Computational Fluid Dynamics

Prof. SreenivasJayanthi

Department of Chemical Engineering

Indian institute of Technology, Madras

Module No. # 05

Lecture No. # 26

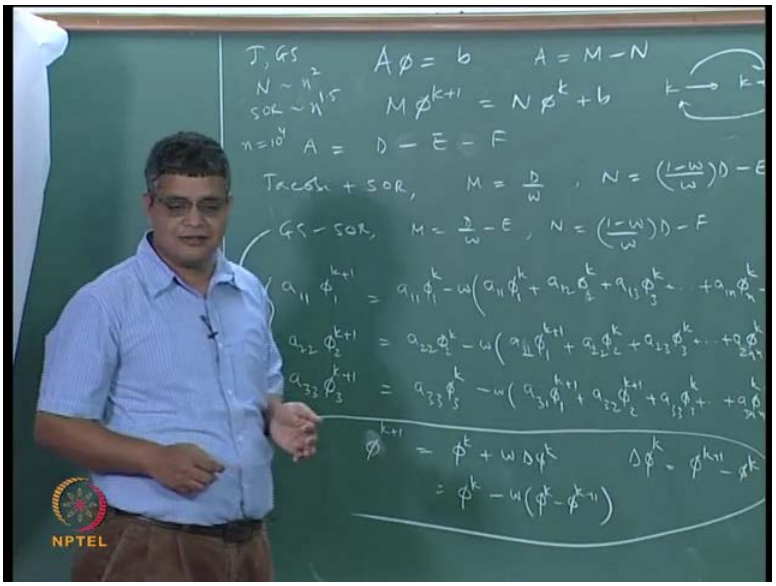
Convergence analysis of basic iterative schemes

Diagonal dominance condition for convergence

Influence of source term on the diagonal dominance condition

Rate of convergence

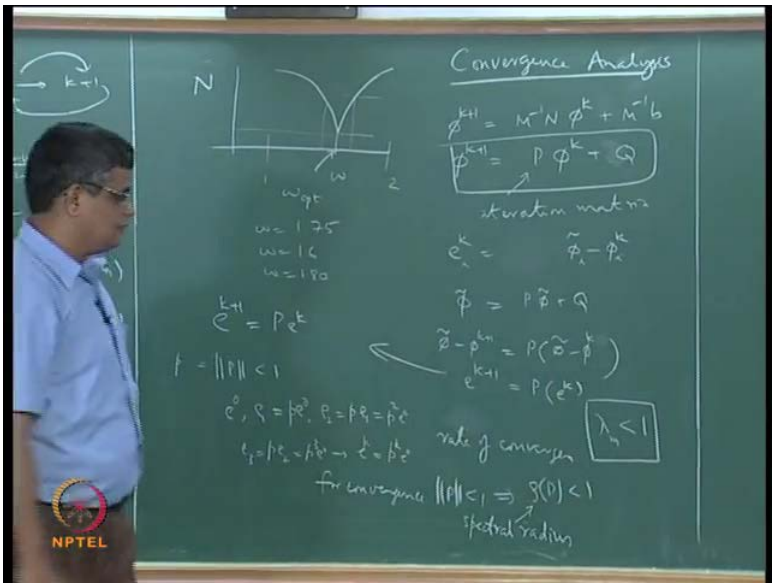
(Refer Slide Time: 00:10)



So, we have looked at three basic methods, the Jacobi method and Gauss-Seidel method and SOR method as basic iterative classical iterative schemes.We know how to use these methods to get a solution, but we still have not answered the two questions which are also important to the choice of whether or not to use these things.

It is what are the conditions under which these methods converge and secondly, what is the rate of convergence?For this, we can formulate a general criterion for the assurance of convergence, but the actual implementations of that will be non-turbulent.

(Refer Slide Time: 01:06)



Let us look therefore, at the convergence analysis of the classical iterative schemes. So, our classical iterative scheme, an iterative scheme is of this particular type. So, we can write phi k plus 1 equal to M inverse N phi k plus M inverse 1b.Although, as we have seen here we never compute the M inverse and then do this.In the actual implementation, it is different from what we have written, but it is good to put this up and in this particular form as the structure of the iterative scheme.

As already mentioned, this is the iteration matrix that determines the rate of not only the rate of convergence, but also whether or not it would converge. So, thisiteration matrix here is the key to the behaviour of this iterative scheme with the specified decomposition of a into M and N ok.

So, now let us say that the error at the kth iteration is phi k, which is the current estimate of the value phi here minus phi tilde where tilde is the exact solution.We can put it as the exact solutions minus the approximation that we have.If you have n number of equations there are N estimations. So, we can say that for the ith variable at the kth iteration, the error is the exact value of phi i which we are denoting by phi tilde minus the current estimate of that particular variable at the kth iteration. So, this is the error here and sowe can we know that the exact solution satisfies this this thing.

So, we can say that phi here is equal to P phi tilde plus Q.For example, if you use a direct method with infinite accuracy, we can get this corresponding to that a phi equal to b, but at

this stage we are not saying what this value is.We are saying that there is an exact solution to a phi equal to b and that exact solution is given by this.

Since,we are since a phi equal to b is equivalent to this for a convergence schemes, then the final solution that we get is is doing this and if you subtract this from this equation, we get phi tilde minus phi k plus 1 equal to P times phi tilde minus phi k plus 1. Q and Q will cancel out and this is nothing but the error at k plus 1 error k, this is error k plus 1 and this error k. So, we can say that from this we have written it like this to show that the error at k plus 1 at k is multiplied by the iteration matrix P and this to give you the error at k plus1. So, this is error at k plus 1 is P times the error at k ok.

This is a Matrix equation is not as simple as that, but with any matrix we can identify magnitude and that for a matrix is given in terms of various norms of the matrix and the norm is an indication of that magnitude of the matrix. So, therefore, we say that for error not to increase from k to k plus 1 to k plus 2 to k plus 3 like that or for error to decrease with increasing number of iterations with successive approximation, the magnitude of P or the norm of P must be less than 1 ok.

So, this is the conditions for the convergence of iterative scheme.If this is the iterative scheme and P is the iteration matrix, the condition for convergence is that the norm of the iteration matrix must be less than 1, so that if let us say 0.9, if it is your initial error is 100, then at the first step it is going to be 0.9 times 100.Next time it is going to be 0.9 times 0.9 times 100 like that.

So, if here initial error is e naught, then according to this e1 and let us put this norm as some small p. So, then this will be equal to ptimes e naught and e2 is p times e1,so that this p square times is 0 and e3 is p times e2, so that is p cubed times e naught and so on,so that we can say that the error, the kth iteration is pk times e naught the initial error. So, that initial error is p raised to power k times this and for this to be decreasing p must be less than 1, otherwise this will keep on increasing.

If p is 1.01, then the error will increase with k and p is 0.99, then error will decrease. So, depending on weather how close p is, we have p is to 1.We have either convergence or divergence of this particular iterative scheme and also if p is very close to 1, then the rate of divergence or convergence is also going to change. So, the effectively what we can say is the rate of convergence for convergence, the norm of p must be less than 1 and we can say for

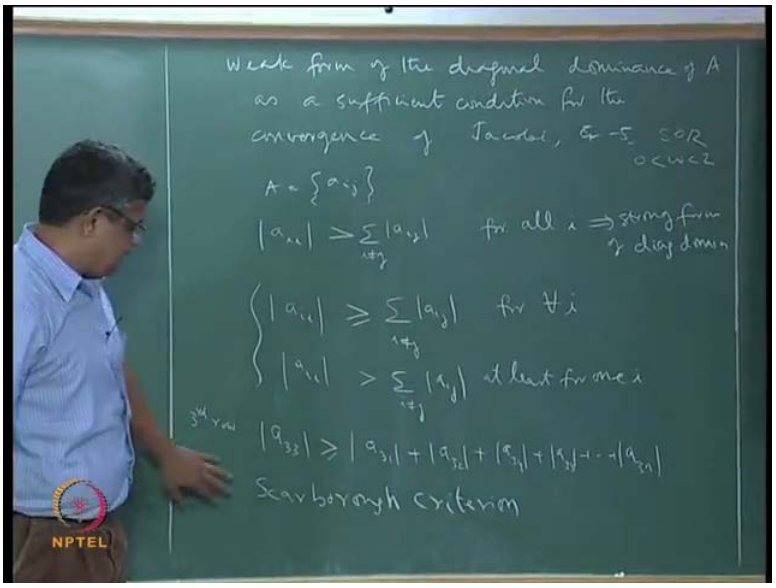
these type of equations when we are looking at constant real coefficients andso on.We can say, we can express the same thing here as the spectral radius rho of p must be less than 1 and rho here is the spectral radius and the spectrum that we looking at is the spectrum of the Eigen values of the matrix of the iteration matrix rho.Spectral radius is the value of is magnitude of the largest Eigen value of the iteration matrix.

So, the necessary condition for the iterative scheme to converge is that the largest Eigen values iteration matrix must be less than 1. So, if you say that lambda, the Eigen value and lambda m is the maximum Eigen value, then lambda m must be less than 1 for convergence. So, once we construct M and N and once we construct p, the iteration matrix even before we go through the iteration procedure,we can find out, we can predict whether or not the scheme would converge by finding out the Eigen values of this.Typically, Eigen values are very difficult to find especially for large matrixes.

We can see that in a typical cfd problem, with say 1000 grid points or 10000 grid points, the matrix a will be large.If you have 10000 grid points, then you have 10000 Eigen values and you have to choose the largest of these things and that largest of these must be less than less than 1and it depends very crucially on whether it is less than 1 and how close it is to 1.

If the largest value is 1.0005, it is going to diverge.If it is 0.99995, it is going to converge. So, we have to determine the largest Eigen values to that much accuracy. So, for large values the determination of the Eigen value itself may be very time consuming and it may take N cube number of operations.

(Refer Slide Time: 12:02)



So, in such a case although, we have a theoretical confirmation here of the necessary condition, this method is is not very practical.We have a less restrictive condition which is known as the weak form of the diagonal dominance as a necessary, as a sufficient condition for the convergence of the Jacobi method,Gauss-Seidel method and SOR method for omega between 0 and 2.

So, a more readily implementable criterion sufficient condition is the weak form of diagonal dominance.What is this is the weak form of diagonal dominance of matrix A, not matrix p because p is the iteration matrix and that depends, that varies for Jacobi method,Gauss-Seidel method and SOR method, but for all these methods your a phi equal to b are the coefficient matrix A is the same.If the coefficient matrix A exhibits the property, as the property of the weak form of the diagonal dominance, then we can be assured that any of these methods will work.

What do you mean by the weak form of diagonal dominance?We have the coefficients. So, A has a coefficientsaij and if the magnitude of the diagonal element is greater than the sum of the magnitudes of all the half-diagonal elements in a particular row for all rows ok, so we are taking row by row. In each row, if the diagonal element has a magnitude, which is greater than the magnitude, sum of the magnitude of the half diagonal terms, then we have a strong form of diagonal dominance.The weak form of diagonal dominance is that aii, that is the magnitude of the of the diagonal element must be greater than equal to the sum of the

magnitude of all the half-diagonal elements is greater than or equal to for all rows and at least for one row it must be greater than at least for 1.

So, if you have M number of flows, for each of the M rows this diagonal dominance condition must be satisfied at least with an equal to sign and at least there should be one row for which this is satisfied with a greater than sign.We should keep in mind that we are talking about the magnitudes for example, if you take the third row, then a33 magnitude must be greater than or equal to a31 plus magnitude that is modulus of a32 plus a34 plus mod of a35 isso on upto a3n.

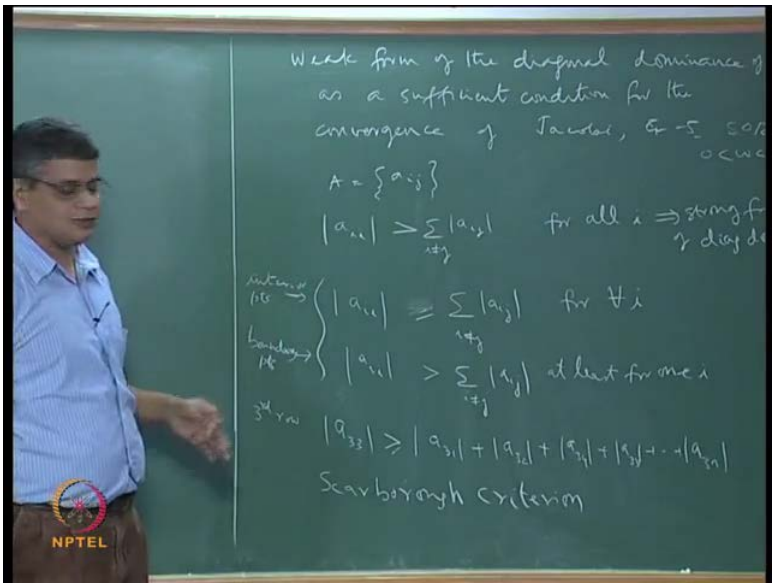
So, this is the condition for the third row.If this is satisfied for all the rows with a greater than sign, then we have strong form of diagonal dominance.If it is satisfied with with an equal to sign for all of them and at least for one of these rows it is satisfied with a greater than sign, then we have the weak form of diagonal dominance.This this weak form of diagonal dominance is sufficient to ensure convergence of the Jacobi method and Gauss-Seidel method and SOR method applied to a phi equal to b with the SOR parameter being between 0 and 2.

This condition here is known as Scarborough criteria and this is an easily implementable criterion for ensuring the convergence because all we have to do is by the time, we come to the solution of a phi equal to b, we already know all the elements of aij.We can very quickly check whether we are imposing this whether, we are satisfying this weak form of diagonal dominance and once we ensure that, then we can make use of Jacobi method or these basic iterative methods.

Since, we want to, if we want to use this as part of calculation procedure at the time of discretization itself; we make sure that the discretized equation which finally, results in a phi equals to b is such that this condition is satisfied.We can we can say that if you are dealing with simple diffusion either one-dimensional, two-dimensional,three- dimensional diffusion with constant diffusivity for a steady state problem, then the Scarborough criterion is satisfied.

For all the rows we have, this being satisfied with either greater than or equal to.For all the interior points, which are away from the boundary,we typically have this being satisfied with an equal to sign and typically, for boundary points we have the this diagonal dominance condition being satisfied with greater than sign.

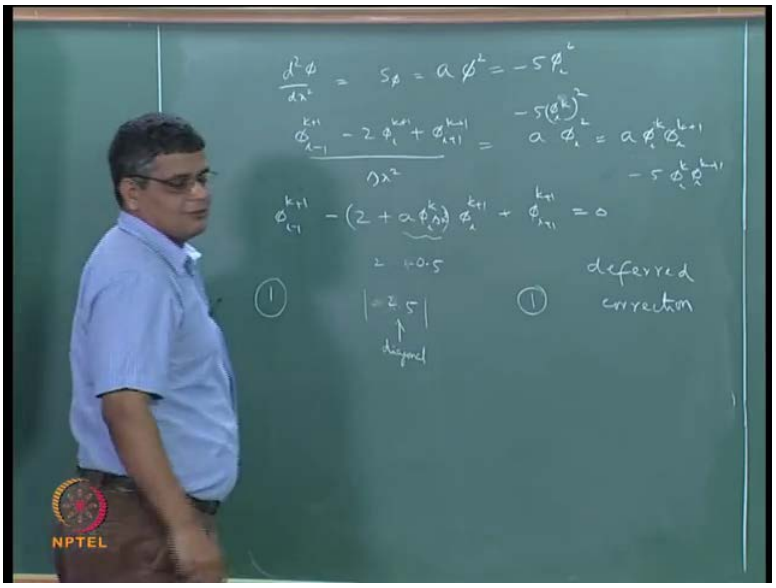
(Refer Slide Time: 12:02)



So, this happens for boundary points definitely for Dirichlet boundary condition this is satisfied and typically for a first order Neumann boundary condition, Neumann boundary condition with first order discretization also we have this form is satisfied and this is satisfied for all interior points.

For all interior points, this is satisfied with an equal to sign. So, for diffusion problems or for unsteady diffusion problems, we can rest assured that this will be satisfied without a source term, but if we have a source term, then depending on whether or not the source term depends on the value of phi, we can either break this condition or we can still honour this condition. So, that is why in the discretization of the source term, we have to be careful for example, if you have a source term which is ok.So, let us consider a diffusion equation with a source term. So, we are one-dimensional diffusion equation, so we are saying d square phi by dx square equal to source of phi and if you put this as let us say a phi square, so the value of this source depends on the constant a and phi square.

(Refer Slide Time: 20:30)



So, if you write this on a one-dimensional grid this will be phi i minus 1 minus 2 phi i plus phi i plus 1 by delta x square is equal to a phi i square. So, this is a non-linear source term. So, we can linearize it for example, in the Picard way. So, we can say that this is roughly equal to a phi star and phi i, where a phi this is some guess value and we can convert this into an iterative scheme.For example, by making this as k plus 1, k plus 1, k plus 1, this as k plus 1, this has the previous value of k. So, we can do it like this. If we do that, then we can bring this term to the left hand side.We can say that phi i minus 1k plus 1 minus 2 minus a phi ik. So, because I put this minus, this becomes this phi ik plus 1 times plus we have delta x square and we have plus phi i plus 1 k plus 1 equal to 0.

So, this is the resulting discretized equation and we can see that this is for the ith term and this is the coefficient of the diagonal coefficient and these are the half-diagonal coefficients.In the absence of the source term,this two here matches with oneandone for all the interior points.At the boundaries at the left boundary that is at i equal to 0, then this term would not be there.You will have minus 2 here and 1 here, which is greater than that.

So, this is satisfying for the left boundary with a greater than sign.The sum of the coefficients without the source term for the left boundary is minus 2 for the diagonal term and plus 1 for the half-diagonal term.Again, at the right most boundary at which point you do not have i plus 1. So, you have only i minus1.

Again, it is satisfying that with the greater than sign, but for all the values for all the interior points between 2 and N minus 1, where N is the last value on the right hand side and i equal to 1 is the first value,you have 1 here, 1 plus 1 is 2 that is the sum of the diagonal half-diagonal elements is 2 and the diagonal element itself is 1. So, in this case without the source term, the weak form of diagonal dominance is assured for us.

Now, the presence of a source term here is adding a times phi k times delta x square to to this.Now, what will this do to the solution?Is this going to be,Is this going to spoil the diagonal dominance or Is going to help in the diagonal dominance?That particular thing will determine, will be determined by a, which may be positive and negative.You have not put any condition on that and of course, the value of phi phi ik.

So, let us say that phi i is always positive, then if a is positive, then the diagonal value is enhanced by adding to this 2, but if a is negative and phi i is positive, then this will be subtracting it.Let us say that this value is this,this thing is minus 0.5 and this is 2,so this becomes minus 1.5. So, this is the diagonal element value and the half-diagonal is 1 here and 1 here. So, the sum of this 1 plus 1 is 2 and the magnitude of the diagonal element is 1.5.

So, that means that we are not able to satisfy even the weak form of diagonal dominance condition from the source term.If the source term is such that this a phi delta x square is positive, this is plus, then this becomes this becomes 2.5 and this is going to be greater than this.So, even for interior points this will be greater than the sum of the half-diagonal elements.

So, in the case where you have a source term,we have to we have to be careful that the diagonal dominance condition may be violated depending on what is happening here.If you have no control over phi phi k, then it is difficult to predict a priory, whether it is going to be enhancing it or or suppressing it, but if you if you know that if you have a condition that phi k is always going to be positive, then by knowing whether a is positive or negative. We can see whether the presence of the source term, which is linearized in this particular way is going to make make us lose the diagonal dominance or enhance the diagonal dominance.

So, we have to be careful in that.If ‘a’ happens to be negative, then of course, this becomes weare going to lose this. So, if a is negative for example, minus 5 phi i k phi i k plus 1, so no matter in that particular case we are always going to lose this provided delta x square is provided phi ik is positive.

So, we can in such a case, we will not be able to apply the Jacobi method or Gauss-Seidel method or SOR method because there is no guaranty that we are satisfying the Scarborough condition. So, in such a case, we have to treat the source term differently.We cannot write it like this. So, in let put this as minus 5 phi i square.

So, in this case what one could dois that instead of writing like this, I can write this as minus phi i square phi ik whole square, where I am using the entirely the previous value and what does that do to my discretization because this is known for the k plus 1th iteration, this remains on the right hand side and the value on the right hand side that is the terms in the b matrix do not have any bearing on on the p p matrix.We do not have to worry about whether it is going to, that is it would not influence the diagonal dominance condition because it remains on the right hand side, but if we express this in terms of k plus 1 and then bring it here, then we have a potential problem in terms of correcting it ok.

So, this type of approach where some of the terms which influence the value of i at k plus 1 are evaluated with the kth value.The previous iteration value is known as deferred correction;correction which is postponed. So, this is one way of accounting for a source term, which is going to spoil my diagonal dominance. So, with this deferred correction approach,I can treat this using this or even some of the values which may be appearing in this.I may be treating them in a deferred correction approach and push them on to the right hand side of the overall equation,so that my matrix still remains diagonally dominant, at least weak form of diagonal dominance. Therefore, I can go ahead and use my Gauss-Seidel method or SOR method and work with the solution, but because we are using an approximate value here,The next time round we have to correct this the next time around we have to correct this.

So, the overall rate of convergence of this deferred correction approach will be will be compromised. So, we have to pay the penalty of that decreased rate of convergence of the overall schemes with the deferred correction and sometimes, if the value of this is going to dominate this, then we may not have good convergence; we may have divergence.

So, the deferred correction approach is is a practical way of dealing with terms which are going to spoil the diagonal dominance condition. So, if we have made a priory decision to solve the matrix equations, let a1ub equal to b1 and a2vb equal to b2 and a3p prime equal to b3, all those things are matrix equations. So, if you have made an a priory decision to solve those things using Gauss-Seidel method or one of these basic iterative methods, then we have

to ensure that in no condition this a1, a2, a3 become, they lose the property of diagonal dominance.

So, in such a case for those terms which are going to create us problems, we have to use deferred correction approach and the deferred correction approach may mean loss of overall conversions.We have to be vary of that and if we have that, what do we do?In such a case, it means that we cannot use the Gauss- Seidel method.We have to go for different kind of solver, which does not have this diagonal dominance condition and what kind of solver.

We have already seen Gaussian elimination method, LU decomposition method.These do not require Gauss diagonal dominance. So, we can use those things, but they have N cubed number of mathematical operations whereas, Gauss-Seidel method may have N square number of operation.

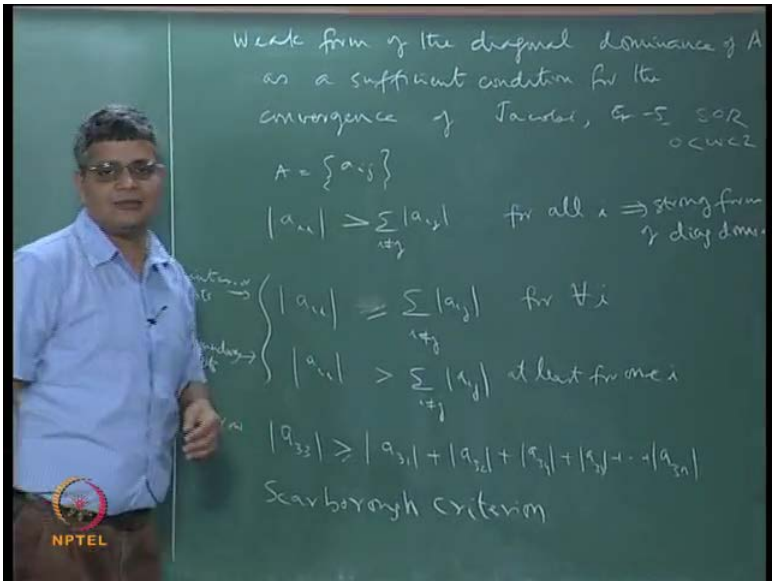
So, we have to pay the penalty of more number of mathematical operations, more computational time in order to get a solution. So, we have to choose between get not getting a solution at all by insisting on a solution method, which depends on diagonal dominance or getting even a late solution using some other method ok.

We do not have only the Gaussian elimination or LU decomposition for this.We have other methods, which do not depend on the diagonal dominance condition. For example, the Conjugate Gradient, but the Conjugate Gradient method is more complicated to program than Gauss-Seidel method. So, we have to we have to at some point we have to pay the penalty for having a problem which is more complicated than the simple cases that we have to do and that is where ones ingenuity, ones perseverance in getting a solution, all that comes into a picture. So, the point that you want to make here is that the basic iterative methods, the Jacobi method,Gauss-Seidel method and SOR method are very simple to work with, but they have a stringent criterion which is that the Scarborough criterion, which is readily verifiable has to be satisfied in order to have the assurance of convergence of this and that Scarborough criterion imposes a restriction on the type of discretization that we have to do.

One example that we have given is a diffusion equation, a steady state diffusion equation with a source term, which is of this particular form.When you discretized it in a straightforward manner using the Picard correction to non-linearize this.To linearize that particular term, then we can get in to problems with the loss of diagonal dominance. If we do it with a,we can therefore, do it with a deferred correction approach and still use the overall

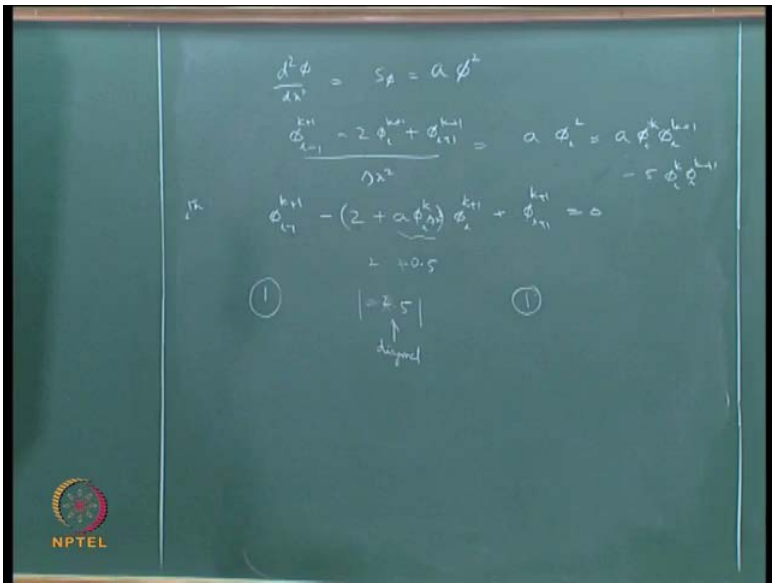
Gauss-Seidel scheme.For that, we have to pay the penalty of delayed conversions ok or we have to go for some other method for the solution of this.We have seen that in this particular case, we are going to get a Tri-diagonal matrix, but in a Tri-diagonal matrix without the diagonal dominance, the Thomas algorithm may not work. So, we have to be vary of it. So, we have to do things more carefully in cases where we have a source term and there is a possibility of loss of diagonal dominance.

(Refer Slide Time: 34:58)



So, this is about the condition for convergence of the basic iterative methods.Now, what about the rate of convergence?We have mentioned that the condition for convergence depends on the iteration matrix p and the largest Eigen eigen value are the spectral radius of that particular iteration matrix determines, it has to be less than one in order for it to converge. When you are dealing with typical cases, then and if you are dealing with a case where you want to have a stringent condition for convergence of the Iterative scheme that is when you want to have an accurate solution for 5, maybe fifth decimal accuracy or tenth decimal accuracy like that. So, when you want to go down to very accurate calculations, then the rate of convergence depends also on the spectral radius.

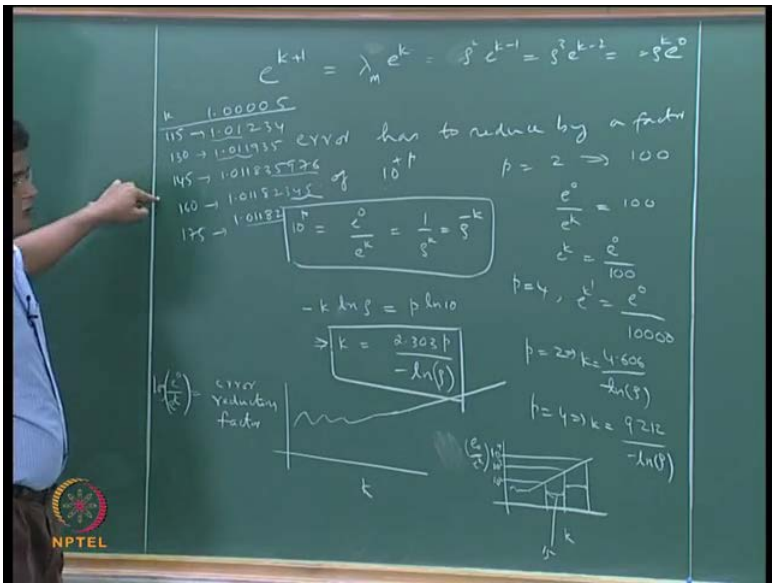
(Refer Slide Time: 20:30)



So, the spectral radius will not only determine whether or not it is going to converge, but it is also going to determine the asymptotic rate of convergence.The asymptotic convergence rate is an important quantity when we characterizein goodness of an iterative scheme and what we mean by asymptotic convergence rate is the number of iterations that are needed to reduce the error byso much quantity.

When we look at any particular solution, we do not know the exact solution apriority. So, we do not know what is the error at n,so we cannot define exactly by how much the error should be, but it is possible to get some idea, some assurance that whatever that initial error is, that is going to be reduced byso many factor.That is based on the Eigen value the Eigen values of the iteration matrix p and for a given matrix p consisting of N variables, we will have N Eigen values.

(Refer Slide Time: 37:36)



So, this can be arranged in the order of increasing magnitude of these Eigen values and the largest Eigen value is going to be the one which determines the rate of convergence or the asymptotic rate of convergence.That we can see because we have said that the error at kth iteration k plus 1 is lambda times k times error lambda times error at the kth iteration.So, that is, as we go from kth iteration to k plus 1th iteration, the error is multiplied by lambda.This lambda here as k tends to large values; this this is the value of the largest Eigen value ok.

So, if the largest Eigen value has a magnitude which is less than 1,one can see that with each multiplication, the error is reducing by a factor of lambda.Let us say 0.9, then for the next iteration it reduces by another factor of 0.9,so that is 0.81 of the value and if you continue to sufficiently large number of iterations, then this factor can be the error can be reduced. So, we can say that this is equal to; we will just denote this as the spectral radius.

So, this we can say is rho square k minus 1 that is equal to rho cube k minus 2 like this upto rho k, the initial error. So, the initial error that we had at the beginning is multiplied by rho to the power k. So, if we say that the error has to reduce by a factor of 10 minus p, 10 to the exponential for example, if you say that p is equal to 2, then that means that the error reduces by a factor of by a factor of 10to the power of plus p.

So, that means that it reduces by a factor of 100, so that means that initial error divided by error at k number of iterations should be 100.Therefore, the error at k number of iterations is

initial error divided by 100 and if you say that p is equal to 4, then error at the number of k prime number of iterations will be the initial error by a factor of 10,000.

So, we can see that if we define it like this, then we can say that the error at certain number of iterations has to be a factor of 1 by 10 to the power p number of the initial error. So, therefore, we can say that 10 to the power of p is equal to e0 by ek and e0by ek is equal to 1 by rho k ok.

So, this is equal to the rho minus k. So, now we can take this relation therefore, we can say that minus k is p lone of 10. So, we can say that k is equal to 2.303 p by lone rho minus lone rho.So, that means that means this implies that the number of iterations required to reduce the error by a factor of 10 to the power of plus p is given by 2.303 times p, where p is the exponent divided by minus lone rho where rho is the spectral radius of the iteration matrix.

So, when p is equal to 2, then this will be 4.606 divided by minus lone rho and when p is equal to 4, then k is equal to this, k will be equal to the times this. So, this 9.212by minus lone rho and what we see from here is that for a given matrix, the spectral radius is fixed. So, if you want to go from an error of 100 times to error of 10000 times,so that is to reduce the error by two orders of magnitude, the number of iterations further iterations is fixed ok.

So, per one order of reduction, we need 2.3 divided by minus lone rho of minus of lone rho is the number of iterations.Therefore, if you were to say that error reduction factor versus k, where k is a number of iterations, we know that as,so this is equal to initial error by error at k.We know that for a convergent scheme ask increases, this error reduction factor should should increase because the error is decreasing.This is fixed value,so this this value is going to increase with k. How this increases is eventually, in the initial cases the error reduction factor does not depend only on the spectral radius or the largest Eigen value, but it also depends on all the N minus 1 Eigen values which are less than the spectral radius which are less than the.

So, in that case the reduction, there may not even be reduction.It may be going through like this, but eventually it will go into a constant slope like this.The constant variation where with increasing number of iterations, then it reaches a constant thing; where we are looking at k versus log of p. So, this we take in terms of logarithm value here and then we get a straight line between k and this.

So, this slope here is such that when we reduce, let us redraw this.After some initial things, it will be a constant line here. So, this if let us say that this is 10 square, this is 10 cubed 104 like this. So, this this error reduction factor is put on a log scale like this.The number of iterations required to reduce the error from by factor of from this to this is the same as the number of iterations required to reduce this from 10 to power of 3 to 10 power of 4.

So, that means that if you are putting in decimal space, the accuracy of the solution in terms of the decimal place accuracy. So, if you say that the value is 1.00005 is the exact value at the end of 100 iterations, so if you say the value spaces is arrived at 0.1234 and let us say that this number is 15. So, at the end of say, 115 iterations you have this value, at the end of 130 iterations that is another 15 iterations, you get a value which is 1.0.This is now fixed and we will have say 1935 like this.

So, and at the end of 145,another 15 iterations, this is accurate only after this thing now.This is accurate up to this; it is only these numbers will change. So, it will be 1.01183596769 and at the end of 160th iteration, it will be 1.0118.Now, only these numbers will further change. So, this is 2345 like this and 175thiterations, the any further variation will only be in this.

So, all these digits are fixed.So, that means that once you reach this kind of stable condition, once the influence of the lesser Eigen values less than the maximum Eigen value is is taken out, then you reach a stable convergence regime in which the errors reduction happens at a constant rate per decimal place accuracy. So, and if you say that we have reached that condition at the end of 115 iterations, then we know that in a computed solution by the fact that from 115 to 130 as you go with 160 and 170 like that, as we see the second decimal place stops changing from here onwards. So, third decimal place up to third decimal place; it remains the same.It is only the fourth decimal place is changing and further on it is only the fifth decimal place that is changing and sixth decimal place like that is changing.

So, that means that one can loosely say that this solution here is accurate up to second decimal place and this is accurate up to third decimal place and this is accurate up to fourth decimal place.This is accurate up to fifth decimal place.

So, we want, if you want to increase the accuracy the decimal accuracy of the solution, we have to have another delta k number of iterations and that additional number of iterations is given by by this.It is given by 2.303 divided by lone minus lone rho. So, this this is the asymptotic conversions rate and this is determined only by for long values of k.

For large values of k, after we have gone through that initial transient period and after we fall into this asymptotic regime,then further accuracy of the decimal place accuracy of the solution that we get depends only on the maximum Eigen value of the iteration matrix. So, if the iteration matrix changes, then the the asymptotic conversions rate of the method also changes. So, if you take aiterative solver like the Jacobi method or the Gauss-Seidel method or the SOR method, in each case the iteration matrix changes and for different values of the over relaxations parameters, in the SOR method, the iteration matrix changes.In such a case, each of them has a characteristic spectral radius ok.

So, that spectral radius will determine how many more number of iteration I need to do with that particular iterative scheme to gain one decimal place accuracy in the solution, but only after we have reached that asymptotic conversion. So, this gives us a method of estimating.What is the number of mathematical operations that are required to get a reasonably accurate solution using the iterative scheme?We know that we cannot directly compare an iterative scheme with with a direct method in terms of the total number of mathematical operations that are required to get a solution.That is because an iterative scheme is only improving the solution and it is asymptotically improving the solution.

So, by these by definition there is no end to the iterative solution, but using this property of the iterative scheme that after a sufficiently large number of iterations, further accuracy of the solution is determined only by the spectral radius of the iteration matrix.Making use of this, we can get an estimate for what is the number of mathematical operations that are required to achieve a certain decimal place accuracy of the solution and that will give us an idea of the total number of mathematical operations that are roughly required to get a defined accuracy.Therefore, it gives us an estimate of the total number of mathematical operations required for the solutions of a phi equal to b. So, that knowledge, that estimation requires knowledge of the spectral radius and the spectral radius depends on the iterative matrix.