Computational Fluid Dynamics

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Module No. # 06

Dealing with complexity of physics of the flow domain

Lecture No. # 38

One-equation model for turbulent flow

And especially the eddy diffusivity or the extramomentum flux, that isarising out of turbulent fluctuations, and these two parameters are obtainedat every point within theflow domain. So, these are two parameters, these will be a functionof x y z and t, and these are evaluatedby their ownpartial differential equations, which represents the conservation of these quantities, these scalar quantities. We have already seen, one such additional equation for the turbulent kinetic energy, and we look at the similar kind of derivation for the other quantity. Now, what quantities are these that wouldreflect properly, accurately and well enough the effect ofturbulent fluctuations on the overall turbulent flux. It cannot be that everythingcan, any two parameterswill work. It should be that, they have some relevance to the turbulenceprocess that is taking place.

And, in the previous lecture, we have derived the equation for k, theturbulent kinetic energy, and we have seen that thesquare root of the turbulent kinetic energy can be a measure of the velocity of the eddies, and these could be related, these could be analogous to the velocity of the molecules, gas molecules which interact with each other and produce a enigmatic viscosity.

So,therefore, the turbulent flux is characterized by theeffective turbulent viscosity, which has a velocity parameter, which we have derived from theturbulentkinetic energy, and it also requires a length parameter, and it is in the length parameter that we found deficiencies in the one equation module, because we had to resort to the only the length scale that we have, which is the mix in length measure, which is typically taken as assert and distance fromthewall, and so, that is nota very good prescription for a length scale, in general case, where you may be quite far from the wall or it may be that you havea significantinfluence from several wallsand so on. So, the idea is to derive alength scale, which is independentof the geometry, and if you wereto have that then you couldwritean expression for the turbulent viscosity.

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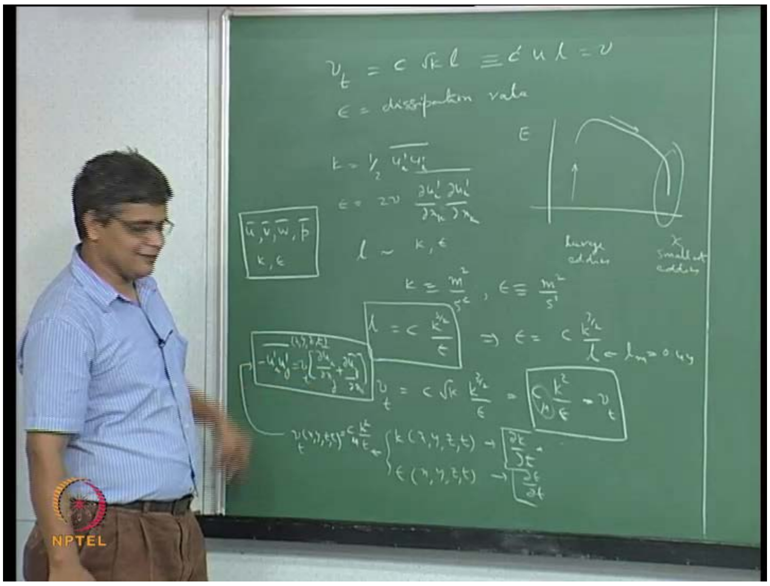

Turbulent viscosity has been proportional to constant, timessquare root of k, times the length scale, which is analogous to u times l times aconstant C prime, which is a characteristic of themolecular viscosity as per the kinetic fluid gases. Now, what can be the length scale here. The point that, we have to keep in mind is, this is a measure of the turbulentquantities, a measure of theturbulent fluxes, and it should be therefore representing a length scale associated with the turbulent fluctuations, and this is where we also. Notice that, we in the derivation of the equation of k, we have come across the turbulent dissipation rate, and this turbulent dissipation rate, is the rate at which the turbulent energy isgettingdestroyed, is beingtaken out of the system and that is avery important quantity in turbulent flow, we have seen a typical spectrum in terms of the wave number k.Let us, notconfused this with the turbulent kinetic energy and the energy, and wehave seen that typically goes like this. This is where energy is beingput into the system, this is the large eddies and this is where we have a smallest eddies.

Turbulent energy is generated by large eddies as a result of destabilization flow, in regions of large velocity gradients, andthis is where one can say turbulent energy is produced, and ascharacteristic of the mean flow parameters, and this energy is casketed down, the eddy sizes, as you go from here towards this, the eddy size becomessmall and smaller, and thiscascading process happens without much of dissipation of turbulence.

So, this is almost like a frictionless transfer ofthe energy that is createdat the large eddy sizes, and this energy is finally dissipatedhere, at a rate which is given by the turbulentenergydissipation rate, sothis dissipation rate is happening, around this smallest eddies and this smallesteddiesare so small, that effectively there is noinfluence of the geometric parameters associated with the system.

So, this isour key feature of turbulence modeling, especially that the turbulentenergy dissipation happens at thissmallest eddies of the length scale associated with the turbulence, and theseare so small, that they do not know whether the turbulence is createdin a flow in a pipe, in a circular pipe or in a boundary layer or in somemixing region or anything like that. So, because those scales associated with thecircular pipe or boundary layer or with pressure gradient and all those things, are very large compare to this small eddy scales that are happening, and this isindependent to the geometry.

Therefore, the length scale that appears here, characterizing the turbulent viscosity is not dependent on the overall features of the flow, but it depends only purely on the turbulent quantities. And wehave seen that we have here, two quantities k, which is defined as half mu I prime mu primebar, sum of all the turbulentvelocity fluctuations, and epsilon which is defined as 2 nu dau I prime by daux k over bar. So, this again is a positive quantitiessquare ofthesethings, these are the parameters which depend only on the fluctuating quantities.

These are the quantities which are purely turbulencedependent, and one could therefore, argue thatthe characteristic length of turbulent flow depends only on k and epsilon, becausethis is also a turbulent thing, this is alsoturbulent and this is alsoturbulent, and over most of the spectrum involving the wave numbers and especially in the regions, where towards a higher scales, these characteristics do not depend onthegeometry of this. So, if one wereto do like this, based on dimensionarguments, because k has dimensions of meter square plus second square, and epsilon has dimensions of meter square andsecond cube.

One can say that, the length here is in constant times k to the power 3 by 2by epsilon, so this iswhat we can use in order toget an expression for epsilon, we can write this as epsilon equal to some constant times, and this is the expression that we usefor theturbulent kinetic energy equation, in the one equation model. So, we say that, we replace this with the mixedlengthmodel, and therefore, we say that this is for example, equal to point 4 times y, where y is normaldistance, and this constant is adjustedfor best fit with the data and then we have anoverall model, and we know that in this way we have problem with specification ofthe mixing length, and we make use of this very factor here. To invert this relationand say thatepsilon is given by, not here,it is already inverted here.

We make use of this relationto substitute this into this, andsay that nu t is some constant times square root of k,times k to power 3 by 2 by epsilon, and therefore, constant let'scall this as say mu, times k square by epsilon.

Now, the turbulent viscosity is given by in terms of constant to be determined, and k and epsilon which are defined as per, in terms of theturbulent flowfluctuating quantities, velocity fluctuations and their derivatives. So, the idea of coming up with ageneral model, a generic model which can account for length scale, which is not dependent on 1 dimension prescription, is toexpress turbulent viscosity in terms of two quantities k and epsilon, for which we can derive the conservationequations, from thefundamentalmomentum equations and their manipulations, and if youdo that then we canevaluate k at every x y z t from itsconservation equations like this, and we can evaluate epsilon at every x y z t, using an equation need to bedevelopedlike this, and from these things we can define nu t at x y z t as constant it to be determined, times k square by epsilon.

And from these we can write u i primeu j primewhich appears inthe momentum equation, as nu t times d u i bar by d x j plus d u j bar by dx i. So, in the two equation model, we developed an equation for k, and then an equation for epsilonby solving these in the usual c f d way, we can get theinstantaneous and localturbulent kinetic energy and it is dissipation rate

From these we get theturbulent viscosity at alocal point and using these, we can evaluate the Reynolds stresses different Reynolds stresses at every x y z t, and this equation here which is in terms of nu t, which is given by k and epsilon which are being evaluated using these equations, and in terms of the time average velocity gradients, is such that, there won’t be any newterms appearing in thetime average momentum equations, thenwe can writetime average equations, in terms of known variables that isu bar v bar w bar and t bar, and in terms of two more time average variables k and epsilon, we noticeall thatwe have not put time average here, by definition they are time average quantities.

So, we have 6 equations and 6 unknowns here, and by putting up all these things together, we can get the description for turbulent flow, whichenables us to calculate each of these quantities, as a function of x y z t without having to give prescribe any arbitrary values, and in this modeling we still have certain constants we determined, like C mu and other constants which we will see, and those constants are basedfound from some experiments and somesemi theoretical arguments.

But, essentially people have been able to come upwith a set of almost universal constants, which can go into theequations, which describe each of these, so that this model will be a generic model which can be used for 3-dimensional time dependent flows, without having toprescribead-hoc kind of courierense for the quantities which are involved.

Solet’sjustsee how we derive these equations, and so that we have a good understanding ofthe model itself, and when we will discuss some of thelimitations of this model and how this can be overcome, how these are overcome in higher level of models.

Now, the essence ofthe two equation model is to be able to derive a conservation equationfor k, and aconservationequation of epsilon, and so that these equations can be solved to get k and epsilon at everyx y zeta t, and we would like this equations to be in the form the generic scalar transport equation, for which we know how to do the discretization, and thenconverting them into a phiequal to b, and thenefficientsolutions, all that template is already fixed, and if we like equations which go into the template which describe theseturbulent parameters, so that from which we canget kind of epsilon and then the turbulent viscosity and then this.

So, how do we derive the equation fork and epsilon, we have already seen thebasic idea for the derivation of k, we notice that k consists of the product of u i primeand u j prime, and therefore we have written earlier.

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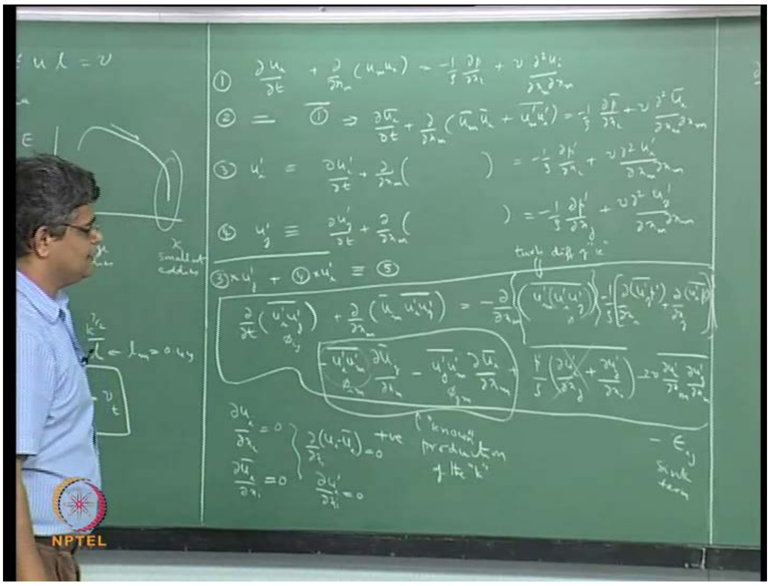

The instantaneous xmomentum equation i thmomentum equation like this.Let us, call this as equation 1, and then we hadderived equation 2, which is the time average of equation 1, and this we can write down as dauui bar bydaut plus dauby ddau x m of u m bar, u i bar plus u m prime, u i prime equal to minus 1 by rhodaup bar dau x i plus nu dausquare nu i bar by daux m daux n.

And by subtracting this fromthis, we are able to get an equation for u i prime, which was of the formdau u i primeby daut plus, dauby daux m ofthis expanded and subtracted in all that thing, equal to minus 1 by rhodau p primeby dau x i plus u dausquare u i primeby dau x mdau x n, so there are terms that are coming here, and this is for the i thcomponent, and what we need is i thcomponent and i th component, and summed over all the 3.Therefore, the kis to, write an equation which is similar for u j primeessentially, wherever you have i here, we place it with j, and then we have an equation for this, in the form of dau u j prime by dau t plus dau by dau x m, of a big (()) is equal to minus 1 by rho dau p prime by dau x j plus nu dau square u j prime by dau x m and dau x n, and now what we do is we multiply 3 by u j prime, and add to thatthe multiplication of every term inthe 4th equation by u i prime, and then take the time average, and then we have to do lots and lots of manipulations.

But, this gives usequation 5 which can be written down like this, after doing these things, and thendoing the time averaging, we get an equation 5, which is of this form,we can see that this is the beginning of anequation for the Reynold stresesisdau by dau x mof u m bar, that is the meanquantity here,Reynold streses, and here we have attempt which looks really humongous, u mprime, u i prime, u j prime,it is a simultaneous correlation, time average of theof all the three fluctuating quantities, and terms involving pressureand velocity fluctuations.

So, here we have dau bydau x i of u prime j by dprime, here we have dau by dau x j of u prime ahp primecorrelation like this. And, then we have minis u i prime, u mprime of, here we have u i prime u m prime do u j, so this is I here and this is j; this is the main quantity and this is a fluctuating quantity, and similarly here we have j here, and then of course, this is the fluctuating quantity, and here we have mean derivative

So, this mean derivative, mean velocityderivativeand this again isthe mean velocityderivative, multiply a turbulent stresscomponent here. So, these terms will appear, plus another special term which is return out separately, because this is something that wouldnot finally, appear inthe turbulentcontinent j equation.

So, this is the resulting equation upon manipulation, which we can get, and it looks permeable, it has been derived by many of graduate student all over the world. So, this has the assurance that it can be derived, but one has tobe patient, it has been put togetherin this form.

So as to give somespecific meaning here, so we can see that this is ofform similar to the standardgeneric scalar transport question, and the scalarthat we referringto here, is this phi, and off course this has anindex phi j, this is a stresshere, and this is the advection term, this is being adverted by the mean flow, that is the mean velocity here, and gradient that is coming here, so thedivergence operator.

And we are missing, I think we are missing this, so if missing a term which is in the generic scalar transport equation we have an advection term, anddiffusion termand then we have a source term here, and so we have all these rest of the term here, which constitute all the unknown kind of things, these are in a way, there are some terms which are known, but this do not fit into the standard diffusion term.

For example, here this is another scalar phi i m, and this is another component of this particular scalar, so in that sense one could say that if there are 6 suchscalars, theseareknownterm and this is also known. So, from that point of to these two terms here, or in a way one can say these are known or movable, when we write all these term see together, but these are triplecorrelationsof 3 different velocitycomponents, and these are not to be known, within context of solution of these equations, these 6 equations along with moment equations and the continent equations.

Again, the simultaneous fluctuation of p and u prime again is not known, this termhere is the simultaneousfluctuation of the pressurefluctuation and the velocity gradient fluctuations is not known, and this is also not known. So, there are so many terms that are not known here, and thesehavebeen given an interpretation. Now, we havesaid in thepreviouslecture, and also right in the beginning, that turbulence is produced in the regionwhereyouhave large mean velocity gradients, and the mean velocity gradient are coming in thisoverall equations right here, so this is a mean velocity gradient and this is another mean velocity gradient, and this is like a Reynold stress term, and we have put it here in this particular way.

So, and we have put it in the way that minus u i prime u j prime here is nu ttimes d yby d x j like this, and in thespecific case of 1 dimensionalflow this thing will become nut du bar by dy, and that is equal to minis u prime vprime bar, and so minis u prime v prime bar is the positive quantity here. So, one could say that, this along with the minis is apositive quantity, and also this along with a minus is typicallypositive quantity, and when this is positive, this is positive and soon. So, this whole thing is a positive quantity, and this is a positive quantity, in which the mean velocity gradients arecoming into picture, and this is therefore, treated as a term production of the turbulent kinetic energy.

And similarly, this term here is when we put i equals j, this becomesapositive quantity, so this becomes a negative term, sotogether one call this as epsilon, and in the present format it will beepsilon i j, but once we in ordertoderivefrom this, the expression for k we have to put i equal to 1 and j equals 1 here, and then writein equationfortheu primesquare, and then youput i equal to 2 and j equal to 2 here, and then to the corresponding thing, so essentially whenwe put i equal toj equal 1 2 and 3 we get the three normal spaces, and we add together to get the overall k equation.

So, essentially as faras, the turbulent kinetic energy equation isconcerned, i isequal to j, so in that sense we go away, and then we have a epsilon here, and at that point this is a positive quantity, this is the rate of dissipations of turbulent kinetic energy.

So, this is simply like a sink term, so this is a source term, and this is a sink term for this.This term is suchthat, although this is not known, once you put i equal to j and this thing goes to 0, because thisbecomes, we get dou by dou x i become dau u i primeby dau x primed u i prime, and that becomes equal to 0, because are instantaneous continuityequation is dau x i by dau x, dui by daux, i equal to 0, that time average form of the continuity equation isdau u i bar by dau x i equal to 0.

So, if you subtract this from this, you get dau by dau x i of u i minus u i bar equal to 0 and this is nothing, but, dauu i prime bydau x i is equal to 0. So, that means that value put i equal toj here, in order toget the normal stresses, and then this term is identically 0, this 1 is identically 0, so this would not come in to picture,that is why we want to decompose this in to this form.

So, of all the terms that are referring here, these two terms together, cross should the production term, as of source positive source term, and this term here, cross woods a sink term which is trying to reduce the amount ofterm kinetic energy, and this is the term these two terms are essentially not known very well, andwhatpeople have try to show is that, this term isbeing neglected in the case of, is canceling of in the case of k term, this is very important in the relative magnitude of the various normal stresses, so in that sense very important term, andbecause of the(( )) between this u prime m and fluctuation and dau x mgradient, that is coming here in this, and if one worth treat this has a scalar that were the dealing with, the scalar is the time average quantity of this,where as this is the instantaneous that quantity of that kind of thing.

So,it is not exact, but in one word to say that, this is phi here and this isdau bydau x m of u prime m o bar, then it become very similar to this, and this representsthe turbulent diffusion of x momentum, so, by the same token, thiswe considered as turbulentdiffusion of this quantity, which is the k like whom quantity here.

So, this time is not very essay to deal with, so together these termsare brought under the umbrella of turbulent diffusionofk. So, this equation ofthe Reynolds stress when we contract the enosis, that is when you make i equals j here, and then right in a equation for dau by daut of halfof u i prime u i prime, which is nothing but k here.

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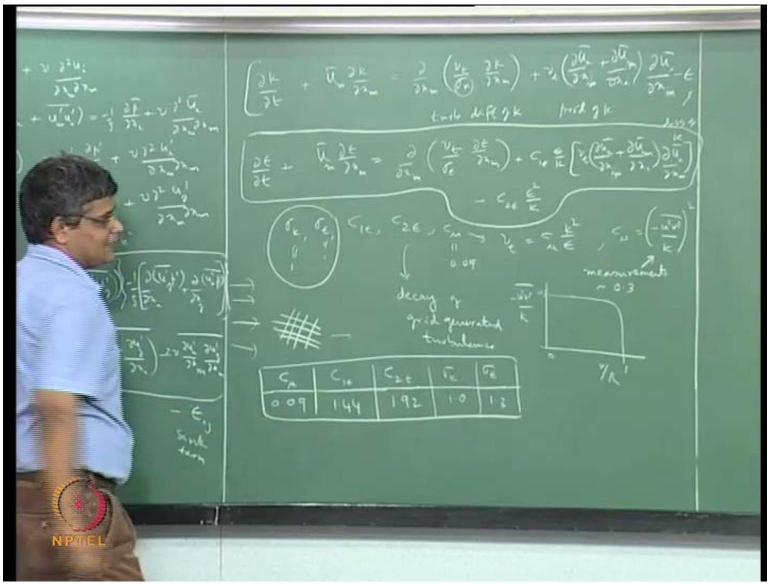

Finally, takes aform like this, this is the turbulent diffusion, and we have turbulent diffusion of momentum, and this can be consider as an equivalent of the frantlenumber of the k, so frantlenumberis the ratio ofmomentum diffusivity to thermaldiffusivity, and that comes into picture when we arelooking at temperaturegradient in the corresponding thermaldiffusivity. Here we are looking at diffusion of k, so this sigma k can becalled as the equine of frantlenumber or the Schmidt number for k, and this is essentiallyby the turbulent mechanism, sowe can put it like this.

So, this is overall effective diffusivityof k, and then we have the production term whenyou put i equal to j here, then this becomes 2 timesthis and 2 times u i primehere, andwe have to divideby half throughout, so ultimately we can get, and this quantity is expressedusing this expression here, so that once you put this into here, and then you do this, we get the overall production termnutdau u i bar bydau x j plusdauu.We are using m here throughout, so we can use m without any lossgenerality, so that is the production term which takes account to this, and this term is 0, and then we have minus epsilon, so this is overall equation for the for k which is derived from some kind of hand waving argument, with respect to the diffusion term, and especially with the triple correlation, but otherwise one can derive thisexactly.

And in the process, we have introduced only one constant sigma k, and we have made use ofah the (( )) type of hypothesis to represent these things as nu t times dau i bar and d x j and all these, when we are dealing with production term. So, in this way we can derive equation for k, and the derivation of the epsilon is also similar, in the case of k equation we wrote an equation for u i prime, and then we wroteanequation for u j prime, and then we multiplied by u j primehere, and then u i prime here, and then added them, and then took time average to get anequation for this, and then we contracted the indices. Epsilon here hasdau u i prime by thedau x k,dau u i prime by dau x k time average.

So, once we do this we have to take du bydau x k, we need to take the divergence of thisand that will give us the equation fordau u i prime bydau x k, and then we have to take again the divergenceof this whole equation to get dau u j prime by dau x k, and then we need toto multiply this by this, and this by this, and then add them together and them take the time average, and then we get horrendous expression involvingmany more correlations andmany moreunknowns, plussome term which we can readily identify. So, just as in this equation, in this exact equation without any assumptionshere, this is derived wholly from the navier strokesequations and continentequations, and in this we can identify this term and this term readily, and we can identify some elements of theseterms, but there are number of terms which are not identified.

The number of terms which are not identifiedby the epsilon equation is much more, and all of them areswept under thesegenericterms, this will be exact, we have diffusion so one can say this is turbulent diffusion of k, production of k, and this is the dissipation of k. So, a very analogous relation is finally derived fromthe mess, that comes out of all these operations here, and then the final equation will be of this form, again a diffusive term, which is very loosely written and analogouslywrittenas nu tby epsilon, sigma epsilon,plus a production of epsilon, which is essentially this term multiplied byepsilon by k.

So, C 1 epsilon,epsilon by k, times this term, and thisterm is also multiplied by another constant C 2 epsilon,epsilon by k multiplied with the epsilon by epsilon square by k. So, thisis the module equation for epsilon and that is the epsilon equation that we talking about.

So this is only loosely and roughly derived fromthe exact navier stocksequations with a lot ofah approximations and simplifications, and when you look at the equations, the additional equations that we have to solve, you see that these are of the standard form generic scalar transport form withan accumulation term, advection term, a diffusion term, and production plus, source and symptoms put together. The only difficulty from a mathematical point of view is some of these terms are non-linear, and this term involves is a function of epsilon, thisterm is also function of epsilon, and in this, this is an unknown, so there iscoupling into just between this and this, and coupling introduce between these and these together to the k term, which is appearing here, and there iscoupling introduce between these two and the momentum equations, in the form of the expression for u i prime u j prime here, which is expressed in thisway, where nu t itself is expressed in the form k and epsilon.

So, we have the overall set of equations, the 6 equations that we have for u bar v bar w bar p bar k and epsilon are very strongly coupledamong each other, through these algebraic expression, and through these expressions for k and epsilon, which not only are inter coupled through the dissipation rate and then the production rate, but also with the mean velocity gradients which are coming in the production term.

So, the source term linearization and their treatment of the source term is very important, especially when we talk about the turbulence model, but other than that the equations look very similar to what we have, and the things that are yet to bedetermined are the constants sigma k, sigma epsilon, C 1 epsilon, C 2 epsilon, and the constant which is coming here we have written it as C mu, which is appearing in nu t be equal to C mu k square by epsilon, these constants are determined partly from some estimates of what is expected, these are effectivelyprantle number for diffusivity of k and epsilon, and in realcase, when we talk about fluids, the prantlenumber of a fluids can be very different from 1,for example theprantle number for gases typically, so the order of point 7, but for water,it is the order of 7, and for some organic liquids it can bevery large, going into thousands, and for liquid metalsprantle number is very low of the order point 0 1, so they can be quite a lot of variations for real fluids.

Similarly, for the diffusivity, are the Schmidt number which represents the ratio of thekinematic viscosity, and the diffusivity, the mass diffusivity, so that can also vary over a very wide range, when you look atthe typical fluids, and in those cases, the mechanism is by molecular interaction.

But, in these case, the diffusion that is coming here, and the diffusivity that is causing the diffusion of k, and epsilon is essentially by the same mechanism which is also the causing the turbulent diffusion of momentum, and that is arising out of mixing of different fluid molecules, by thefact that we have turbulent fluctuations, and by the fact that we have these eddying structure.

So, the rate of this mixing, and the rate of this effective flux that is generated is determined essentially by the intensity of turbulence, which is causing the eddies to be present, and also the size range of eddies that are presented.

So, in that sense, themixing of k and epsilon, these are different scalar quantities is not govern by the intensive property of the k and epsilon, but it is govern by the rate of mixing, and it is the size of the eddies that represent. So, whether we talk about the momentum diffusivity, the turbulentmomentum diffusivity, orthe turbulent viscosity orthe turbulentprantle number, or the turbulentprantle number for k, that is the sigma k and sigma epsilon, all these are depend on the same quantity.

Therefore the ratios ofthe turbulent momentum diffusivity to turbulentk diffusivity, which is what is represented by the sigma k, these should be of the order of one, and this is also of the order of one, and the precise values of these are adjusted bycomparing the overall results for velocity of profiles in turbulence levelsin experiments, but these two are expected to be of the order of one.

Now, this quantity here is determined under conditions, where the k equation, the k evaluation and epsilon evaluation are determined wholly by the source terms. For example, when you are looking atcases very close to the wall, where under the study conditions, where u m is very close to 0, and the diffusion term can be neglected, and it is the rate of production of k and the rate of local dissipation of epsilon, these two factors dominate, so under those conditions one canshow that C muis minus u prime v prime square bar by k, whole square, and measurement show that very close to the wall at which point these equilibrium between the rate of production of k and the rate of dissipation, under those conditions, measurements show that, this ration is of the order of point 3, andso if you were to plot along the radial distance r by R, and these quantity minus u prime v prime by k, we note that k is alsoa turbulence quantity.

So, we find typically,it is like this around point 3, and close to the wall, it goes down like this, over a large portion of this r by R is equal to 0 and 1, so over a large portion this has a value of point 3, based on that you put these here, you get C mu equal to point 0 9.

Now, under the conditions of decay of grid generated turbulence that is you have a grid, you have a mesh, and you have flow going over it, and you have essentially uniform flow going over it without any significant velocity gradient. The fact that is flowing over the wires will produce some velocity gradients and turbulence, and some fluctuations of k and epsilon and as you go pass these grid, there is no more generation of turbulence, because at thispoint d u bar by d x n is going to be 0, because there is no mean velocity gradient, so this term will be 0.

So,this is 0, this term is there, and so this term is usually a negligible, you have set in rate of decayof k with respect to distance, and based on that andwe can get thisvalue here, and so based on these considerations, you can fix the overall values for the constants these are vary from person to person C mu is point 0 9, C 1 epsilon is 1 point 4 4, C 2 epsilon is 1.92, sigma k is 1.0 and sigma epsilon is 1.3. So, these are the typical values and one can see that these two are of the order of one here, and these are fix by the grid turbulence, and other measurements, and this is based on the velocity measurementsclose to the wall here.

So, once these values are specified, then in this equation we have only epsilon, which is the parameter of importance here, and this equation determines the value of epsilon at every x y z t, and similarly in this equation this value is known, and everything else is in terms of knownvariables, and therefore this can be used to calculate k. So, inthe two equation model, of which this is one particular form, in the two equation model, we are solving two equations, that turbulent kinetic energy conservation equation, the conservation equation for the rate of dissipation of turbulent kinetic energy, so big quantity, and these two equations are solved for the instantaneous local values of k and the epsilon.

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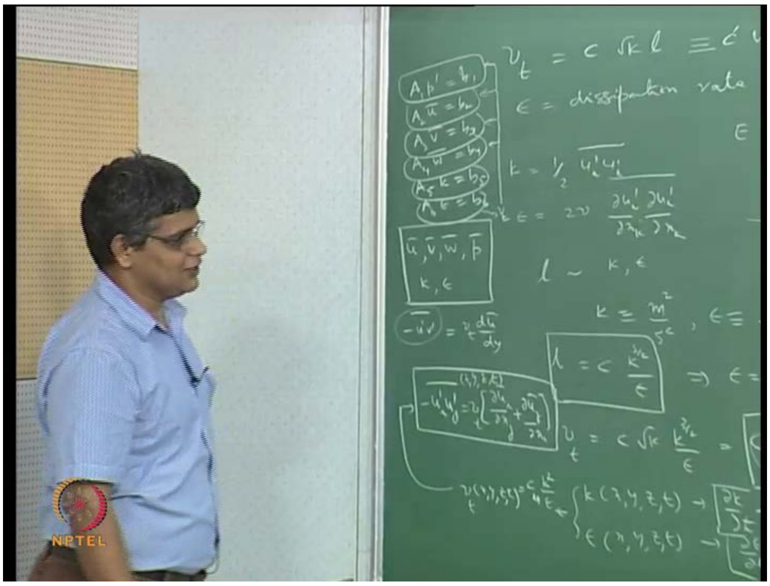

And from these things we evaluate the instantaneous local value of the turbulent viscosity, using the constant of point 0 9 and k square by epsilon. And, using this instantaneous local value of the turbulent viscosity, we evaluate the Reynold stress component in terms of (( )), where you have themean velocity gradients that are coming here.

So, this value, in each of the momentum equations there will be gradients, appropriate gradients of 3 of these fluxes, and these fluxes revaluated, and these come into as source terms, in thesource terms or effective diffusion term inthe momentum equation.Therefore, in the momentum equation we no longer have to write these u i prime u jprime, we need to only this, but in order evaluate this term we need to know this, and in order to evaluate this, we need to know these two, and these two are obtained only fromthese equations.

So, whenwe talk about a turbulent flow calculation, using the two equation model, we have the continuityequations, which will be put in the form of A 1 p prime equal to b 1 in its discritized form, together pressure fluctuation, and pressure correction and from thiswe getthemean value of pressure. Please note that,this is not pressurefluctuation, this is a pressure correction, and then we have the discritization of the time average x momentum equation will give us A 2 u bar equal to b 2, and the discritization of the time average form of the y equation, will give us A 3 v bar equal to b 3, and similarly, the time averagez momentum equation, it will give us A 4 w bar equal to b 4, and the discritization of the k equation, right here, will give us A 5 k equal to b 5, and the discritization of theepsilon equation will give us A 6 epsilon equal to b 6.

So, we have a big loop in which; we evaluate this, and thenwe evaluate this, and then we evaluate this, and then we evaluate this, and then wecan evaluate this, evaluate this. We get nu t, and using nu t and available estimate, we can evaluate the u i term and u jterm which will fill into these things, and then once we update all these things, along with the updates which is going into this, we come back and do one more iteration.

So, we can do each of these things sequentially, that is one after the other, andupdate the values frequently with an under relaxation, andget an overallevaluation of the time average velocity, components and the pressure, and the local time average quantities represent in turbulence k and epsilon, from which we can determine the diffusivities and so on.

So these is how two equation model of turbulence works, and what we notice is, that these conservation equationsdo not require specification of the mixing lengthor turbulent viscosity or anything like that these are self consistentcontained equations.

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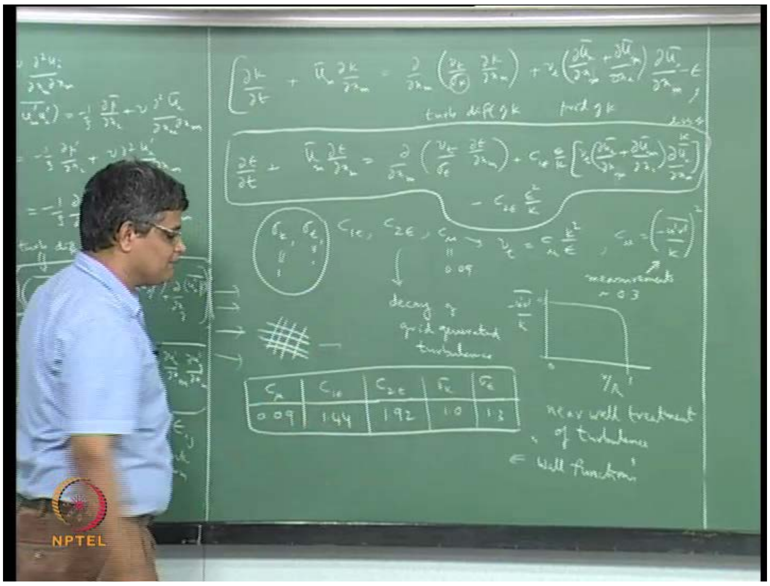

And there is no more, need to say anything other than the fluid properties, theflow domain, and these constants. The only thing that remains is the boundary conditions, there are certain boundary conditions very close to the wall, specifically for k and epsilon, and we have to give somespecial treatment at the wall, and that forms asubject of a large variety of turbulent models, the near wall turbulence thing, we don't have that much time in this course to deal with the turbulence modeling approaches, butwe need to do something close to the wall, near walltreatment of the turbulence, I would refer you to standard books on turbulence modeling and advance fluidmechanics, butone approach, one simplistic approach is to usesome kind of wall functions, which roughly capture thevariation of k and epsilon and the velocities very close to the wall, and they give rise to algebraic prescriptions for what the u v w and k and epsilon wouldbe close to the wall, and that way we can overcome the special effects associated with near the wall, there aremuch more advance models like that.

So, along with these, wall functions, wall prescriptions for this, we have complete description ofall the parameters that described fairly addictively and in a fairly general way a turbulent flow, and one wouldhave to solve these extra equations, in order to come up with the calculations for turbulent flows.

So, in the next lecture, we will see how we can extend this, when we have, for example the calculationof the non-isothermal flows, that is what we can do forthe energy equation, and also for the typical scalar transport equation, where we are looking at mass transfer or flows with the with chemically reactions and so on, with that we will be able to say that, we would now have equations, which describe turbulent reacting non(()) thermal flows, and all of these are put together in the standard form, and they can be readily appendedto the method that already developed for a typical C f d calculation.