Computational Fluid Dynamics

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Module # 06

Dealing with Complexity of Physics of Flow

Lecture # 36

Reynolds Stresses in Turbulent Flow

Time and Length Scales of Turbulence

Energy Cascade

Mixing length model for eddy

We have seen that for turbulent flow we can time average out the very rapid fluctuations resulting in the Reynolds averaged Navier Stokes equations, which we can write in this form: dou u i bar by dou t plus dou by dou x j of u i bar u j bar equal to minus 1 by roué dou p bar by dou x i plus nu del square u i bar minus dou by dou x j of u i prime u j prime over bar.

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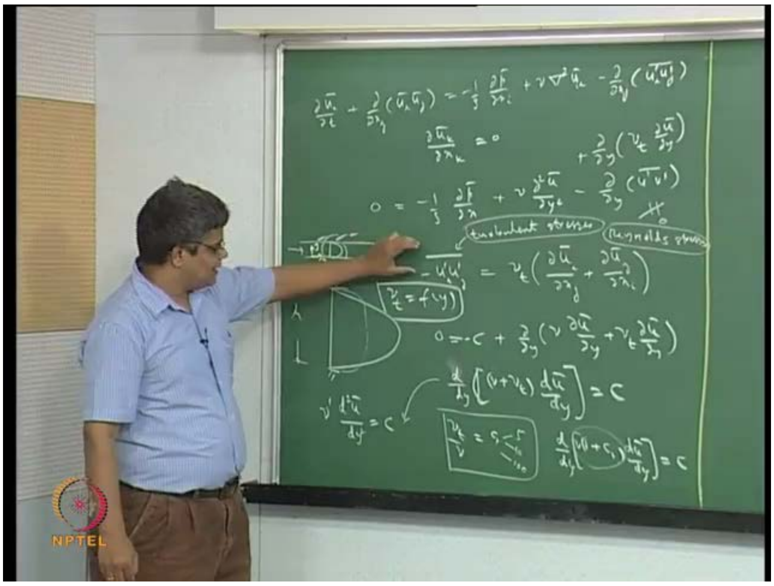

So, when we compare this and also the time averaged continuity equation is dou by dou x k of u k bar. So, when you look at these Reynolds averaged Navier Stokes equations we find that these are expressed in the form of time average velocities and the time average pressure; even the diffusion term here contains the time averaged velocity (Refer Slide Time: 01:32). So, that if one were to neglect this we would be expressing the momentum equation in the continuity equation in terms of time average quantities and it is these time averaged quantities which enable us to make assumptions of steady state or one dimensionality or two dimensionality of the flow.

Now, the set of momentum equations is not complete without this extra term and there are for each equation there are three such terms that are appearing in the momentum equations there are no additional terms in the time averaged continuity equation. We have also made the point that these additional quantities involving the flu the product of the fluctuating quantities cannot be neglected straight away because, for the case of steady one-dimensional fully developed flow the x momentum equation leads to 0; that is, all the left hand side quantities becomes 0 equal to minus 1 by rho dou p bar by dou x which is a constant when we have fully developed steady flow in 1 direction plus nu dou square u bar by dou y square coming from this the other quantities are 0 and the only non zero quantity here will be dou by dou y of u prime v prime bar and this is the case of fully developed flow through two infinitely long parallel plates which are separated by a constant distance under turbulent flow conditions.

So, the flow is taking place and you have velocity profile like this; this is our y direction this is our x direction and if we were to neglect this term which is the additional term which is coming here then; that means, that this equation is very similar to the laminar flow part laminar flow form of the equation we would be getting a profile which is parabolic with maximum velocity which is 1 point 5 times the mean velocity for the case of a flow like this. But, we know that under turbulent flow conditions through the same things if you expand this thing here (Refer Slide Time: 04:18) and then draw it you have the 2 plates separated by the distance h under laminar flow conditions we expect the velocity profile which is like this.

Under turbulent flow conditions experiments show that the velocity profile is more uniform significantly more uniform than laminar flow and if we were to neglect this and then if we were to solve only this then we would be getting a laminar like velocity profile not like a turbulent like velocity profile. Therefore, this cannot be neglected (Refer Slide Time: 04:56).

So, this not equal to 0 in turbulent flow and therefore the extra terms that are coming here are not are not are not necessarily 0 and not only that they have a tremendous influence on the turbulent transport of momentum. In this particular case, if you were to take the energy equation and then time average it you would find additional term comes out of it and those addition term represent the tremendous influence of turbulent transport of energy and similarly the turbulent transport of mass. So, we cannot neglect these things and we have to account for these.

So, that we get turbulent flow like behavior of the flow parameters from our time averaged equations and how can we write how can we write this we need to know more about these fluctuations and 1 of the early attempts at modeling this was to write this u i prime u j prime thing minus of this term average of this u i prime u j prime as being equivalent to a turbulent viscosity times mean sheer based on the sheer rate based on the mean velocity components that is like this and if we were to do this then this particular term here would become minus dou by dou y of new t dou u bar by dou y ok.

So, then under these conditions the flow, the equation governing the velocity profile in this particular case can be written as minus 0 equal to pressure gradient is constant. So, some given constant c plus dou by dou y of undo u bar by dou y which is this 1 plus this is minus here. So, this because we are saying minus of u i prime u j prime equal to nu t times this; this minus is... this becomes this becomes plus like this (Refer Slide Time: 07:20). So, plus nu t dou u bar by dou y or we can say and we notice that u is the function only of y we can replace the partial derivates with the total derivate we can say d by die of nu plus up nu t du bar by d y equal to constant.

If we put minus here then this becomes a constant. This constant is what is given for driving the flow and this is what we get from an example from the boundary conditions. Pressure at this end is this much; pressure at this end is this much and therefore, it is the pressure difference between the 2 which is driving the flow. So, this is a given constant u is the property of the fluid. So, this is known and in order to solve this we need to know nu t that is the turbulent viscosity turbulent kinematic viscosity which is being used to model these additional terms that are representing the turbulent momentum flux.

Now, what can we say about this new t we know that the turbulent momentum flux adds to the momentum flux coming from viscous forces and therefore, 1 can say that the overall turbulent transport in turbulent flow is greater then what we have and we know that from increased friction factor. So, 1 possibility is to say that nu t by nu is some constant c 1 which may be for example, 5 or 10 or 100 whatever it is that it is the transport is increased by multiplication factor c 1 and this c 1 is what we want to do what you want to input to this. So, that. So, that the pressure gradient computed from this or the volumetric flow rate for a given pressure gradient matches with the experimenter 1.

So, 1 possibility is to prescribe a value a constant value for the turbulent viscosity such that the pressure drop versus volumetric flow rate for example, in flow through turbulent flow through a pipe is on it. So, essentially we are increasing the fraction factor by a constant by a certain multiplicative factor.

That kind of thing is for the prediction of either the total flow rate or for the pressure gradient given that the other component is given. But, if you were to put this in to this then, what this equation would become? d by d y of this will become 1 plus c 1 where c 1 is nu by nu t and we have nu here times du bar by d y equal to c and all this is a constant it is not a function of y here. So, this is come out and we have essentially another equation this is nu prime d square u by by d y square equal to c.

So, this equation gets transformed into a equation like this which will give us a velocity profile which is exactly like a laminar velocity profile although of a lesser magnitude because of new prime is greater than nu. So, for a given constant a prescription of a constant multiplicative factor of turbulent viscosity it will give us a lower flow rate which we can match with the experimental volumetric flow rate by adjusting this constant, but the velocity profile that we are predicting will be laminar like and it would not be turbulent like therefore, prescribing a constant value constant multiplicative value of the turbulent viscosity is not correct, but because it cannot gives us a turbulent like velocity profile which is more uniform than the case of laminar flow which is more peaking at this center. So, the simplistic approach of prescribing a constant higher viscosity is not correct.

So, we have to make it in order to go from this kind of profile to this kind of profile we have to make nu t a function of y in this simple case it has to be a function of y. So, that when this function of y acts on this profile will bring it back in to this through this equation now what kind of functional variation can be ascribed to nu t.

So, for this we need to know this new t is some kind of modeling of these terms here this additional terms we need to know more about these quantities instead of saying these quantities let us from now on call these as turbulent stresses these are also called as Reynolds stresses for obvious reasons these arise from time averaging as per the Reynolds decomposition of the velocity of mean velocity and the fluctuating velocity. So, these turbulent stresses or Reynolds stresses being represented through this turbulent viscosity and through this kind of thing.

So, we need to know more about these turbulent stresses how can we know more about it is through experiments because that empirical information of how to replace this as a function of the other variables can only come from empirical results unless of course, we take recourse to direct numerical simulation and that is also being done to a great deal

So, now what kind of we can we say about these turbulent stresses u i prime u j prime which are 6 in number we are looking at Cartesian coordinate system. So, we have u prime v prime u prime u prime and of course, u prime w prime and then we have v prime w prime and v prime v prime and w prime w prime.

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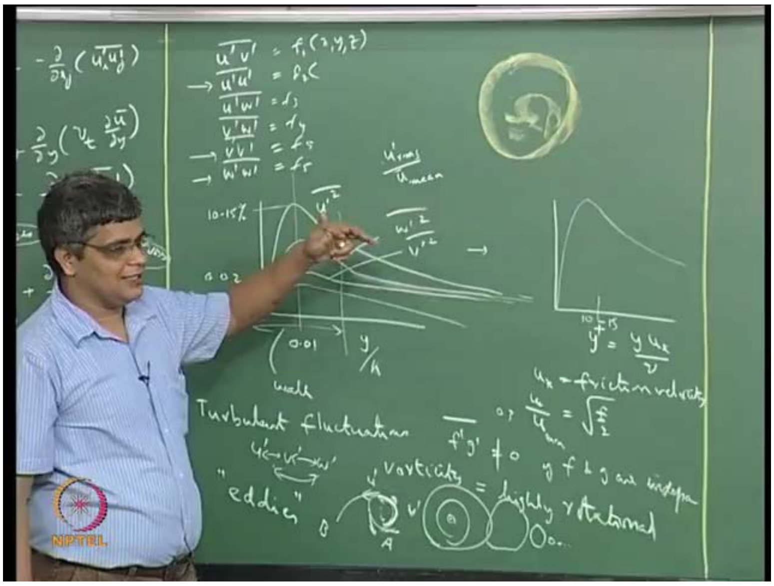

There are 6 Reynolds stresses or turbulent stresses and we would like to know how each of these things varies in the general case with x y z for a steady flow because, these are time averaged quantities. These will be steady if the turbulent flow is steady in the sense of turbulent flow time averaged flow parameters are steady; so, these functions of x y z only if the flow is steady. We will consider the steady case and there is no reason to believe that all these are functions of x y z. So, you have f 1 f 2 f 3 like this and these are what we need to know in order to replace these things coming in the in the moment equations.

Measurements tell us that the variation of these quantities even in very simple flows is very complicated. So, the spatial variation of this for example, if you take the simple case of flow between this kind of flow where we have fully developed steady flow between 2 parallel plates separated by distance in which the flow is driven by a constant pressure gradient.

We can plot these turbulent stresses as a function of y the distance from the wall and typically the stresses will exhibit the variations like this most of these quantities this the normal stress components these three will exhibit a variation like this and the variation we can see that it is not a simple variation like linear or second order or exponential. So, those kind of simple variations is not there because it is going through a peak and then it is coming down and what is also very important is that if you were to look at the spread of this as compared to the channel width y by h if you were to plot this in terms of y by h then this peak occurs very close it depends on Reynolds number, but it may be as low as point 0 1.

So, it is within this small distance from the wall you may have a large peak of this and then it behaves like this and this is for 1 of these things and you another thing may be like this. So, this may be u prime square bar, where u is in this direction. So, this is x and this is y and this is z and we are talking about a distance a region which is very close to the wall here because we are talking about y by h of order of point 0 1 to point 0 5 like this and this is a variation not as simple as this may be the variation of w prime square and this is of v prime square.

So, that means, all of them go through a the peak and towards as you go further and further away from the wall they achieve an almost constant value the magnitude of these things on a non dimensional basis may be fairly small this may be order of 10 to percent 15. So, if you where to say that what we are looking at is u prime r m s by u mean where u mean is the average velocity through the through the duct is often 10 to 15 percent and this may be lower and this is even lower and what we find is that here is the strong effect of the wall in this region, but as you go further and further away from the wall the wall itself does not play any it makes the turbulence more homage igneous-the more isotropic and in the sense the 3 normal components have more or more less the same magnitude which may be of the order of point 0 2 or something like that.

So, the presence of the wall will fundamentally alter the way the individual stresses are changing and we can see that from here that the component of the variation normal to the wall that is in the y direction is v prime square, is suppressed and the component in the flow direction that is u is enhanced and even w prime square is enhanced to some extent, but not as much as same as this.

So, we have a strong variation which is like this and this is true even of flow in a boundary layer very close to the wall and so on (Refer Slide Time: 20:00). So, for even for the simplest cases like one-dimensional steady flow very close to the wall, we find a variation which is fairly complicated and what we are interested in what are coming into the momentum equations are not the value of this u prime square and all these things itself, but it is the spatial gradient of this. So, it is the dou u by dou by dou by dou x and dou y of these things.

Dou by dou x is coming in like this and other things will be coming. So, you have also this and very close to the wall and all of them tend to 0 here to go down to 0 and. So, in that sense they exhibit a complicated variation the same plot is usually put in the form of plot y by h.

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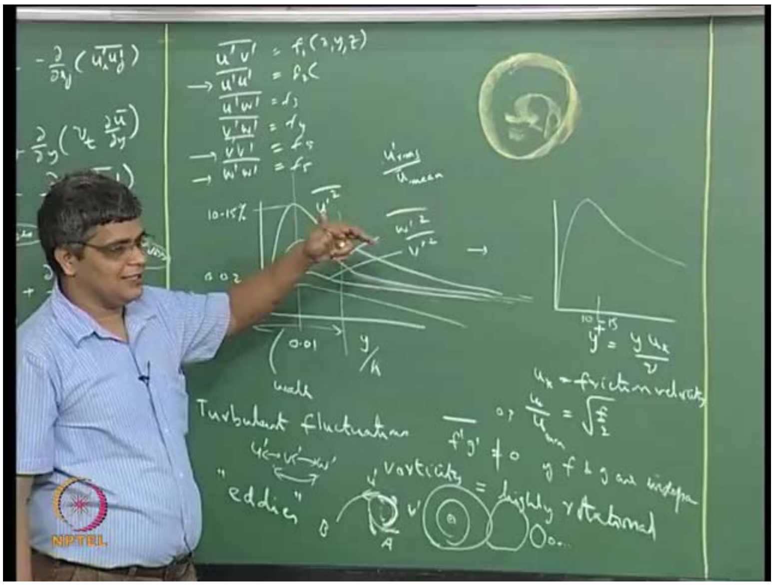

But in the form of y plus which is defined as y u star by nu where y is the normal distance of the wall where u star is known as the friction velocity and this is essentially square root of f by 2 where f is friction factor such that f is equal to 16 by Reynolds number in the case of laminar flow and I would refer you to standard books on fluid mechanics and turbulent flow to get this and in such a case, what we find is that the peaking of this these stresses occurs for y plus of around 10 to 15 it happens and then it goes on like this and the typical maximum value of y plus as you that is as you go from the bottom wall to the center that depends in the value of the friction factor and u star by u mean is the square root of f by 2. So, as the Reynolds number increases then the u star increases, but typically, this maximum value may be around 5000 or several 1000 may be two 2000 to 10000or even higher.

So, you have a very strong gradient of these stresses very close to the wall and they exhibit a highly non-linear behavior very close to the wall and that kind of non-linear behavior in a small region is very difficult to capture in a c f d type of solution because we normally use first order accurate or second order accurate method and the accuracy is very sensitive to the actual gradient of the quantity.

So, the this is, but what we draw from this is that the turbulent stresses are not 0 they exhibit a very complicated variation and they do not exhibit a variation which is similar although the for example, the sheer stresses components in this will be more linear not on the same scale,, but it may be like this very close to the wall.

Of course, this may be, this will again, I have to come back to this (Refer Slide Time: 23:45). So, you have a variation which is fairly complicated and it cannot be predicted for the general cases for example, if you looking at turbulent flow within this room then a turbulence at this point is going to be influenced not only by distance from this wall, but also from this wall and also from the bottom and also over the prevailing wind flow pattern in this.

So, it is a fairly complicated thing and it is not possible to come up with simplistic expressions for the spatial variation of these turbulent flow quantities in the general case. So, we have to come up with the models which are not necessarily simple because the underlying flow parameters do not exhibit a simple certain variations and the other thing about turbulent flow the nature of turbulent flow and the nature of turbulent fluctuations is that we have said that f prime g prime the correlation is 0 if f and g are independent.

Measurements show (Refer Slide Time: 25:05) obviously, that the normal stress components are not 0, but even the off normal stress components are also not 0. So, turbulent flow fluctuations are such that f and g prime and g prime are not 0 and in a non dimensional way they can be as much as even point 3 like that the cross correlation coefficient and be as high as point 3 under certain cases. So, this means that there is some linkage between the fluctuation in u and the fluctuation in v and similarly the fluctuation in u and fluctuation in w. So, there is some sort of linkage and this linkage has to be captured by us in order to make this model here. So, what is this linkage? The linkage is such that a turbulent flow typically has a high degree of vorticity. So, it is not irrotational it is highly rotational is a fluid mechanics way of describing this.

So, turbulent flow is highly rotational and in a pictorial way 1 can imagine as the flow is going through this duct here (Refer Slide Time: 26:49) it is not going in stream lines like this in straight paths. Like this the fluid is it has this kind of eddy kind of structures. So, it has this strong small whirlpools spread throughout the flow domain and the particular particle instead of going straight through it may be going like this and then it may go off and then it go like this and then it will go off like this.

So, a fluid particle is not following a straight path in turbulent flow and it is following a path which it is taking a path which is a highly irregular, but which has a strong degree of rotationality and this rotational element is not there all the time and it is not always small. It may be part of a bigger rotation it may be a part of a smaller rotation it may undergo the rotation only for a short time or for a long time.

Typically, what we express is that there are eddies or small whirl whirlpools that fluid particles encounters as it flows through in a turbulent flow and these eddies type of objects that are present in turbulent flow are the cause and result and sum and substance of this turbulent flow correlations because these eddies are fluctuating eddies in the sense that they are formed and then they grow and then they decay.

So, they represent in a way these turbulent fluctuations they characteristic of turbulent fluctuations and for an eddy to form a fluid particle here must go like this and the fluid particle here go like this and a fluid particle must go like this here like this. So, that you can have a eddy like structure so; that means, that the v prime fluctuation here and a u time fluctuation here must be correlated. So, that as this moves this moves.

It is not exactly perfectly correlated, but on the whole 1 can see that there is for in order for eddies to be present there has to be some sort of correlation between u prime and v prime at several points at points within this and also at the same point from time to time and that is why these turbulent stresses which are these f prime g prime over bar are not equal to 0. So, this is the fact that this is not 0 is for the substantiation of an eddy like structure in turbulent flow and we must capture this and eddy means that that the fluid which is here is being mixed by convection with eddy from here. So, with the fluid here, this element will go quickly into this then it will mix and it will exchange the properties that it has. And, if so, the property exchange between the fluid particle between this location and this location can now come about because, these 2 full particles are brought together by convection by the macro movement of the of the of the fluid particle from here to here in the turbulent eddy if this were not the case as per molecular mechanism.

If the property of the fluid element at this point has to be exchanged with the property of this fluid element here it is only possible through viscosity which means that this particle will exchange with this particle and this particle will exchange with this particle and this like this.

So, it goes through several particles and several interactions before it can it can effectively interact with this. So, there are a lot of middlemen between this trader and this trader in exchanging goods in trading goods whereas, in turbulent flow you are creating a railway path. So, that these people can travel up to this and then exchange goods. So, the exchange and trading of the goods between this particle and this particle is faster because you have laid a railway track and it is easy for them to go through you do not have to go through lots of middlemen here who are exchanging between and then who are taking a long time and so on.

So, this is why that exchange of momentum and exchange of heat and all those things the flux that is associated because of these differences is much faster in the case of turbulent flow and that arises from the fact that this eddy type of structures are present another characteristic of turbulent flow is that not only are there eddies like this, but the eddies of different sizes are there and it is this eddies of different sizes which actually gives as to effective mixing.

For example, if you are looking at point a here and point b at this point these 2 has to be mixed and we are talking about for example, chemical reaction which happens at molecular rates molecular distances then this can mix. So, all the fluid particles in this band along this sort of railway track will mix easily, but there is no mixing of this fluid particle with this fluid particle ok.

So, if you had only eddy of one size then mixing would happen only up to a certain length scale and beyond that it has to depend on molecular mixing arising out of molecular collisions. Whereas, in turbulent flow you have eddies of this size eddies of this size eddies of smaller size, smaller size, right down to very small lengths here (Refer Slide Time: 33:09) and smallest length is determined effectively by the rate of dissipation of turbulent energy it is determined by the action of viscosity on preventing these kind of a very small fluctuations and the largest size of the eddy is determined essentially by the physical dimension - the characteristic dimension of the flow domain. For example, the pipe diameter in the case of a circular pipe or the distance between the plates in the case of flow between parallel plates and typically the ball diameter and flow over a sphere. So, these are the characteristic sizes characteristic linear dimensions and the largest eddy size is typically a fraction a significant fraction maybe 10 percent, 15 percent of the characteristic size of the overall domain whereas, the smallest eddy size is characteristic of the amount of dissipation of this kinetic energy associated with these fluctuations that is this plus this plus this is a sort of kinetic energy of the turbine fluctuations and at what rate that is been dissipated will govern the smallest eddy size.

So, what you want to say is that because you have an eddy of this size,, but also eddy of this size like this mixing will happen not only at this length scale, but also at this length scale also at this length scale and also at this length scale so, that means, that mixing can happen right down to very small scales and that is when we have true mixing otherwise we will only see if you have mixing only b caused by 1 big eddy then mixing will happen only in that band there would not be mixing between this particle and this particle, but if I have mixing not only like this, but also like this and like this I can see mixing happen over this and this eddies are being converted by flow these are not stationary eddies.

So, this mixing that is happening here will transport further downstream and then that is also that also causes mixing and not only that eddies do not, of a particular size do not remain like that they grow to a certain size and then decay. So, they whether there are particular mechanisms by which eddies of certain size are created and these eddies will slowly decay in size and then they go away. So, you have a characteristic lifetime of an eddy and characteristic size of an eddy and these are random type of eddy kind of structures which are continuously being created and destroyed and which are exchanging energy which with each other and so, this kind of thing is expressed very nicely in a pictorial form in the form in the form of energy cascade which is very characteristic of cascade characteristic of tubular flow.

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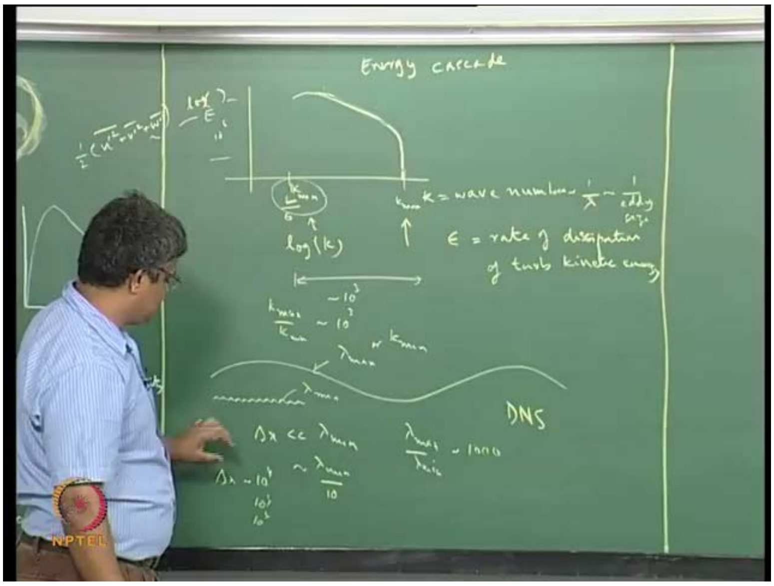

What we are plotting here is the energy and this energy is turbulent kinetic energy and what you mean by tubular kinetic energy is this energy proportional to half u prime square bar plus v prime square bar plus w prime square bar. So, that is the kinetic energy associated with tubular fluctuations.

So, 1 can say that what we have on this side is either the wave number which is proportional to 1 by lambda. So, this is proportional to 1 by eddy size these are very not mathematically rigorous definitions and one can look up standard text books to get much better idea, but the essence of this is that. So, when you have large value of k that means, that very small eddy size and when k is very small then you have large eddy size.

So, the largest eddy is fixed by the characteristic dimension of the domain and if l is the characteristic length of the domain then typically n by 6 is the largest eddy size and we have the k minimum this is fixed by the characteristic size of the domain and then at the upper extreme you have k max which is fixed by the rate at which this turbulent kinetic energy is being dissipated and. So, that is usually given this symbol epsilon.

So, this is the rate of dissipation of turbulent kinetic energy there are proper definitions for this and we will get back to these things, but the higher rate the higher is the value of the k max or the smaller is the eddy size that is possible. So, in between this the characteristic energy it varies like this.

So, you have turbulent energy is produced at by these large eddy sizes by the large eddies which are characteristic of the instability mechanisms that are happening in high sheer regions that is high sheer regions where you have a large gradient of mean velocity which typically happen close to the wall.

So, that is why very close to the wall you have large values of this turbulent kinetic energy of the order of say, point w within one percentage of the total characteristic length of that size. So, that is where most of the turbulent kinetic energy is produced and then it is cascaded down in a characteristic variation; this is a log plot log of e and log of k. It is cascaded it is transferred from the large eddy side to the next size next size next size right down to the smallest eddy sizes like this and when you come close to this k max, it is no longer possible to sustain these eddy sizes and you have too high too much of viscous dissipation of this fluctuation and beyond this you do not have any fluctuations or energy associated with this. So, this is the energy cascade.

So, the energy associated with this turbulence fluctuations is produced by eddies which are characteristic of length size which are which are of the order of the length size characteristic length of the flow domain through mechanisms instabilities which are created very close to the walls these are called ejections and sweeps are those kind of phenomenon which are taking place in turbulent flow although this is an instability there is also a self regulating mechanism by which this instability does not go in to uncontrollable system behavior. So, it is produced here and then it goes down and then goes down to the smallest length here.

So, it is this presence of these eddies and the exchange of momentum between eddies of different sides which is characteristic of the energy cascade and one can therefore, say if one were to characterize turbulence then one possibility is look at this lowest eddy size that is possible. So that can survive in the turbulent flow against viscous dissipation and on the other size of the spectrum we have the largest eddy size which is characteristic of this. So, these are two reference points which essentially determine the range of the eddy sizes and the characteristic features which cause the high turbulent a turbulent transport of mass momentum and energy in under turbulent flow conditions and typically for turbulent flow for fully developed turbulent flow for the Reynolds's numbers of the order of 50,000 of the higher this variation here can be a few decades. So, this can be something like an order of 10 power 3 and this can be 10 to the power of 6 or even higher.

So; that means, that k max by k mean is of the order of the power 3 and similar this may be order of to power 6 energy max and energy mean associated with essentially with measureable things is like this and what is especially of importance is this range here that is fluctuations are possible of this kind of we have said that the this is the wave length of this fluctuation and this is also in a way loosely representative of eddy size. So, means that we can have wavelength of fluctuations like this associated with lambda max or k min and we also have fluctuation associated with the smallest (()) this is this is associated with lambda min. Now, if we want to a proper turbulent flow calculation which captures the full range of this turbulent fluctuation from the smallest size to the largest size you must have a grid spacing which is much less than this lambda min ok.

So, delta x must be much less than lambda min and we know that from this lambda max by lambda min is of the order of 1000. So, that means, that the number of grid points to capture these variations may be is of the order of 10,000 that is because, if we say that delta x must be less than this.

So, if you say that delta x is lambda min by 10 then the number of grid points that we have in one direction itself is 10 to the power 4 to capture the faithfully all these fluctuation and we know that turbulent flow is 3 directional.

So, you have delta x in the x direction you have 10 to the power 4 points y direction 10 to the power 4 points and z direction 10 to the power 4 points. So, that is the number of points is humungous to capture a highly turbulence flows and. So, that is why this kind of computations very difficult to do to be done in for highly turbulent flow calculations with very large re Reynolds numbers and the d n s that is attempted the direct numerical simulation where we solve the unsteady form of the 3 dimensional Navier Stokes equations with small delta x in all these things is typically carried out for rainouts numbers of order of 30,000 I think it has gone up to 30,000 or of this order may be 50,000 like that.

So, now, we want to come back to prescription of how these things vary the prescription of the functional variation of these things. So, having seen the complexity of turbulent flow in the form in the sense that there are highly spatial variations based on measurements and highly temporal variation in the form of eddies of vast range of sizes which are produced and being which are continuously getting bigger and smaller and exchanging with all kinds of things (Refer Slide Time: 47:10); therefore, exhibiting highly temporal fluctuations we can say that it is not possible to come up with a very exact description of this turbulence process.

So, people have tried to capture the essence of these things of the nature of the turbulent flow and the mixing that it causes through simplest prescriptions in the form of this effective viscosity effective turbulence viscosity which is causing the turbulence process. So, in this particular case they are abandoning any attempt to make this fluctuations realistic in the sense of this turbulent flow characteristics that are present here in the form of eddies and all that. So, that particular thing is essentially being is being subsumed.

In the definition of an equivalent turbulent viscosity and a very simplistic picture for the justification of the turbulent viscosity is presented the presence these eddies of different kinds is equated to the presence of molecules as per the kinetic theory of gases and. So, just as in gases the molecules interact collide with each other and then therefore, exchange momentum it is also said that there is a possibility of similar of kind of mechanism in the form of interaction of this eddies that gives rise to a viscosity that kind of mechanism is what is thought for this turbulent viscosity.

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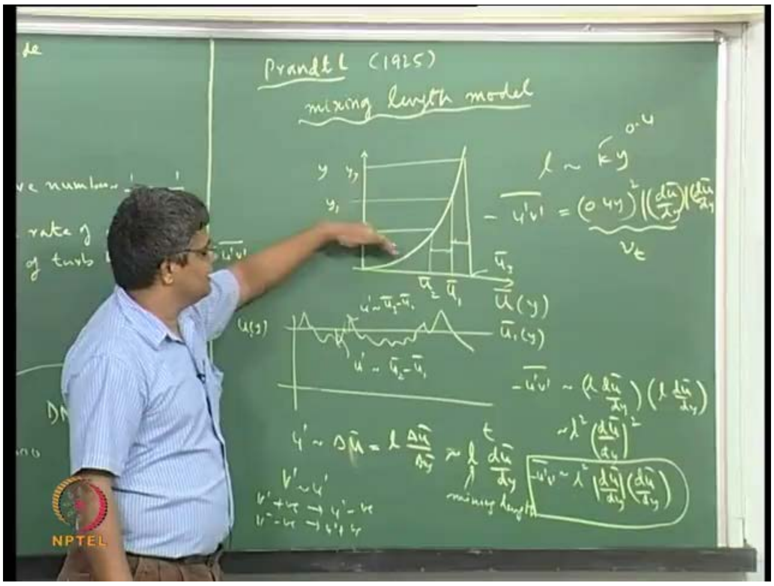

So, this is where for example, the simplest generic model for this has been proposed by print in 19 25 and for the simple case of one-dimensional flow that we have here (Refer Slide Time: 49:14), he proposed what is known as mixing length model there are probably several interpretations to this, but the simplest way to say is to say that the fluctuation that are absorbed at particular point is because fluid molecules from different parts are being brought to that particular point.

So, if you have a flow which is this is the channel dimension you know that very close to the wall it has to have very low mean velocity and as you move away from the wall you have higher and higher velocity now the nature of turbulent flow is such that there are eddies because of which mixing of fluid elements from different heights takes place and that is the that is the reason why the fluid with the lower velocity here can come to a region of high velocity and exchange momentum.

So, if you are measuring the velocity at a particular point sometimes you are measuring the velocity of a fluid element which is originally there which has a mean momentum associated with that height (Refer Slide Time: 50:46). Sometimes, you are measuring the velocity of the fluid momentum which is brought from a different place and then it is then it is coming here and if it is brought from a different place where the velocity is lower you record a lower velocity and if you if that fluid molecule is bought brought from a higher zone of higher mean velocity then you record a higher velocity so, that means, that the velocity fluctuations that you are recording at a particular point are because fluid elements with different velocities e are being mixed by the turbulent motion by the turbulent eddies that are present and therefore, we can say that if you were to take a velocity profile like this we have the mean velocity as a function of y is plotted here and this is the mean distance from the wall at the wall you have 0 velocity and as you move away from the wall the velocity is increasing like this let us say you are making a velocity movement at this particular y let us say at y 1.

So, the expected velocity is u 1. So, if you are now plotting as a function of time the measured velocity at u 1 then it is going to show oscillations like this and the mean velocity is this u 1 bar y.

So, this is the expected velocity, but at different times you are missing this velocity, this velocity, this velocity, this velocity, like this (Refer Slide Time: 52:36) and the idea is the velocity here is lower because the fluid element at that particular point has come from a lower height which had a lower velocity and at this point it has got a higher velocity or meshing a higher velocity because this particular fluid element has come from maybe a higher velocity gradient and therefore, you are meshing a higher velocity.

So, the facet in turbulent flow the fluid elements are allowed are enabled to move across different layers by these eddy structures can give rise to various structures. So, you're saying that if a fluid element has come from a lower position here then the velocity gradient that you're instantaneously measuring is roughly the velocity at this position u 2 and if it is coming from the y 3 here then the velocity that you are measuring is the mean velocity that is measuring at that position that is u 3.

So, the fluctuation that you are measuring is a characteristic of the distance from which these fluid molecules are being brought here and also the velocity difference velocity defect or the extra velocity that is there for the fluid molecules at those distances.

So, we can say that if the fluid molecule is coming from y 2. So, instantaneous u prime that we are measuring here this is u prime is u 2 minus u 1 bar and the velocity that you that you measured here is characteristic.

So, the velocity fluctuation, the mean velocity that is expected here is u 3 minus u q 1 bar. So, the fact that there is the velocity fluctuation deviation from the mean value is because the velocity that you are measuring belongs to the velocity from a different place and it is being brought here and getting mixed.

So, you can say that u prime is delta u. So, delta u bar where delta u bar is this 1 and this 1 and it can be positive or negative depending on whether it is coming from the bottom or the top. So, essentially you can say that on a time averaged basis you can measure points from the boo coming from below or above. So, and this is characteristic this delta u bar can be given by a mean distance over which the ma the fluid elements are being brought forward. So, it is a characteristic size of the eddy times the velocity gradient.

So, we can write this as l du bar by dy where l is the mixing length. So, he attributed the velocity fluctuation to a characteristic mixing length which represents the characteristic eddy size a single eddy size from among which is representative of the net effect of this large range of the eddy sizes and to the velocity gradient that is present here and of course,, we have u prime and also v prime,, but we know from measurements that this u prime and v prime are not very different. So, u prime is of the same order of v prime,, but we notice that when v prime is positive; that means, that a fluid molecule has a vertical velocity so; that means, that it is coming from a lower height to this height; so that means, u prime will be negative because here delta u is measured is negative.

So, when v prime is positive u prime is negative and v prime is negative. So, it must be coming from above when it comes here you record a positive delta e. So, when v prime is negative then u prime is positive so; that means, that the correlation u prime v prime is going to be negative.

Roughly speaking it is not an exact thing. So, there based on these things we say that minus u prime v prime is l du bar by die which is the velocity fluctuation u prime and this is also of the same order. So, again l du bar by dy. So, this is l square du bar by die whole square and to make all possibilities we put this as l square du bar by dy modulus of this and du bar by die like this. So, that if the velocity profile is not like this, but in different way then even then this can work.

So, essentially you have an estimate for u prime v prime which is expressed in the form of a characteristic mixing length times the velocity gradient and he suggested based on fitting and all these things mixing length is proportional to k times distance from the wall his proportionality constant being of the order of point 4. So, you have an overall expression for u prime v prime which is point 4 times y whole square times du by die modulus times du by dye and this thing here is of characteristic turbulent viscosity.

So, you can say that minus u prime v prime is u t times du bar by dye where u t is point 4 y square du by dy the modulus of the velocity coefficient and we can see that that expression satisfies the requirement that u t has to be a function of y it is a function of y because its contains y itself which contains y square and it contains also the local velocity gradient and also we can see now that we have an expression for nu t which does not require any more information it is involving y which is an independent variable and it involves gradient of the known velocity.

So, once you substitute into this expression here we have a single equation and a single unknown we have closed we have overcome the turbulence closure model in the simplest case.

So, this mixed length model is one model which is based on the, to some extent, on the physics of what happens in turbulent flow; which is that mixing which is caused by the fact that fluid molecules and enabled to move across different layers and that process they bring around an exchange in momentum is then captured in this. It has limitations and it has advantages which we will examine in the next lecture.