Computational Techniques

Prof. Sreenivas Jayanthi.

Department of Chemical Engineering

Indian institute of Technology, Madras

Module No. # 05

Lecture No. # 24

Gauss-Jordan method

L U decomposition method

TDMA and Thomas algorithm

Let us look at the specific variation of the Gaussian elimination method. This is called the Gauss-Jordan elimination method. And in this particular method, we continue with the elimination of the variable phi 1, phi 2, phi 3, phi 4. All these things, not only in those rows which are below the diagonal, but in those rows which are above the diagonal. And what we get therefore is, in the first elimination, we have not eliminated anything; in the second equation, we have eliminated phi 1; in the third equation, we have eliminated phi 1, phi 2; in the fourth equation, we would have eliminated phi 1, phi 2, phi 3 and so on.

In the nth equation, we would have eliminated phi 1, phi 2, and phi 3 all the way up to phi n plus 1 so that for the lower triangular elimination matrix and we have an upper triangular elimination method in the Gaussian matrix.

So, in the Gauss-Jordan matrix when we eliminate phi 1 from the second equation and phi 2 from the third equation and so on, we also continue with the elimination of these things from the first equation, second equation onwards so that the variables that are eliminated get eliminated also from the first equation.

So, when we eliminate phi 2 from the third equation, we simultaneously eliminate phi 2 from the first equation. So, now, at the end of the step three when we eliminate phi 1 and phi 2 from the third row, we would have eliminated phi 2 also from the first equation. So, at that point, the first equation will have a 1 1 phi 1 plus a 1 3 and so on, it would not have phi 2.

When we go to the fourth row by then we eliminate phi 1, phi 2 and phi 3 from the first equation and we also eliminate from the second equation. So, when we do this, when we by the time we go to the last row, we would have eliminated all the variables from the first equation, except phi 1 1, and in the second equation we would have eliminated all the variables, except phi 2 and so on. So, in each row will have only one non-zero coefficient that is a diagonal element.

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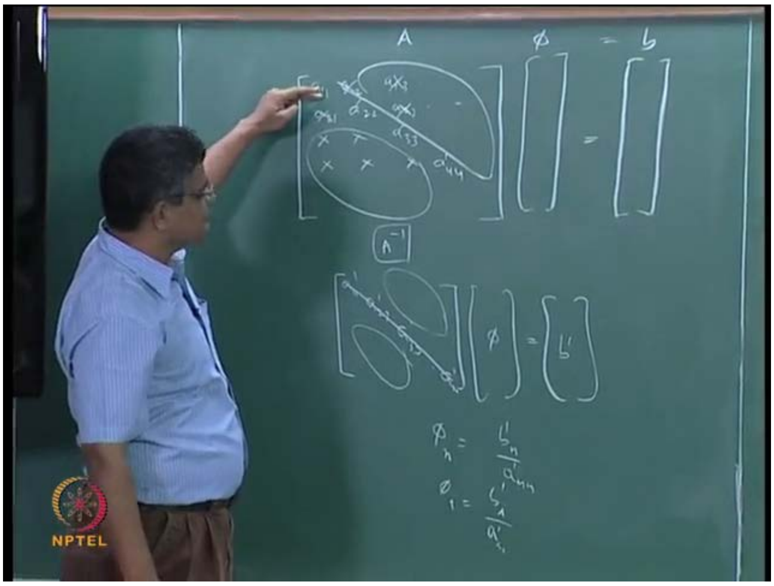

So, the matrix structure, A phi equal to b, we have a1 a 12 and so on and a 21 a22 a 23 and so on like this. If we eliminate phi 2 variable from this, then this becomes 0 and this becomes 0 0 and we have a 3 prime type of thing. So, when we do this, when we have eliminated phi 2 from this equation, we also eliminate from the first equation so that this becomes 0.

When we come to the fourth equation, we would have eliminated this and this we will have a prime 4 4. So, at the same point, we also eliminate phi 3 from this equation and from this equation and this equation. So, at the end what we will have is zeroes on this side and zeroes on this side and we will have just the diagonal elements here.

So, a 1 1 prime a 2 2 prime a 3 3 prime a 4 4 prime all those things as non- zero; all the others as zero. So, now, this is equal to b prime. So, we have made this in to diagonal matrix and the solution is, this is equal to this and this is equal to this and in order to phi n, we say that this is equal to b prime n divided by a prime n n and phi i is b prime i that is the i th row divided by a i i prime like this.

So, the back substitution is very simple; each evaluation of each variable will involve only one variable like this and we will essentially this will be A inverse. So, this method, where we not only eliminate this element variable phi 1, phi 2 and phi 3 in the variation below, but also in the rows above is called Gauss Jordan elimination and as a byproduct of this, we not only get this, but we also get A inverse here.

So, this is one method for finding the inverse of the matrix; a very efficient method of finding the inverse of a matrix. And obviously, doing this row elimination elimination of row variables above and below will be costlier; it will involve more number of operations than in the simple Gaussian elimination process, where we are simply eliminating these things.

So, there are more number of operations that we have to do here, but the total number of operations still varies as n cubed; it is more than what we have for Gaussian elimination, but as a byproduct we are finding the inverse.

And the back substitution is simpler in this; this is only n number of operations, whereas we had only n square number of operations for the back substitution in the case of Gaussian elimination. But the elimination of these things will include n number of operations here. So, the overall cost of computation using Gauss-Jordan elimination is more than the cost using Gaussian elimination.

So, if we are solving a single method A phi equal to b, then this method is not more efficient than Gaussian elimination, but if we have a number of these equations to be solved, then we can use this, because as a result of this, we have A inverse.

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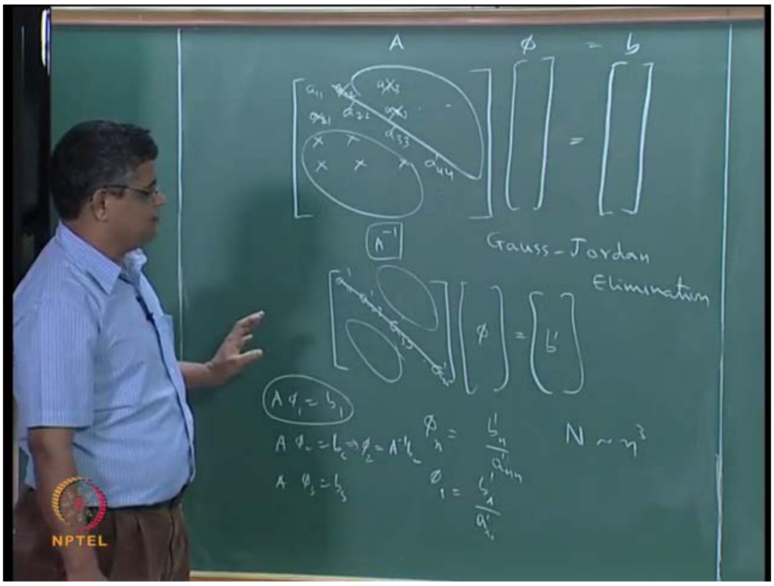

So, if we have a situation, where you have A phi 1 is equal to b 1; A phi 2 is equal to b 2; A phi 3 is equal to b 3 like this, then we can write this as phi 2 equal to A inverse b.

So, in the since this A inverse is made a diagonal thing, the computation of this is very simple. So, for this for the solution of this, we use more number of operations than the Gaussian elimination, but for the subsequent elimination of phi 2, phi 3 and all the other variables, then the computation of the solution of this equation with the same A is made very easy with this.

So, in that sense, the Gauss-Jordan method would would work, but otherwise this method is this method is not specially useful in the context of finding a solution to A phi is equal to b. As long as A phi equal to b is a single equation, there is no need to go for Gauss - Jordan method, but this is a general purpose method for the finding of A inverse in an efficient way.

So, this is Gauss method Gaussian elimination method and Gauss-Jordan method, they follow essentially the same type of approach, a different approach is what is known as L u decomposition.

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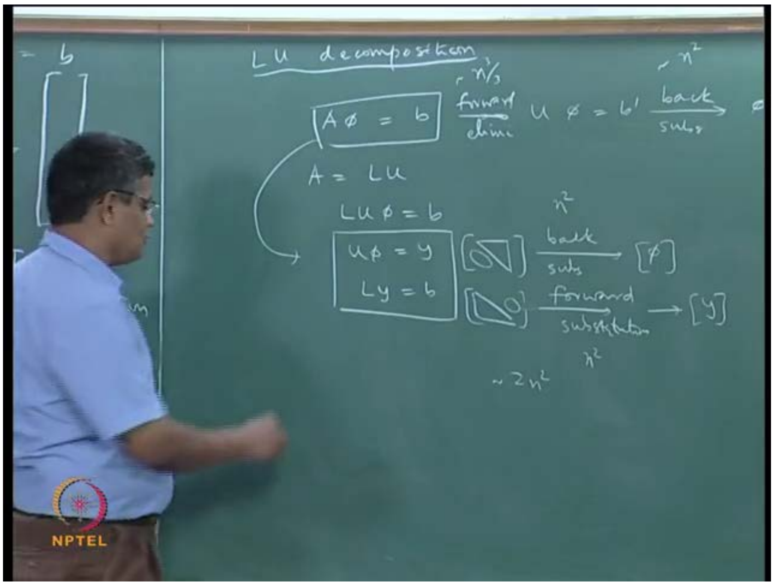

Let us discuss the basic idea. We have A phi equal to b and in the case of Gaussian elimination, we are converting this in to u phi equal to b prime, and then, we solve this by back substitution. So, this by the forward elimination, we convert A phi into u phi equal to b prime, then we solve this by back substitution to get phi. In the L u decomposition method, A is decomposed into L, the product of a lower triangular matrix and an upper triangular matrix.

Now, what is the advantage of this? If we now substitute this here, then this equation becomes L u phi equal to b. So, now, we can each of this is, this is a lower triangular matrix and this is an upper triangular matrix. So, we can put u phi equal to y; if we do that, then if we substitute, this becomes L y equal to b and u phi equal to y.

So, the problem of finding A phi equal to b is now decomposed in to the problem of finding two equations u phi equal to y and L y equal to b. So, now, the question is what is the advantage in this? Now, the advantage is, we know if we come to this process f in the Gaussian elimination, this forward elimination takes n cubed number of operations and the back substitution takes n square number of operations. So, the back substitution is easy. So, each of this this is a lower triangular matrix L here and this is an upper triangular matrix.

So, this particular thing u will be like this; all these are zero and here we will have like this. So, we can solve this using forward substitution and this will give you y and we make use of this y here and solve this using backward substitution to get phi.

So, the solution of A phi in to b is now rendered into two steps, first a forward substitution to get y in which b and L are known and once we know y here, then a backward substitution to get phi from this. What is the advantage? We know that backward substitution takes n square number of operations and this also takes n square number of operations, because the basic idea is the same; this you get one multiplication and this is one multiplication plus one more multiplication, two operations and three operations like that, this also n square number of operations.

So, we are getting a solution of A phi equal to b in 2 n square operations, not n cube operations, because here this is n cube number of operations. So, the method would work efficiently would be much better than Gaussian method if we restrict ourselves to these two steps, but there is a catch in this. How do you get? How do you do this decomposition? How do you make it into L and u? If obviously, what we are writing here is a matrix here a i j and this we are saying as the product of l i j, where we have these as nonzero times product of u i j. So and a i j, a particular component here is given as the product of l i k with u k j.

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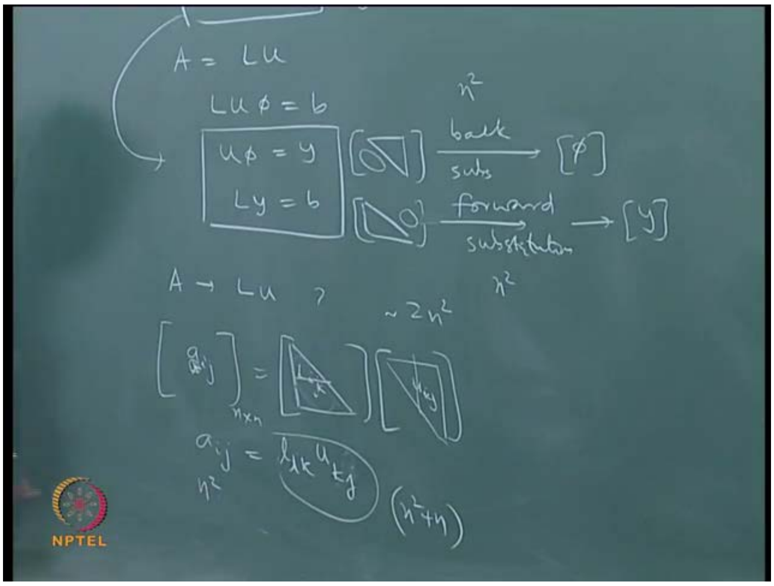

So, the product of l i k and the u i j is given is equal to this a i j. So, you have a i j is equal to l i k u u k j. So, from matrix multiplication, we have this kind of relation and here we have, if you say that if we have n rows and n columns, then there are if you have a total of n variables, then this is n by n matrix. So, there are n square number of a i j's. So, we have n square points here and here we have n square by two number here and n square by two number, when you add these two, your diagonal is added twice.

So, the total number of unknowns in this so, the unknowns in this are n square plus n number of unknown. So, each of this has n square by 2 and this has n square by 2 plus n type of thing, one can see that when you superimpose this, you have all these things as unknown and all these things as unknown and this is already included in this. So, you are over counting this and there are exactly n diagonal elements. So, the total number of unknowns l i j and u k j to be determined n square plus i and the total number of elements that are known here as part of the specification of a is n square.

So, we have a system which is over specified. So, what people do is that, you either assume that all these diagonal elements of the lower triangular matrix as being equal to one. So, these are not determined; so they are already known. So, that like extra things known or you can also make these as unknowns or you can make all these things as known. So, you will specify that the diagonal values of either the lower triangular matrix or the upper triangular matrix you can specify all these things also; conventionally specify either these things or these things to be 1 and that determines that leaves us with n square number of unknowns and n square number of equations to be determined and therefore, there is a certain possibility of determining l i j and u k j the elements of all the elements of lower triangular matrix and upper triangular matrix from the knowledge of a i j. So, in that sense, the decomposition of a into l u is unique provided, you specify n number of elements in this; by convention we specify the diagonal elements of either l or u. So, there is a possibility of determining the l i j and u k j from a j a i j itself, but at what cost?

Special easy to use algorithms e and easy to program algorithms have been programmed for this. (Refer Slide Time: 18:29) there is a Crouts algorithm is there in which you sequentially determine l i j, these elements and these elements and these elements and these elements like that; so through some process and this process enables us to sequentially find the sequentially and alternately find the elements of l and the elements of u as per this algorithm.

So, finding l i j and u k j is not difficult and there is a certain method and once you find that, then you can substitute, you know now the l elements here and the d here are anyway known put the y here and u is known from this. So, you solve for phi. The difficulty is that the determination of l i j and u i j given value of a i j itself takes n cube number of n cube by 3 number of computations.

So, the decomposition of a in to L u itself is almost as if equivalent to the forward elimination process of the Gaussian elimination method and that is where we find that the L u decomposition, although it seems to be advantageous in this, this is advantageous only if we know L and u, but if we start with the matrix a here and you have to determine L and u as part of the solution, then it is no more advantageous than Gaussian elimination method

In fact, slightly more expensive, because you are solving two back or forward substitution, we are solving an extra forward substitution here, but in fact, there is a relation between the Gaussian elimination and the this decomposition; the elements of these things are none, but the coefficients that we have used to multiply the pivot equation.

So, there is some relation between the some of the multiplicative elements that we used in the forward elimination process are also elements of these things. So, the two are not very different, except that it is going to cost as much as the L u decomposition. So, the L u decomposition, if you have a single A phi equal to b type of equation, it is not any more advantageous, but if you have a situation, where you have a number of equations to solve with the same a with the same coefficient matrix, then this advantage is superior to Gaussian elimination.

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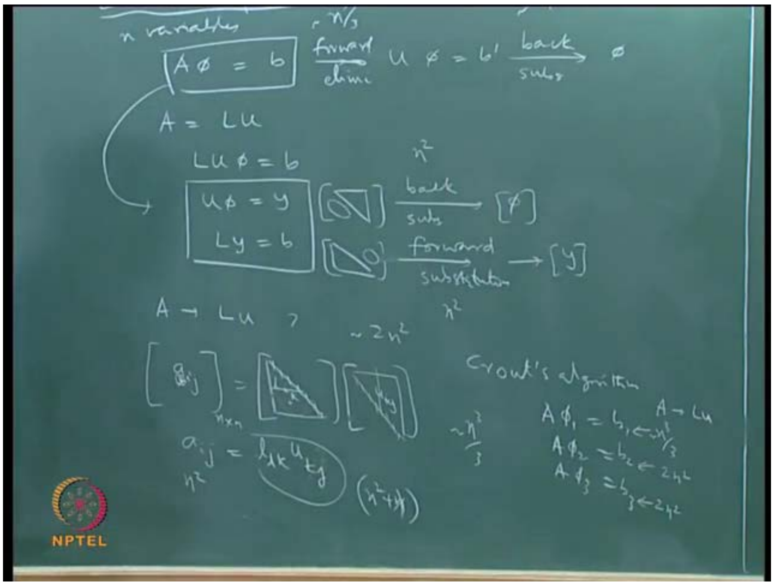

Because same way as before, you have A phi 1 equal to b 1; A phi 2 equal to b 2 and A phi 3 equal to b 3 and so on; it is the same matrix here. So, you use this n cube here by 3 number of operations to decompose A into L u and this decomposition is unique once you specify the n values that are superfluous that are extra, that you can fix and once you fix them to be once here, then this decomposition is unique. So, we can use this same L and u elements in this and L and u elements here. So, although the solution of this will take n cube by 3 numbers of mathematical operations, this will take only 2 n square and this will take only 2 n square number of operations.

So, in that sense, this is more advantageous when you have a number of equations to be solved with the same coefficient matrix, but the right hand side matrix- the b matrix - is different. And in terms of Gauss-Jordan method also, this method is superior, because we are not spending the extra n cube number of operation to do the additional eliminations which are not really necessary. So, the Gauss-Jordan method which results in the A inverse, although the back substitution is easier, the forward elimination process to eliminate not only these, but also these will require another n cube by 3 number of mathematical operations.

So, we are substituting the n cube by 3 number of operations here for the elimination of the elements above this with two back substitutions. So, if you had a system of A phi, these type of equations to be solved with the same coefficient matrix and with different b' s here, then the L u decomposition method is better than the Gauss-Jordan method which itself is better than the Gaussian elimination method.

So, in that sense, you have to do it three times with the Gaussian elimination method, whereas we need to do only once, and then, we have the inverse of the matrix and we can proceed much faster with the Gauss-Jordan method. Here we do once the L u decomposition method and then we can use this. So, this method is complete once we specify how to determine the L and u.

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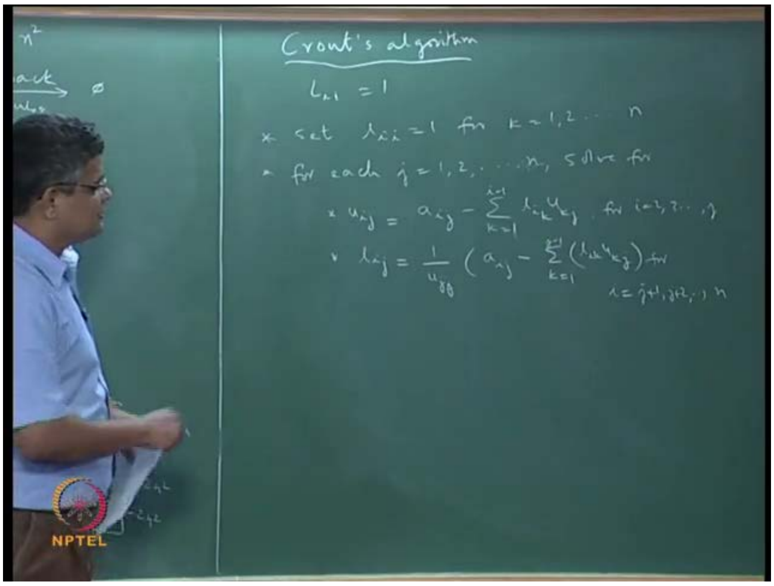

For this we have what is known as a Crout's algorithm; this is the case where we have assumed that L i i is equal to 1. So, that is the case, where we have identified we have specified that all the diagonal elements of the lower triangular matrix to be 1. So that we have n square number or extra unknowns belonging to n square by 2 minus n number of variables here, and then, n square by 2 plus n number of variables here and we have n square here and n square number of things and in such a case we have Crou'ts algorithm and in the case where you specify the upper triangular matrix diagonals to be all 1, then we have another algorithm called do little's algorithm or do little's composition. So, you set for k equal to 1, 2 up to n, and then, for each j equal to 1, 2 up to n, solve for u i j equal to a i j minus sum over k equal to 1 k equal to 1 2 i minus 1 l i k u k j for i equal to 2, 3 u k up to j. So, we use this step to do this summation here, and then, l i j is evaluated as 1 by u j j for i equal to j plus 1, j plus 2 up to n. So, this is Crout's algorithm.

We are setting all the diagonal elements to be 1 in the first process, and then, we sequentially evaluate u i j upto elements up to i equal to j and after that, we switch over to l i j evaluation, from i equal to j plus 1 to n like that.

So, although it looks a bit complicated, the beauty of this algorithm is that the values of L and u as required to be substituted here are obtained in a sequential way provided that we follow these limits. So, this method is known to work and unfortunately this will take n cube by 3 number of operation it looks simple, but it takes that much and… So, the resulting, these explicit way of evaluation of at the value of l i j and u i j without having to do this multiplication here to solve n square number of simultaneous is avoided by using the brute force method. So, the algorithm is evaluated at u i j and l i j in an explicit way for given a i j like this. So, using this, we can find l i j, u i j then we can go for back substitution.

So, this is the method that can be used using the L u decomposition. So, this is a direct method which is almost as expensive as the Gaussian elimination method for the solution of a single equation, but for a sequence of iterations for the sequence of computations using the same coefficient it is better, but still it is n cube.

So, we have to keep in mind that it is a generic method; it is as widely applicable as the Gaussian elimination method, that is, the decomposition of A in to L and u, the product of L and u is can be done for any non singular matrix. Now, one difficulty with respect to the Gaussian elimination is with respect to pivoting strategy, in a Gaussian elimination we can do more number of pivoting strategies than what is possible with a L u decomposition.

So, in a L u decomposition typically since we are doing in a certain sequential way, we can really do partial pivoting, whereas full pivoting can be done in a Gaussian elimination.

So, when we are faced with a problem, where the round off errors accumulation of round off errors is a problem, then one can say that Gaussian elimination is superior to the L u decomposition method. But as mentioned earlier, for problems with A phi equal to b type of things arising in c f d computations in c f d interest, we do not have we have sparse matrices. So, because of the sparseness, we do not have to do so many multiplications. So, in such a case, we do not usually have round off errors and we can safely use this. But we should keep in mind that the using L u decomposition is still not the best way, because it goes as n cube and we have mentioned that, there are iterative methods which go as n square.

So, in that sense, L u decomposition in not generally used in c f d applications by itself, it is used in certain cases, where for example, a simple iterative method like Gauss Seidel method cannot be used; what kind of application, what kind of a scenario, where a is not diagonally dominant in such a case, we said that Gauss Seidel method cannot be used; so in such a case, we can use L u decomposition method and when can a b be not diagonally dominant?

If you have an equation which is not posed in orthogonal coordinates this typical scalar transport equation is not posed in orthogonal coordinates if a typical scalar equation is not done in orthogonal coordinates, then we have cross derivatives influence of cross derivatives makes the overall structure of a to be not diagonally dominant.

So, in such a case, we can use L u method, but even there L u method is not used in itself; it is usually clubbed in some form of iterative method in the process of incomplete L u decomposition kind of methods, in that context L u decomposition is used. So, typically in large c f d problems neither L u decomposition method nor Gauss elimination method is used, it is only the iterative methods are used, but the influence of these methods in the development of coupled or combined methods, where we take elements of the direct method and elements of the iterative method and put together and get newer methods. So, in that context, we need to understand how L u decomposition starts. So, that is why we have spent time on this.

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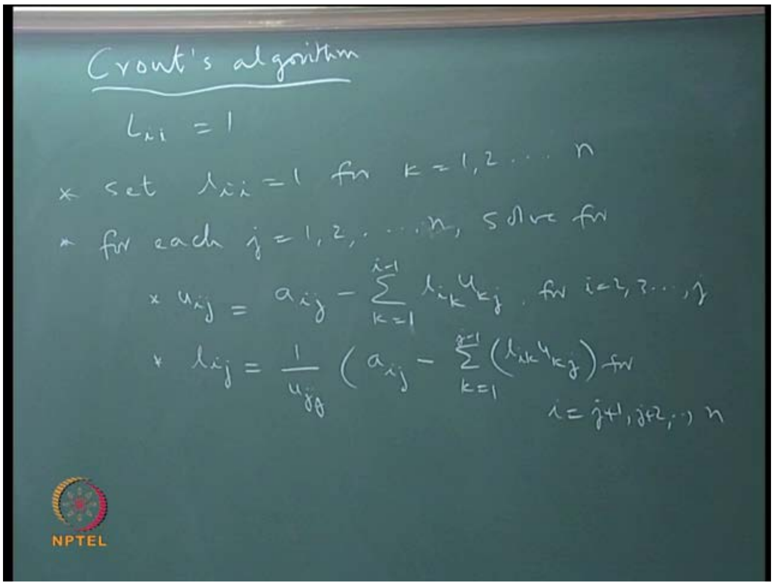

Before we close the discussion on the direct methods, let us look at a special method for the banded matrices. Banded matrices are matrices in which non-zero coefficients in the coefficient matrix A occurs along certain diagonals.

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So, if you are looking at the matrix A be like this, then typically you have the diagonal elements are non-zero and you have those immediately adjacent to the diagonal also non-zero then. So, this is one the simplest form of the banded matrix and this is a tri diagonal matrix and we can see where it occurs.

If we take the case of one-dimensional fully developed flow u d square u by d y square equal to a pressure gradient equal to d p by d x, which is constant; so in such a case, if you use central differencing for this, then we can write this as mu times; we can take the mu here and so we can take this here and write this as 1 by mu. So, this is a constant and we can write this as u i minus 1 minus 2 u i plus u i plus 1 by delta x square delta y square is a constant and this constant... So, we can write this as (Refer Slide Time: 35:37) and this is our b side and we can see that in this difference equation, we have i here and i minus 1 and i plus 1.

And we have to write this in order to determine the u i value, you need to know both the neighboring values. So, in such a case, it is not possible to march forward from i equal 1 to i equal to n; you have to solve all of these simultaneously and we also note that, this is our y and you are starting at y equal to 0 to y equal to let us say h here and you have this is i here and so this value is given by the two neighboring values and this value will be given by the two neighboring values and so on. So, at any point is given by only the two neighboring values and the coefficient of all the other variables will be 0 in this. So, when you put this together, we get this kind of banded matrix and with the Dirichlet boundary condition, it will be like this kind of thing.

So, these coefficients here correspond to i and these coefficients will correspond to i plus 1 and these coefficients correspond to i minus 1. So, in in this particular way, we can envisage a structure of A resulting in a tri diagonal matrix.

If you take again a different case, for example, dou u by dou t equal to some nu dou square u by dou y square, this is known as the stokes first problem and this is the case of an equation which represents the variation of velocity in the y direction, when a plate that is set in this infinite expanse of fluid is suddenly made to move at a constant velocity. So, if we were to do this using explicit differencing implicit differencing, we can write this as u i n plus 1 minus u i n divided by delta t equal to nu times u i minus 1 n plus 1 minus 2 u i n plus 1 plus u i plus 1 n plus 1 divided by delta y square.

Again you have i plus 1. So, this whole thing simplified here, will give you 1 by u by delta y square u i minus 1 n plus 1 with the minus sign and 1 by delta t plus 2 nu by delta y square times u i n plus 1 and minus nu by delta with a minus sign and 1 by delta t plus 2 nu by delta y square times nu by n plus 1 and minus nu by delta 1 square 2 i plus 1 n plus 1 is equal to we take this to the other side; so we take u i n by delta. So, we have coefficient of i minus 1 coefficient of i here and coefficient of i plus 1 coming here.

So, this is also of a similar form and this represents the time dependent one-dimensional diffusional equation and done in an implicit way; if it is done in a explicit way of course, we have we have no need to solve this kind of matrix equation, but if it is done in an implicit way which gives us improved stability, then we have to solve a matrix type of equation and this matrix will be of this tri diagonal form. So, in cases like this, when we apply the standard Gaussian elimination, then we convert this A matrix into an upper triangular matrix, but we can take advantage of the fact that we have zeroes all around here and convert this into convert A phi equal to b in to u phi equal to b prime, where u here has only the diagonal and then the upper diagonal till here.

So, we have only two diagonals. So, from three diagonals when we convert to upper triangular matrix and we have two diagonals and the solution of this we have becomes very simple. And the overall procedure we have in doing this is what is known as Thomas algorithm which is which is published in 1949 and this is a special form of Gaussian elimination which is used without any pivoting strategy only to deal with the three rows here and convert them in two rows here. So, we do not do any kind of multiplications or other things and additions involving these zero elements.

So, the resulting algorithm is something that we can derive and we can derive it like this. So, if we say that all these coefficients are A coefficients. So, this is coefficients of a i and this is bi and these coefficients are c i. So, if we take… So, what we mean by b i, c i is that, if we take the i th equation, then c i represents this coefficient corresponding to the i minus 1th row and a represents the coefficient corresponds to phi i and b i represents the coefficients corresponds to this. So, we can write the general equation as c i phi minus 1 plus a i phi i plus b i phi i plus 1 is equal to and let us say that the right hand side is (Refer Slide Time: 43:06) let us put this as d here equal to b i and let us say that this is the coefficient e i and we have taken all these things to be 1.

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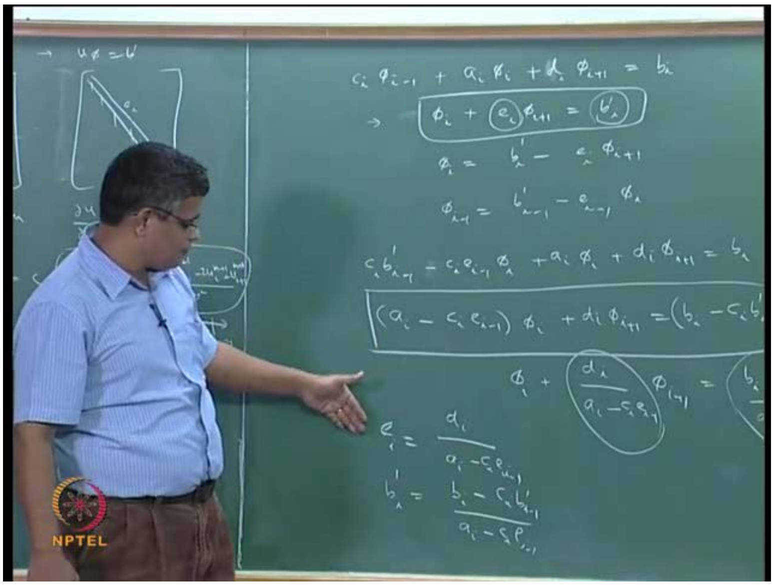

So, this is converted into phi i plus e i phi i plus 1 equal to b prime phi i. So, the process of elimination goes from tri diagonal matrix in which an equation is written like this with c i, a i, d i and b i known from the equations here is converted into something like this.

So, if we can find out what this e i and b i prime are, we can call this b i prime we will keep it like this. So, we can if we know these things, then we can solve it easily by substitution back substitution involving only one product each.

So, this is the form of the equation. So, we can write this as phi i equal to b i prime minus e i phi i plus 1, this is an equation which is valid for all the points. So, from this we can also write phi i minus 1 equal to b prime i minus 1 is e i minus 1 phi i.

So, we can now substitute this in to this. So, that gives us this c i b i minus 1 prime plus c i e i minus 1 phi i plus a i phi i plus d i phi i plus 1 equal to b i . So, we can take this on to this side, this is a constant and we can club these two things. So, that gives us this as we have a minus. So, this is that gives us a i minus c i e minus 1 times phi i b i minus c i b i minus 1 prime.

So, we have this equation here and we have this equation, which is of the desired form. So, this equation has phi i and phi i, and i plus 1 i plus 1 like this. So, we can compare the coefficients, and then, we can also divide by this. So that we can put this as phi i plus d i by a i minus c i e i minus 1 times phi i plus 1 equal to b i minus t i d i minus 1 by (Refer Slide Time: 47:25).

So, if we compare, this coefficient must be equal to e i and b i must be equal to this. So, we can get a relation which is b i is equal to b i by a i minus c I e I minus 1 and b i prime is equal to c i b i minus 1 divided by a i minus c i e i minus 1. So, if we examine this, this gives us the coefficient in terms of e i minus 1 and a b c and what are a c d? These are known coefficients here.

So, the value of i is expressed in terms of the value of i minus 1 and other known coefficients. Similarly, b i prime is unknown and this is expressed in terms of b i, which is known, a i which is known, c i which is known and e i prime i minus 1 prime i minus 1 is something that we expect to know from this, because we are dealing with the i th and this is again e i minus 1. So, if we are going through a sequential solution from i equal to 1 to i equal to n from i equal to 1 to i equal to n, then when we come to i here, we already would know i minus 1 here, and i minus 1 here. So, in that sense the value of i is expressed explicitly in terms of i minus 1. So, this gives us the possibility of doing this, provided we can start and when we look at the actual structure of this, which when we look at the first 1, we do not have c i.

So, for the first one this c i is 0, because the first element the first equation will have only two constants here. So, this is not known. So, we can find out that this particular one will not give us any problem and in that way, we can start this equation start this process.

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So, the overall solution method is like this. So, we have e 1 is d 1 by a 1 and b 1 prime is b 1 by a 1 and we know d a, and b a and so the solution of this is straightforward and for i equal to 1 to n minus 1, you can say that e i plus 1 is equal to you can make use of this d i sorry and we can use d prime i plus 1 is equal to b i minus c i i plus one. So, we have…

So, we can there are total of n equations. So, the first ones are given by these expressions, so e 1 and b i prime. So, e 1 and b i prime are the things that that are to be determined so that we convert this into two diagonal matrix. So, the first ones are determined by these simplified equations specifically for this case, and then, subsequently from i equal to 1 so this becomes e 2 which is divided by d 2 which is known divided by a 2 known minus c 2 known and e 1 which is already known here.

Similarly, b i 2 is given by b 2 known here c 1 c this is c i plus 1 known c i plus 1 is known and b 1 which is already known and this is already known e 1 is known and then... So, from ones we go to twos and from twos, we again go to threes involving these values which are already here. So, we can go through this process, and then, find out all the elements of the upper bi diagonal matrix, only having the element which is here and once we have this, then last equation will have only this one, and then, we can solve this.

At any row this will contain phi i plus e i and phi i plus 1 like this. So, we can use this this equation here to solve from the bottom of it. So, we have the last one is coming from the i plus 1 b prime here, and then, we can substitute that value here, and then, get this and this value here, and then, get this value here, and then, get this like that. So, the back substitution will involve only one multiplication.

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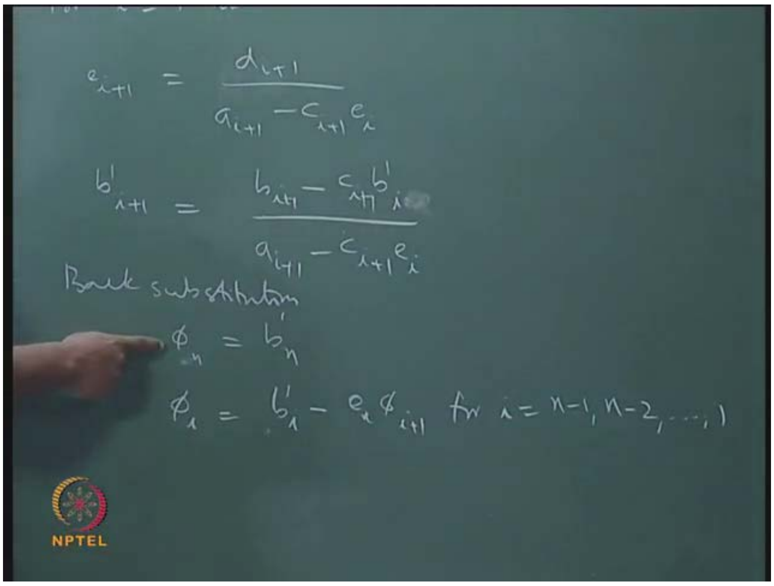

So, the back substitution is done like this. So that is what we have phi i is equal to b i prime minus e i times minus i plus 1, this is for i equal to n minus 1 n minus 2 like this, whereas for phi n this is simply given by the right hand side.

So, we can see that each back substitution involves 1 pro one multiplication; so there are n multiplications here or n minus one multiplication in the back substitution process and in the forward elimination process, we have one multiplication here and one division here, again one multiplication, two multiplications and one more.

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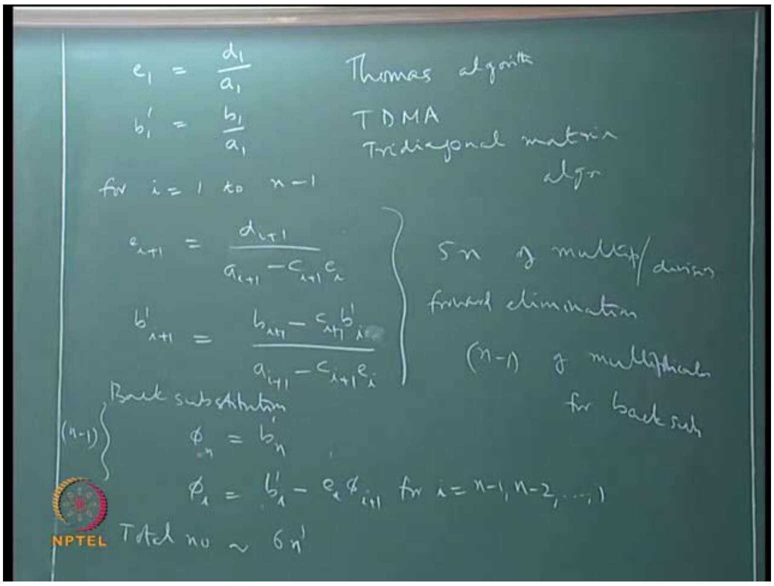

So, this is two multiplications here two multiplications or divisions here, and then, three, four, five. So, there is phi n numbers of multiplications slash divisions for elimination for forward elimination and n minus 1 number of operations of multiplication for back substitution.

So, the total number of operations here is about 6 n number. So, this varies as n raise to the power of 1. So, this algorithm essentially contend to solve c i, a i and d i is given here and this is known as the Thomas algorithm. So, the tri-diagonal matrix algorithm it is also known as TDMA- tri-diagonal matrix algorithm, this is very efficient, because this involves 6 n number of operations and scales linearly with increasing matrix size. But it can be used when you have a matrix of the tri-diagonal form, which is obtained only, for example, elliptic equations in one dimension or these kinds of parabolic equations again in one dimension involving the diffusion involving a diffusion term.

So, the moment you make it like this in two dimensions, you do not have three; you have two more dimensions which will be somewhere here with zeroes here and zeroes here.

So, when you have five diagonals which are separated by some zeroes here, then we cannot make use of the pentadiagonal matrix algorithm. You can do a similar kind of simplification of the Gaussian elimination process for a pentadiagonal matrix, where a matrix is like this; so five adjacent diagonals representing, for example, i minus 1 i minus 2 i plus 1 and i plus 2.

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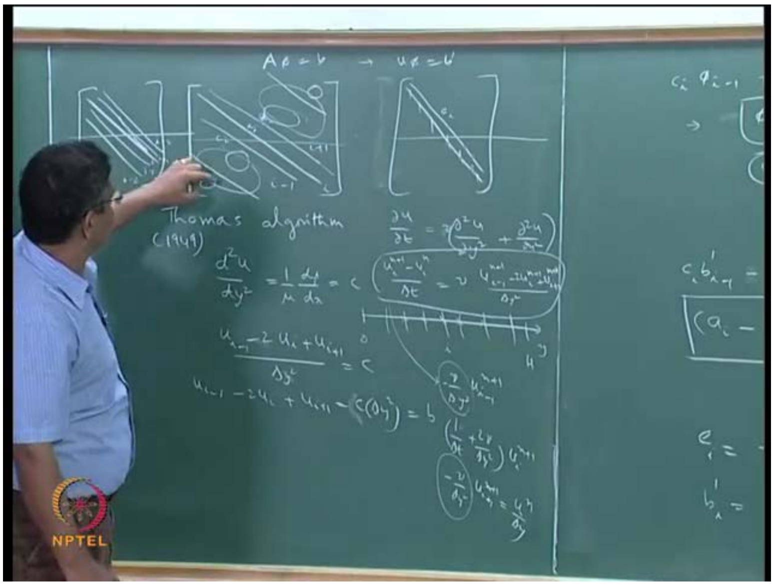

So, five succeeding elements, five successive elements are coming in to the difference equation that is what we have when we have, for example, a term like this a 1 dimensional term which is differenced using a fourth order accurate central differencing for the second derivative will give us five points centrally spaced like this.

So, in such a case we have pentadiagonal matrix and this pentadiagonal matrix method can also result in this kind of explicit method like what we have here which is also very efficient, but we cannot make use of the pentadiagonal matrix algorithm for solving these five diagonals that appear when we do second order differencing for a two-dimensional diffusion equation. So, this diffusion equation will also have five non diagonals, but these diagonals are not adjacent diagonals, they need three adjacent diagonals which are coupled with this and this.

So, for such cases we cannot make use of this. So, although this method the TDMA method is very efficient, it cannot be used for the general case, because for the general case we have more than one dimension, but still it is used as the basis for coming up with an iterative scheme which takes advantage of the fact that we have a 1 dimensional flow a one-dimensional algorithm which is very efficient. So, in such a case, for example, you can do some operative splitting kind of methods or alternating direction methods which are based on the efficacy of TDMA in solving one-dimensional diffusion equation very effectively.

So, as a method in itself for the whole problem it does not come in handy, because we do not have those kind of specialized cases, but as a combination method, this definitely finds its use in the c f d (()).

One last thing about the TDMA scheme is that, we have not made sure in any way that any of this division by error does not occur in this. Round off error is not a problem, but division by error is a possibility, but if we have matrix a here (01:02:00) the tri-diagonal matrix here which is diagonally dominant then we know that there would not be any division by zero like this. So, in such a case we use the Thomas algorithm.

So, Thomas algorithm can be applied blindly to any tri-diagonal problem which has the diagonal dominance as the condition, otherwise we have to be careful in terms of doing this. And especially in c f d problems in which the diffusion problem is diffused using central diffusing in such a case, we do have diagonal dominance as almost as almost as a corollary for the differencing formula.

So, in such a case we can make use of Thomas algorithm. So, with this, we have completed description of some relevant direct methods of use of relevance to the c f d type of problems.

We will then describe some types of basic iterative methods, and then, we will look at what what kind of computation time is required for them, and then, we will look at combinational methods which will give us complete solution. Thank you.