluid. So, we need to have some sort constitutive model. A model which describes the constitution, of the, of the fluid in terms of what kind of stresses and one such constitutive model is this. This stress is one of the stresses that we considering, and although we have put it as tau here and we have put it as sigma here, one can say that the tau, this is also a stress which is a viscous stress and this is been related here through a viscosity and a velocity gradient.

So, this gives us a hint has to how you can go about it, because if you were to identify with this and this as one of the stress components, and if we know the viscosity of the fluid, then we can substitute for tau the corresponding stress here with this expression, and this expression has two components that viscosity which is a property of the fluid, which can be measured, and this du by dy, which does not introduce any new variable because u is already there. So, we need to describe the stresses in terms of known quantities.

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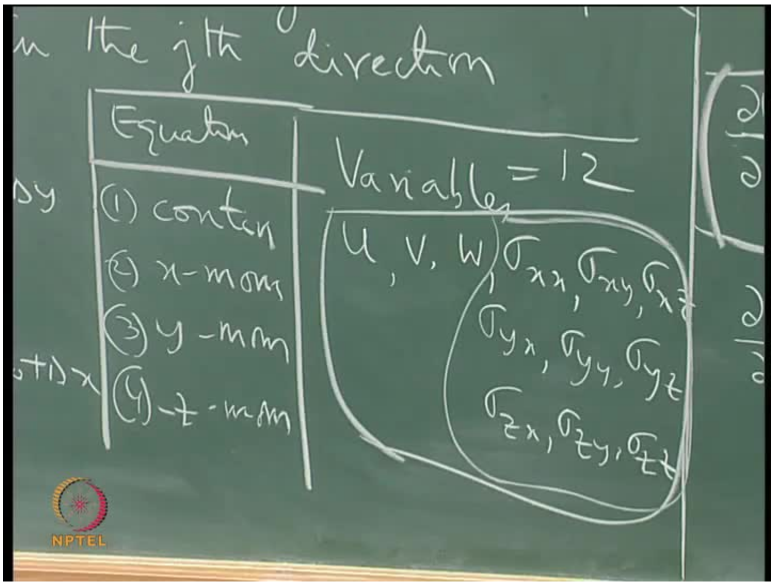

And this is a kind of constitutive equation which, for example, here says that the shear stress induced by relative motion with in the fluid is propositional to the strain rate. So, that is the constitute law which is being described mathematically here. So, this kind of relations are needed to describe each of these stresses so that these stresses can be replaced by an equivalent velocity gradients in viscosity and all that. If we have that, then all the stress would disappear and we will have much fewer number of variables.

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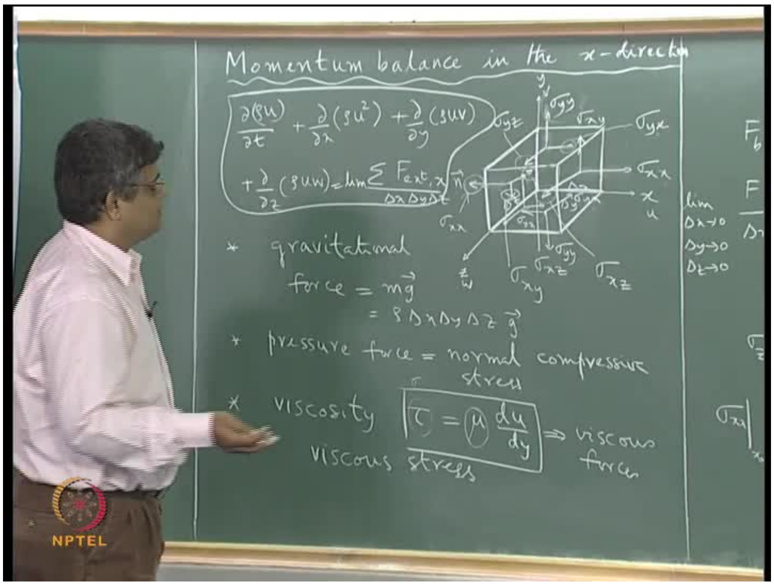

So, we are looking for a constitutive expression for what these stresses are. We can go back to this Newton's law and say we can replace, but this that this is not enough for us, because what is the stress? So, this stress can be identified as tau x y or tau y x something like that, that, what about the other stresses? So, this is, this is for a specific case; this is for a specific case of one-dimensional flow where you have a single non-zero stress, that is, there in a general case, this particular relation is not valid. So, we have to look for a more generic expression, which is, which is able to identify all the stress components in an equivalent way.

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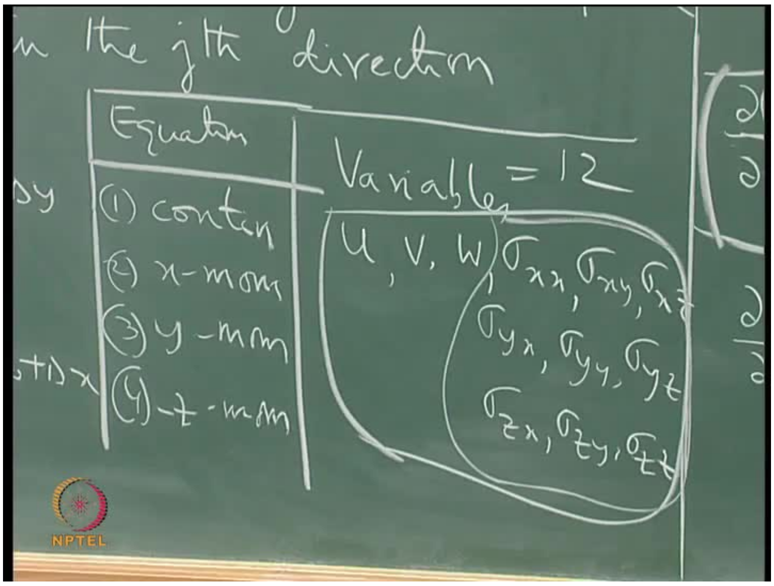

So, we now search for a constitutive law which describes the stresses within the fluid, which are part of the momentum balance equation. We do this by drawing analogy with solid mechanics, because solid mechanics faces is face with the same situation, and then, you bring in the equivalent hook's law where you say that the stress is propositional to strain.

So, here, we bring in a same kind of argument. We say that the stress is propositional to the rate of strain and why it is the rate of strain is something that we have to see, because fundamentally, a fluid is different from a solid, in the sense that a fluid will continue to deform as long as there is a stress acting on it, whereas, a solid will deform initially, and then, it will reach a condition of equilibrium with the force applied on this which is crossing the stress and the deformation which is resisting that force that force so that you have distortion and a distortion which comes into equilibrium with the force that is applied.

Whereas, a fluid by definition is something that continuous to deform as long as a forces applied and which stop deforming immediately as the force is taken away for a simple Newtonian fluid and it does not try to regain its original shape. Whereas, the solid, for example, if you take an eraser and then try to press, it will deform, and the moment you take it away, it regain its original position.

Whereas, a fluid does not regain its initial position; it will only stop deforming further. So, what is important in the case of fluid is not the strain itself, it is the rate of strain. So, we will try to relate the stress that is arising in the fluid by relative motion to the rate of strain, that is generate in this. So, and we would like to have a generalization of this stress verses strain relation which is applicable for three-dimensional flows.

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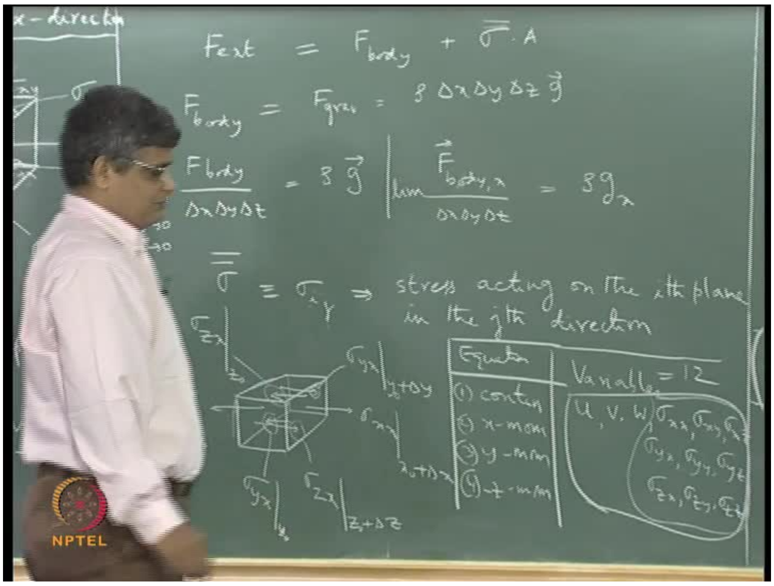

So, that is what we will consider here. To derive extra relations to this ah stresses, so that we can ultimately have a situation where we have the same number of equations as the number of variables. So, that will do now. This is now making a generalization of the newton law of viscosity which expresses relation that the shear stress is mu times du by dy or shear stress is propositional to the strain rate. In the process of generalization, we also want to understand better what exact we mean by strain rate and what exactly we are modeling in this.

So, we would like to consider an expanse of fluid, in which, we identify four particles - a b c d, which are such that at time equal to t naught. They form a rectangle and each of this, they are at the corners of these rectangles and they all have their own velocities. As a result of which, they are moving along the this flow domain, and after a small time delta t, the particles, the four particles will move to a different place, and we see after a small time delta t, how this initial rectangle has become deformed.

From this deformation, we try to find out the strain rate which expresses in a way deformation, the rate of deformation, and we compute this strain rate and we will see that the strain rate is related to the velocity gradients, and from these velocity gradients, we will attribute these velocity gradients to be linearly propositional to the shear stress, and thereby, get the general expression for the relation between generalized stress in terms of the generalized strain rate. So, that is the strategy and we will see how it can be done.

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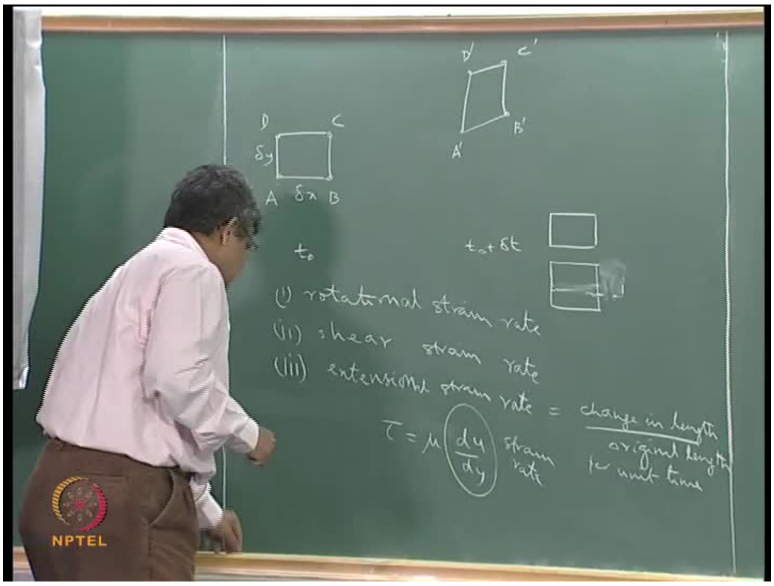

So, we are considering a wide expanse of fluid, in which, we have four particles - A B C D which form initially a rectangle. So, let us call this as A B C D. These are particles, all the four particles are within the expanse of the domain, and this particle a and b are horizontally separated by a distance delta x and these are vertically separated by delta y. So, this say delta y and this is also delta x, and this is the situation at time t naught after a time t naught plus delta t, this particle will be moving around here and this particle will have to move this particle and all of them will have moved. So that, now, have A here and B here and C here and D here. So, we will call this as a prime, b prime, c prime and d prime. These are the new positions of the particles at A B C D after a small time delta t. If we now join this, you will see a shape here which is different from the rectangle that we have considered.

We can see that and this result in the shape of an initially rectangular thing to a more general quadrilateral thing has come about because the points A B C D are not all moving at the same velocity, that is, if there is velocity variation within the fluid domain, that is, between a, and particle located it A and B and C and D, then there is a corresponding there is a an expectation of a change of shape of this particular initial rectangle, that, that, that we get by joining all these things. So, there is a change in shape if there is a velocity difference, and this change in shape can be identified as being resulting from three components - one is rotation, the other is shear and the third one is extension. In addition to this, we have a fourth one which is translation.

Now, we can imagine that this is a plane; this is the initially the rectangular plane and we can identify each of this components by looking at this. If all of them are moving have the same horizontal velocity, then after some time, they come like this. They are translated horizontally with at a certain velocity, and if horizon have only vertical velocity which is the same, then this is a translation - pure translation - in the vertical direction, and if they have the same, if all of them have the same u and v, then they can be moved into this position which is a pure translation, in which, there is no relative that relative disposition of the four particles remains the same.

So, we can say that near translation or pure translation does not cause any relative disposition of the particles within this fluid. So, we can leave this out as a contributing to the relative motion. So, now, what we mean by pure rotation is - if we have it like this, then the particles are rotating, this whole plane is rotating like this, so that this particle is relative to this, it has moved, and the rotation need not be about this particular axis, it can be about this axis or it can be about the center.

So, one can see that there is a, if there is a different velocities of these particles, you can have something like rotational motion. If this velocity is 0 and if this has velocity which is aligned in this tangential direction, one can imagine that this particle is moving like this. So, if there is a coordinated motion of all this four particles, you can have pure rotation. Shear is where, the, this particle and this particle they have different vertical velocities. For example, if you focus only on this particle and this particle, you see that this particle has less velocity, so it will have move this much, and this particle has more velocity, because which it will have gone like this.

So, the side which was originally horizontal, it has become incorrect. Similarly, this side here you can also have different velocity in this direction and different velocity in this direction, in the same direction for these two particles. So, instead of moving like this, this this thing could have moved like this. So, because of which you can see that this side which is horizontal has shifted slightly inclined and this is vertical has become inclined like this. This is again vertical, it becomes slightly more inclined. So, that kind of thing is possible when you have different velocities in different directions.

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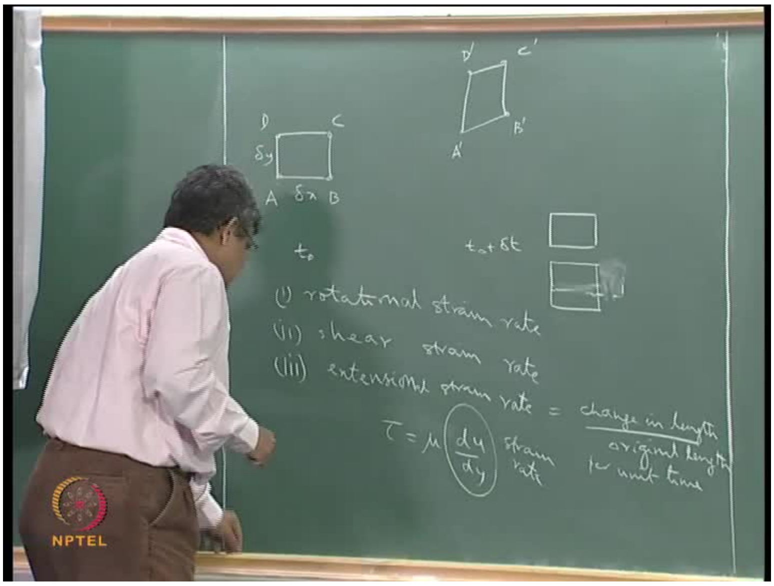

Now, the third one is extension which is essentially either the stretching or contraction of this fluid element. So, that is, you, this whole thing, the whole area in a two-dimensional says the whole area of this rectangular domain, it changes between time equal to t naught and time equal to t naught plus delta t. If there is a change in the area of this rectangle and this quadrilateral, then we have extension here. If there is just rotation, pure rotation, without the change of the angles between these two sides, then we say it is pure rotation.

If there is no rotation but only the change of angles is there, then we say that it is a shear. So, and in the general case, the overall shape may be a combination of rotation and shear and extension. So that over all change that we see from A B C D to A prime B prime C prime D prime is a combination of rotation and shear and extension that has taken place in the time interval delta t. So, we would like to quantify these rotations in terms of the strains, we will, we will identify rotation strain, shear strain and extensional strain and we will see that these strains are related to the velocities that particles A B C D have, at particular, at t naught and the gradients of these velocities are actually related to the strain rate, shear strain rate and extensional strain rate.

And so, therefore, we can attribute the change of relative dispersion of these four particles to different kinds of strain rates and we do it in two dimensions but the extension three dimension is similar, and thereby, we can identify the change of shape to the gradients of velocity which is coming in our Newton's law, which is du by dy; the shear stress is propositional to du by dy. This is a strain rate.

What strain rate? It is something that we will understand by analyzing the kinematics of fluid motion in this particular way. Now, we have to identify, we have define, we have to quantify, in order to quantify these rotational strain rate and shear strain rate, we have to define what these things are, and we say extensional strain is when the length A B is different from length A prime B prime or length A D is different from A prime D prime. So, the extensional strain is relatively easy to understand. So, this is the change in length divided by original length.

So, that is the strain per unit time is the extensional strain rate, and we can see extension in the x direction only or extension in the y direction only or extension in both. For example, extension in x direction only, this is the shape initially and after some time it becomes like this.

So, the height remains the same but the length has changed, and if you consider a strain like this, a change from here to here. This is strain in the extensional strain in the y direction. So, one can see extensional strain can happen in x direction or y direction and there is a corresponding change in length in that particular direction by the original length in that direction per unit time will give us the extensional strain rate.

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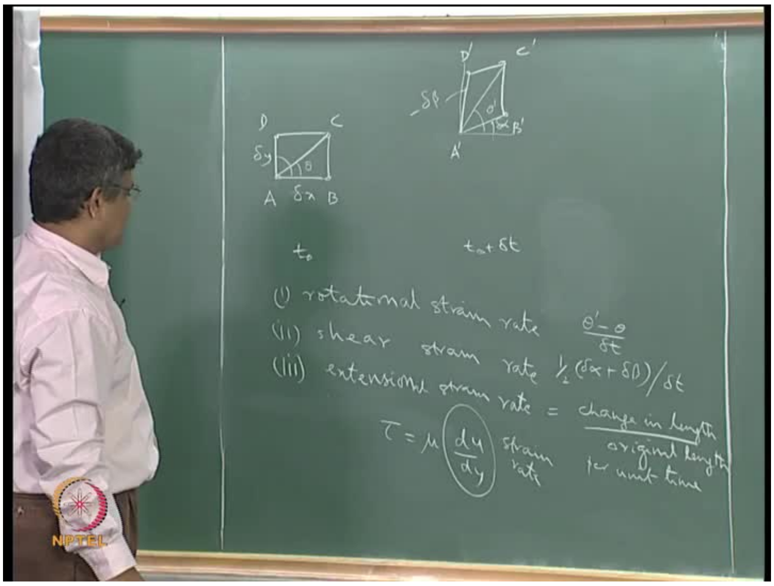

What we mean by rotational strain rate is that we consider the diagonal and we see if the diagonal has rotated, this initially, for example, this may be making a certain theta here with the horizontal. Now, the diagonal here is making at theta prime. So, this strain rate is theta minus theta prime theta divided by delta t - where theta is the angle that the diagonal is making with respect to the horizontal. Shear strain is by which this particular angle here has decreased and you can see that if you say that, this, this particular side a b has shifted so much and this a d here has shifted so much with respect to this.

Then the overall initially ninety degrees minus this minus this will give you the new angle. So, that is also related to the change, in the, in, in the movement of the angle of the A prime B prime with respect to horizontal, and the angle of A prime D prime with respect to the vertical will give us the shear rate. So, this is with respect to this angle say delta alpha and delta beta. We will see what these things are. So, we can say that this is half of delta alpha plus delta beta divided by delta t. So, these are the ways that we try to quantify extensional by the change in length per unit length per unit time and, rotational by the, rotation of the diagonal with time and the shear strain is by how much these things have changed over this thing.