Computational Fluid Dynamics
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Module No. # 04
Solution of Navier-Stokes equation
Lecture No. # 21
Topics

Pressure-correction approach to the solution of NS equations on a staggered grid simple and its family of methods

Before we look at the pressure correction method, which would we say is the recommended method for many steady state flows, because of the robustness that it has demonstrated, and the wide application that it found finds, in many computed codes and calculations, let us look at the idea of a staggered grid.

Now, we know that in the momentum equations, we have a pressure gradient term, which appears in each of the three momentum equations. In a general compressible flow, the pressure is linked to the density; therefore, when the density changes, the pressure changes; you have a linkage that exists in the compressible flow, but in incompressible flow, there is no linkage between the pressure and density.

Density is a given quantity in the Navier-Stokes equation and pressure is a property of the flow itself; it is not a property fluid. And when you consider this form in incompressible flow, pressure becomes a variable that does not appear in its absolute sense; it appears only as a gradient. So, when we evaluate the local derivatives at a particular point, we have to make sure that the velocities velocity components, in the for example, advection term and diffusion term if they are evaluated at a point i j, the corresponding pressure gradient is also evaluated at the same point and this brings us difficulty.

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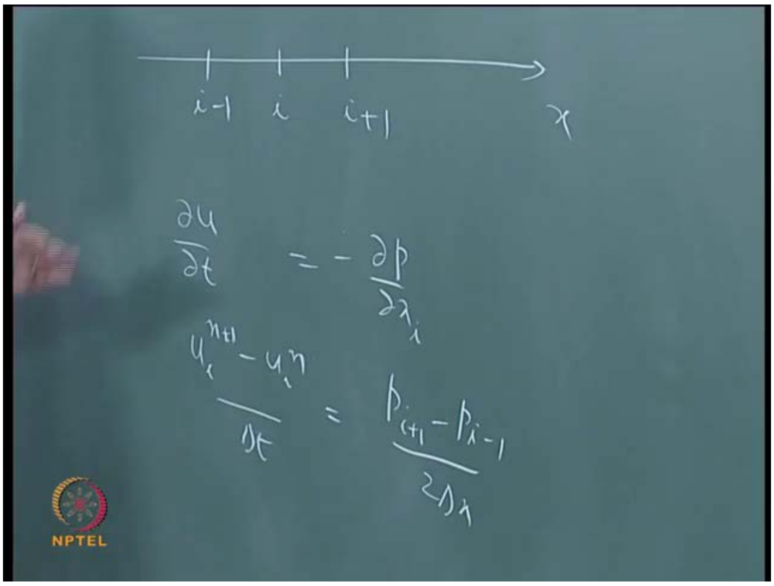

Because in a standard notation, for example, If we consider a one-dimensional grid, where x is changing like this and if you have this is i i plus 1 and i minus 1, if you are talking about dou u by dou t equal to minus dou p by dou x i like this, then we write this as u i n plus 1 minus u i n by delta t and this has to be evaluated at at the point i and this can be represented as p i plus 1 p i minus 1 by 2 delta x.

So, what we there see in this, that the pressure at this particular point is not coming in the equation for the momentum equation at the point i. So, that is the velocity at point i spatial location is not influenced by the pressure at point i and that arises, because the pressure gradient is coming as an absolute variable not the pressure itself.

So, to get around this particular idea… and this has the consequences of what are known as chequerboard oscillations. (Refer Slide Time: 03:30)

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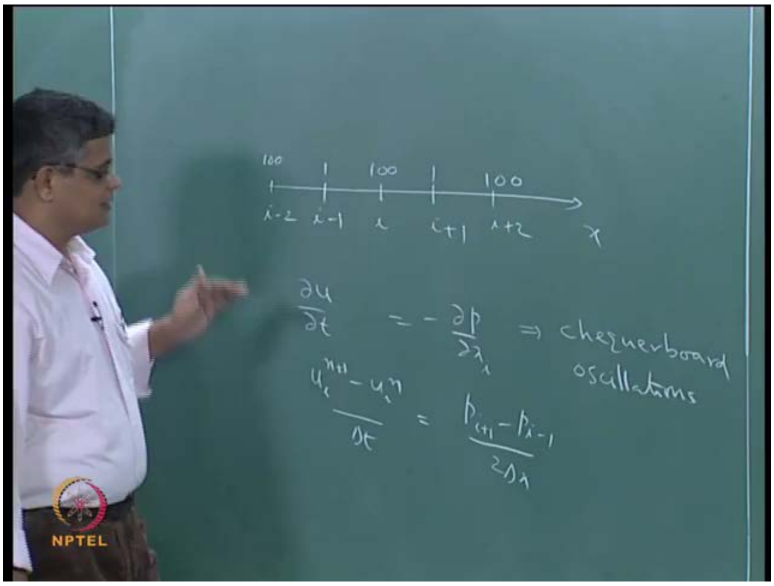

For example, as for this particular formula, if you have a variation of pressure here as 100 and here as like 1, 100, 1, 100 this, when we evaluate this term at this point, the pressure gradient term the right hand side will be 1 minus 1 divided by 2 delta x, which is zero; when we evaluate the velocity this term at this point, again the pressure gradient term will contribute nothing.

So, in in this sense, this is a kind of chequerboard oscillations in one direction. So, because of this possibility because the velocity at point i is not linked to the pressure at point i here; we have the possibility that the pressure gradient in it is fastest oscillation one can say that it is varying like this is not contributing to the velocity variation at that particular point and this kind of thing can happen to get around this this difficulty which is associated in incompressible flow with that fact that pressure appears only as a derivative not as an absolute quantity. We have the idea of pressure and velocity being evaluated at different points at alternate points.

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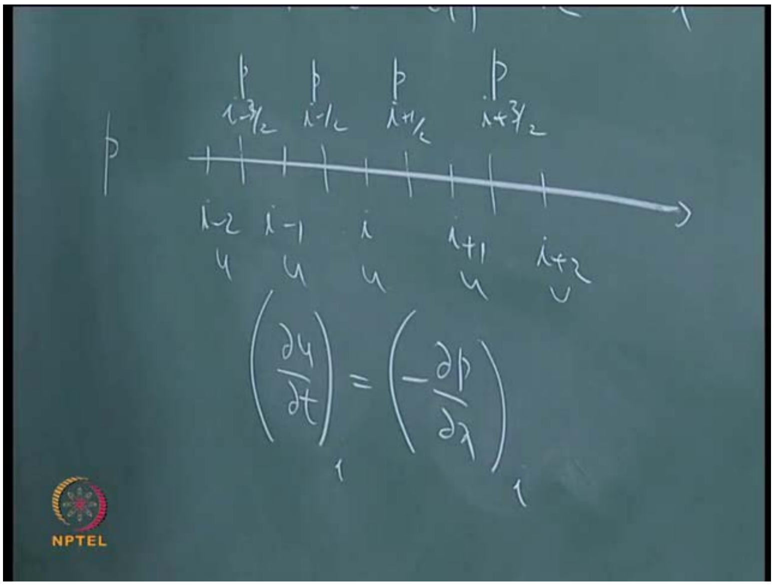

So, if this is where the velocity is evaluated, pressure will be evaluated; if this is i here, pressure will be evaluated in between, this is where we have u is evaluated and pressure is evaluated at these points. So, this is you can say i plus half i minus half i minus 3 by 2 i plus 3 by 2. So, the we evaluate the pressure at these locations therefore, now when we talk about dou u by dou t equal to minus dou p by dou x and this is the simplified form of the momentum equation. So, we do this at point i, because that is where we are evaluating the pressure and this is also evaluated at point i. Now, this becomes u i n plus 1 minus u i n by delta t and now we can evaluate the pressure gradient at point i here by writing it as p i plus half minus p i minus half divided by by delta x.

Now, in this particular case, we are evaluating both the quantities at i, but the pressure gradient is evaluated based on the smallest scale that is possible. Now, if you say that you have 100 here and 1, 100, 1, like this for successive things, when we evaluate the velocity term at this point, this will be 100 minus 1 1 minus 100 by delta x and here it will be 100 minus 1 delta x. You can see that this kind of possibility, that the pressure gradient term is always zero on the chequerboard oscillations is not possible here, because here it will be minus 99 by delta x and here it will be plus 99 by delta x. So, in that sense it will be reacting.

So, there is a contribution of the pressure gradient to the velocity which is coming in when we evaluate the pressure at the smallest distance that is possible. So, this kind of thing will eliminate the chequerboard type of oscillations.

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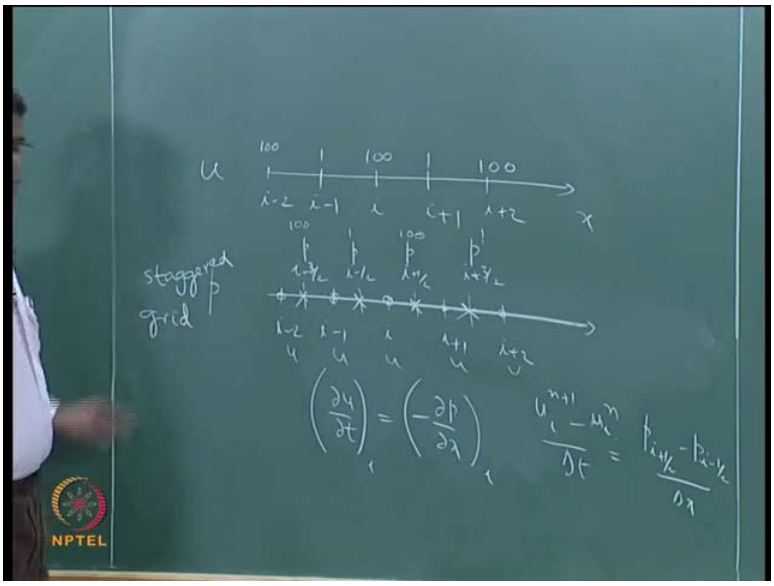

So, this approach, where pressure and velocity are evaluated at alternate points not alternate points at slightly staggered points, if you look at the overall variation of x here, these are the points at which velocity is being evaluated at i, i plus 1 and i plus 2 like this and pressure is evaluated by half a grid point staggered . So, this is called a staggered grid.

The staggered grid enables us to get rid of the chequerboard oscillations. So, we use this approach - the staggered grid approach - to derive the pressure correction equation and then, derive the overall discretization momentum equation in the pressure correction method.

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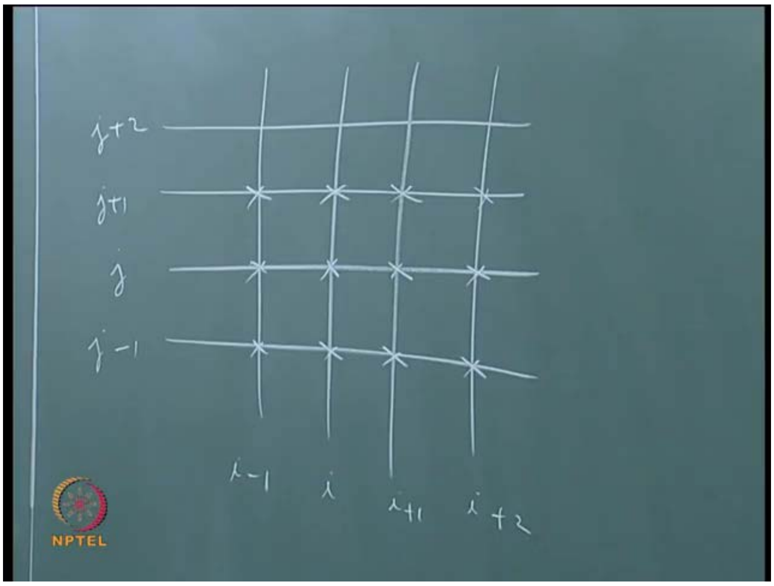

So, let us just see what we are up to. Let us do this in two-dimensions; we have, let us say that these are points i, i plus 1, i plus 2 and i minus 1 and this is j, j plus 1, j plus 2 and j minus 1; we are concentrating on this particular point and let us say that pressure is evaluated at this pressure is evaluated at these points; velocity u velocity is evaluated by with respective to the pressure by staggered by half a grid point.

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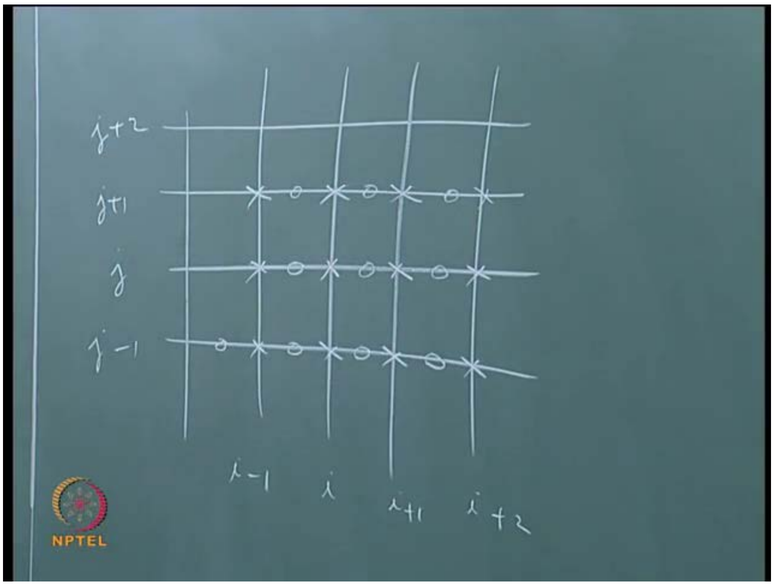

So, u velocity is evaluated here; similarly, v velocity is evaluated staggered in the y direction by half a grid point with respect to the pressure.

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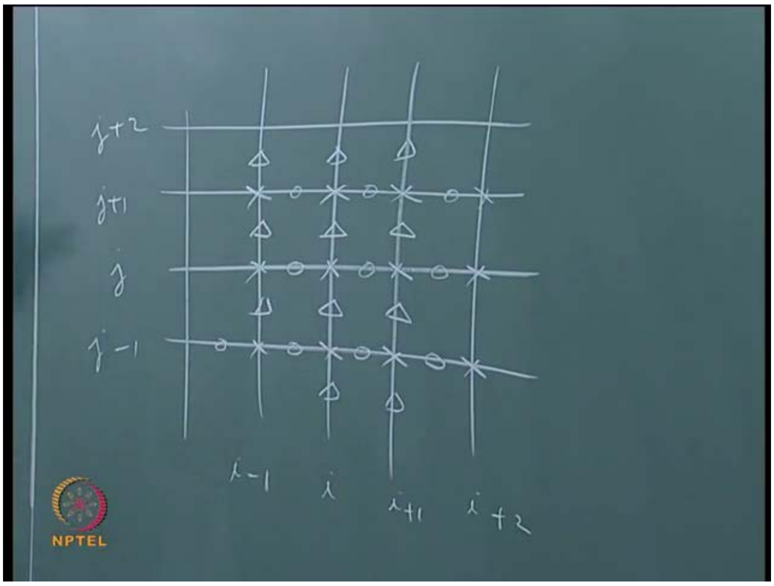

So, v velocity is evaluated at these points which we represent by a triangle. So, this is the kind of staggered grid that we are looking at, where each of the three quantities in this particular case in the two-dimensional case, we have u, v and p these are not evaluated at the same i j; these are evaluated at slightly staggered points with respect to each other. So, velocity with respect to p is is evaluated half a grid point along the x direction along the x equal to y equal to constant line here and the vertical velocity component v is evaluated with respect to staggered with respective to half grid point in the x equal to constant line in this direction. So, the three components are evaluated slightly in a staggered way and in the case of three dimensions, even the w velocity is staggered with respective to half a grid point in the z direction.

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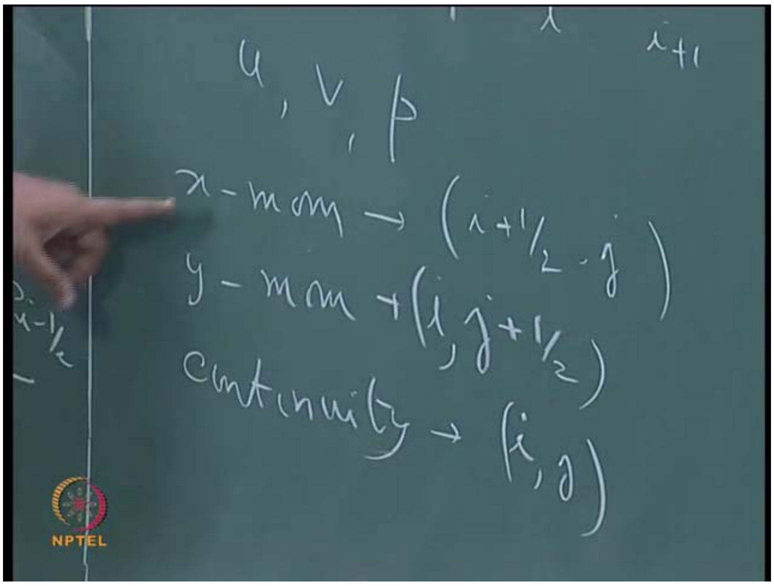

So, that means, that when we discretize the x momentum equation, x momentum equation is discretized at, for example, i plus half and y momentum is discretized at j plus half and the continuity equation which is what is used for deriving the pressure equation or the pressure correction equation is evaluated at i.

So, when we talk about two dimensions, x momentum is discretized for i plus 1 j i plus half, j; y momentum is discretized at i, j i, j plus half and con continuity is evaluated at i, j. We can see that y momentum is displaced in the j direction by half at the same i; similarly, the x momentum is discretized at the same j as the pressure, but discretized at displaced half a grid point in along the i direction. So, this is staggered grid.

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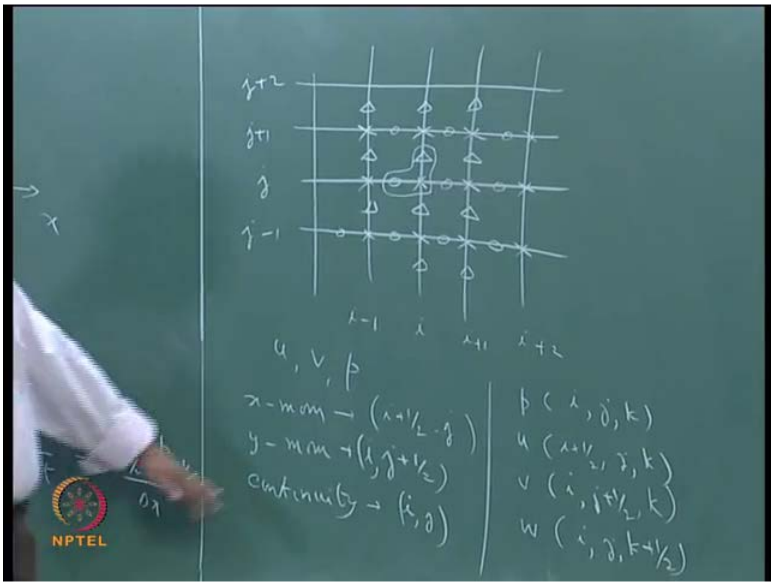

And when we go to three dimensions, we have pressure evaluated at i, j, k; u velocity at i plus half, j, k; v velocity component i, j plus half, k and i, j, k plus half is the point, where w velocity is evaluated. So, this is the kind of staggered grid that is used especially in early days, because in a rectangular kind of domain in a Cartesian mesh, it is as easy to discretize at i, as it is at i plus half.

So, when we are looking at simple grids, it does not matter where you discretize; you can easily discretize evaluate at these points and then, we can run you can find the corresponding finite difference approximations. When we come to more complicated domains, where the grid lines are not linear, they are not straight like this or they are not along the fixed fixed domains for orthogonal points, at that particular case then you may have a complicated mesh for each of these things and it becomes nontrivial thing.

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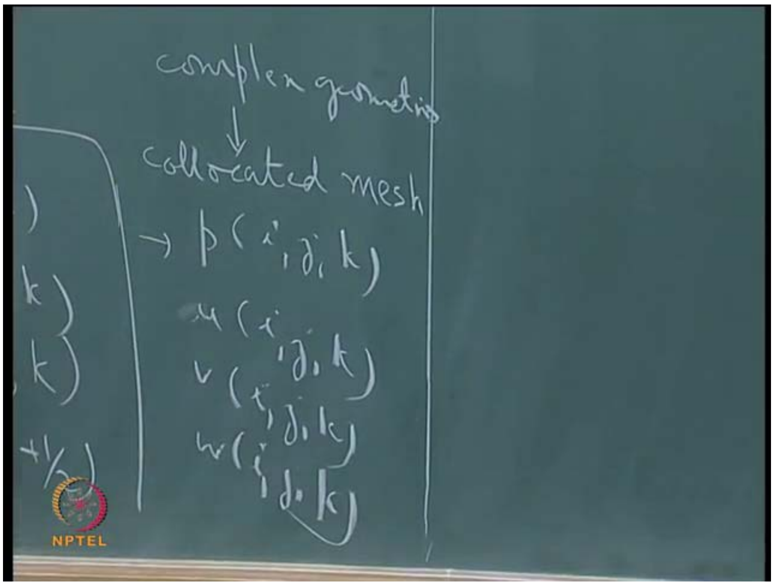

So, at which point we it is when we come to complicated flow geometries, then we go back to pressure and velocity components all being evaluated at the same location v i, j, k and w i, j, k.

(Refer Slide Time: 15:33) So, when we are dealing with simple geometries, we use this; when we are dealing with complicated geometries, then we use this approach. This is called collocated mesh or collocated grid, where all the velocity all the variables are evaluated are located together, whereas this is a staggered mesh.

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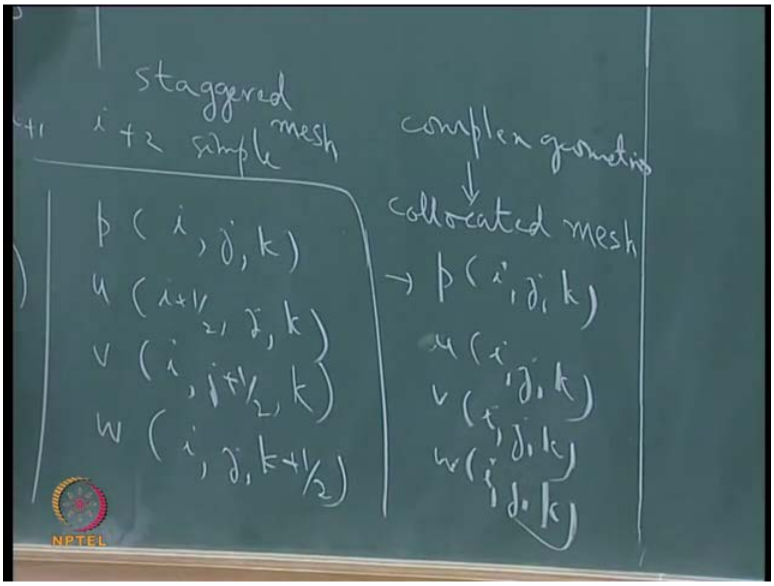

So, you can have a non-collocated mesh and you can have a non-staggered mesh describing this; so you can have all the things. Now, staggered which is useful primarily for simple geometries and this is used primarily for complex geometries complicated geometries, because when you when you are dealing with the complicated geometry, the mesh generation itself becomes a tedious task. So, if you are using something like a staggered mesh, you have to do four times the mesh generation and you have to have each mesh point will have to be evaluated and information regarding the phases and distances all that thing will have to be evaluated and stored for four different meshes and it is much simpler to go for a collocated mesh. But we illustrate the concept of the pressure correction and evaluation starting with the simple geometries; at a later stage when we deal with complex geometries, then we look at the same (( )) for a colla collocated mesh.

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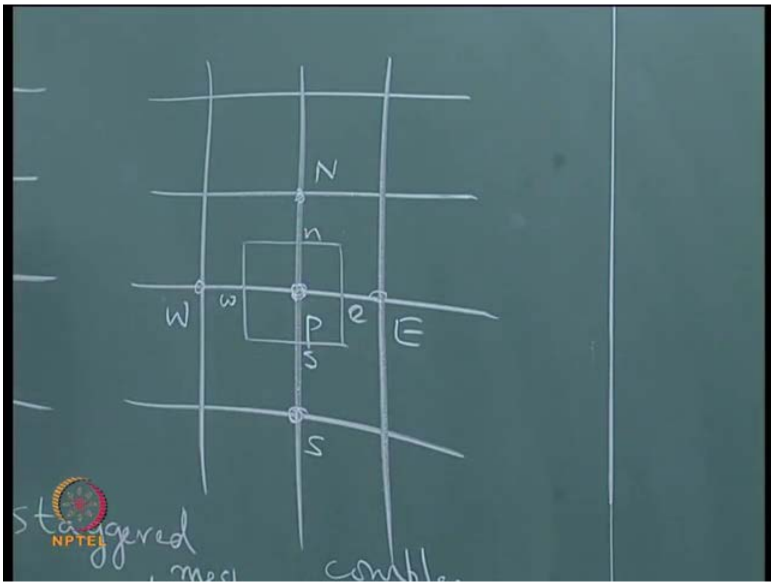

So, right now we are looking at a staggered mesh at which the velocity components and pressure and all those things are evaluated at different points surrounding each pressure. If pressure is the point here, then velocity is here v velocity here and u velocity is here. So, the same thing can be represented in a much simpler way.

In this particular way, if you say that p is the point at which pressures evaluated and if we imagine a control volume, which is extending half grid point to in either side x direction y in positive x and negative x, and negative y and positive y like this, then this control volume has four faces in a in a two dimensions and this is the east face; this is the west face; this is the north face and south face. So, the nearest east point neighbor is represented like this; this is the nearest east and this is the nearest west and this the nearest north and this is the nearest south; when we compare this and then if this is i j, this is i j minus 1, i j plus 1; i plus 1 j; i minus 1 j.

So, with respect to this notation, where the nearest neighbor is associated identified as a capital E, capital W, capital S, capital N, we can also represent the face in the east direction typically with the small e east face with small e and west face with small w. So, when we type out things we must distinguish between the capital W and the small w and similarly small s and capital S and like this.

So, with respective to this notation, which we will now continue use to derive the equations discretized equation at a particular point.

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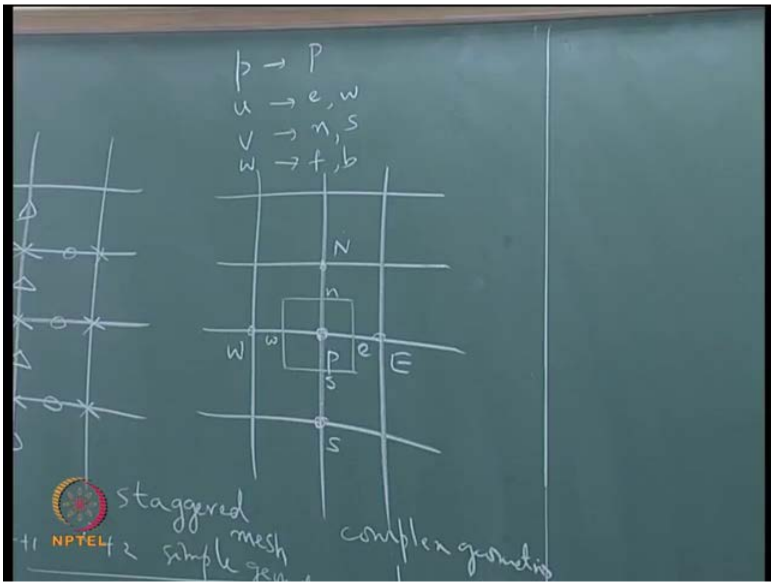

With respect to this, we can say that pressure is evaluated at capital p; u is evaluated on the east face and the west face that is at small e and small w; v is evaluated on the north face small n and the south face small s and if you have w, then this will be evaluated on the front face f and the back face.

So, we can have this kind of notation, where the velocity components are evaluated on the faces of the control volume surrounding the midpoint at which the pressure is evaluated. So, pressure is evaluated at the center and the velocities are evaluated on the corresponding faces displaced by half a grid point.

So, with this, we can we can have a much simpler notation without having to worry about i plus half and j and j plus half and all this. So, with this notation let us now look at what we are dealing with the pressure correction equation, we have written down the momentum equations many times and the continuity equation.

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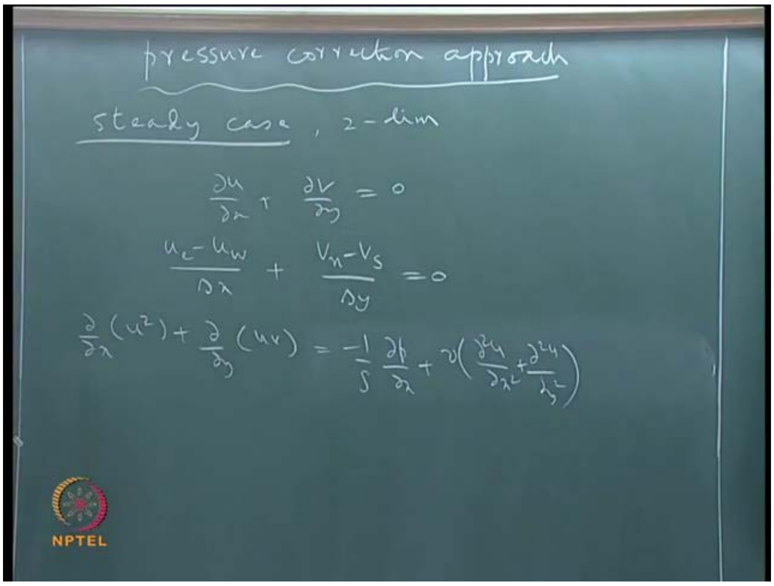

So, let us now rewrite those things, but let us also first illustrate the pressure correction method taking a steady case, for which the continuity equation… and let us also do for two dimensions and extension to three dimensions is straight forward, it would not pose any special difficulties . So, we can have dou u by dou x plus dou v by dou y equal to 0 and if you were to discretize this equation term in this notation around point p, then we can write this as u e minus u west divided by delta x, where we are using small e and small w to indicate the velocity evaluated here and here and similarly, this will be v north minus v south divided by delta y equal to zero, we have a plus.

When we write down the x momentum equation, so we have dou by dou x u square plus dou by dou y of u v plus dou by dou z of u w equal to minus 1 by rho dou p by dou x plus nu dou square u by dou x square plus dou square u by dou y square plus dou square u by dou z square rho we are dealing with two dimensions. So, we can simplify by not considering the z derivatives the x square plus dou square u by dou y square.

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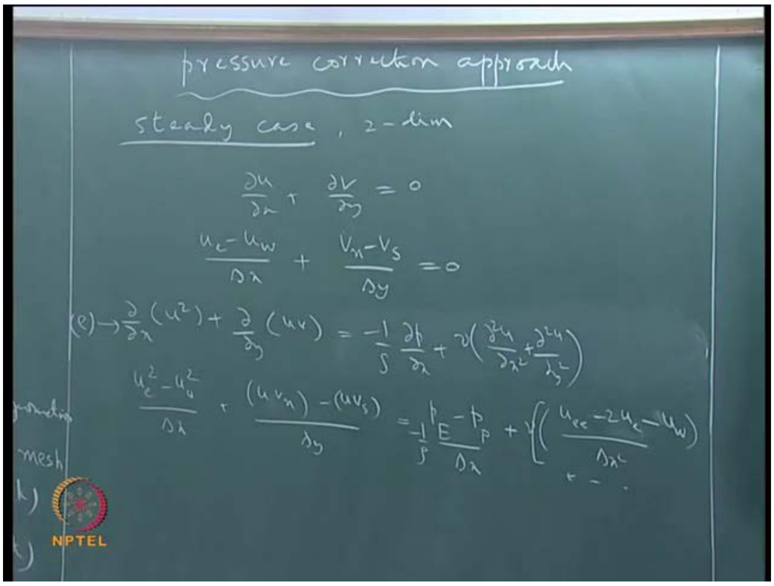

So, we can similarly write down, at this point we can see that there is a difficulty with with this term, because this is a non-linear term. So, we will have to write this as, for example, u e square minus u w square divided by delta x plus u e and here, we have a difficulty what is the we have a difficulty with a this term here; u v n minus u v s divided by delta y equal to p e minus p p divided by delta x. We will have we will have the approximations for this involving we are looking at we are looking at evaluation of this is evaluated at location e.

(Refer Slide Time: 25: 32) So, when we come to the pressure gradient here, the pressure gradient at e is now expressed in terms of pressure at e minus pressure at p that is why we have this divided by delta x and we have minus 1 by rho plus the expression of second derivative of u at this point which will expressed in terms of… this is the east side to the point closest to the east side. So, we can write this as e u e e plus minus 2 u e minus u w by delta x square and so on.

So, let us not worry about the details of this discretization, but the point that we are trying to make especially when we are looking at the evaluation of pressure is that, the in the discretized momentum equation here, the pressure gradient is appearing is being evaluated as the pressure at capital E minus pressure at capital P divided by delta x for velocity in which we are when the when we are evaluating the velocity at eastern face here.

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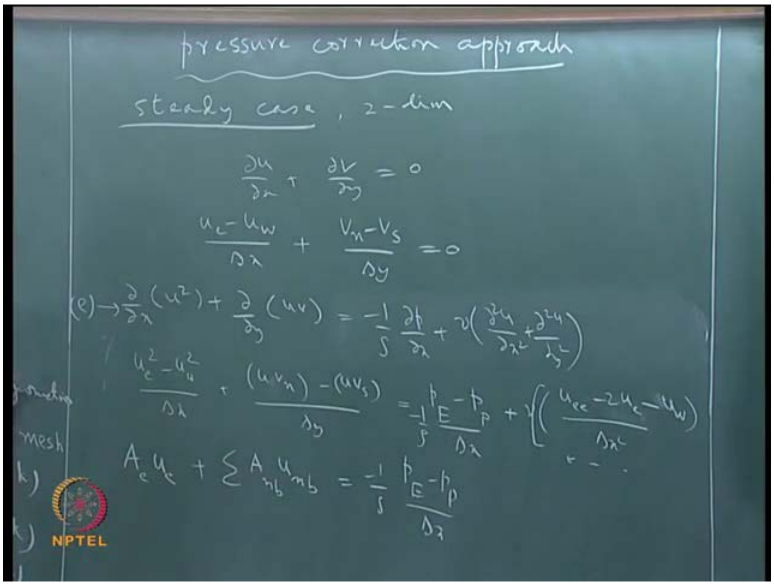

So, we can do some linearization which we can discuss at a later stage and then, we can write this as A e u e and we know that just as we have here this u e expressed terms u w and u eastern neighbor and all these things, we can we can say that the velocity at this particular point will have the four neighbors which will come into picture.

So, the sum of A n b u n b, so the contribution of all the neighbors and this will be equal to the pressure gradient term which is minus half minus 1 by rho p e minus p p by delta x.

So, we can write down the discretized form of the momentum equation for the velocity at e in this particular form, where coefficient of a e, for example, coming from this term will be minus 2 by delta x square and we will have minus by 2 by delta y square coming from this and we will have contribution coming from each of these things and it depends on how we do the linearization and all that.

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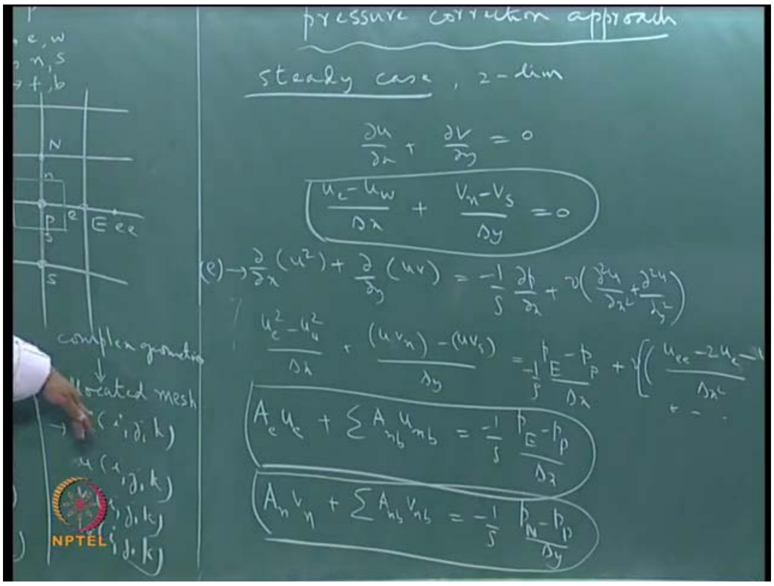

But one can say that the discretized form of the x momentum equation at point small e, where point small e is to the eastern face side of point p, where pressure is being evaluated will be like this and similarly, the discretized form of the y momentum equation will have A n v n plus A n b v n b; n b here refers to the neighbors will be expressed in terms of 1 by rho.

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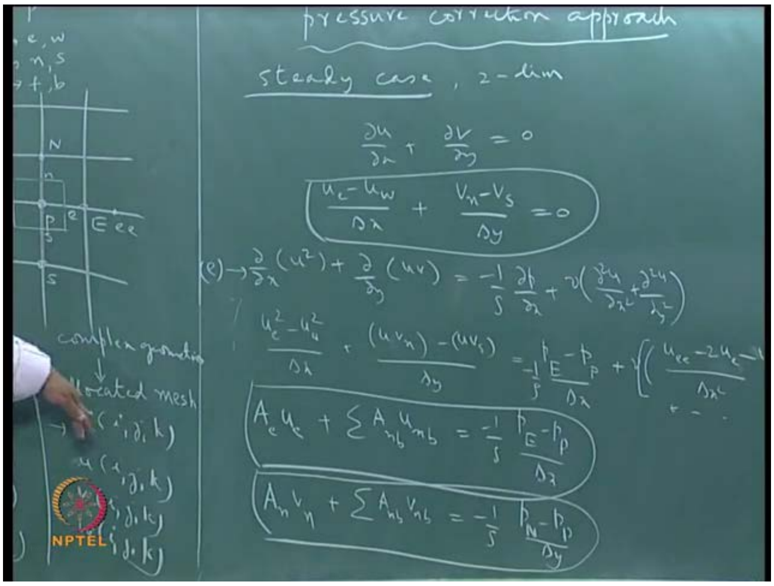

We are talking about velocity evaluation here; the pressure gradient at this particular point will be evaluated as p n minus p p divided by delta y minus p north minus p p divided by delta y; we are concentrating mainly on the evaluation of pressure in this method. So, that is why we are using this short notation here.

So, now, we have this discretized form of the x momentum equation, discretized form of the y momentum equation and discretized form of the continuity equation and these are the three equations which we need to solve in order to get u. (Refer Slide Time: 30:21) So, this will give us u e; this will give us v n and this should give us p.

So, the three variables u, v, p are evaluated at is slightly neighbor staggered points and p at point p is the pressure at point p is the unknown variable, then the correspondingly we have u at small e is the corresponding unknown variable and v at small n is unknown variable as of course, for this these are also these unknowns, but they will also will be represented associated with their own pressure gradient pressure evaluation point like this.

So, the three components here, the three unknowns u e and v and p are evaluated from these three discretized equations. Now, the question is how we can solve this. Because the solution of this will involve the neighboring velocities, so we need to know this; we need to know velocity which is v velocity which is coming in in this equation here and we also need to know the pressure distribution here; the pressure on the along the x axis and pressure variation along the the y axis is there and in this equation as usual as you we are familiar with, we do not have pressure coming in this; we want to solve this for pressure.

So, the approach that we adopt here is that let us assume pressure, let us assume pressure at every point. So, let us assume pressure at capital P, at capital E and capital north and capital south whatever it is.

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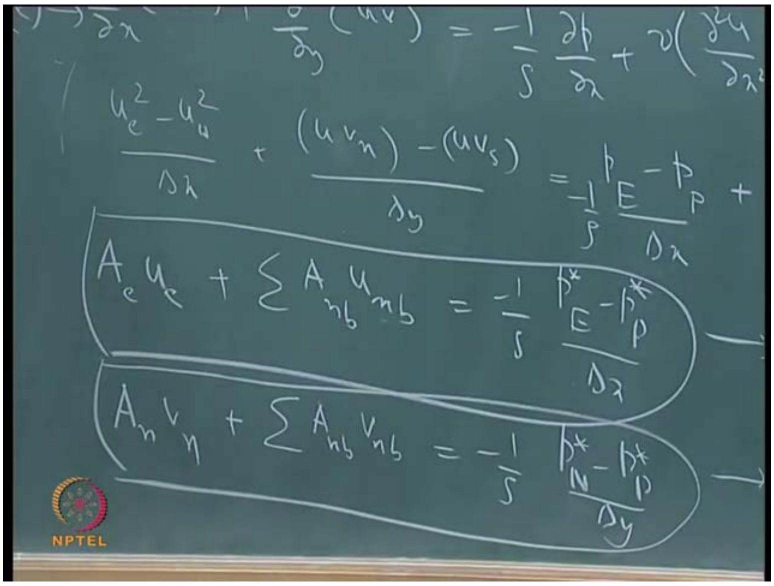

So, we assume a pressure which is p star throughout the domain. So, at all i, j, at all intersections, wherever pressure is going to be evaluated. Now, once this is known, then this equation can be solved.

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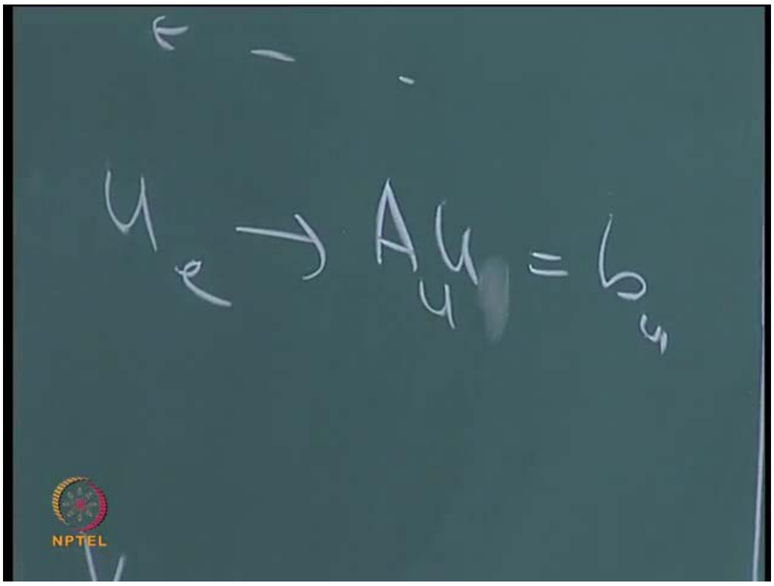

So, this equation once the right hand side is known, this can be written as some A u e equal to b, where each of this is a is a matrix.

So, when we write this kind of discretization for all the points at which the u velocity has to be determined, we have a matrix equation and by solving this, we can get the velocity at all the points. And similarly, this also becomes converted into another matrix form a v v equal to b v, where this subscript b indicates that this is these are a matrixes for the y momentum equation and this as of now cannot be converted into a p equal to b, because the pressure is not coming as a variable here.

So, if this is known, then we can convert this in to this and because we making use of a guest pressure field, we will put this as b star; now once we evaluate this, we get only a velocity which is based on the guest pressure field.

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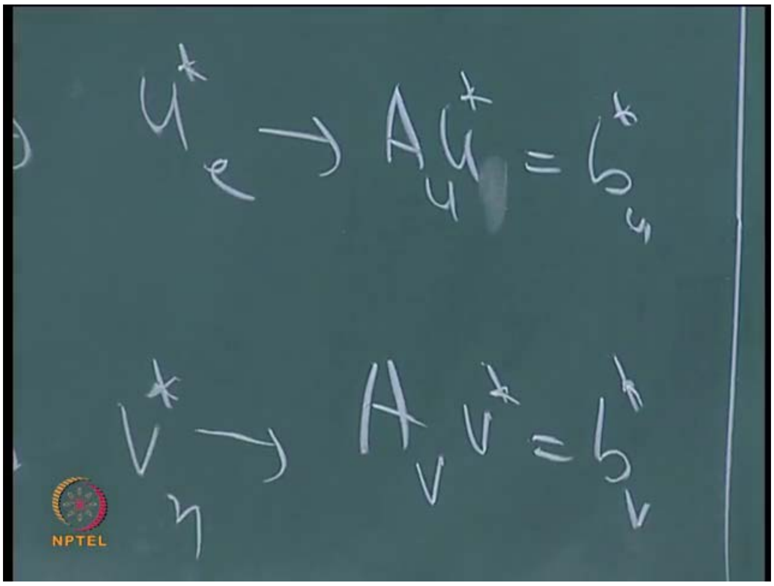

So, we will call this as u star. So, we get an estimated velocity v u here and also v star which is based on the guest pressure field. Now, we want if the pressure field is correct, then the corresponding velocity field that we getting that is u and v at different locations will be such that this would satisfy the continuity equation.

So, it will satisfy the continuity equation. So, if this were correct, then this would be exact this would be exact this would be exactly equal to zero, but it is not necessarily so, because we have gotten it with. So, this is in general this is not equal to 0.

So, and that is where lies the problem that, if we make any pressure field here and using that guest pressure field we get velocity field; the corresponding evaluated velocity fields do not necessarily satisfy this. So, they are not an exact equation of the for Navier stroke equations exact solution of the Navier strokes equations. So, this is a problem that we have and this also presents an opportunity, because given that these do not satisfy this, now we can come up with a correction to the pressure in such a way that this is satisfied.

So, we say that this guest pressure field is is giving us this result; the guest pressure field is giving us these estimated velocity fields which do not satisfy this. So, I want to make a correction to this pressure correction which are represent by p prime, such that this will be equal this will be equal to zero, such such that this will be equal to 0. And I know that if I change my pressure here, if I introduce a pressure correction here, then the velocities will also change; if I change this to p p star plus p prime, then this will also change; when this change, the velocity will change and when this changes, the v velocity will change. So, the moment I change my pressure my correct pressure, this will also give me a u which is now u star plus u prime and a v which is equal to v star plus v prime.

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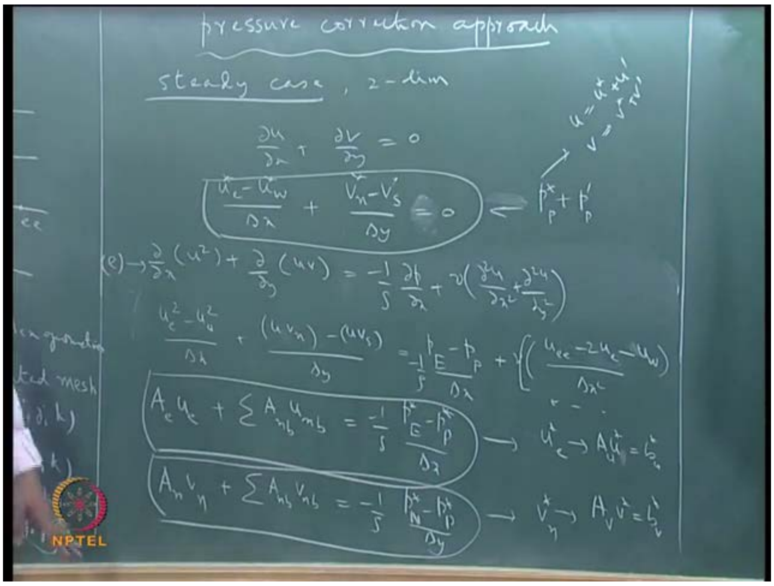

So, what I am trying to say is that, when if I want to change my pressure, if I want to correct my pressure by a small quantity p prime, then that will induce changes in the velocity in the estimated u velocity and the estimated v velocity from these things. So, and I can make a rough approximation, for example, I can say that I am after all this is the equation which is representing the velocity variation and the linkage with the pressure here and these terms represent the contribution from the convective term and the diffusive term in to some extent the in to they in incorporate some of the contribution from those things and if I say that to a first degree of approximation, the corrected pressure will not have significant component from this; I can use this equation here to get an estimate of the velocity correction that is introduced by the pressure correction in this and similarly, I can use this to get an estimate for a corrected velocity velocity correction here in the y direction from this.

So, and those velocity corrections u prime v prime when added to this will give us will me will give me my new velocity components u new and v new such that when they are substituted into this, I will be satisfied.

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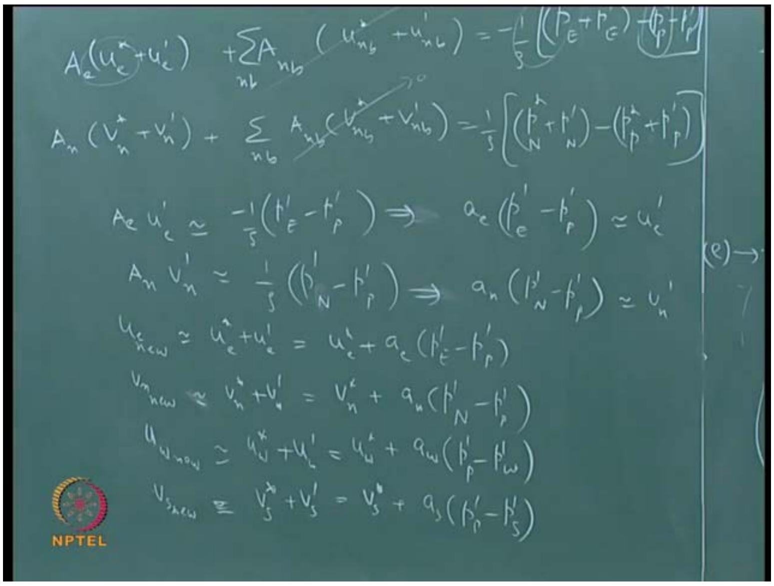

How to get the new velocity u new and v new? Once we have made the pressure correction if we knew that, then we would have a method by which we can find out the pressure correction and then we can get we can get the solution. Now, when we change the pressure we have said that this changes. So, this must be the basis for evaluating the u velocities here. So, we can do that here. So, let us say that we are now changing p star to p star plus p prime. So, when we evaluate, wherever we have p start for example, at e we change from p star p 1 prime at e. So, there is a pressure correction which is evaluated at each point e n p and all that.

So, we again similarly we have we are going from p star to p 2 p star p plus p prime p. So, when we substitute that into to this equation, now we are saying that the coefficient here is unchanged u e star plus u e prime that is the velocity correction is such that this will be satisfying this equation with the new pressure here. So, that is what we are writing plus A n b u n b star, so that is the new old value of a neighboring point velocity plus the correction in the neighboring velocity that is coming summed over all the neighboring points is equal to minus 1 by rho p e star plus p prime e minus p star p plus p prime.

So, once we have changed the pressure by a small correction p prime which may be different at different points, because just as p is different, now pressure correction will also be different. So, the corrected pressure field here will give us a corrected velocity field which is given by this and this u n b here will not in include u points, but also the v points. And similarly, we can say that from the y momentum of equation from the discretized y momentum equation we can say that we have a corre new velocity filed which is v n star plus the correction at n to this plus sum over the neighboring points of a and b, v and b star plus v and b prime equal to minus 1 by rho p north star plus the corrected pressure at that point minus p p star plus the pressure correction at this point.

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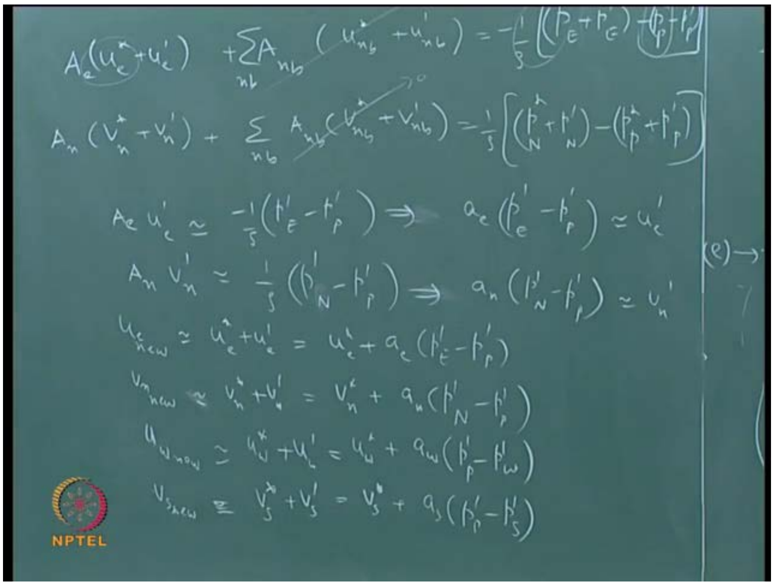

So, we have to if you are able to solve this, then for a given pressure correction we can get the velocity corrections and we can substitute them into this. The difficulty is of of course, how to do solve this, because we notice that this velocity neighboring velocity points will be will mean this becomes the fully implicit solution. So, one assumption that is made in the very first proposal of this pressure correction is to completely neglect this and completely neglect this. So, we have an estimate for the velocity correction.

So, we have A e prime and we notice that this particular part is cancelled out by this particular part from from this equation, because your u star such that that satisfies this relation here. So, from this we can say that A u e prime is equal to minus 1 by rho p prime e minus p prime p it is not equal to, it is only roughly equal to and similarly, from this once we neglect this, we can say that a n v n prime is roughly equal to minus 1 by rho e prime e minus p prime p p prime north minus p prime p.

So, we have this kind of thing and we can subsume this we can take it on to this side and we can say that this is equal to some let us say a p capital P prime capital E minus p prime capital P and this is a n p prime capital N minus p prime capital P. So, this is equal to

So, from this… So, we have just rearranged the coefficients here such that u e prime the estimated velocity correction for a given pressure correction is given by this and the estimated velocity correction in the v y direction is given again by the estimated pressure correction coming from this like this and in which this these coefficient are known from the discretization. So, in that from that point of view, there is there is no further approximation done from this stage to this stage; there is approximation done from this stage to this stage, because we are neglecting this.

So, now, knowing this, we can now say that u e new is roughly equal to u star e u e new u star e plus u prime e which is equal to u star e plus a e the known coefficient times p prime e minus p prime p and similarly, v n new is v n star, which we have got by solving this equation therefore, this is known plus v n prime this is roughly equal to. So, this equal to v n star plus a n times p prime north minus p prime p. Now, what we are saying is that these new velocity fields must be such that, they they will be satisfying this.

So, just as we have evaluated u e and v n here, we can also evaluate u west new as being roughly equal to u west old. Now, how do we get to this u west old? The simultaneous solution of all these equations or the matrix equation here, the solution of this will give us u east, u west similarly, v north and v south for all the points essentially the velocity at all those points, where it is evaluated it is known.

So, this is known and… So, u west prime and this u west prime will be given by a west some coefficient which is known times p prime p minus p prime w, where w is the capital w and similarly, v south new is roughly given by v south star plus v south prime that is v south star, which we would have gotten by solving an equation like this equation with the guest pressure field plus a south times p prime p minus p prime all this.

So, the solution method is such that we start with a guest pressure field, we write down these equations for each point at which u velocity is evaluated, each point at which v velocity is evaluated; when we put this whole thing together, we get a matrix equation for u star, where u star is the velocity field obtained with the guest pressure field when we solve this, we get u velocity at all the points at which it has to be evaluated.

Similarly, when we solve this matrix equation, we get v velocity all the points, where it is evaluated. Now, when we come to the continuity equation we can evaluate this making use of this starred velocity fields, so which is not going to be zero. So, therefore, we want to introduce the pressure correction at every point and this pressure correction will give us a velocity correction of both u and v at every point and we have estimated them here like this, using this.

So, for a given pressure correction at e and p, you have the velocity correction given by this. So, these can be calculated if we knew these things; right now we do not know what are this things are, this is still unknown, but we are we have got a approximate relation for this also for this and for all the velocities. Now, what we are saying is that the pressure corrections at all these points should be such that when we substitute these velocities in the discretized continuity equation, we should be getting the continuity equation to be satisfied.

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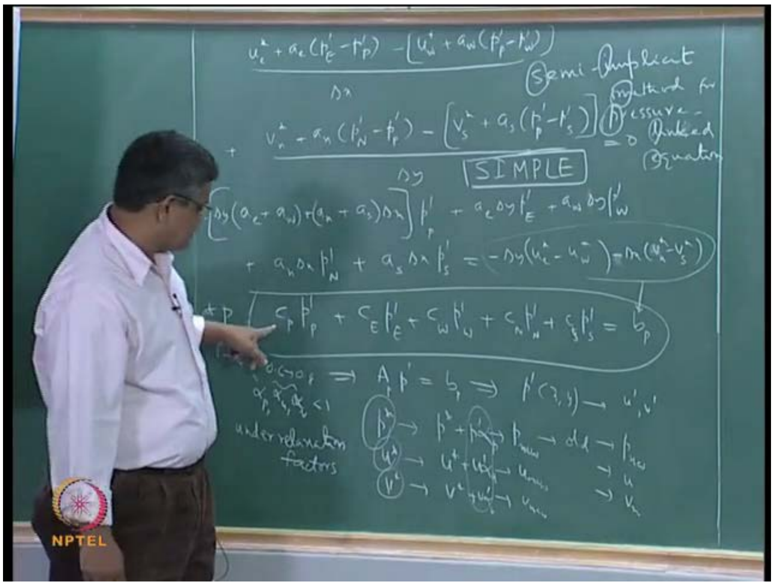

So, let us substitute this in the discretized equation. So, we get u e star plus u prime which is a e times e prime e minus p prime p. So, that is this 1 minus u w new that is u w star plus a w p prime p minus p prime west divided by delta x that is this part, except that instead of star values we have now the new values and where in the new values we have substitute it for the estimated velocity correction in terms of the estimated pressure corrections plus v n new. So, that is v n star plus a north times p prime n minus p prime p minus v south star, so that v south u, so that is v star plus a s p prime p minus p prime s by delta y is equal to 0. And we can see that in this, this is already known these are known and what is not known are these things and we can write this whole thing in terms of now all this things are not known.

So, we can write this has from this here a e plus a w plus and this is with minus plus a w and then, we have plus a n plus a south, let us say times delta y delta x this whole thing minus p prime p is what we get by simplification and we are essentially multiplying both this by delta y delta x. So, that this cancels out; this gets multiplied by delta y and this gets multiplied by delta x and we can also see here, plus a e delta y p prime e plus a w a w delta y p prime w plus a n north delta x p prime north plus a south delta x p prime south is equal to all these left hand side these starred quantities.

So, that is delta y times u e star minus u w star plus delta x times v north star minus v south star, the maybe plus or minus change here; so this is actually minus and minus. So, this is an equation here in which all the delta y delta x and all the small a's are known and u star and all these things known.

So, we can rewrite this expression as, for example, c w p prime w plus c north p north plus c south p prime south is equal to b b p something like this, where this b p is obviously, what is there on on this side. So, this whole thing b p and c p is the corresponding coefficient here and c is the coefficient for this like this. So, each of these coefficients here can be evaluated and the right hand side can be evaluated.

So, this is for the continuity eq equation applied at point p. So, that is corresponding to i j and so there are several such points for every point at which we want to evaluate the pressure; we apply we derive a similar equation. So, when we do that, we will have several equations and we can see that the pressure correction at point p is now expressed in terms of this own thing plus the pressure corrections at the neighboring points.

So, just as here we have the velocity at small e is expressed in terms of its own thing plus the neighboring points here, we have an equation like this. For a particular point when put together all these things, then we can put this in a matrix form equal to b. So, where a p is matrix and b p is a matrix corresponding to the equations that appear in the pressure correction equation. So, this is a matrix equation which can be solved using several methods this will give us p prime.

So, p prime essentially at x and y now with this p prime we can evaluate u prime and v prime. So, once we have p prime, using that approximations we can roughly get u prime and v prime. So, what do we have? We have corrected we started with pressure and then, we have improved upon it by adding pressure correction subject to this pressure guest pressure field, we have u star and we have improved upon this by adding u prime and similarly v star is improved upon by adding the v prime.

So, we have from a guest pressure field we have an estimated velocity field and from these things, we evaluate the pressure correction using the condition that the corrected pressure will induce a corrected velocity filed which will now satisfy the continuity equation.

So, using that using that condition, we have derived a new estimates; this is new pressure new velocity in each direction. So, these are only approximate, because we have made the assumption that the velocity correction is given only by this and the contribution of these things does not come in to picture.

So, this is not an exact solution. So, using these new values, we go back to the momentum equations and then, we get new estimates for the velocity field such that the new velocity field will satisfy the discretized momentum equations from these and once we get this new velocity field, then we come back to the pressure correction which involves these things. So, from when when we substitute the new velocity field into the continuity equation, again this does not satisfy completely.

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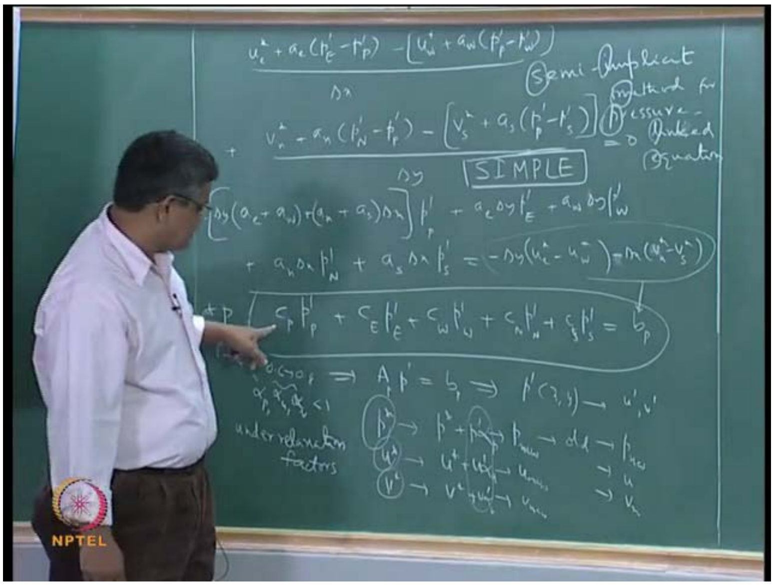

So, we have to introduce another pressure correction. So, we come back here, then we evaluate all the coefficients that appear in in the pressure correction equation and then we assemble them here and then, we solve for a new pressure correction and from the new pressure correction, we get new velocity corrections and then we get we generate another new.

So, this becomes old and from this, we get another new velocity fields and then, we go through the cycle several times and if we are lucky and most of the time we are lucky especially when we use some under relaxation, we get velocity field which is convergent.

So, successive corrections to the previous pressure correction, velocity correction will be small and smaller and eventually we get to a stage where the corrections are so small that we can say okay we have got a converged velocity field. So, in at that point, that point the pressure that we have at that point will be satisfying the discretized momentum, the velocity will be satisfying the discretized momentum equations and the obtained velocity field will also be satisfying the continuity equation.

So, this whole series of methods, series of steps will enable as to go from a guest pressure field to the estimated velocity field and from the estimated velocity field using a pressure correction equation, we get the pressure correction and from these, we get estimated pressure velocity corrections and with these we update and then… So, using this pressure correction velocity correction, we now get a new pressure end velocity fields and starting with this, we again go back to the discretized momentum equations and get new velocity fields and then from that, we get new value of pressure correction, then we go through this in in successive sequences.

So, when we do that, we ultimately reach a converged condition, where we have a pressure and velocity field, which satisfy all the three equations that we tried to solve. So, that will be the ultimate equation. So, we can this method resembles in a way the implicit pressure equation method, except the fact that in two respects it is different; we are not solving directly for pressure; we are solving for a pressure correction, but the pressure correction is still such that we are trying to satisfy the continuity equation. So, in that sense, it is it resembles that and it is also not fully implicit, because we are only making it partially implicit through this approximation.

So, that is why this is this is called a semi implicit method and the whole procedure as given by proposed by (()) is called semi implicit method for pressure linked equations.

So, if we take the first letter of each of these things, we get S I M P L E. So, the acronym is the simple method or a simple scheme for the solution of the momentum equations and the continuity equations together essentially for incompressible flows.

So, this is the method which is specifically targeted to extract the pressure field from the continuity equation with the condition that the pressure is used to satisfy the continuity equation.

So, in that sense, this is this is a method totally focused towards solving the incompressible form of the Navier-Stokes equations and because of the implicit nature semi implicit nature, this has a better possibility to convergent than a purely explicit method and because it is fully implicit, it is not fully implicit then it does not also converge for all cases and… So, we what we use is, we use under relaxation factors the basic idea is that with starting with the pressure estimated pressure and estimated u star and v star we evaluate corrections to the pressure and velocity and then we add the corrections to the original values to get a new field.

So, when we do the under relaxation, we do not add the entire amount of the correction; we use only a fraction of the correction that fraction being denoted by alpha and that for each of these corrections we can have a fraction. So, alpha p here and alpha u here and alpha v here, where alpha u is the fraction of the correction that we are adding and the fraction of this that we are adding here and typically, alpha p, alpha u and alpha alpha v must be less than 1 for them to be called as under relaxation factors and typically these are taken to be something like 0.6 to 0.8 and alpha p is essentially 1 minus alpha u. So, if this is 0.6; this will be 0.4; this is 0.8 and this is 0.2 like this; these values have shown to give very good results for large number of cases.

So, in using the simple method, we start with the guest pressure field; we discretized the momentum equations and all the x momentum equations are put in the matrix form a u equal to b, where b contains the information of the guest pressure field and we can solve this to get an estimated u velocity field and then, we solve the we discretized the y momentum equation and then with the guest pressure field we can put that into a matrix equation and get an estimated velocity y velocity and using these estimated velocity fields, we evaluate the pressure correction the components of the pressure correction the coefficients of the pressure correction using this this approach and this pressure correction is again put in the in a matrix form like this and the coefficients of these things are evaluated a p and b p and we solve this to get the pressure correction at every point and we using these pressure corrections and using the simplified momentum equations like this, we get the velocity corrections also. And we use an under relaxation factor for each of the terms to get the new velocity new pressure field and new velocity field like this and we use these new velocity fields to linearize some other terms that coming here and then we go back to the drawing board.

We can go back to the original step here and with now the new velocity new pressure field as the guest pressure field and using the new velocity fields to non linearize and do this kind of thing we again get u velocity v velocity and from that we get new pressure correction and then, from that we get new velocity corrections; we under relax them a bit and then we we get a new set of pressure velocity. So, in that sense, we go through a cycle of pressure evaluation forward by velocity estimations pressure correction estimation, velocity correction estimation, under relaxation finally, get new estimates of pressure velocity.

So, v this takes us from one step of velocity velocity and pressure to another step of velocity and pressure and we will go through many such steps in order to get the final solution and all this is done for steady equations.

We are looking at a steady equation; if it is a unsteady equation, at every time step we have to go through this procedure. So, that becomes complicated, but this has proved to be robust method and in this form, it can be used for three dimensional methods although the we have derived it for two dimensions and we straightaway extrapolated it to three dimensions and although we have derived for steady state things, we can also do it for unsteady things.

So, this is the a generic method it can be used for wide range of incompressible flows in under transient conditions, under three dimensional conditions and the same principle can also be extended for turbine flow calculations and other calculations.

So, this is what we can use for this. In the next lecture, we will we will make a flow chat of this to outline the method and then, we will also touch up on some improvements which have been made to make the iterative process of going from old values to new values and then, new values like that, that iterative method will converge, essentially by relaxing some other assumptions made in estimating a velocity correction from a pressure correction. So, by doing this we can we can get an improved convergence and that improved convergence rate has made has formed a series of variations of the simple scheme.

So, we will go through that and that will give us a generic method for the solution for the coupled solution of all the equations; at that stage we will have completed almost the possibility of a solving the Navier-Stokes equations or the equations which covered in the fluid flow using numerical methods. The only thing that will be left is how to solve a matrix equation like what we have here.

We can see that if we are using the simple method for the solution of momentum of Navier stoke equations, we have to solve one matrix equation here, another matrix equation here and another matrix equation here and we have to do this three times only to go from old velocity of pressure and velocity old estimates of pressure to new estimates and then we have to do it again and again and again many times.

So, in every step we have to do at least three solutions of these equations. So, and we have to do many steps many such iterations. So, we must have a really robust way of solving these matrix equations. Once we describe such methods, then we will be armed with a set of techniques which we will be able to which will able to use efficiently for the solution of the fluid flow equations.

So, that is what we are going to do it the next few lectures. (01:10:43).