Computational Fluid Dynamics
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Module No # 04
Solution of Navier-Stokes equations (Lectures 16 to 21)
Lecture No. # 17
Topic
Illustration of application of the template using the MacCormack scheme for a three-dimensional compressible flow

We will first look at the MacCormack method for the simple case of linear wave equation first, and then, we will add the diffusion term, which will make it burgers equation, and once we understand how to discretize according to the MacCormack method, - the advection term and diffusion term - then we will go back to the, the, vector form of the conservation equation and we will apply this method to those.

So, when we consider linear wave equation, we have dou phi by dou t plus c dou phi dou x equal to 0 in one-dimensional form, and we can, according to the MacCormack method, we can go from n to n plus 1 in two steps and this is a strategy which has been used by many people to go from step n to n plus 1 in two steps wise to increase the accuracy of time difference.

We have seen that normally we have used forward incline for many of the dicretization and that is only first-order accurate. If you want have times accurate, time accurate solution of the velocity field and pressure field, we need to have second-order accuracy at least, and if we therefore use a second order center in times, then it is a not self starting method apart from stability considerations.

So, the two step methods where you have predicted step and corrected step will give us an algorithm which is self starting, which can be started from n equal to 0 or times equal to 0 with a initial boundary conditional specified to time equal to any time t of our choice. And we also make use of the predictor and corrector series of calculations to make it second-order accurate in time. There are many methods and MacCormack method one of these things, these methods.

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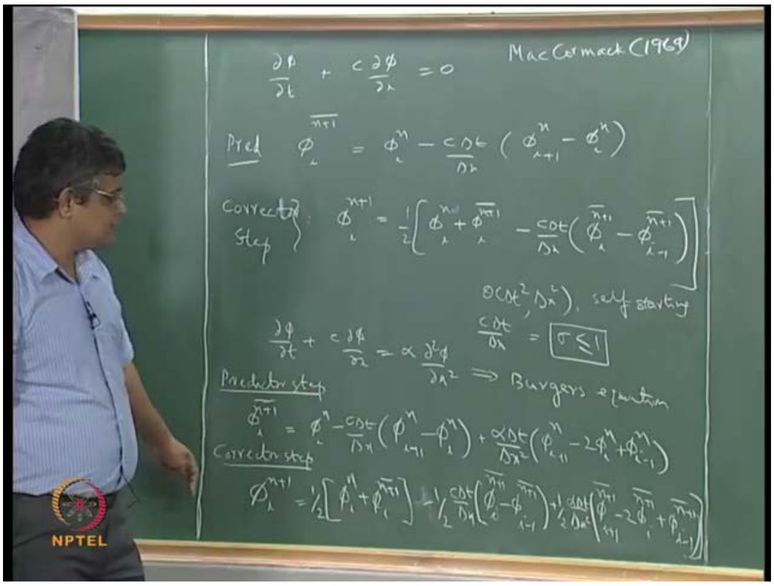

And in this, the predictor step involves forward in time for this and forward in space for this. So, we know that forward in space and forward in times are f t f s scheme is unstable, but using that as a predictor step, and then, a corrector step which cures the instability associated with the f s, forward in times is that total algorithm that we having this.

So, we have the MacCormack method proposed in 1969 as in intermediate evaluation of the value of at phi at n plus 1 bar step, which is given as phi i n minus c delta t by delta x. We have taken this to other side and delta t, and this is based on forward differencing of the space; this is called predictor step.

The corrector step evaluates phi i n plus 1. Now, we actually get the value of phi and n plus 1; this 1 intimate value. Takes this 1 to be average of the two; so, we have half of phi i n n plus phi i n plus 1 bar and we have minus c delta t by delta x, which was, this is in the predictor step, it is evaluated is forward in times. Now, we will make it backward in time phi i minus phi i minus 1 and this is evaluated now at n plus 1 bar.

So, you go from phi i n to phi n plus 1 in two steps. In the predictor step, you make use of this explicit f t f s scheme to evaluate phi i values at the intermediate thing for all i, and having known the phi i at n plus 1 for all i, we come to the corrector step, in which, we evaluate phi n plus 1 as the original value plus the estimated predicted value here, and a backward in space discretization or the space derivative making use of the predictor values of phi i at that intimated step.

So, the overall scheme here is of second-order accurate in time and second-order accurate in space and its self starting in the sense that, if we know initial conditions, then you can start this and then go on to this and it is conditionally stable for sigma less than or equal to 1.

So, it is not unconditionally stable, it is conditionally stable with sigma equal sigma - the courant number, which is c delta t by delta x being less than 1. So, this is one method which has proved to the popular. Now, in our governing equation, not only have the advection term, but we have also diffusion term. So, when we have an equation with advection term and diffusion term, this model equation with constant c and constant alpha is known as the burgers equation.

MacCormack in 1971 extended to this method to include this case also and the evaluation of this is relatively straightforward, because we normally use central scheme for this. So, and MacCormack also use the same thing. We have a predictor step, which is given as phi i n plus 1 bar; all the predicted values are given with n plus 1 bar.

So, this is phi i n minus c delta t and delta x. We make use of forward differencing here plus central differencing for the diffusion term phi i n plus 1 n minus 2 phi n plus phi i minus 1 n and the corrector step is very similar phi i n plus 1. This is the value that we are actually now seeking, is the average of phi i n phi n plus 1 n and, the, the contribution coming from this. So, that is minus half c delta t by delta x.

Now, this is evaluated as backward in space, that is, phi i minus phi i minus 1 and this also evaluated using the predicted values n plus 1 bar n plus 1 bar and this is evaluated again plus half of alpha delta t by delta x square. This is evaluated using center differencing, but with predicted values, it is phi i plus 1 minus 2 phi i plus phi i minus 1 at n plus 1 bar.

So, now, this is the way to go from phi i n to phi i n plus 1 in two steps, each of which is an explicit step and each of which is used to evaluate for all i points at n plus 1 bar and then at n plus 1, which makes the overall solution as an explicit and it is still second-order accurate in time and second order accurate in space, but since we made use of the diffusion term, the stability is not only given by sigma value, but also the diffusion term will come in the picture in term to determining the stability. Let us not worry about the stability for the one-dimensional case, because what we are interested is the general three dimensional case. So, and we will try to describe what is the stability condition for general three-dimensional case.

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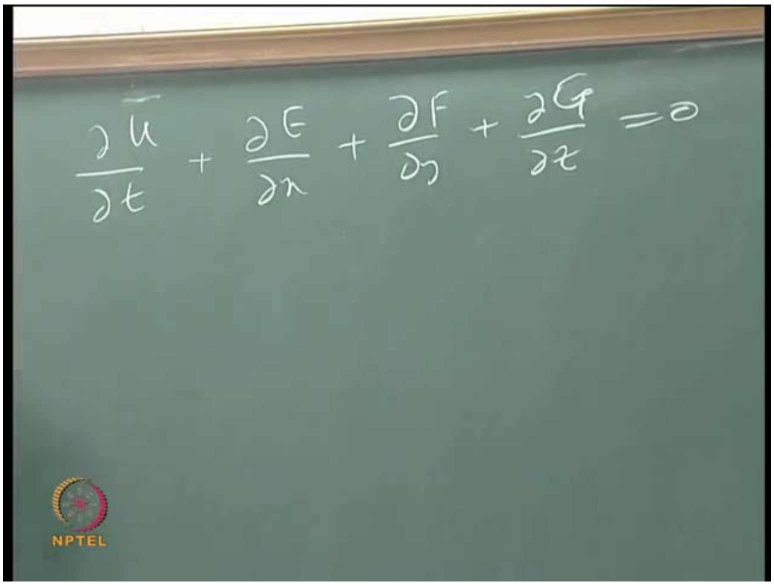

And this method has proved to the very popular and we will see how this method can be applied to are governing equations, which are now written as dou u by dou t plus dou by dou x plus dou f by dou y plus dou g by dou z equal to 0 - where each of these u e f g or column matrices containing four terms each representing the terms which appear in the mass concern equation or continuity equation, x momentum equation, y momentum equation and z momentum equation.

So, there are four terms coming in an each of this column vectors and they represent four equations. Now, you have an equation like this and we can readily apply the MacCormack method to this. Keeping mind that the e term f g here, they contain both the advection term and also the diffusion term.

So, although we have written it as purely this method this like this, it also includes both the advection and diffusion term and we have to discretize them if we are using a MacCormack method in the proper way, that is, the advection terms as the combination of forward and backward, and the diffusion terms using central differencing making use of the concurrent values, and the corrector step having in a average value of this and an average value of the advection term and also average value of the diffusion term.

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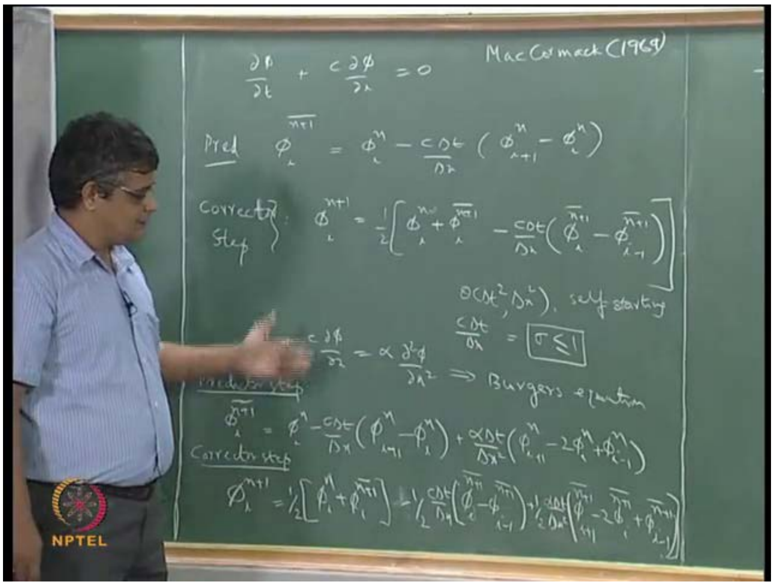

So, in the overall phi n plus 1, there is some contribution coming from phi i n obviously here, and part of the contribution from the advection term is evaluated in terms of n and part of the contribution is evaluated in terms of n plus 1, and similarly, for the diffusion term, you have part of this is evaluated there and part of this is evaluated here. So, this is seen as a correction to the value that is predicted using only the n values.

So, together this predicted valuation of the diffusion term and the corrected one together will give the overall value of phi n plus 1. So, you have prediction and the correction; that is incorporated for diffusion terms in a certain way and advection terms in a certain way and we have to make use of these recommendations in evaluating this.

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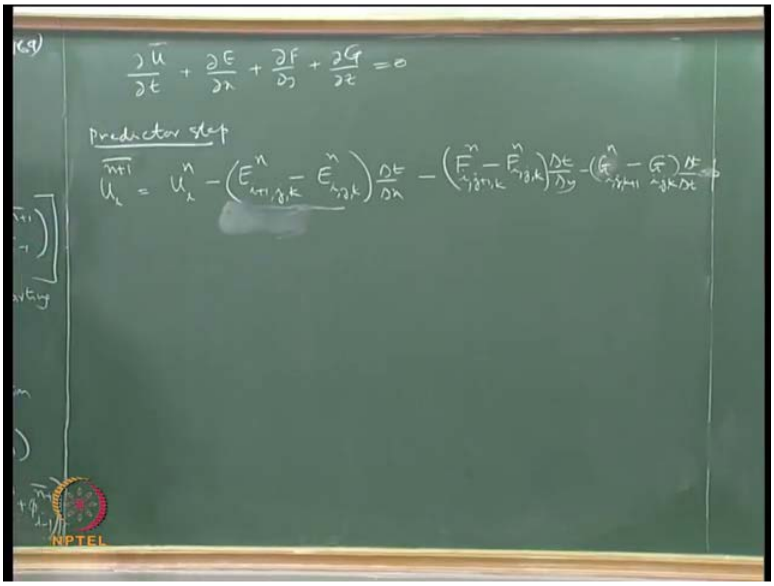

So, as per this, we can write down the predictor step in the following way - u i n plus 1 bar is the predicted value; this is u i n minus we have this term, this is evaluated in the predictor step using forward differencing. So, that is E i plus 1 j k minus E i j k n divided by delta x, and of course, we have delta t going here. So, we will just put this like this

And this is also evaluated in forward differencing in the y direction. So, this is f i j plus 1 k minus f i j k evaluated at n times delta t by delta y and this is also evaluated in a forward differencing in the k direction, so, i j k plus 1 n minus g i j k times delta t by delta z equal to 0. We have taken them on to right hand side, we have. So, this is the predictor step which looks like a straight forward application, of this, of this method.

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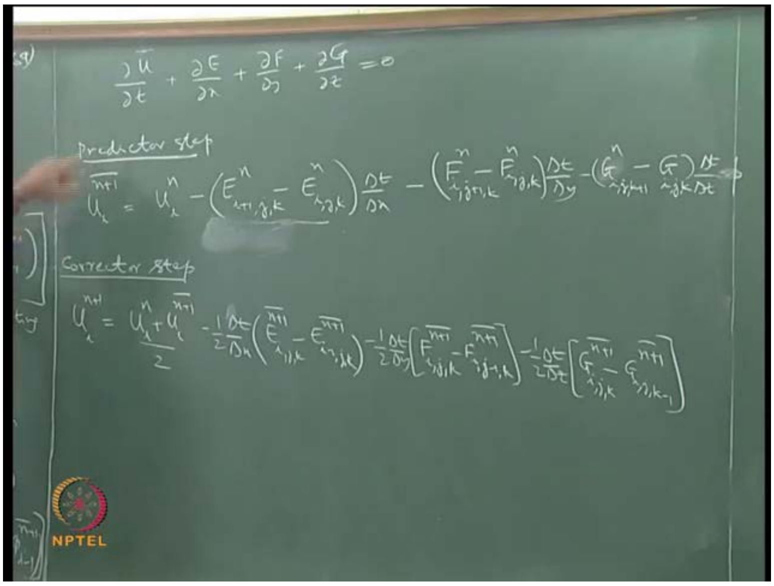

Now, we come to the corrector step, and here, now we have u i n plus 1 is equal to u i n plus u i n plus 1 bar by 2. We note that this u i n plus 1 bar is evaluated for all of them, and now, we have minus half of half delta t by delta x of this term evaluated using backward differencing. So, that is E i j k minus E i minus 1 j k and this is evaluated making use of the current known values minus half delta t by delta y. Again we make use of backward differencing i j k minus f i j minus 1 k at the intermediated values minus half delta t by delta z g i, j, k j, k minus 1 at the intermediated i. So, we evaluate the value here, in, in this way.

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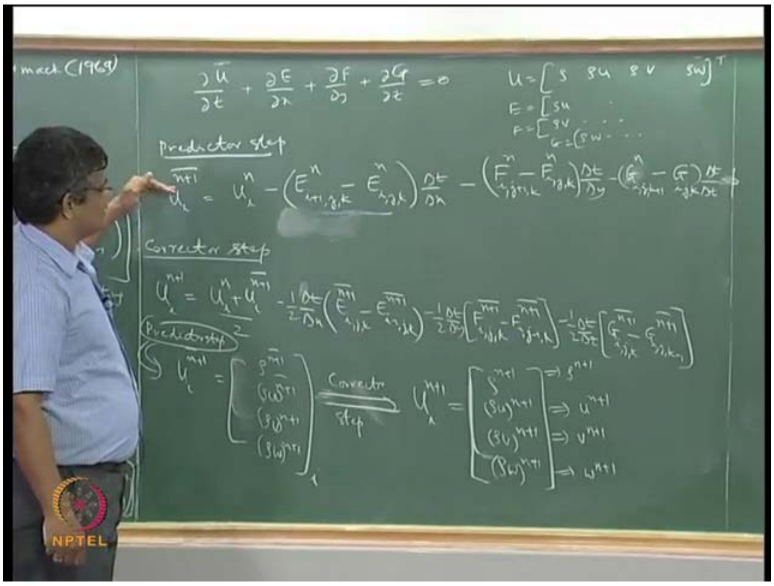

Now, this is a vector equation. Before we go to the valuation of each this e terms which have the mixture of advection term and diffusion term, let us look at the overall scheme, and so, this step has four equations to solve, because u has rho rho u rho v rho w transports, and similarly, e has the corresponding term in each of those things. So, using these steps, we first evaluate u i n plus 1 bar. So, that gives us rho n plus 1 bar rho u n plus 1 bar rho v n plus 1 bar and rho w n plus 1 bar.

So, the predictor step will give us these and we notice that the variables that are appearing are rho rho u rho v rho w, and once we have these values for all i, then we go from this in to the corrector step through that corrector step to get u i n plus 1; that means, we get rho n plus 1 rho u at n plus 1 rho v at n plus 1.

So, if we get this, so, we know rho at n plus 1 and this is, so, from this, we get u n plus 1, because in this, this is rho terms rho n plus 1 u n plus 1 and rho n plus 1 is known from this. So, you get n plus 1 and this is we get rho n plus 1 v n plus. Again, we know rho n plus 1 here. From this, we get v n plus 1, and from this, we get w n plus 1 and we have a… So, this gives us rho n plus 1. So, the implementation of this particular method is that, we first solve the continuity equation as per this discretized equation, this discretization with taking the corresponding values of e f and g from the continuity equation.

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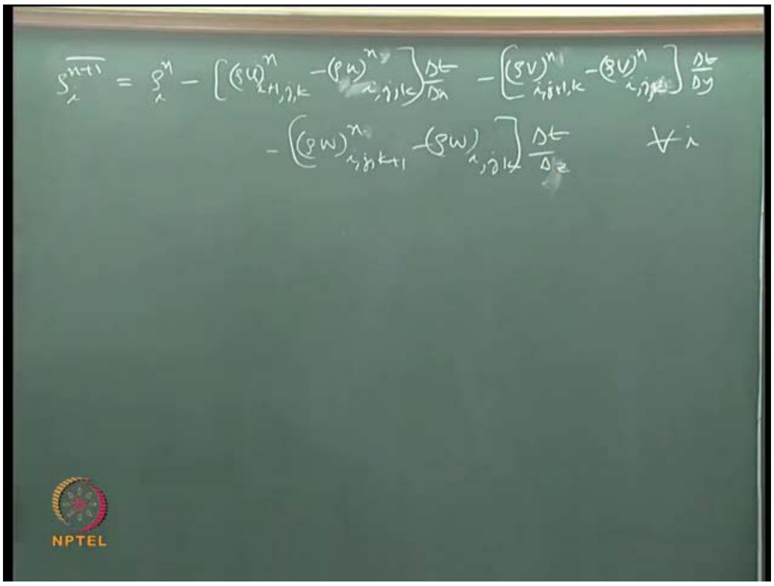

For example, we have e will have rho u and f will have rho v and g will have rho w like this. So, if you want to the first thing that we do in a predictor step is to solve the continuity equation as per this. So, let us just write it down. So, we get rho n plus 1 bar is equal to rho i n plus 1 bar equal to rho i n plus 1 minus the term that is appearing here is rho u i plus 1 times delta t by delta x minus, in f, we have rho v. So, this is rho v at n at n i j plus 1 k minus dou v at i j k minus rho w at i j k plus 1 n minus rho w at i j k.

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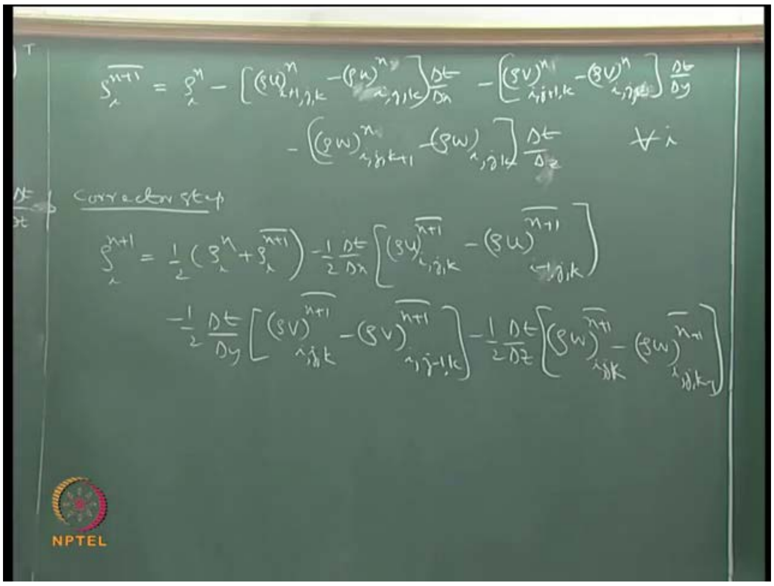

So, we solve this equation knowing the values of rho u v at nth times step for all points, that is, i plus 1 i j plus 1 j and k plus 1 k. So, this is in explicit from; this is delta t by delta z. So, this is a evaluated for all i and then we come to the corrector step as rho i n plus 1 as half of rho i n plus rho i n plus 1 bar minus half of delta t by delta x rho u n plus bar, which is already calculated i j k minus rho u at i i minus 1 j k n plus 1 bar minus half of delta t by delta y rho v at n plus 1 bar i j k minus rho v at finally, minus del half delta t by delta z rho w at n plus 1 bar. So, you can see that this, these are the equation that we solve to get rho n plus 1 bar and then rho i n plus 1.

Now, but here, we see that there is a difficulty, whereas, we can execute the predictor step readily in order to get the corrector step, we need to know rho u n plus 1 bar at i j k is rho v n plus 1 bar i j k is like that.

So, the way that we implement this equation is that, the predictor step is applied not just to the rho the equation, the continuity equation first. This is applied to all the four equations. So, from these things, we get rho n plus 1 bar rho u at n plus 1 bar rho v n plus 1 bar and rho w n plus 1 bar.

So, having applied the predictor step for all the four equations and having now evaluated rho n plus 1 bar and all these values all i j k, then we come to corrector step, and then, now, we know these things, because we have already solved the predictor step of the x momentum, y momentum and z momentum; then, we go in to this.

So, there is a sequential process here that, the predictor step is evaluated for all the variables and we get the predicted values of all the variables here and then we make use of this to implement the corrector step for the all the variables, and then finally, get the values here, and at each stage of implementation, there is a hierarchy; there is a sequence. First we evaluate; first we applied this rho and then we applied this rho u and then rho v and then to rho w. So, in that way, we go through to the whole equations, the coupled equation, and then, solve them sequentially once through the predictor step and then through the corrector step in an explicit way, and finally, get the solution for rho u v at nth plus 1 times slip for known values of nth times slip.

So, this particular method is a, it makes a close linkage among the equations. That is why, it, it, it works even though you have nonlinearities. So, we are starting with known values of u v w and rho at all points.

So, after we know them, we do not go and straight away evaluate rho n plus 1, we go through the intermediate evaluation of rho n plus 1 bar here. At that point, we are making use of u v w; the all values of u v w, and when we come to n plus 1 bar, we need the intermediated values of u v n bar u v n plus 1 bar which we solve by again by applying predictor step to other equation.

So, that is, as we go from rho n to rho n plus 1 bar to rho n plus 1, we are not only updating rho, but we are also updating u v w and n plus 1 bar. So this intermediate updating of all the variables allows the nonlinearity of a the equations to be properly accounted for and that is why, it, it seems to work for the combined set of coupled equation. We notice that when we want to solve the continuity equation, we need know u v w, and when we want solve the x momentum equation, again we need to know not only u, but also v and w and the same thing goes for y equation and z equation.

So, in that sense, for each equation, we need know all the values of the all variables at the neighboring points, that is, at i plus 1 i minus 1 j plus 1 j minus 1 k plus 1 k minus 1. So, this need to know simultaneously everything makes the solution very complicated and very prone to divergence, and this way of doing it in two steps of a delta t separated by small delta t. That is going from n to n plus 1 bar to n, in which, all variables are evaluated and then corrected will make the method work. So, that is the advantage of this.

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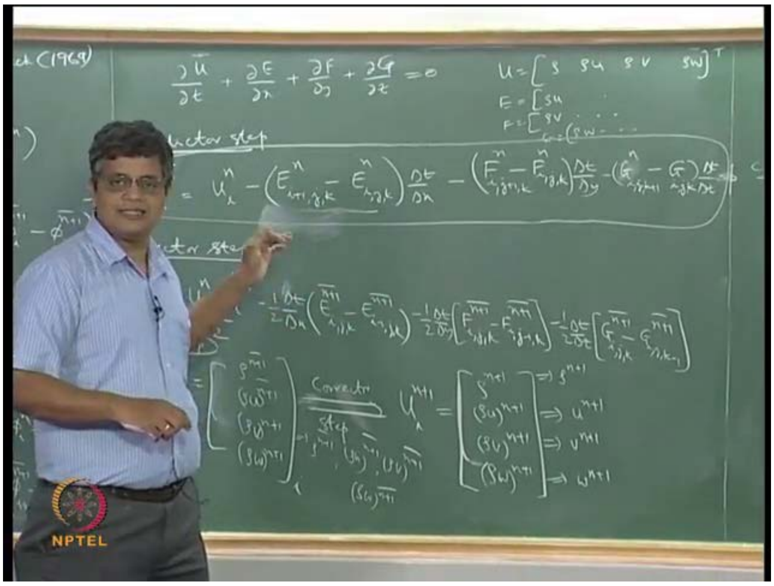

This is an explicit method for the solution for the all partial differential equations together and it is second-order accurate in time and space and it is conditionally stable. The stability is not only determined by the advection term, so, that is by the something like the courant number, but also by the diffusion alpha delta t by delta x square and it is also determined by the fact that now we have three dimensions since space.

So, that is we have delta x term and delta y term and delta z term, and we have seen that when we go from one dimension, two dimensions for the pure diffusion case, then the allowable times steps is reduced. Now, we are going to three dimensions and we have to get an overall stability criteria, which is not possible in that in the general case.

So, there are some numerical experiments done and approximate analyses done, which gives an idea of the possible times step that we can have for the combined solution of all these things using MacCormack method. Before we look at the discretization of each of the terms in e and f and all those things, let us take an overview of the method and then make a complete description of the method.

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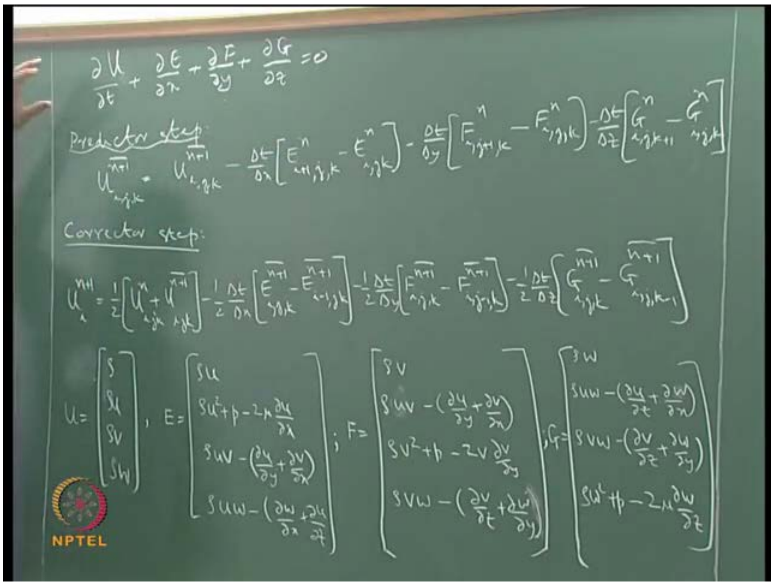

We have written the governing equations, the Navier-stokes equations in the form of du by dt plus dE by dx plus dF by dy plus dG by dz equal to 0 - where U E F G are column vectors containing four terms each. We will come to those four terms, and once we have an equation like this, we can look at how the u which is variable of interest evolves with time by going through a two step process to go from n to n plus 1 in the MacCormack method, MacCormack explicit method, and in this, we write, in the, in the predictor step, we evaluate the value at i j k n plus 1 bar.

This is intermediate times step in terms of the value, at, at the times step n minus the discretization of this using forward differencing. So, that is, delta t by delta x; the delta t coming from here will go there, and this is forward differencing, that is, e i plus 1 j k minus e i j k. So, this makes it forward in x derivative evaluated at nth times step, and similarly, the dou f by dou y is discretized using the forward differencing in the y direction, that is, f i j plus 1 k minus f i j k.

So, this makes it forward in delta y evaluated again it nth times step, and dou g by dou z is again discretized in the forwarding differencing in the z direction, so, that is, g i j k plus 1 minus g i j k, were g is evaluated and nth time step.

So, this enables us to get the value of u i j k at the intermediate step. This is in explicit step because all the values here are evaluated at nth times step. Therefore, we know them, and we go through the evaluations of u at the intermediate times step for all the points of i j k of interest, and then, we come to the corrector step, in which, we actually update from n plus 1 bar; that is the intermediate times step to nth n plus 1, and this is evaluated again explicitly has half of u i j k n, which is the previous times step plus half of u i j k at n plus 1 bar, which is evaluated like this, and then, here, we go back to back to backward differencing for each of this space derivatives.

So, we have delta t by delta x; we also the half term, because we are making a correction to the estimate that is made here, and this is e i j k minus e i minus 1 j k. This makes it backward differencing, in, in this, in x direction evaluated now at n plus 1 bar times step. So, these are values which are obtained, as a, as a result of the predictor times step, and when the terms within e are functions of the variables, then these are evaluated at n plus 1 and we will see that presently.

Now, again the dou f by dou y is evaluated again using backward in y so that it is f i j k minus f i j minus 1 k making this backwards; evaluated at n plus 1 bar and the dou g by dou z is also evaluated using backward n space so that g i j k minus g i j k minus 1 evaluated at n plus bar and n plus 1 bar. So, together these steps will enable us to go from u i n to u i n plus 1, sorry, it is has to be two i i j k n plus1. So, we complete the predictor step all the points and all the variables and then come to the corrector step. By the end of which, we get the values of interest.

Now, what are the components of u e f g so that this represents a Navier-Stoke equation. We are looking at case where tau i j is expressed as dou u i by dou x j plus dou u j by dou x i times viscosity plus the second coefficient of viscosity terms dou u k by dou x k and we are neglecting the second coefficient of viscosity has being unimportant and also as being undetermined.

So, we have a tau i j given like this that is for a Newtonian field, and under those conditions, we can write down the components of u, which is main variable of interest.0 As in the continuity equation, we have rho. In the x momentum equation, we have rho u; y momentum equation, we have rho v; z momentum, we have rho w.

Now, e term which represents the advection component coming from the x spaces, the positive x spaces, the negative x space. It has in the continuity equation, you have rho u. In the x momentum equation, we have rho u square pressure and the sheer stress, that is, minus tau x x which as per this is evaluated as minus 2 mu dou u by dou x, and the component arising out of the y faces will have the advective component rho u v and tau y x which is the shear stress, which is now given by mu times dou u by dou y plus dou v by dou x with the minus sign. So, this whole thing is tau by x. And here, we will have tau x z.

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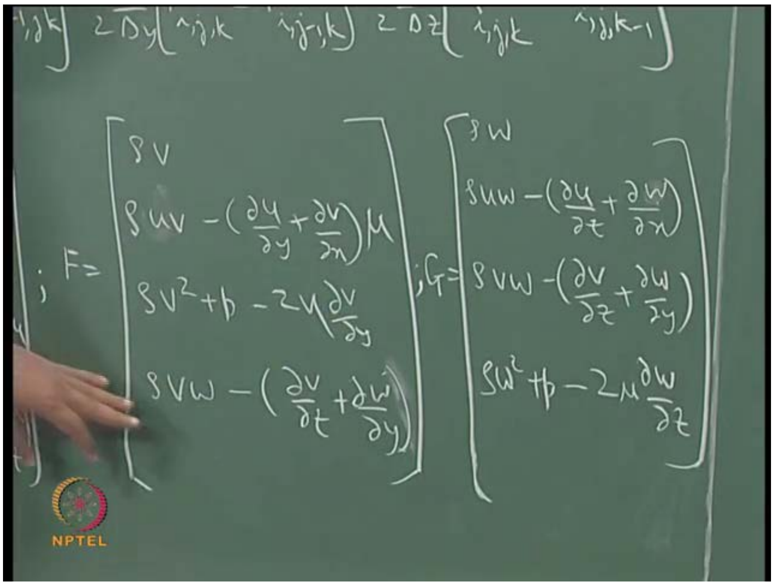

So, we have rho u v minus mu mu times dou w dou x plus dou u by dou z. So, this, these are the stress components which are appearing in the Navier-Stokes equations, which are also components of the corresponding elements of the e vector here.

This representing the component coming in the continuity equation, x momentum equation, y momentum equation and z momentum equation, and similarly, when you come to the f vector, this contains the that advective pressure and viscose stresses components acting on the two y phase, that is, the top phase in the bottom phase.

So, for the continent equation, we have rho v; for the x momentum equation, we have rho u v and then tau y x, that is, minus mu times dou u dou y plus dou v by dou x, and in the y momentum equation, we have rho v square plus p minus 2 mu dou v by dou y, and in the z faces, we have rho v w minus mu times dou v dou z plus dou w by dou y, and finally, the g component here is the sum of the term is the set of terms appearing from the z directional faces, that is, the front and the back face and we have rho w the continuity equation and rho u e minus the sheer stress in the z direction. So, that is the tau z x which is mu times dou u by dou z plus dou w by dou x, and in the y momentum direction, we have rho v w minus mu times dou v by dou z plus dou w by dou y, and in a the z momentum direction, we have rho w square plus p minus 2 mu dou w by dou z.

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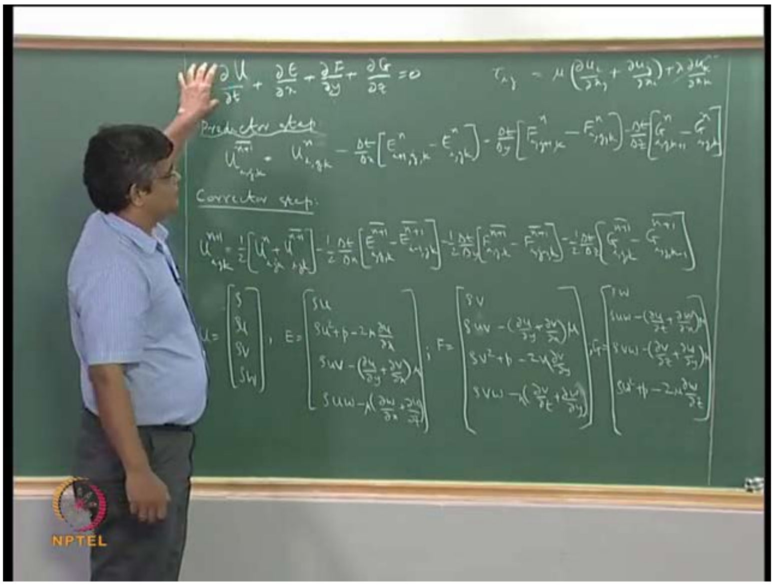

So, the elements which are appearing in this u e f and g, when put together in this form, will give us the Navier-Stoke equations for a Newtonian fluid, in which, the second caution to viscosity component has very little significant. We have as, no, no contribution to make to the momentum equations. So, and the solution of these things will proceed as we have discussed, each equation is solved using this method for the all the points i j k and then we go to the next equation, next equation, next equation. We complete the predictor step for all the points and all the four variables, and once we have these things, then that means rho n plus 1 bar is evaluated for all i j k; rho u n plus 1 bar is evaluated for all ah n plus 1 i j k, and similarly, rho v and rho w.

Having these things, now we go to the corrector step and then we employ these things, and for example, when we say e i j k n plus 1 bar, so, the e here corresponds to this, so, this whole things is evaluated at i j k n plus 1 bar, and so, the evaluation of u here and rho here is not the problem, because these are not particular points and p i j k is also not a point, but we have to be careful in that in evaluation of derivatives here. We have dou u by dou x here and dou u by y here and dou v by dou x, and if we look at any of these e f j components, we have either this same derivative or the cross derivative. The same or cross referring to the way that they appear, in, in the governing equation that is written here.

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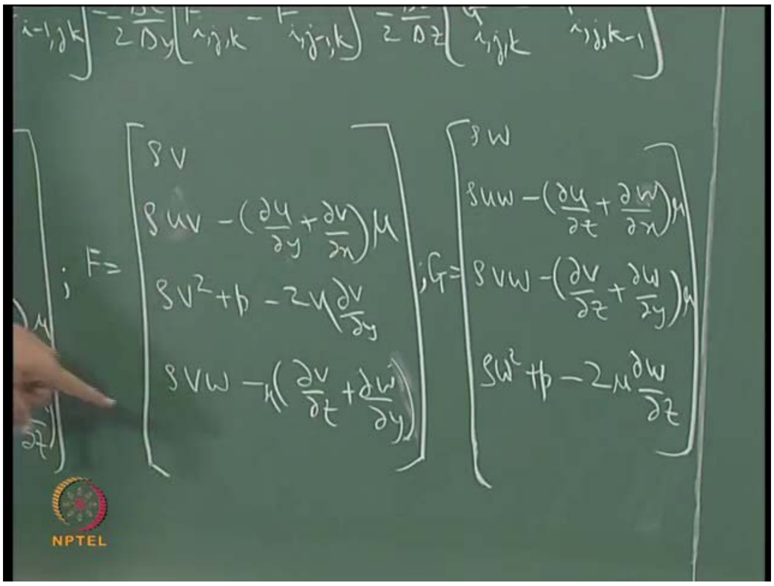

For example, when you consider e term, this is being differentiated with respect to x. So, a term like dou by dou x which will be, because of this whole things being differentiated here, will give dou by dou x of dou u by dou x. So, that is in normal derivative, and if u say dou u by dou y which is also appearing in the e here will be differentiated as dou square u by dou x dou y. So, this is the cross derivative. Again this is normal derivative and this is cross derivative.

So, normal derivative and here this will be normal derivative and cross derivative, and when we come to f here, anything with respect to y here will be a normal derivative, and anything with respective x or z will be a cross derivative, and the same way here anything with respect to z is a crossed is a normal derivative anything with respect x and y will be cross derivatives.

So, that since we have to make sure that the consistency of the evaluation of these derivatives, cross derivatives and normal derivatives is maintained, in order to maintain an overall second order accuracy of this discretization, because the overall evaluation of u i j k is second-order accurate in both time and space.

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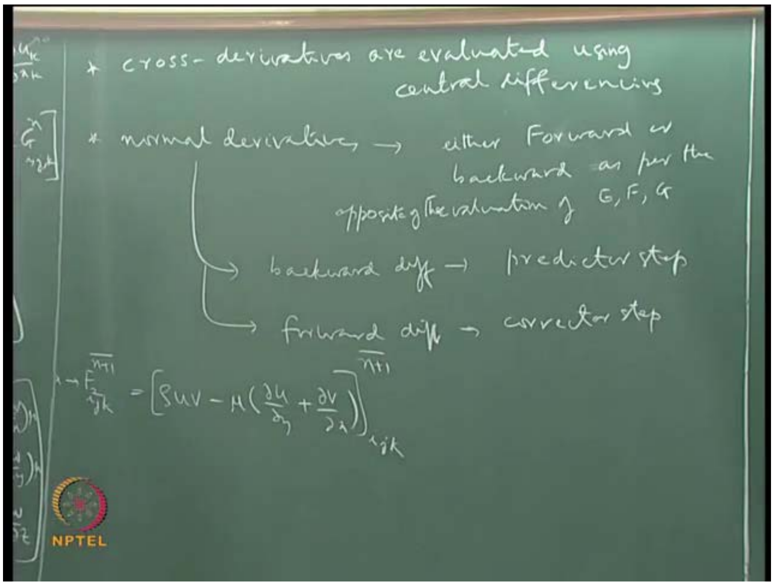

So, in order to do this, we have a prescription in order to maintain the cross derivatives are evaluated using central differencing, and normal derivatives are evaluated using either forward differencing or backward depending on how the main derivatives is, is evaluated.

For example, in, in, in this term here dou f by dou y is evaluated using forward differencing. So, in the predictor step, the normal derivatives will be evaluated using backward differencing. In the corrector step, the same f is evaluated using backward spacing. So, here, one will use a forward differencing for term like dou u by dou y.

So, forward differencing as per the evaluation of e f g, and when I say as per, you should say as per the opposite of the evaluation of e f g, that is, the normal derivatives are evaluated using backward differencing in the predictor step and using forward differencing in the corrector step. So, it is only for the normal derivatives that this will shift the cross derivatives are always evaluated using central differencing.

So, if, for example, we take a term like, let say this one here. So, this is, is a f and this is appearing at a, let us say i j k in the x momentum equation, this is appearing x momentum equation, and this term will be, let us say in the predictor step, we want to get f n plus 1 bar. So, we will just call this as this the second component here, because this is the second component and we want to evaluate dou f by dou dou y component here.

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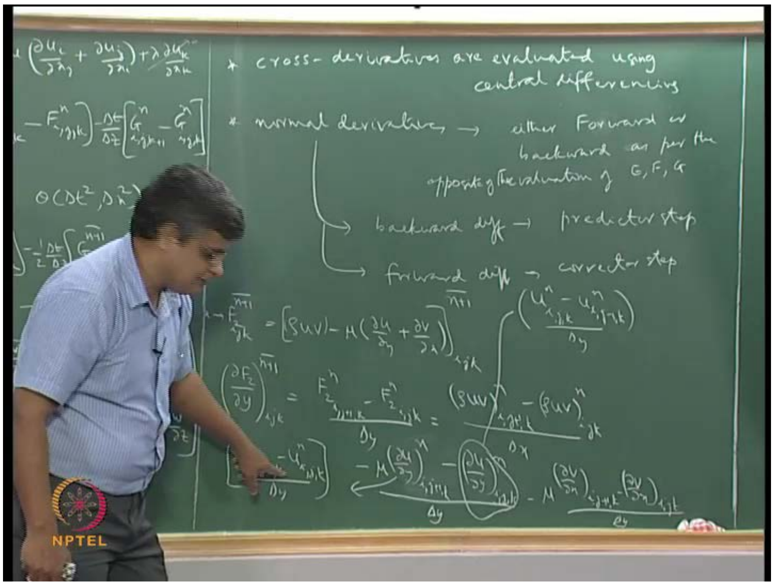

So, we want to evaluate this as it applies to the evaluation of this u i n plus one. So, we have to evaluate this at n and so i j k. So, essentially what we looking at is what is dou f 2 by dou y as it appears here; i j k is evaluated as f 2 n i, j plus 1 k minus f 2 n i j k divided by delta y.

So, that is what we have here it is forward differencing. Now, this f 2 here has each of these components. Now, for these things, it is straight forward. So, we can write the correspondent component of each of this and we can say that this means that rho u v i j plus 1 k n is this 1 if we consider only the first term here minus rho u v i j k n by delta x.

So, the first term of this is written like this in a straight forward. Now, we come to the minus mu du by dy. Now, we are looking at dou by dou y of this. So, that becomes a normal derivative, and if the normal derivative has to be evaluated in such a way that it opposite to the way it is discretized for f, and here, we have made use of forward differencing. So, we have to evaluate this using backward differencing.

So, we can write this as assuming that mu is constant minus mu times. So, dou u by dou y at i j k using backward differencing would be we will take the mu out and we can write dou u by dou y dou y at i j plus 1 k n minus dou u by dou y at i j k n, ok.

So, now, this is the component arising out of this. And so, this is evaluated using backward differencing and this also evaluated using backward differencing, and, and we have this term here minus mu times dou v by dou x. So, again we can write dou v by dou x at i j plus 1 k minus dou u by dou x at i j k divided by delta y, ok.

So, this is the total expression of f 2 i j k plus 1 of dou f 2 by dou y term coming in this, and since f 2 has three components, each of three components has to be evaluated. So, this has to be evaluated in this way, that is, straight forward. There are no derivatives here, but we have derivatives here. So, this term has to be evaluated this term is, this term here, and again, this term at i j k.

Now, here and here, we have to use backward differencing. So, we will write this as u at i j plus 1 k j plus 1 k n minus u i j k n by delta y. So, this is the backward differencing of this term at i j plus k, and similarly, this term here will evaluated as u i j k n minus u i j minus 1 k n by delta y. We can see that here, this is a backward differencing approximation for dou u by dou y at i j k. This is backward differencing approximation for dou u by dou y at i j plus 1 k, ok.

So, because this is j plus 1 here, we seem to be getting a forward differencing, but this actually backward differencing when we evaluate this at i j plus 1 k, and this one here has to evaluated using central differencing and this one also using central differencing.

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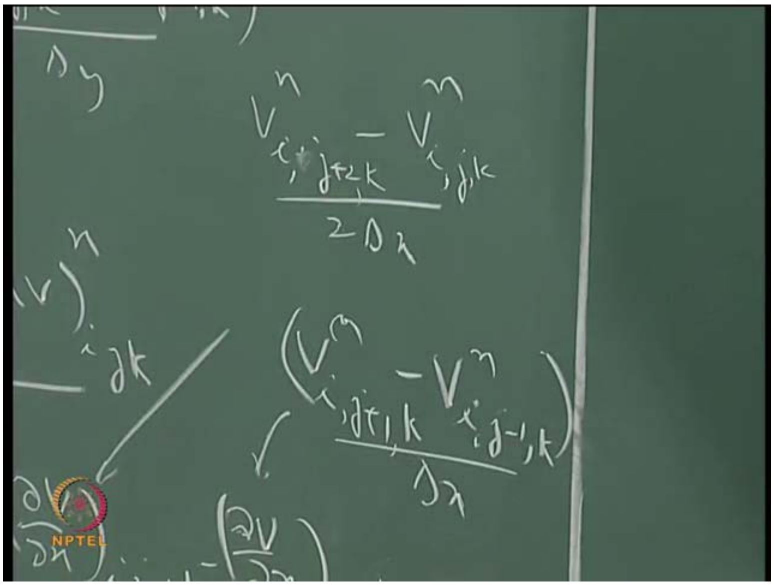

So, we can write this 1 here as v i j plus 2 k n minus v i. So, j plus central differencing around j plus 1 is that j plus 2 and j will come by 2 delta x, and similarly, this expression here will be v i j plus 1 k n minus v i j minus 1 k n by delta x, ok

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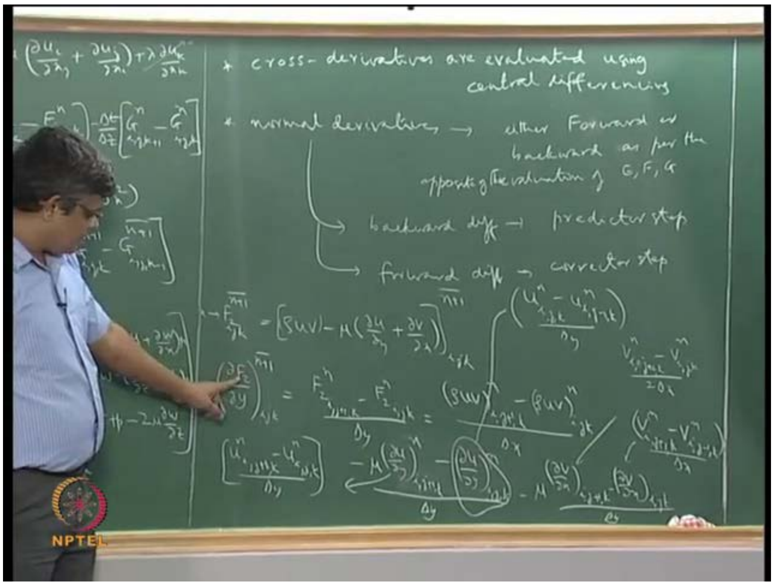

So, in this way, we can evaluate the individual terms, which are appearing in this making use of central differencing for the cross derivatives and using either forward or backward as per the opposite of the evaluation of e f g for the normal derivatives. So, in this particular case, dou y by dou f by dou y is being short and this is being evaluated using forward differencing here. So, that the normal derivatives which appear in this, that is, this is normal derivative because this dou by dou y. So, that will be become dou square u square by dou y square. So, that is being evaluated using back ward differencing at each of these points and this term is being evaluated using central differencing.

So, the cross derivative is being use to evaluated using central differencing. So, that central differencing or backward differencing has to be evaluated for each f evaluation at that particular point.

So, here, we are applying backward differencing at j plus 1 that gives us j plus 1 minus j, and here, we are using backward differencing at j and that gives us j and j minus 1, and here, we are using central differencing centered around j plus 1.

This is dou v by dou x, so, this should have been j plus 1 and central differencing around dou v by dou x. So, this is v i plus 1 j plus 1 k minus v i minus 1 j plus 1 k by delta x, and again, this is dou v by dou x at i j k. So, when we apply the central differencing around i here, so, we will get v i plus 1 j k n minus v i minus 1 j k n divided by 2 delta x and here also we have two delta x here.

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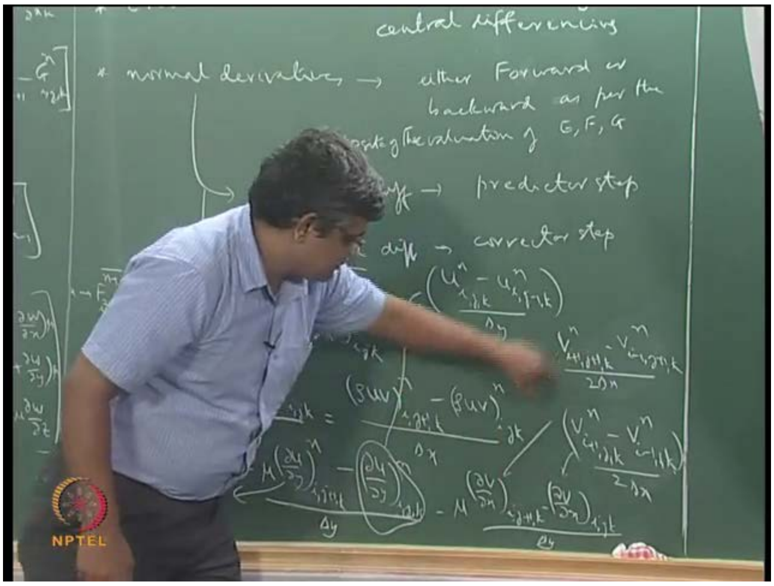

Now, we can substitute this here and substitute here and get an overall expression, and substitute this here and substitute this here get an overall expression and that will give us the evaluation of dou f 2 by dou y at the predictor step in for the x momentum of equation, because we are considering the x momentum equation, and if we were interested in the corrector step for the same term, if we are interested not in n plus 1 thing but as appearing in the corrector step, so, that is n plus 1. Then this would have been evaluated in using backward differencing j and j minus 1

So, here, we would have to use forward differencing and forward differencing, and here, we would be using still at this central differencing. So, by doing this consistently, we can get a set of equation using which we can march forward in time in the predictor step through all the points i j k to get n plus 1 bar and then come to the corrector step, and again, march forward in time, march forward in i j k directions to get n plus 1 values, and from these, we can get these things.

Once we have rho u and so on, we get rho first, and from rho u, we divide by the rho value to get u and we divide by the rho value here to get v divide by w value to get here, and pressure is obtained as a function of density, and for example, temperature and we have assumed isothermal flow here, but for a general compressible flow, we will have to also solve for energy equation which we have not derived yet and the incorporation of the energy equation to the calculation is very straight forward. We will have an additional variable here and additional set of terms coming in the energy equation and those can be solved together with this.

So, this is how we can make use of the template, a given template for the simultaneous solution of the Navier-stokes equations or extended for Navier-stokes equations when we consider even the energy term. So, and this is how we can solve set of the governing equations using one template. Now, will this work? Will this work in all cases. So, that is the question that we will address next.