Computational Fluid Dynamics

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Module No. # 2

Governing Equations

Lecture No. # 3

Topics

Eulerian approach

Conservation equations

Derivation of mass conservation equation

Statement of the momentum conservation equation

Having seen CFD in action, where we are able to get a velocity field for the case of fully develop flow through a rectangular cross section duct. Now let us see, what we need to do, to make it more general ,and something which is also useful in practical context. Let us now ,discuss the course outline, and what we are going to do as part of this course .The first thing that we need, for any CFD solution, is the set of equations .We need to know what equations, we want to solve ,and we need to have a clear identification of the flow domain. We also need to have the boundary conditions, and if you have an unsteady flow we need to have the initial conditions.

So, the first phase of CFD solution, is to have a mathematical description of the flow, in terms of the equations which describe the variation of the relevant parameters .These in fluid mechanics, are none but the Navier-Stokes equations. So, as part of this course, we will be first deriving the Navier-Stokes equations ,for which we assume a first level course in fluid mechanics ,as a prerequisite, and maybe a school-level mathematics should be sufficient .And once we derive the equations ,then ,we get into a solution using an analytical method ,or the CFD approach .So then we come to the CFD approach, and we know that the CFD approach ,means that you have to identify the grid points, at which we want to get the desired velocity variables ,and so on, so the discretization of the flow domain is one of the steps.

So, instead of going through that particular route ,what we do is ,that we will see, that the solution of a fluid flow, for the general case ,requires not the solution of a single equation ,but a set of coupled equations. For example, typically we have three momentum equations, and one continuity equation together ,which have to be solved for the four variables of interest ,which are the three velocity components ,and pressure. Now, how to solve these together is a very complicated issue?

So, what we do, is first of all ,we generate a template which gives us a satisfactory numerical approximation to a generic partial differential equation, which describes the type equation, which we will have to repeatedly solve in fluid mechanics. So ,we will at the end of the derivation of equations, which are needed for CFD ,we come up with a generic form of a scalar transport equation. And for different values of diffusivities ,and source terms ,we can show that this scalar equation can represent the momentum balance, in the x direction, or y direction, or z direction or energy transport equation and all these things.

So ,for this scalar transport equation ,we derive a template which would give us a satisfactorily accurate solution, without any convergence problems ,if implemented properly. So ,we generate the template for the solution of the generic scalar transport equation. Having established this ,in the next module, we will look at how to use this template, to solve the coupled equation, which are necessary for the solution of all the equations which govern the fluid flow .That is all the four equations ,in the case of isothermal flow ,may be along with the energy equation and may be along with equations which represent turbulent flow ,and so on .So that kind of solution of coupled equations, is going to be done ,in the next section .Now, this will give us a framework ,for the solution of all the equations, which govern fluid mechanics

Now, we will try to make it more efficient. So, we look at what methods , that are available, for the solution of the discretized algebraic equations , which ,when we consider a general flow are typically non-linear. So ,we have to come up with, a solution of non-linear coupled algebraic equations. So ,we will look at those issues and we look at the generation of the grid for an arbitrarily three-dimension geometry .Not something, which is as simple as a rectangle or a square, for which we can describe the whole flow domain in a Cartesian coordination system or a cylindrical coordinate system.

If you have a body, which is irregularly shaped, we still have able to discretize this into tiles. In the case of two-dimension, such that all the tiles when put together ,will give you the overall thing. So there are some specific methods, for a structured mesh generation or an unstructured mesh generation. So we look at these methods ,and then come up with algorithms. This will enable us to discretize any arbitrary shaped flow domain into small tiles, which when put together ,will give us the total area ,or the total volume. So this will be the next module .And finally, we look at a solution procedure, which will work not only with the rectangular coordinates ,but also with these generally arbitrarily shape tiles, which will enable us to discretize the governing equations on these control volumes ,and then convert the partial differential equations into algebraic equations .Then whatever method, that we have developed in earlier modules for the solution of these algebraic equations , can be used to get a solution to the general arbitrarily shaped flow domain involving fluid plane. And all those things, and as a final module ,we will look at more number of equations which represent the complexities of industrial flows.

So, that is ,what we would have developed, until this is necessarily for the Navier-Stokes equations. That is ,which represents laminar flow for any tunneled fluid ,we will consider the possibility of turbulent flow ,which is most often the case in industrial things. We will, also consider the energy equation, when we have any kind of heat transfer .We will also extend the equation set, to the case of chemical reactions reacting flows .So that at the end of this module ,the final module we will have a set of equations, which describe unsteady turbulent reacting flows, which can also be laminar .And for this kind of complex case, we have a set of equations. We will show that these set of equations has equations , which can be put in the standard generic scalar transport form, for which ,we would have made by then a template . So using the methods, that we would have already covered in the earlier modules, we will have the capability to solve for generic unsteady turbulent reacting flow in a flow domain of arbitrary shape. So that will cover, the basic set of techniques ,which will ultimately lead us to attempt solutions for industrial flows or complicated flows.

So ,the outline of the course is having done the introduction derive the Navier-Stokes equations ,which govern the flow of fluid .And extract from these equations, a generic scalar transport equation form, and generate a template. For the numerical solution ,of this generic scalar transport equation form ,using the principles ,that have been evolved as part of the CFD subject development ,and use these capabilities to solve a single generic scalar transport equation .To evolve methods, for the solution of the coupled equations ,of the momentum and continuity ,so that we have a complete solution procedure, for the Navier-Stokes equations. And then, we come into the demand of mathematics, in the solution of simultaneous algebraic equations. And we look at a variety of methods ,using which the discretized equations ,can be solved efficiently .Then, we look at the practical case, of having irregular shaped fluid domains, in which we want to know the flow variables.

So ,we will look at methods for algorithms .For a decomposition of the flow domain ,into small tiles, and small bricks ,in such a way ,that the we can discretize ,the whole domain systematically i.nto small tiles .Then we finally develop the method ,by which, even an arbitrary shaped tile, can be taken for the discretization equation .This is where ,we use the finite volume method .So using these, set of techniques, we will be able to tackle real industrial flows. So that is outline of the course ,and the first part of this is the development of the equations.

Let us now, look at the equations, which govern the fluid flow .We are all familiar with these equations. These are nothing. But, the very same equations, which govern the solid mechanics also . By this, we mean the equation of mass conservation ,the equation of momentum conservation , and the equation of energy conservation .So, these are the same equations, which also describe the flow of a fluid .What is different between those ?For example ,the Newton's second law ,and the equations ,that we are going to derive, is that those equations are described for a system of particles . Whereas, in a fluid mechanics scenario ,we are interested ,not in a system of particles , on which forces are acting .Because of which the system is moving ,at a certain velocity, in a certain direction ,but we are interested in a specific domain in which things are happening.

For example ,you may be looking at a fluid flow ,involving a soup making reactor, in which you put different components of the soup .You mix them, you heat them, and then they cook ,and then they go out .So we are not interested ,in a single soup particle ,or a pea, or something like that ,which is going through this .We are looking ,at how different reactants ,are coming together into this domain .How they are exchanging heat with the surroundings, and other sources, and how they are going out .So we are interested, in a fixed domain. And within this domain, we have to write equations, which describe the inflow of reactants ,and outflow of products and so on.

So, we distinguish this, in the fluid mechanics community ,as a Lagrangian viewpoint ,versus Eulerian viewpoint .Lagrangian viewpoint ,typically refers to system-related equations ,like what we have in solid mechanics. The rate of change of momentum ,is equal to sum of all forces ,acting on the system of particles .Whereas in an Eulerian approach, we are looking at a control volume-based derivation of the equations .

So, in fluid mechanics literature ,we also have some theorems like the Reynolds transport theorem ,which show the equivalence between a Lagrangian way of formulating the conservation equations. And the Eulerian way , of formulating conservation equations, slightly mathematical. Given the viewpoint ,that this particular CFD course, is intended for engineers ,whose mathematical skills ,we do not want to challenge too much. So, we would like to take a much more physically appealing route ,to the derivation of the governing equations.

So ,what we do is ,that we take a control volume .A brick with six sides, and we say that in this control volume ,we want to describe the mass conservation .We want to describe the momentum conservation ,or energy conservation ,and then how do we describe it mathematically .And ,how can we derive the corresponding equations ,which describe this .so that is what our objective is.

For example, when we talk about mass conservation, we say ,that we take a six-sided control volume .We say , that the mass conservation equation for the fluid, within this control volume ,can be stated as the rate of accumulation of mass within this control. Volume is equal to rate of inflow of mass, into this control volume, minus rate of outflow of mass, from the control volume ,plus any sources of mass. Now ,most of the time, we are not considering any nuclear reactions, and therefore we say that there are no sources of masses itself.

Keeping in mind ,that this particular control volume, will be ultimately made very small .So that the equation ,that we derive from this , is ,applicable at almost every point within the flow domain. We are not saying , that the point is of infinitesimally volume. But it is so smaller volume ,that at every point ,this conservation equation ,which we derived from this control volume analysis is applicable.

So, we cannot think in real times of sources of mass in a reactor. So which are which are so small, that they are applicable at every point. So we restrict ourselves, to a mass conservation equation ,which can be stated like this:

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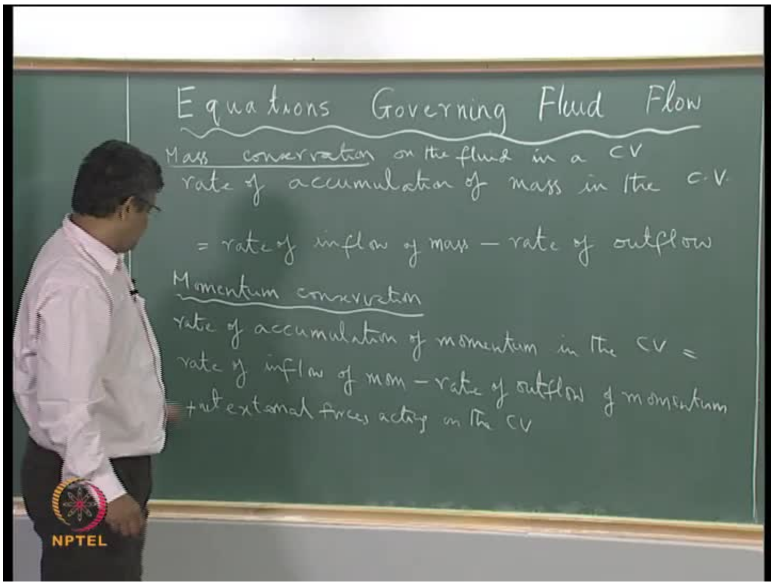

Rate of accumulation of mass in the control volume ,which I am abbreviating as ,a c v, is equal to rate of inflow of mass, into the control volume ,minus rate of outflow . I take this ,as a statement of mass conservation equation, for a control volume . I write these verbal statements, into a corresponding mathematical expression and from that, I derive the equation representing the mass conservation equation .So I can say ,this is mass conservation on the fluid in a control volume.

Similarly , I can write a momentum conservation on the fluid in the control volume .It can be stated as rate of accumulation of momentum ,within the control volume, is equal to rate of inflow of momentum ,minus rate of outflow of momentum, plus other factors .We know that momentum ,can be changed by the application, of forces momentum, can be changed by a rate of work done.

So, these are the kind of things ,that appear as plausible sources, or sinks of momentum. And of these given ,that we are looking at a control volume, which is ultimately going to be reduced to a point, which is prevalent in every part of the flow domain, we consider ,only forces .No work done by a turbine or a or a pump, which is which is traditionally done ,in a in steady state energy balances ,and momentum balances, in fluid mechanics

.So ,we say, that the rate of accumulation of momentum ,and the control volume is rate of inflow of momentum ,minus rate of outflow momentum, plus momentum changed ,due to external forces acting on it on the control volume. So , we can put net external forces acting on the control volume.

So ,we say that a change in momentum,.a change in momentum within the control volume, can take place, because you have excess of inflow or excess of outflow .Or you have some forces acting on it, then and what these forces are something ,that we have to specify. When we translate this verbal statement, into a mathematical statement, and then from ,which we can get the equation .Similarly , we can write about the energy conservation ,and the species conservation ,in the form in the case of reacting flow .That we will see ,at a later stage ,but right now we are concerned with these two statements . We will try to derive the corresponding equations from this:

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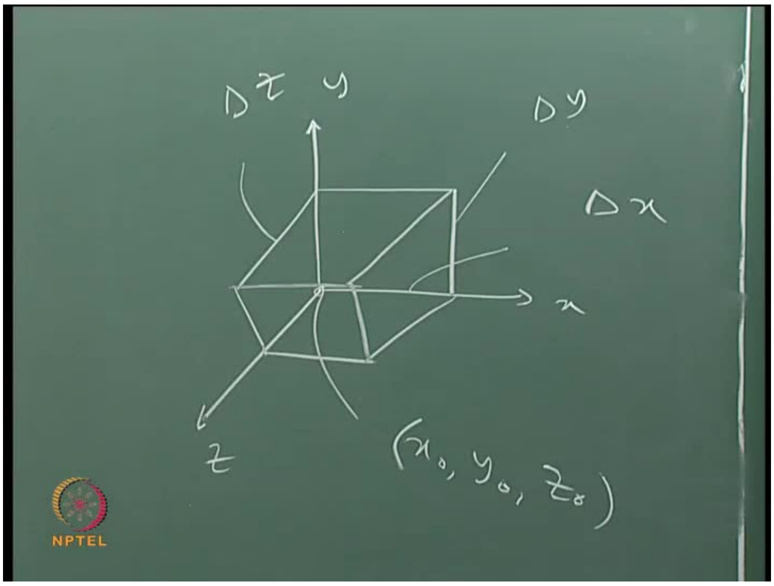

So, for this, let us consider a control volume. When we talk about a control volume ,first of all let us be simplistic .Let us take a Cartesian coordinate system ,with x axis, in this direction y axis ,and z axis ,coming out to represent a right handed coordinate system. For the sake of convenience ,if this is our control volume, we have x, y .z, the three directions . You can see that ,this has six phases .This phase, the left phase ,right phase ,bottom phase, top phase, front phase ,back phase. There are six phases .So we assume, that each of these phases, the bottom phase, or the front phase, or the left phase ,like this is aligned, in a particular plane of the coordinate system.

So, that is why ,we have a rectangular shape in this. And, then this is the control volume, and it has a length in the x direction of delta x ,length in the y direction, of delta y ,length in the z direction ,as delta z ,and we fix the origin here .And this origin is at x zero, y zero, z zero . For the sake of simplicity ,we assume that ,we have a stationary fixed coordinate system . So ,we want to write, we want to describe the statement of the mass conservation, on this control volume.

Now ,this control volume, is immersed within a large fluid domain. So, this is the basic control volume .It is like the discretized domain, in our CFD, and many such bricks, will comprise together the flow domain .So we want to know ,what changes are taking place, as fluid is flowing through ,and out of this control volume .So that is the description that we are looking at:

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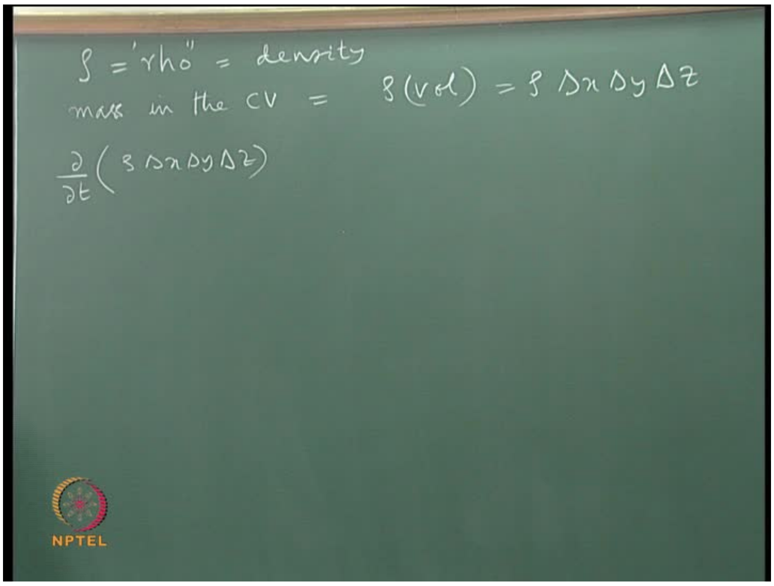

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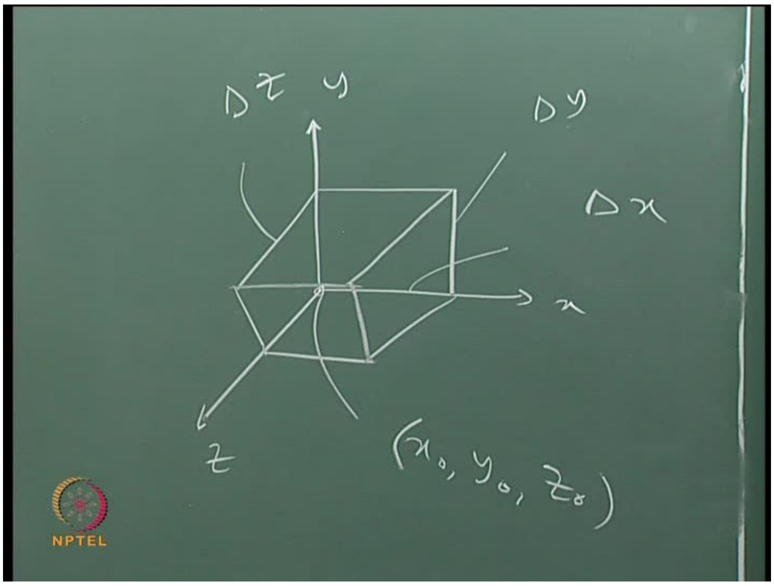

So ,we have a fluid, and the fluid has a certain density rho. This is a Greek letter , rho, describing the density of the fluid . And so that the mass, within this, the mass within the control volume, can be given as rho times volume , and volume for this rectangular parallel pipe is rho times, delta x, delta y delta z. Now we can say ,rate of accumulation of mass ,within the control volume ,can be represented as variation, with respect to time of the mass, within the control volume ,will give us the rate of accumulation ,rho times delta x ,delta y, delta z .This particular term, represents the left hand side of the mass conservation equation. Now this is ,we are saying is equal to rate of inflow, of the of the of the fluid into the control volume and rate of outflow.

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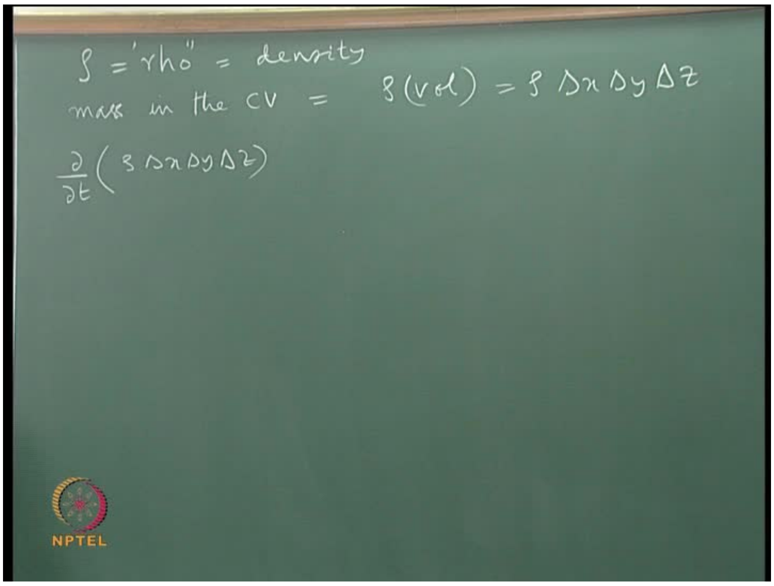

Now ,when we talk about inflow and outflow, what we are saying ,is that there is a velocity field ,u v, w representing the velocity components in the three directions. This because of this particular flow, because of velocity . Then flow is coming through this phase ,and then going out, through this phase, coming through this phase, and going out through this phase. So in that sense, we have to ,we can say that the phases, the surface of this control volume ,is the means ,by which fluid can either enter ,or leave the control volume .And if there is a net imbalance, between the inflow and outflow ,then there can be an accumulation. So the mass conservation equation, is saying that the rate of accumulation ,is also equal to the rate of imbalance between the inflow and outflow .We have to evaluate ,the fluxes through each other six phases .We can say ,that the flow rate through a particular phase, can be seen as the velocity normal ,to the phase times the area of the particular phase.

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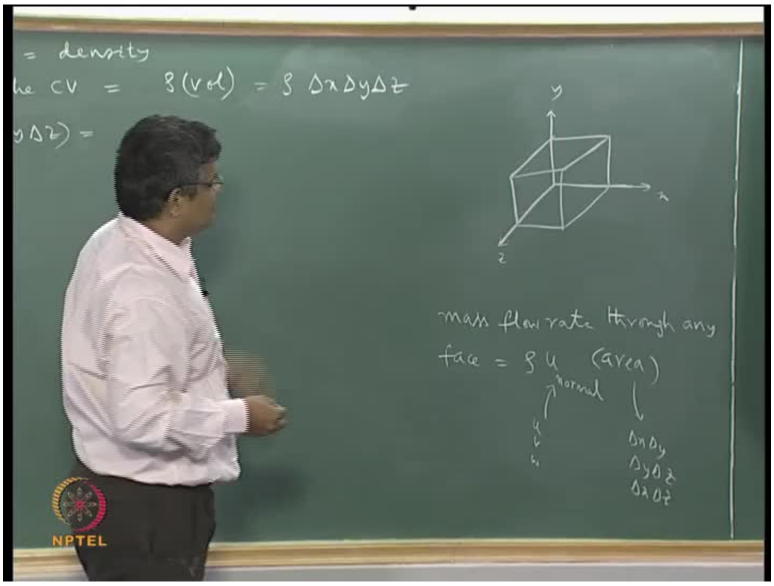

So ,since we have aligned the six phases of a control volume along the coordinate planes .If we take any particular phase ,we can find out the area ,and multiply the velocity component, which is normal to that particular phase. In order to get the volumetric flow rate through that particular phase.

So ,we can say that mass flow rate through any phase, is equal to density times the velocity, which is normal times, the area of the phase .And by our choice of these phases, which envelop the control volume ,to be aligned in the particular coordinate directions ,this component velocity ,will be either u ,v, or w which is specific to each phase. And this area here ,will be delta x delta y, or delta y delta z ,or delta x delta z .So this is a simplification, that we can achieve by choosing a control volume, whose planes are enveloping planes are aligned with each flow direction.

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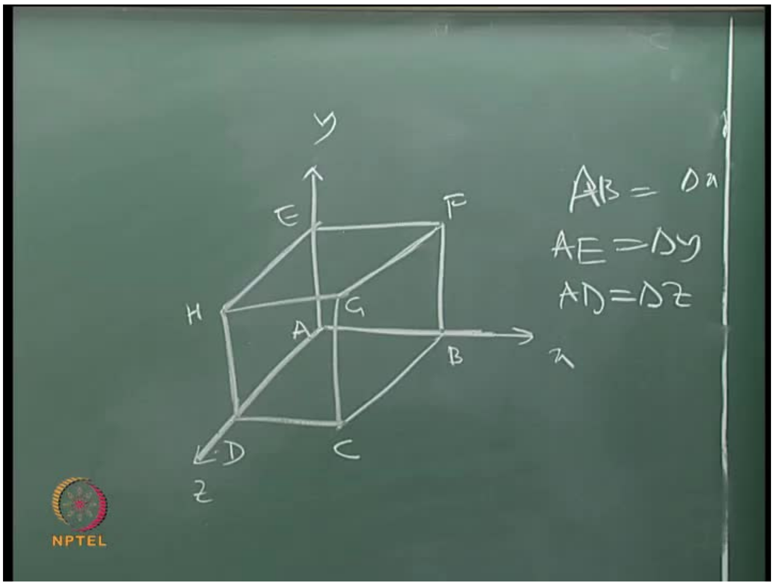

So, now, let us put some put some identifiers for these vertices .We can start with this origin here, as point A. Here, this is B ,C, D, and again E, F, G and H .We have already identified AB as being delta x, and AE as being delta y , and AD as being delta z .So ,these are the lengths, associated with each of these. The corresponding for example ,length EF ,is the same as AB, which is as same as CD ,and similarly the AD is same as BC ,as is same as FG , because we are considering a rectangular parallel paper.

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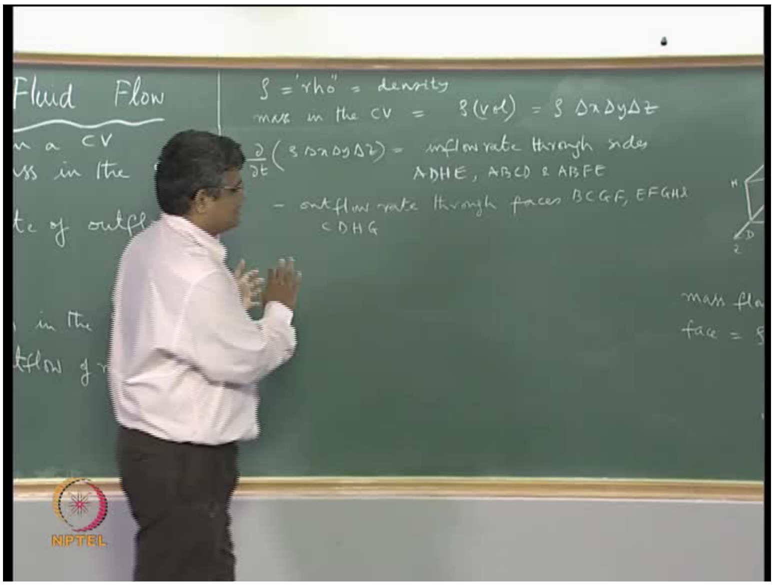

So ,with this we can identify the inflow areas .As the three phases , corresponding to inflow rate through sides ADHE, bottom phase ABCD, and the back phase, which is a b f e minus outflow rate. Through phases the right phase ,which is BCGF ,the top phase which is EFGH ,and the front phase which is CDHG .We are doing in a simplistic way here, without worrying too much about mathematical rigor, in a in an intuitive way .So, let us just take it like that.

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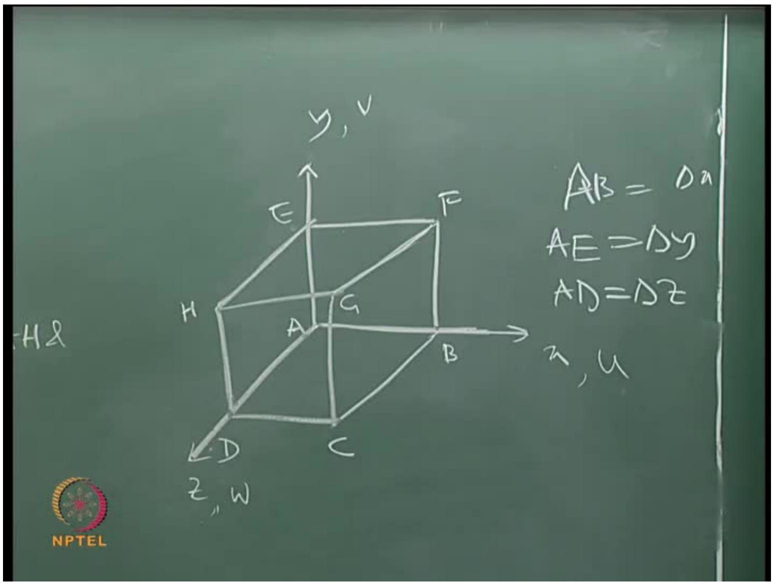

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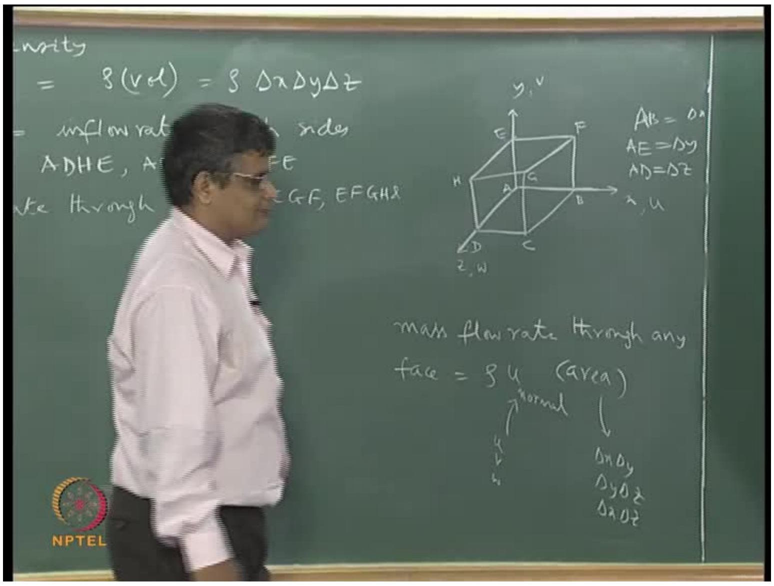

Therefore ,we are identifying visually what is an inflow, and what is an outflow .And associated with this inflow, and outflow is the idea ,that there is a velocity component u ,is always associated in the positive x direction, and the velocity component v, is always associated in the y direction ,positive y direction . The velocity component w ,associated always in the positive z direction .So, if velocity in the negative z direction, w has to have a negative value. For example ,w of minus 20 meters per second ,is velocity going in the negative z direction .x of minus 15 meters per second, is the velocity going in the negative z direction .So, by the fact of associating ,u, v w always with positive coordinate frames ,positive coordinate directions ,we can identify an inflow and outflow from a visual identification of this .It can be done more rigorously ,mathematically .So, through each phase we evaluate the mass flow rate ,by multiplying by the density ,and the velocity component the appropriate normal velocity component ,and the appropriate area of this particular phase.

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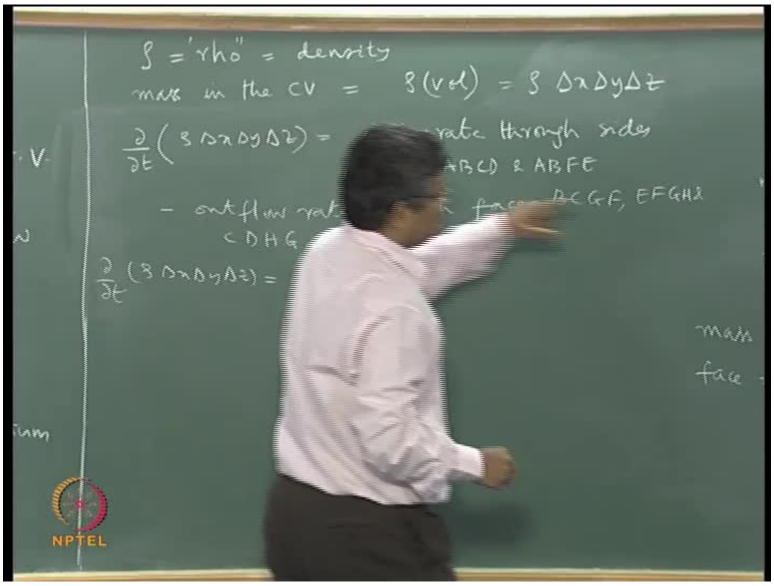

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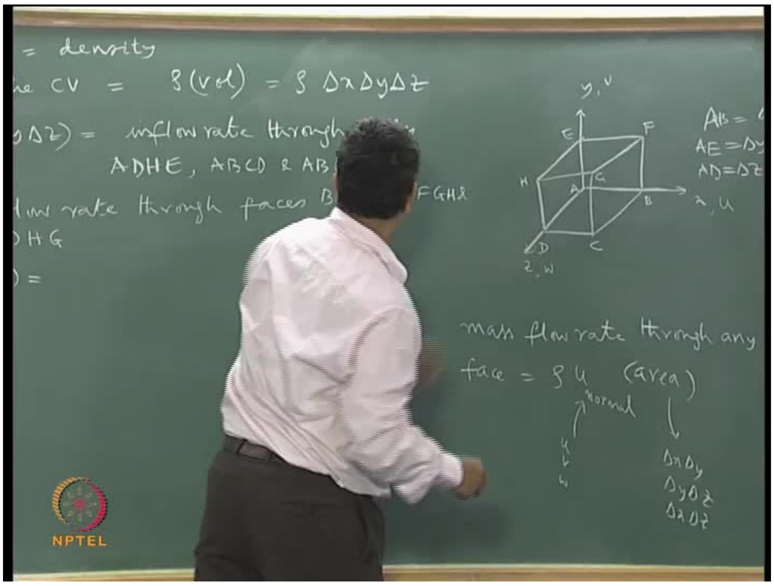

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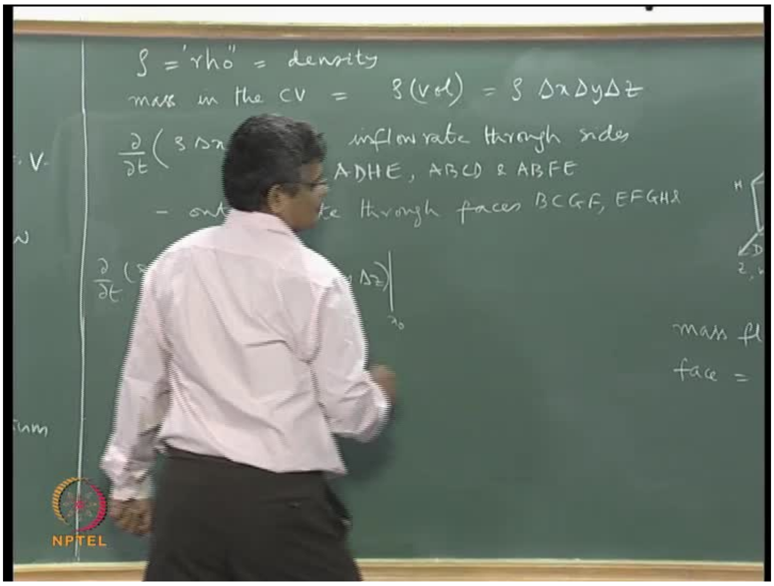

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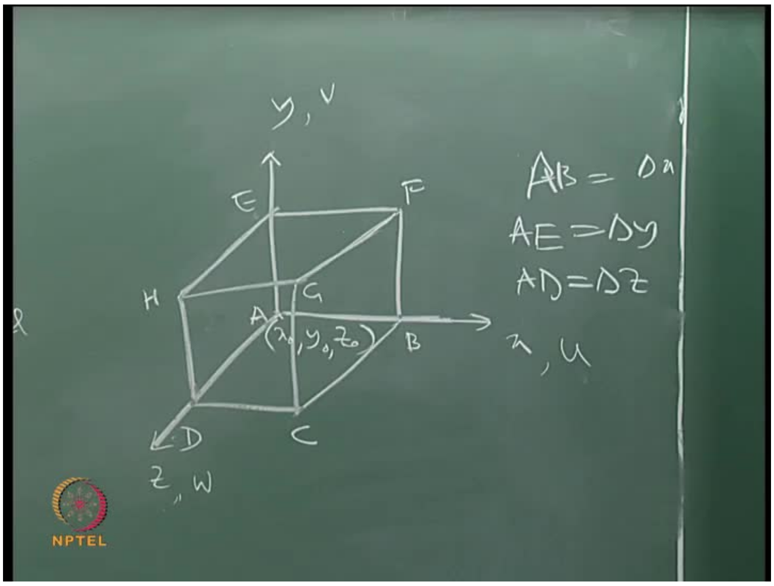

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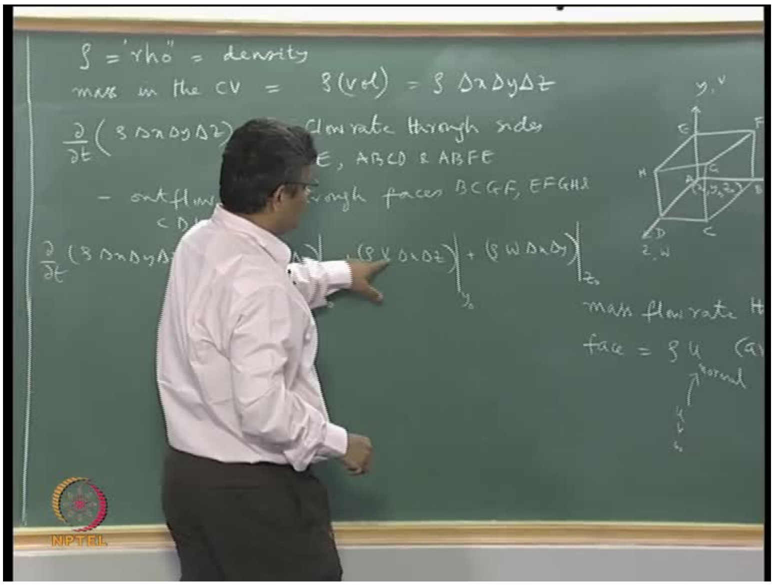

So ,for example side ADHE ,so we can now evaluate this thing. So this rate of accumulation of rho, delta x delta y delta z, is now through ADHE. We have the velocity component as u, which is normal to this plane. So that is rho times u ,and the magnitude of the area is, delta y times ,delta z delta y delta z .These are evaluated at, x naught where this is x naught .The origin is at x naught y naught z naught. So , the flow rate through ABCD which is the bottom phase will be the rho times, the velocity component ,which is normal to this plane is v. And the area of this, is delta x times delta z, and this whole thing is evaluated at the bottom plane, located at y zero, plus through the back phase which is ABFE ,which is this one. So that is the normal velocity component , to this is w ,and the area is delta x times delta y ,and these are evaluated at z 0.

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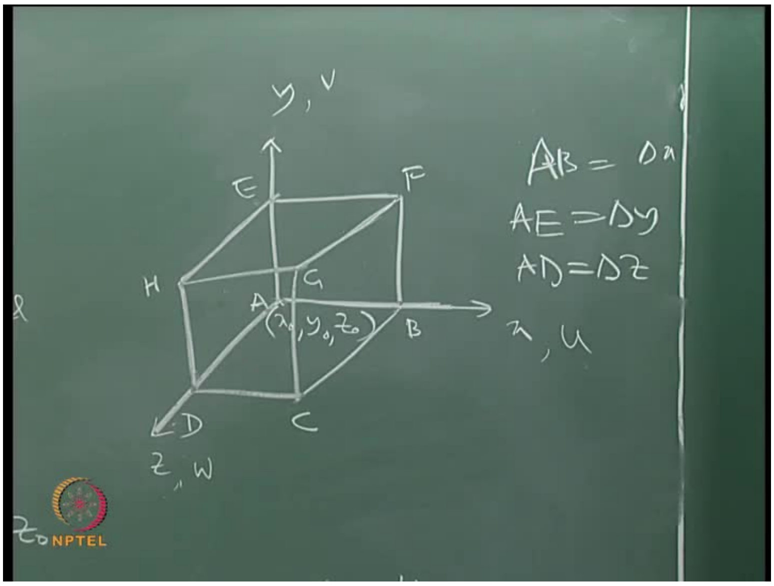

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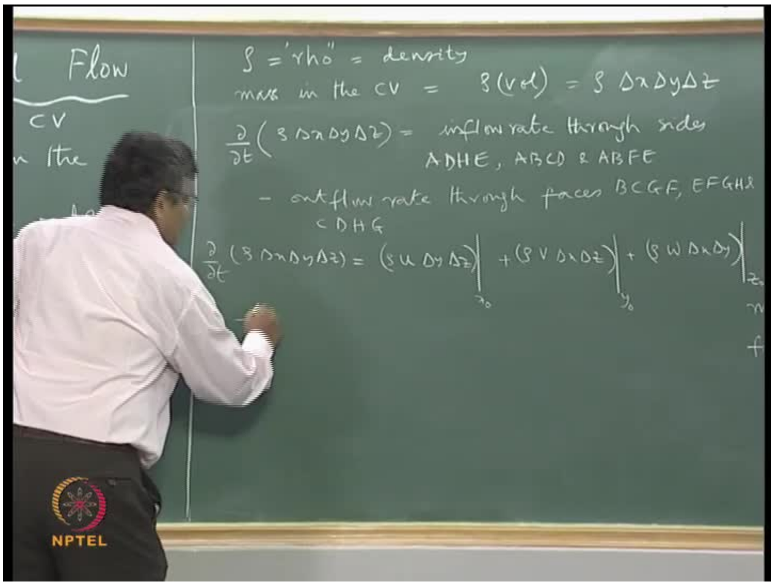

We ,can see when we talk about the mass flow rate ,been given by velocity component. One of these three velocity component ,u ,v ,w and one of this area components. For example, here we are taking u velocity, and delta by delta z, and here we taking velocity, and delta x delta z here, and w velocity like this, and each of this ,the rho and u are evaluated for this particular thing ,at the center of this plane located at x naught .

Similarly ,the rho and v appearing in this particular term ,are evaluated at the center of this particular plane, which is at y naught x 0 ,plus delta x by two a,nd z 0 plus delta z by 0, like that. So these, are the three inflow phases, and the outflow rate mass flow rate through the three outgoing phases.

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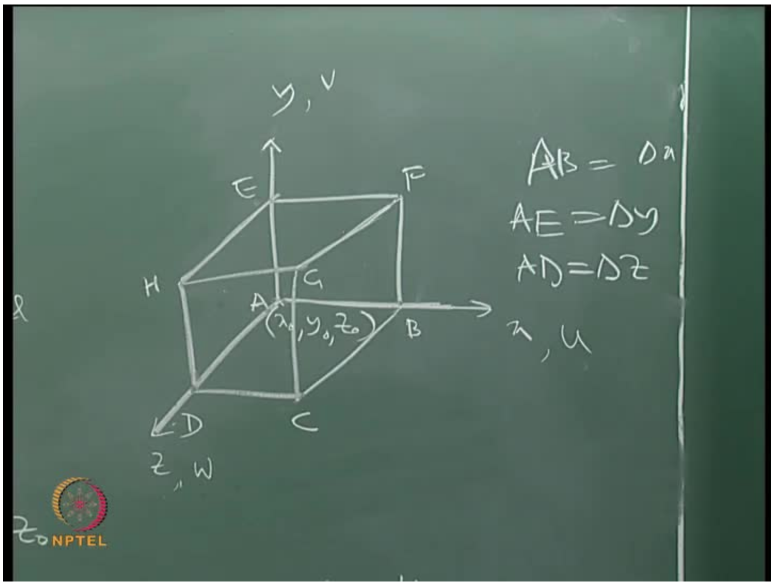

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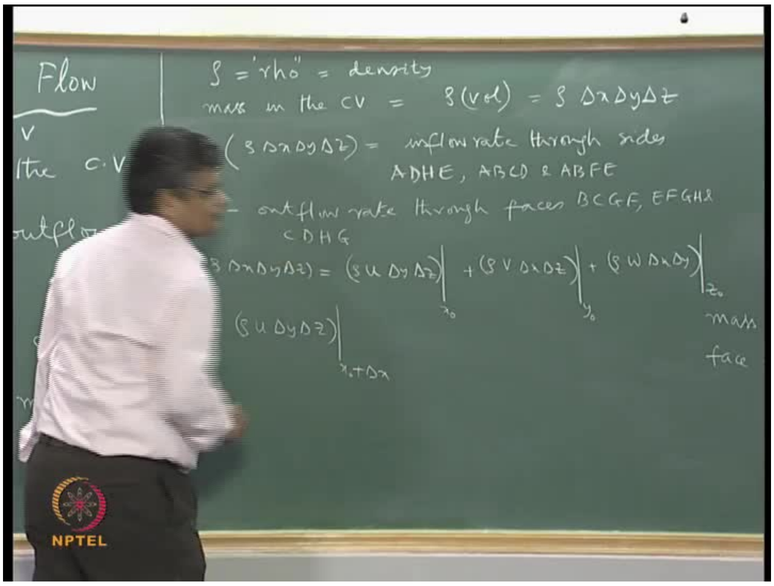

So, that is BCFG .So this is this ,so this again is aligned, in the positive x direction ,within area of delta y times, delta z. The velocity component is u ,which is normal to this. So minus rho u delta y delta z. Now this is evaluated at ,x naught plus ,delta x, because the this plane has a centroid, which has coordinate length of, x naught plus this delta x here.

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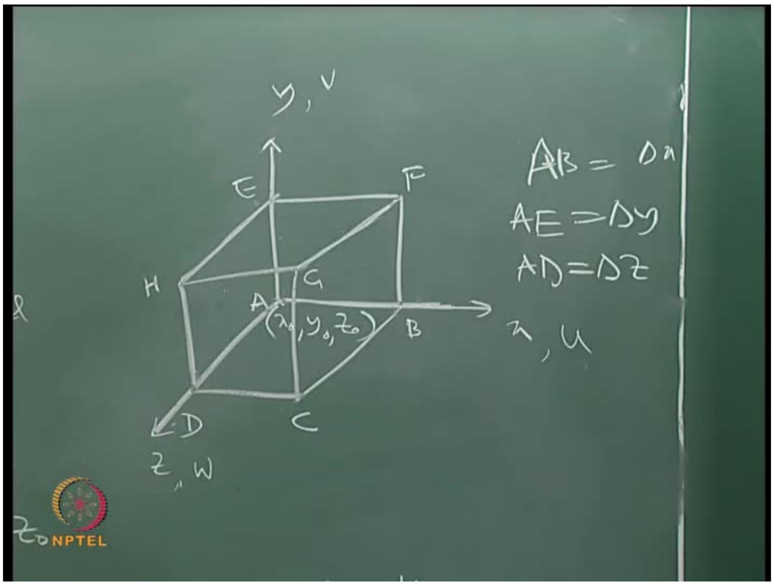

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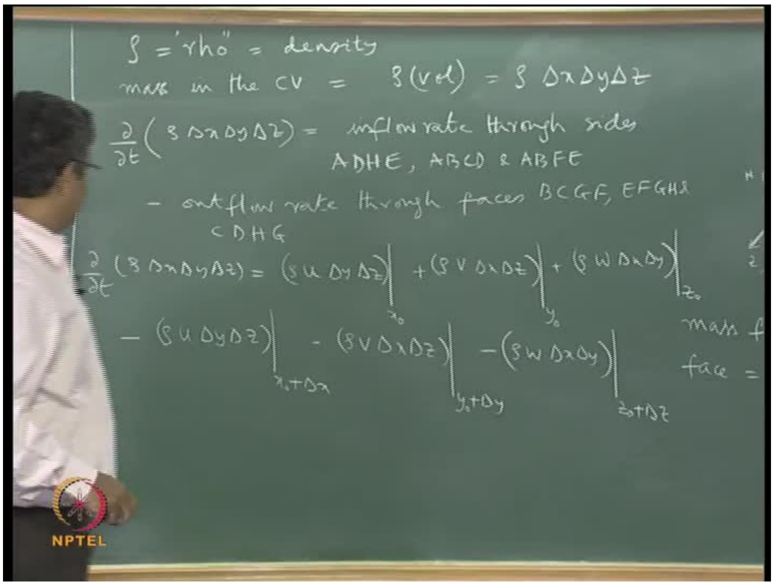

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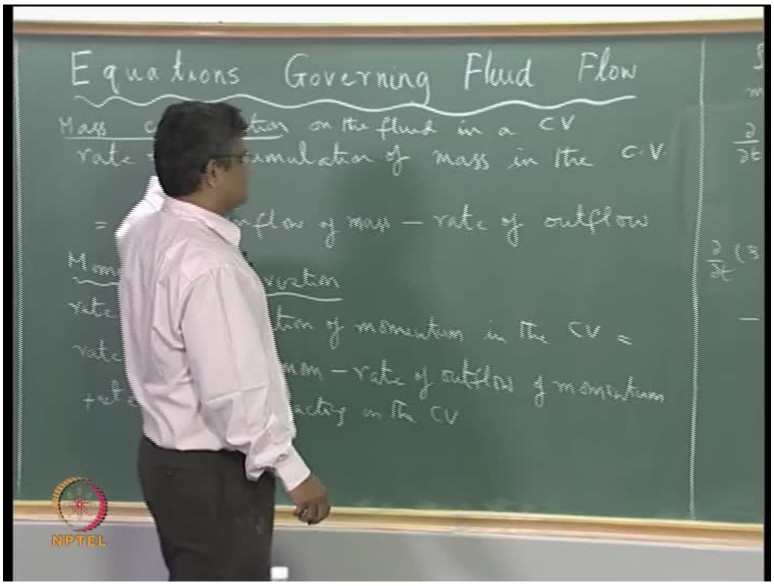

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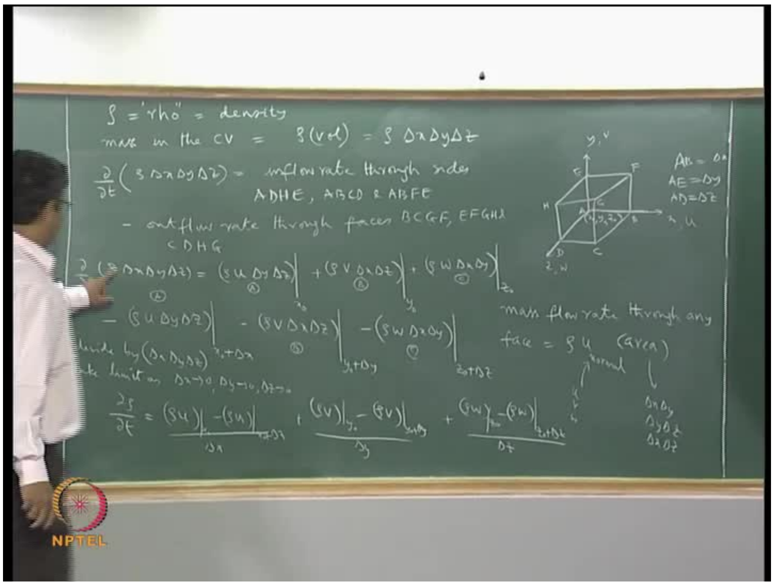

Now, through second phase which is EFGH ,the top phase here, and the velocity component normal to this, is the v velocity .So ,we say minus rho v , the area is delta x, delta z and this is located at y naught, plus delta y .The final phase is CDHG ,which is that front phase. So the velocity component rho w times ,the area which is, delta x delta y centered at z naught plus delta z. So these three, represent the mass flow rates ,through the three surface identified as outflow. And these represent the mass flow rates ,through each of the three phases identified as inflow phases .So this equation, this verbal statement of mass conservation ,is now represented mathematically like this . This is an equation, now it has become mathematical equation .We can do all allowable mathematical operations .So what we do, is that we divide each of this terms ,by the volume divide by delta x delta y delta z,the product of this. So , this is equal to volume, and take limit as delta x ,tends to 0, delta y tends to 0, and delta z tends to 0 as appropriate.

So, if we do these things, divide by delta x delta y delta z ,and take limit as delta x delta y delta z ,individually tend to zero ,then we get an interesting equation. For example ,if we take the first term here, and divide by delta x delta y delta z ,this cancels out, and we get d rho by d t .So the first term ,becomes d rho by d t, and in this we have to divide each of them by delta x delta y delta z .It will take a combination, and put them together .We take this one here , and the corresponding thing ,at x plus delta x ,this term here , and then we take them together.

We divide these two terms ,by delta x delta y, delta times delta z .So that this ,and this cancel out ,and we get rho u at x naught ,minus rho u at x naught, plus delta x ,divided by delta x .From these things ,now let us take this term at y naught , and this term at y naught plus delta, y together divide by delta x delta y delta z, so delta x cancels out ,delta z cancels out. What we are left with, is plus rho at y naught ,minus rho v at y naught, plus delta y divided by delta y ,and we take the remaining two terms together c and c .If we divide these terms, by delta x delta y delta z ,then delta x delta y cancel out ,and we will have plus rho w at z naught ,minus rho w at z naught plus delta z, divided by delta z .So, by dividing each term, in this equation by delta x delta y delta z.

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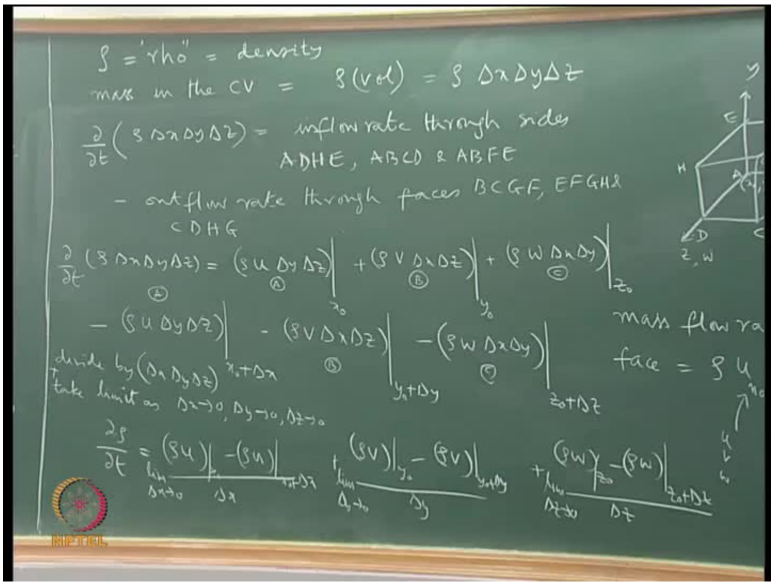

Then. we have this . We take the limit as delta x ,and delta y delta z tend to zero ,so there is no delta x here, but here we have delta x coming here. So limit as delta x, tends to 0 ,and here delta y appears .So limit as delta y , tends to 0 ,and here delta z appears, so limit as delta z tends to 0 . Now you ,can see some derivatives appearing here, so we can say again this:

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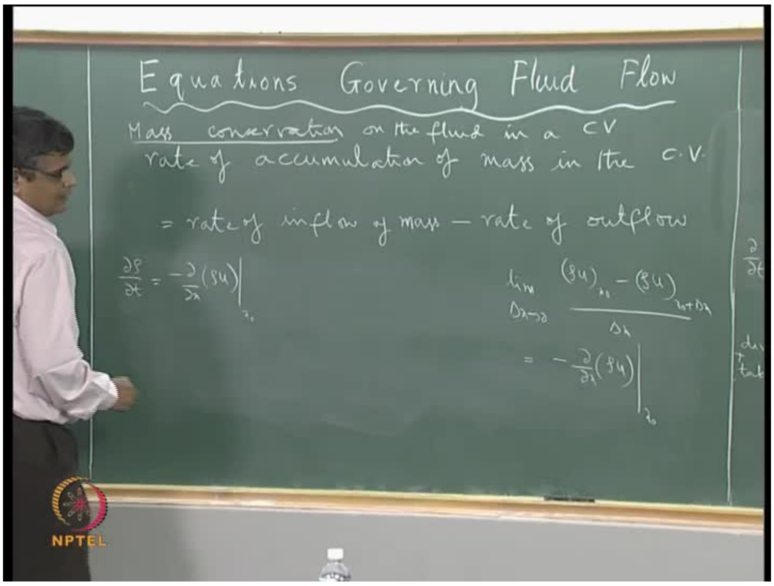

You, can see that limit as delta x ,tends to 0 rho u at x naught minus rho u , at x naught plus delta x ,divided by delta x. We consider this term limit as delta x ,tends to 0 rho u ,at x naught minus rho u, at x naught plus delta x . This whole thing divided by delta x ,is nothing but minus d by d x ,of rho u evaluated at x naught. By the definition of derivative ,partial derivative of rho u with respect to x I,s given by this, with a minus sign .Because we are doing x naught ,and minus delta x naught so using this definition.

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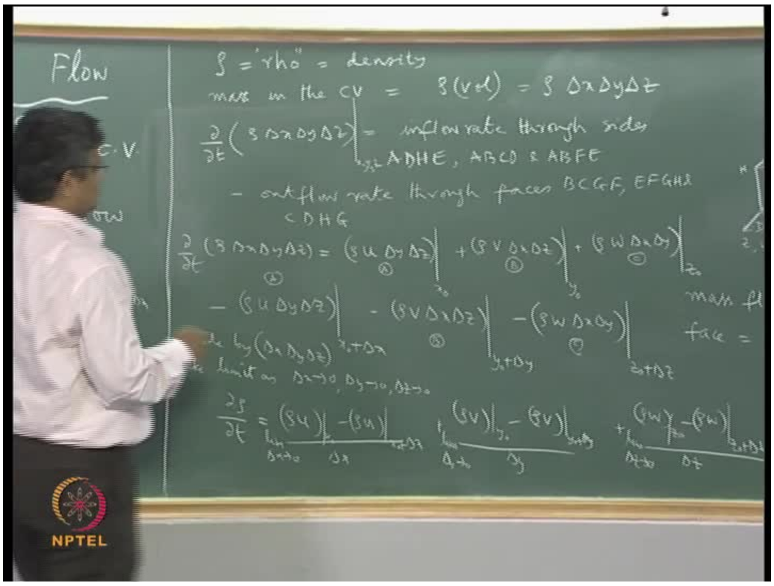

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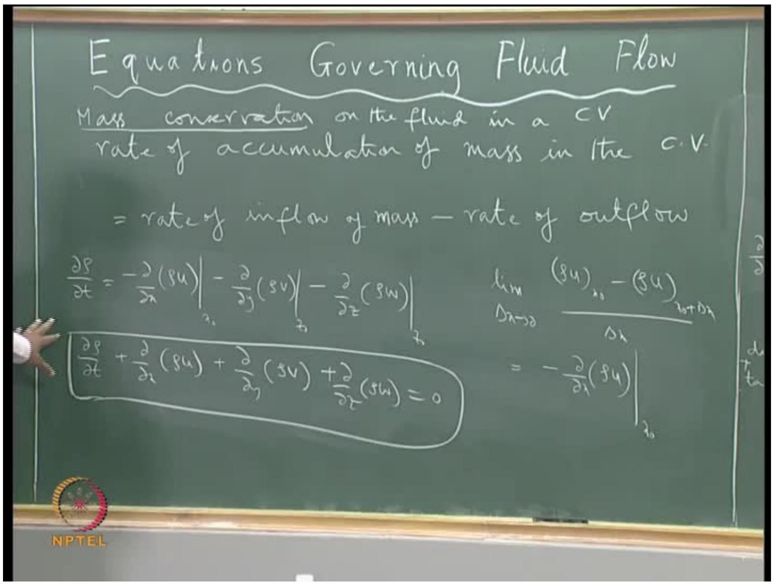

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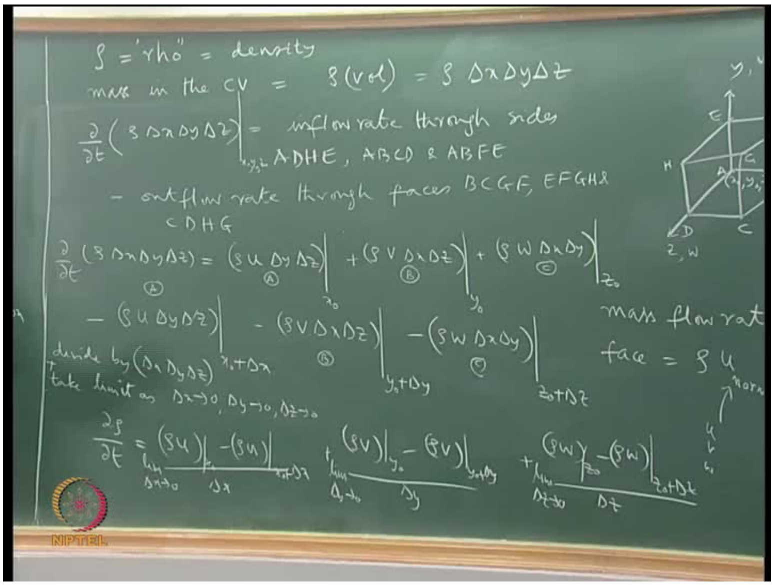

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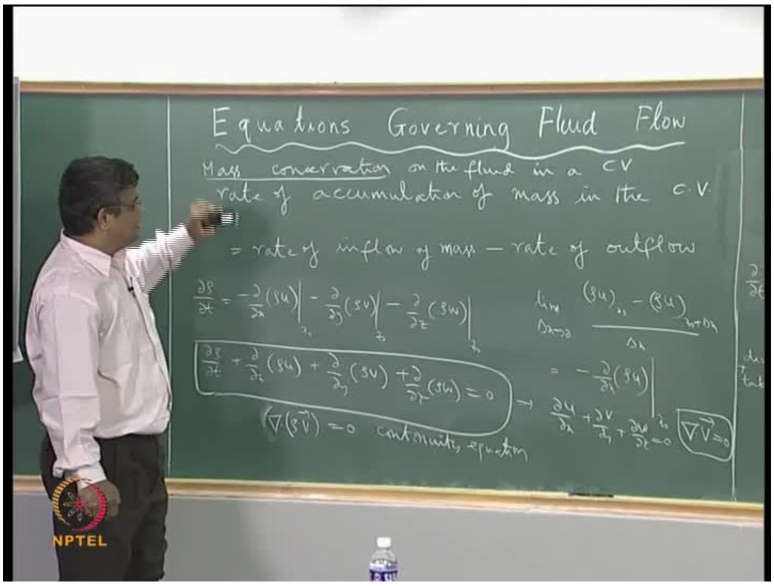

We can write this equation, as partial of rho with respect to t as minus partial derivative, with respect to x of rho u, at x naught .We should remember ,that this is for the control volume centered at x naught y naught z naught roughly .So we can do this ,and minus the second term .Here is nothing ,but partial derivative ,with respect to y of rho v, and the next term will be partial derivative with respect to z of rho w .So we can put this together ,and say that d rho by d t and we can bring this to the left hand side, plus dou by dou x of, rho u plus dou by dou y of, rho v plus dou by, dou z of rho w ,equal to zero. As being a mathematical statement of the mass conservation equation for a fluid ,which is applicable in the limit ,as delta x delta y and delta z tend to 0 ,which as we approach a particular point .So this is a mass conservation equation, which is now appearing in the form of a partial differential equation involving time ,and three spatial directions .As the spatial derivatives, as the independent variable, now the density as well as u ,v, w from this equation, can be a function of x y z. In the special case , where we are dealing with an incompressible flow, then rho is not variable . Then we can say that this thing goes to zero , and the rho is constant .So we take this out , of the derivatives. Since the right hand side is zero, this equation reduces ,to the simpler form, of dou u by dou x ,plus dou v ,by dou y plus dou w ,by dou z equal to 0 ,or del dot v ,is equal to 0. This is also known as the continuity equation , and the more general case we have del dot rho v equal to 0 this is the continuity equation.

So ,the variation of u ,v, w and rho, which are the properties of the fluid , and the flow at any point within the flow domain , must be such that they satisfy this equation .Because it is a fundamental statement ,of the mass conservation equation .So ,this is one of the equations ,which describe the variation ,how the flow properties can change in a particular control volume.

This is one equation, and we can also apply the other fundamental equation . That we know this is momentum conservation, so whatever be the fluid flow it has to obey the mass conservation, and momentum conservation ,as well as the energy conservation equation.

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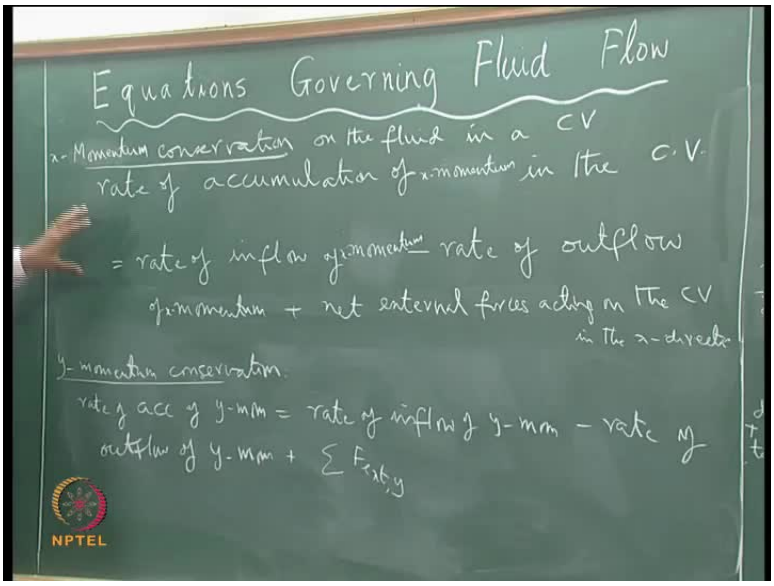

So, let us now take a very brief look at the momentum conservation .We have already written a statement on that .So momentum conservation ,can be written as rate of accumulation of momentum I,s equal to rate inflow of momentum, plus rate of outflow of momentum ,plus net external forces acting on the control volume .We say that if there is a force ,and it is acting on a control volume, that it introduces a rate of change of momentum. It leads to a rate of accumulation of momentum ,so that is what we are saying here, and this can be developed analogously like this. We can make advantage of, what we have already derived, and modify this slightly. We note the fact, that momentum is a vector quantity, unlike mass.

So ,this equation has three components ,and in each of the three components this equation is valid .So ,we can say, that rate of accumulation of momentum in the x direction. So which we call as x momentum ,is equal to rate of inflow of x momentum, and rate of outflow of x momentum, plus net external forces, acting on the control volume in the x direction .Similarly, we can say ,that will be the x momentum conservation . Similarly we can write the y momentum conservation ,it is a short hand notation for saying that conservation of momentum in the y direction ,it can be stated as rate of accumulation of momentum in the y direction, is equal to rate of inflow of y momentum ,minus rate of outflow of y momentum, plus sum of all external forces acting in the y direction. So this can be a statement on the conservation momentum in the y direction ,and similarly conservation momentum in the z direction.

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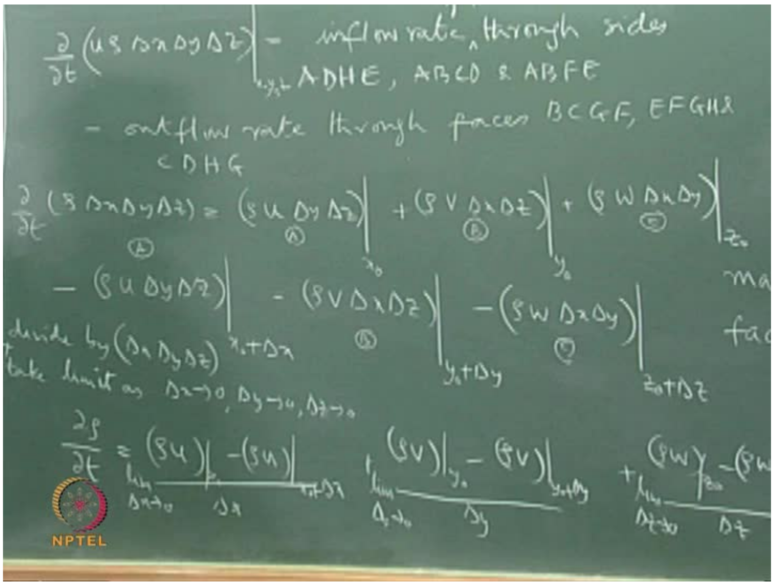

So ,using the same procedure as what we have adapted for the mass conservation, we can convert this verbal statement into a mathematical expression .By essentially following, the same way rate of the total x momentum, we know that momentum is mass times velocity ,and we know that the total mass in this ,is rho times delta x delta y delta z times v .

Now, we can say that momentum in the x direction, is which we are calling as x momentum ,is velocity component in the x direction. So this will become rho times ,delta x delta y delta z ,times velocity in the x direction. This is nothing ,but u so that is rho delta x delta y delta z times u is the momentum in the x direction contained in the control volume. So, we can say rate of accumulation of x ,momentum in the control volume is nothing but rho times .This times the velocity component u ,u is the inflow rate of x momentum through the sides the flow is coming in, so whatever it has it brings in it ,brings in the enthalpy, it brings in the entropy ,it brings in x momentum, y momentum, z momentum.

So ,whatever it is it , it brings in a concentration of particular species .Because we are considering a flowing system ,there is a convection or advection component associated with rate of change term. That is what is being described in this ,and outflow rate of x momentum, plus the sum of all of external forces acting in the x direction. So, this is a statement of the x momentum conservation

So, now we can do it in the same way ,dou by dou t of rho times u times .This is equal to, we have already evaluated the flow rate through each of the phases . So just as we have defined x momentum ,as this x momentum flow rate is mass flow rate times , the velocity in the x direction. So the x momentum, that is being carried by the fluid ,which is entering through the left phase, which is this is u. So ,we can put u here ,and the x momentum being carried by the fluid, which is entering through the bottom phase is mass flow rate times ,the velocity component in particular direction in representing the x momentum which is u. The x momentum being carried by the fluid ,which is entering through the back phase is the mass flow rate times t,he velocity component in the x direction . Again here ,this is the mass flow rate ,leaving the control volume through the x phase. So multiplied by the u component ,will give us the rate of x momentum .Flow rate through that particular out phase ,and rate of x momentum flow rate through the top phase ,will be the mass flow rate through that particular phase times u .At that particular phase, and the mass flow rate leaving through the front phase will be the mass flow rate through the x momentum flow rate, will be the mass flow rate times ,the x momentum velocity component which is u.

Now we have external forces, acting on it. So for the time being ,we will just put it as sigma f ,external acting in the x direction .We will not right now talk about it. Just as we have done earlier ,by dividing delta x delta y delta z ,and all these things. We can write this term ,as dou by dou t of rho u here then clubbing these two ,like earlier ,we will have rho u square at x naught minus ,rho u square at x naught plus ,delta x . Here ,you have u and v here ,so we have already, u v .So rho v u ,at y naught minus rho v u ,at y naught plus delta y a.And here ,we have rho v rho w u at z naught, and rho w u at z ,naught plus sum of all external forces, acting in the x direction divided by, delta x delta y delta z times limit, as delta x tends to 0 delta y tends to 0 delta z tends to zero .

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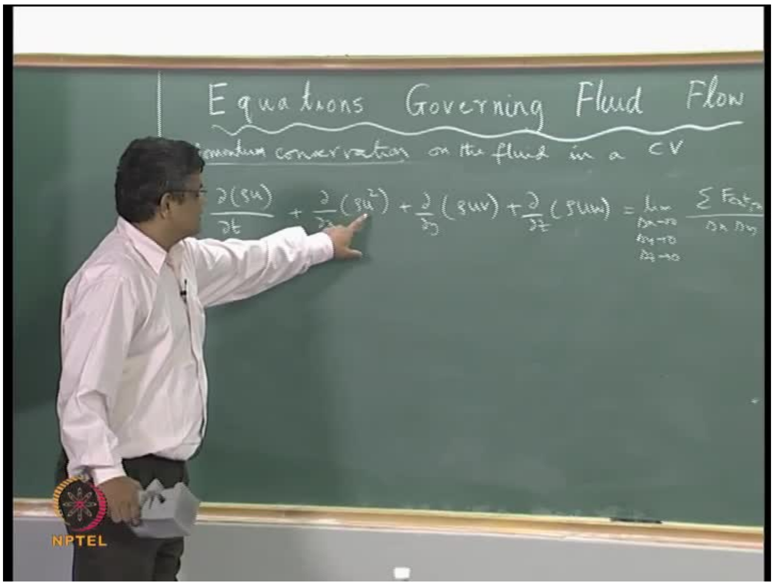

So, this is the statement ,and using the same argument ,as earlier we can derive a mathematical expression ,which is d by d t of rho u ,plus dou by dou x of rho u square ,plus dou by dou y of rho u v, plus dou by dou z of rho u w, equal to sum of all external forces acting in the x direction ,divided by the volume, delta x delta y delta z in the limit ,as delta x tends to 0 ,and ,delta y tends to 0 delta z tends to 0.

So, this is the x momentum equation. We can also derive the y momentum equation .And all these things what we see here is, that we still have the u v w components coming in this .This statement is not complete, until we describe what these external forces are.

So , in order to complete a statement of the momentum conservation, we have to describe, what these external forces are . We will consider that in the next class.