Computational Fluid Dynamics
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Module No. # 03
Template for the numerical solution of generic scalar transport equation
Lecture No. # 11
Topic
Numerical solution of the unsteady advection equation using different finite difference approximations

Having looked at various ways of writing finite difference approximations for any derivative ,of any order, to a given degree of accuracy, and having looked at possible complications arising from cross derivatives ,and also from time derivatives.We can claim now ,that ,we are in a position to take up our governing equation, the template generic scalar transport equation ,and then write a finite difference approximation for each term .Then put it, convert it into an algebraic equation and proceed with the CFD solution.

So at this stage, we may be able to take a claim, that we know how to do a CFD solution, for a generic transport equation .But is that confidence justified, or is it misplaced? Let us test ourselves, by taking a few simple examples, and see how much ,we can claim to know that we are capable of discretizing properly.

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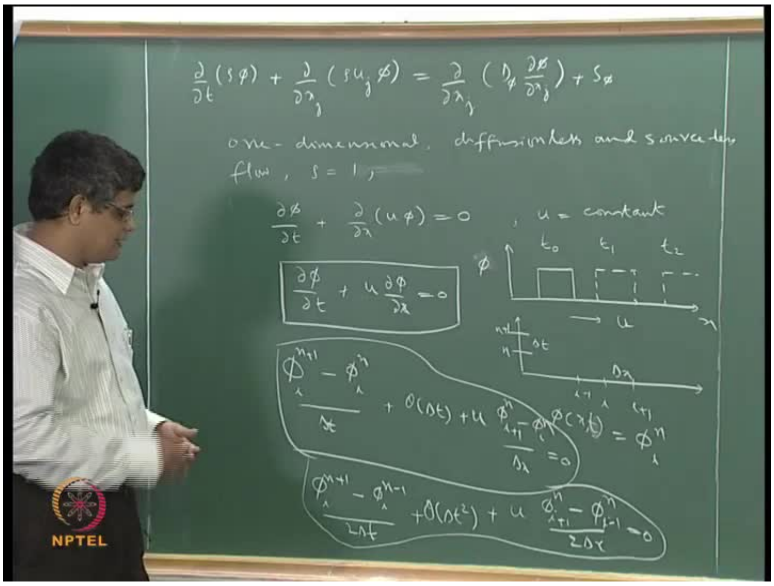

So ,we go back to our scalar transport equation, containing the accumulation term, the advection terms ,the diffusion term ,and the source term .And out of this , we make a simple approximation, which is one dimensional, diffusion less, and source less flow . For good measure, we say that rho is constant, and u is constant. We say that rho is 1, in which case, we can write this equation. Because it is diffusion less, this term goes to 0 .Because the source is less ,it is source less that is 0 ,and because it is 1 dimension ,we have the governing equation, reducing to with rho equal to 1 dou phi by dou t, plus dou by dou x of u phi ,equal to 0 . We also say that ,let us take u to be a given constant, and therefore we consider the very simple equation .One can say, that this has some resemblance to the governing equation ,and it captures some of the physics, that is contained in the original equation .The physics that contains ,is a change of scalar because of advection is, what is contained in this .So it is a diffusion less, source less ,1 dimensional ,situation where u is not changing significantly, with respect to x during that time of interest.

So under those kind of approximations, we can come up with this simple form .We want to test our discretization of equation, finally writing finite difference approximation capability and knowledge against this simple equation.

Now the simple equation is not complete, without a statement of the boundary conditions and initial conditions. We can see that,this is a 1 dimensional equation, and it is like hyperbolic equation, because it has a velocity with which a scalar is being transported in the x direction. Therefore the it has. If you consider the 1 dimensional case x, and as a function of time, if you had an initial scalar as the variation of phi with respect to x is, if it initially, you had for example a square pulse ,here ,then after some time ,this would be moving in the horizontal direction retaining the same form .Because it is there, is no diffusion in this, and then with further progression of time, for example, t ,t 1 and t 2 it will be something like this

So it is moving in the x direction, at a speed of u. This is what is being represented by this .As if you had this is the highway, and you had a lorry ,which is containing that particular scalar ,and the lorry is moving at a constant speed. So wherever the lorry is, that is, where the scalar is, and outside the lorry, the scalar value is 0 .The scalar is actually contained in a tightly packed container, so that it does not diffuse out.

So, on this highway, the position of the of the scalar, is associated with the lorry, and as the lorry moves, along at a certain speed, the scalar position also becomes nonzero at the position, where the lorry is present. This is what is contained in this equation, and that is the true solution. So we want to get a numerical solution for this equation, using the finite difference approximation, and the corresponding thing.

So we have to write a finite difference approximation .For it in the last class, we have seen that ,when you are dealing with a time derivative ,and a space derivative ,when your governing equation, contains both time derivatives and space derivatives, you have an option of choosing an explicit method or an implicit method .Where in the explicit method, the value of the variable at a time n plus 1, is expressed explicitly in terms of ,the values of the variables at neighboring locations ,and also at the same location at earlier times. So that you can march forward, from one point to another point in the positive x direction and get a solution. So that is a simple solution .Whereas, an implicit method will require us to at a particular time level. It will require us to assemble all the equations, for all the points in the x domain, and then form a matrix, and then we have to do a matrix solution.

So obviously, the explicit method is much simpler. So ,we will try to use an explicit differencing for the time derivative .And ,again we want to use a forward differencing, because that is most obvious thing. So, we can write, we can discretize the domain in the x direction, into i ,i plus 1 ,i minus 1 with a spacing of delta x.

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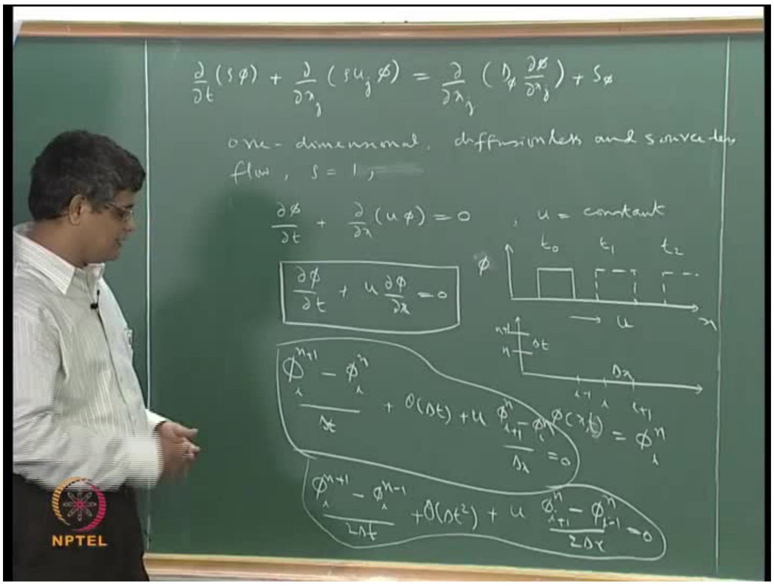

And ,similarly the time domain, is also discretized in delta t, with n ,n plus 1 and so on, in this way .So using the standard notation that, phi of x y ,x t is represented as phi i n ,where i stands for, x i ,which is x not plus i delta x ,and t n stands for, t not plus n delta t n minus 1 delta t. So using that notation here, we can write forward differencing approximation, phi i n, plus 1 minus phi i n,by delta t plus terms of the order of delta t .We know that this a first order scheme and here we are considering u to be constant u times ,and here we can use space derivative ,so we can make use of central differencing.

We can make use of, for example ,the same forward differencing for this .So we can write phi i plus 1 ,minus phi i by delta x equal to 0 Since we are making it explicit ,these are evaluated at n th times .So this is a discretization of our governing equations ,simple governing equation using forward differences for both.

Now the reason why we have chosen forward differences, in the time domain, is that we know that this is an initial value problem. So we know that at n equal to one ,we know the entire solution as part of the initial condition. If you know, that then we can get n plus 1 in terms of n values .If we used a central differencing time ,phi i n plus 1, minus phi i n minus 1, by 2 delta t plus, which is second order in space .In time, second order accurate is time, plus we can again use central differencing here, phi i plus 1,minus phi i minus 1, by 2 delta x equal to 0

So, this is another possibility and using implicit method, we can come up with. This we understand, that this is first order accuracy, and this is second order accuracy .This is another possible way of doing this. But if you look at this solution ,this is not a self-starting solution .Because in order to get, phi i n plus 1 you need to have, you need to know ,the value of, phi i n minus 1. So that means, you must know the value 2 time steps in advance

So, when you want to compute for the first time step, this formula cannot be used. So for n equal to 1, we know the solution and n equal to 2 .If you want to apply this, then we do not know the value of this 1 .So this is not a self-starting thing. We have to use something like this, in order to get a solution here .And then for the next time step, we have to do .So we will for the time being, we will avoid that complication, and stick to forward differencing in time .

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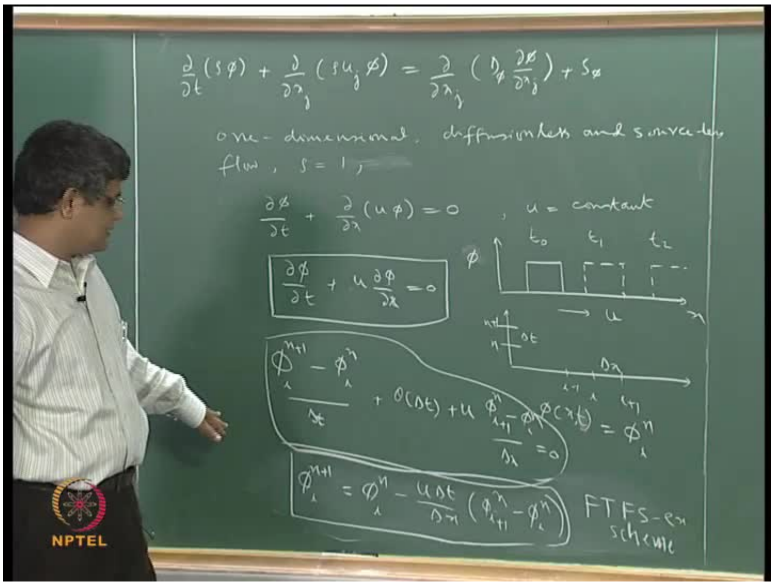

So, now we can rewrite this, in terms of phi i n plus 1 ,and u is given by phi i n minus .We have u delta t, by delta x times phi I, plus 1 n minus phi i n .So this is the resulting template, or the formula .A recurrence formula ,by which we can evaluate the value of phi i ,at n plus 1 th time step, in terms of the values at n th time step at different spatial positions The factors which come into the equation that is u, which is the velocity ,at which the lorry is going ,and then the delta t and delta x .These are the discretization points, and we can say that, this is we can call this method ,as forward in time ,forward in space scheme.

So, we have for this simple equation. We have come up with the forward in time forward, in space scheme, to evaluate phi i n plus 1 .In terms of all the other things and since this is an explicit scheme ,we will put as x FTFS explicit scheme.

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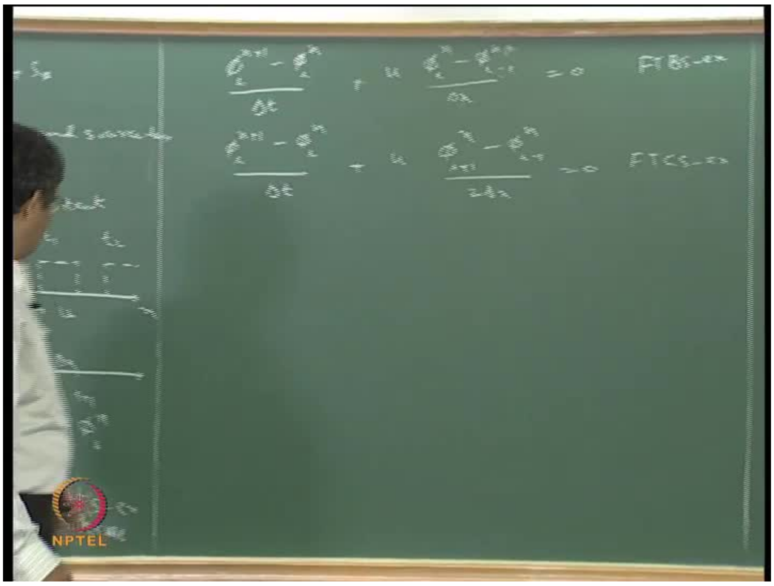

Now, this is not the only way that we can do this .We have just now seen the central in time ,and central in space .And for the time being, we have abandoned that. But we can also make changes here, without affecting the rest of the thing . For example ,we can say that, we will still retain, phi i n plus 1 ,minus phi i n by delta t ,because this seems to be the most logical way of going forward in time .Given that we have an initial condition and we can go forward with that .And for the space derivative ,we do not have any constituents ,that we have to use only the backward space forward in space.

We can also make use of backward in space, phi i n, minus phi i n i minus 1 n, divided by delta x ,and this is equal to 0. We can call this as forward in time, backward in space explicit. We can also, make use of central in space, plus u times, phi i plus 1 n, minus phi i minus 1 n, divided by 2 delta x equal to 0 .And we can call this as, FTCS central in space explicit.

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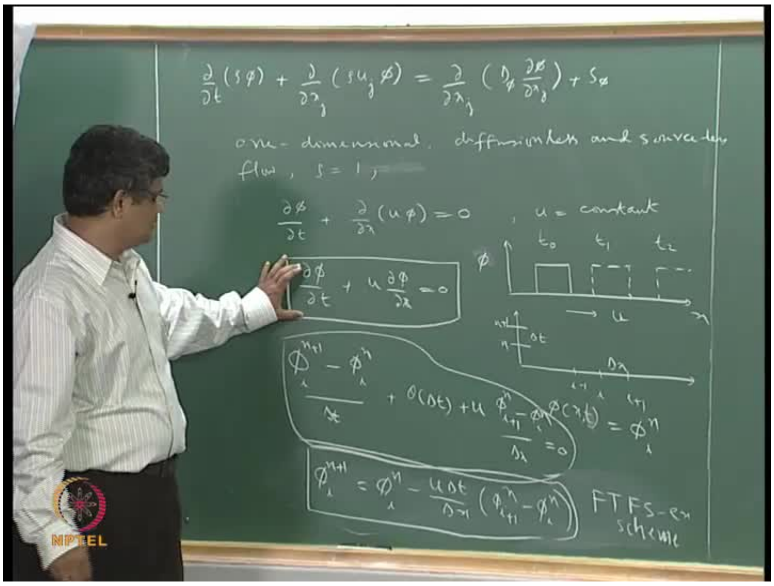

So for the same equation, for the same simple equation, we can readily write down 3 different schemes ,and they share the advantage .These are very simple schemes ,and we can march forward, hop from one point to another point. In time, it does not require much of computation ,as such and you can do this in any data base program ,like even ,excel type of Microsoft of excel ,or any other equivalent program .It can also be used to do this without having to write a program ,so it is as simple as that.

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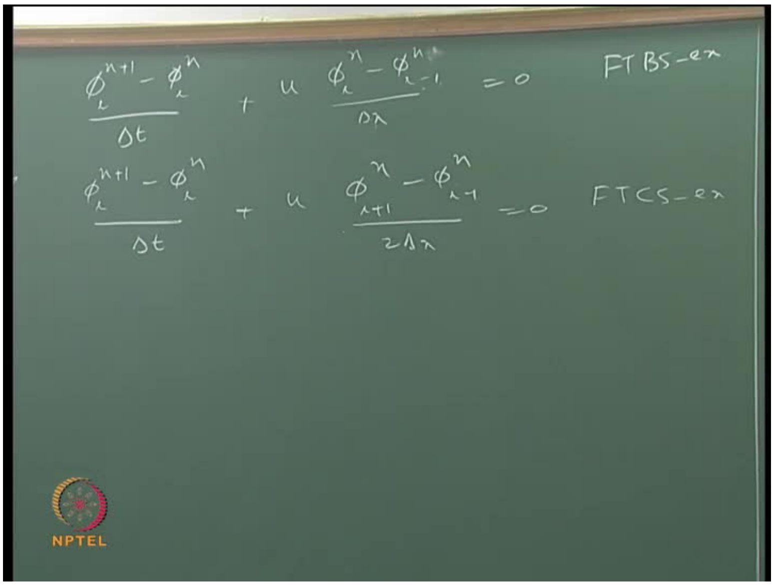

Now, what is the difference between these things? These are different ways of evaluating the derivatives. There is a forward in time ,here for this .And for the space derivative ,the three of them ,differ in the way ,that you treat the space derivative .Here, we are making use of forward in space, which is first order accurate, and here we are using backward in space ,which is also first order accurate .So one would say from the accuracy point of view ,either the FTFS or the FTBS scheme ,are not different ,and this is third one ,is central in space. So this is second order accurate. So, one would expect more accuracy from this method as compared to this method, or other method.

So in that sense from what we know of finite difference methods ,and the approximation of a derivative ,in terms of finite differences ,we are expecting an equivalent solution from this method ,and this method, and a better solution from this method for any, of the for the parameters we are looking at.

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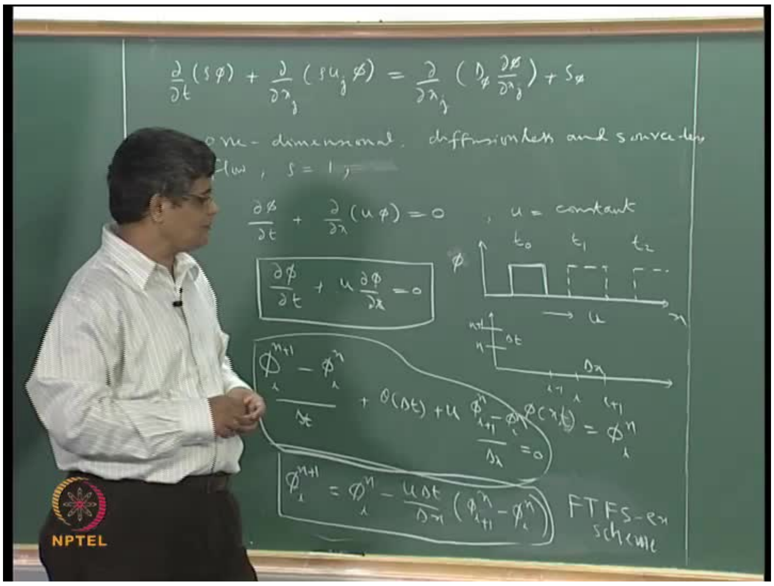

So, and the parameters are u delta t, and delta x u is fixed for us. delta t and delta x are our choices, and for a given initial condition and boundary condition .So with these things ,we can now test ,what kind of solution we get .And as a boundary condition ,here this is a hyperbolic equation. We need to give the boundary condition at x equal to 0, and we can say that x equal to 0, phi is always equal to 0, so here it is 0. And even at this time, at t 1, it is 0, and t 2 this is 0 .So we have, the and the initial condition is a pulse like this. So this is the solution that we are expecting .And if this is the initial condition, we are expecting our numerical computed solution, to match with this expectation, and that this pulse will move in the x direction at a speed of u.

Now, we want to see to what extent the 3 different schemes, will match the expected solution .When we want to attempt a numerical solution, we have to specify everything .So we cannot say u here, and we cannot say a pulse, we have to give values to this.

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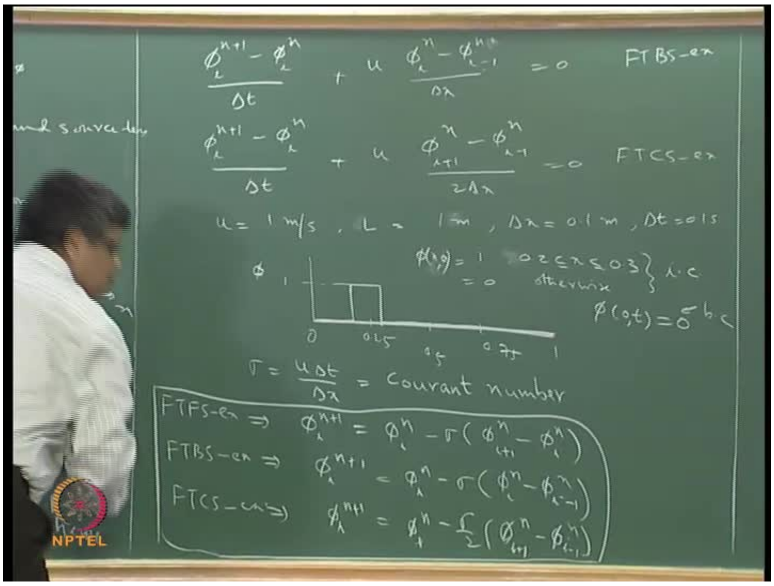

So, we will take u to be one meter per second. We need to take a total length of the domain, for example, it may be some, let us say 1 meter. We can take delta x to be say point 1 meters, so that we have 10 spaces, within this, or we can take it point 0 five. it is up to us ,how much we can take, and delta t is also a variable thing, and we can take delta t to be point 1 second. Just as we can vary, we can vary these parameters, and examine what kind of solution, we are getting. We also have an initial condition, and we say that the value of phi is ,if you say that this is 0 and 1 here, and this is point 5, point 25, and point 75 ,and this is equal to 1 .So phi is equal to 1, between point 2 and point 3 ,and it is 0 throughout everywhere else.

So ,phi is equal to 1 ,for point 2 ,less than or equal to x, less than or equal to point 3 ,and it is equal to 0, otherwise ,at t equal to 0 So this is the initial condition. We say that phi 0 at any t is 0, this is the boundary condition. For the sake of ease of computation, we can define. So we can see, that we are getting this parameter here .We can say that sigma here, is this parameter, which is u delta T by delta x . We will see, that this is called the courant number, after a famous mathematician ,who worked on this in 1920s, and published a paper ,on something similar to this in late 20s ,and we can see that this courant number, and will also come here.

So, let us write down a simplified formulas for this .We can write FTBS, FTFS explicit as phi i n plus 1, is equal to phi i n ,minus sigma times phi i plus 1 n ,minus phi i n.FTBS explicit, will give us ,we take it to the other side, this becomes minus here, this is minus ,this is plus here .

So this is phi i n, plus 1 is equal to phi i n, minus sigma into ,phi i n minus ,phi i minus 1 n .So we take this to the other side ,so that becomes minus u delta t by delta x ,which is our sigma phi i ,minus phi i minus 1 . FTCS explicit will give us phi i n plus 1 ,equal to phi i n .Same as this ,1 plus u delta t by 2 delta x ,so that is minus sigma by 2 times phi i plus 1 n minus phi i minus 1.

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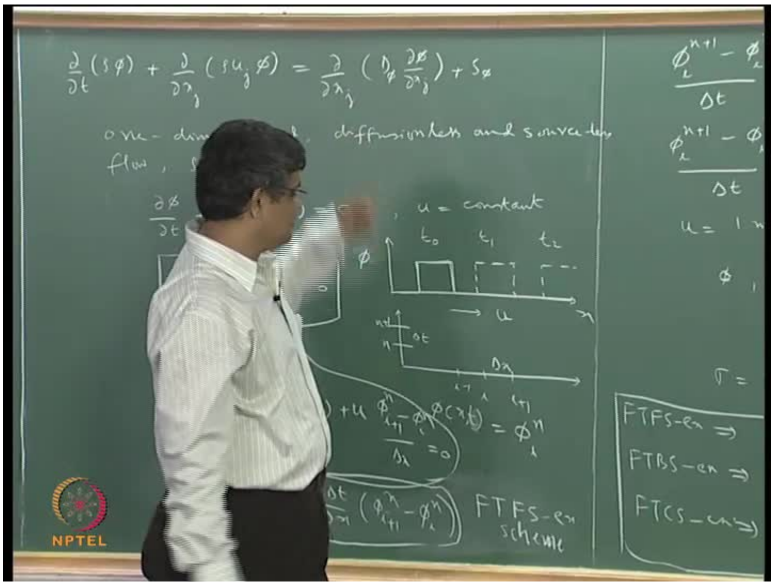

So these are the formulas, using which we can calculate phi at different i locations. At any given time, starting with an initial condition, which is given by this, and we want to see at different times, whether or not ,this initial square pulse representing ,for example, a lorry is moving in the positive x direction ,whether or not moving in the positive x direction ,without changing its shape. Because there is no diffusion, and also at the speed which is given by u, which is equal to 1 meter per second.

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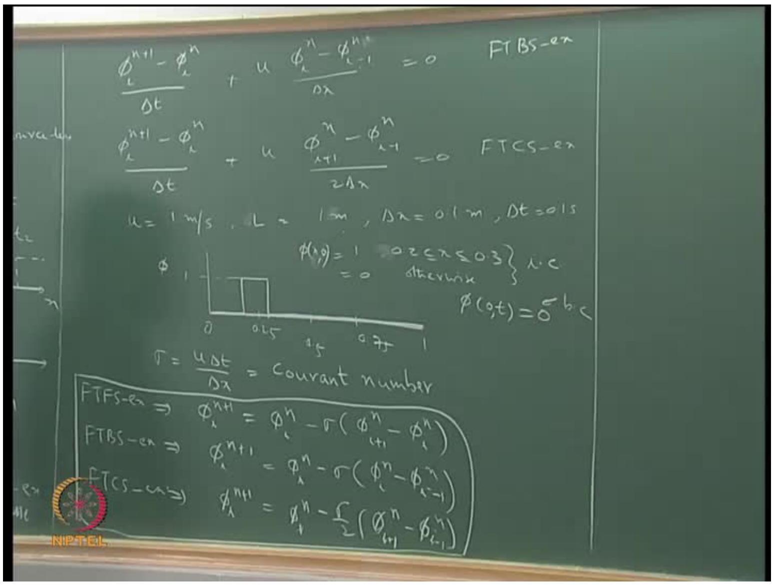

So ,if the solution is correct then it should be this pulse here, which is centered between point 2 and point 3 ,should be moving at the rate of 1 meter per second in the positive x direction .So that at for example point 5 seconds, it will have come from point 2 to point 3, to point 7, to point 8 .That is a kind of solution that we are expecting.

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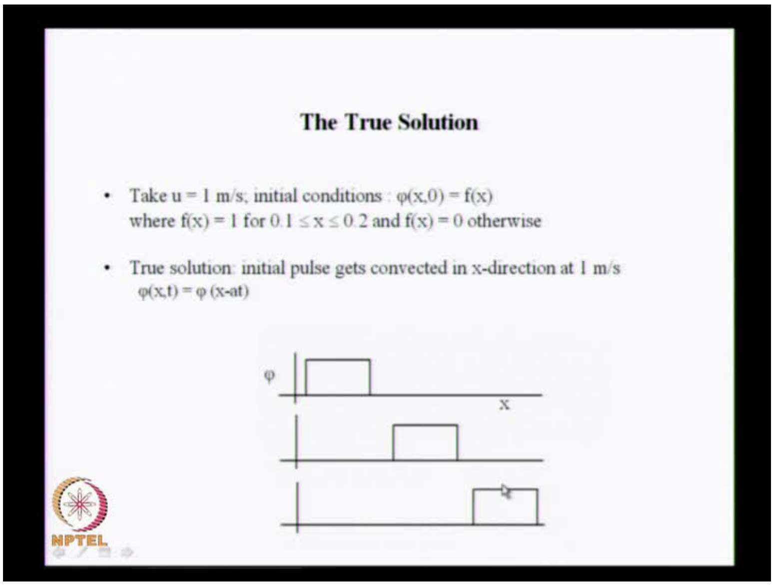

We can ,we need, to have a computer solution. We will look at what the computer solution, shows for these 3 different cases .So, this is the problem that we are considering. We have taken u equal to 1 meter per second ,and the initial condition that is phi for all x at time equal to 0 ,is given by a function f of x ,where f of x ,is equal to 1, for x between point 1 and point 2 ,and it is equal to 0 otherwise. And the true solution ,is at the initial pulse gets converted in the x direction, at a speed of 1 meter per second ,and the solution at any time except t is equal to solution of x phi at x minus a t ,where a is one meter per second ,which is the speed .

So, and that is what is shown here phi versus x .If this is the pulse then it travels at different times, in the forward x direction. Now what is the solution that we get from the 3 different methods?

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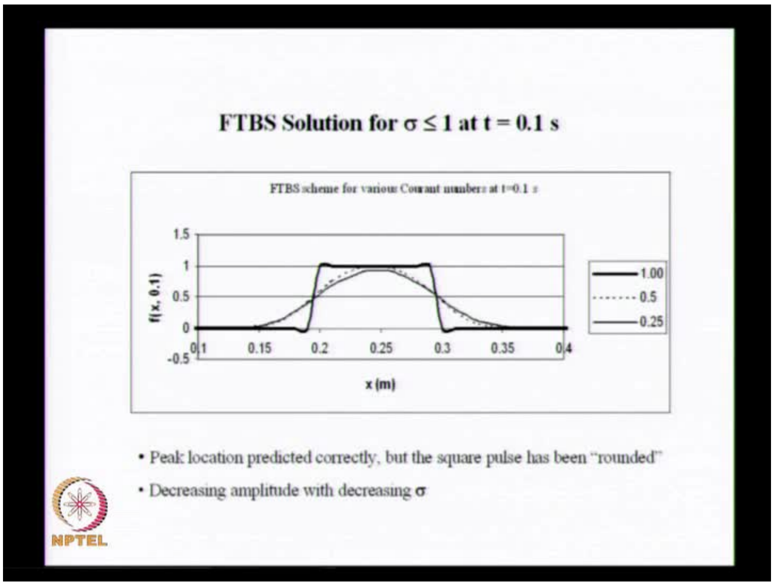

So this is an FTBS solution, that is forward in time and backward in space solution for different values of sigma, at a time of point 1 seconds. So in getting these solutions ,we have taken delta x to be point 01 ,so that is 0 point 01, so it is one centimeter. We have adjusted the value of delta t ,such that the sigma the courant number, takes a value of 1 here, and point 5 here, and point 25 .Given the courant number is,u delta t by delta x and u is equal to 1 .When your sigma is equal to 1 the delta t is equal to delta x numerically ,so that is point 01 seconds .And in this particular case ,when sigma is point 5 delta t, is reduced by 2 ,and it is point 005 seconds .And for sigma equal to point 25 ,we are going even lesser values of delta t ,so that is point 0025 seconds.

So for these 3 different values we are using smaller and smaller delta t .Now what is the implication of that? We are doing a first order differencing in time ,so the forward difference in time is first order accurate in time . We know that, the accuracy of the approximation increases, as delta t is reduced. So we expect a priori ,that when sigma is reduced, while keeping u and delta x constant ,which is what we have done .Therefore as delta t is reduced, for lower and lower values of courant number, we expect more and more accurate solution.

So that is, what this particular trial is about and we are looking at the shape of the pulse, the predicted shape of the pulse at a time of point 1 seconds .So this point 1 seconds, is arrived at for a courant number of 1 ,it has taken 10 time steps to reach this .In the case of point 5 courant number, it has taken 20 time steps to reach this. And for point 25, it has taken 40 time steps to reach this. So we are not comparing the solution at, so many number of time steps, but at the same time elapsed from the beginning of the pulse.

So, we expect, we are comparing apples, and with apples., by doing this ,by comparing at a fixed time ,and not at a fixed number of time steps. Because for each case, for each sigma value, since we have fixed delta x and u, delta t is changing. So we should not compare at different time steps ,but we should compare at the same elapse time, from the beginning and that is point one seconds.

So, what is the solution looks like this ,f of x at a time of point 1 second ,is what is plotted here. So that is the value of phi over the x ? We have zoomed in on the region, between point 1 and point 4, because before point 0, and beyond point 4 it is 0. Any changes happening over this particular range, and we also notice that initially ,we had a pulse ,which was centered between point 1 and point 2 .So it is centered about point 15 and it is moving at 1 meter per second .Therefore in point 1 seconds ,it would have moved point 1 meters to the x direction.

So,now we should be expecting a pulse ,which is centered, which is displaced by point 1 meters from the original solution .So the original solution, had a pulse centered at point 15 .Now we expect it to have moved by point 1 ,to reach at point 25 .So we are expecting the pulse to be centered at point 25, which is what we are actually getting.

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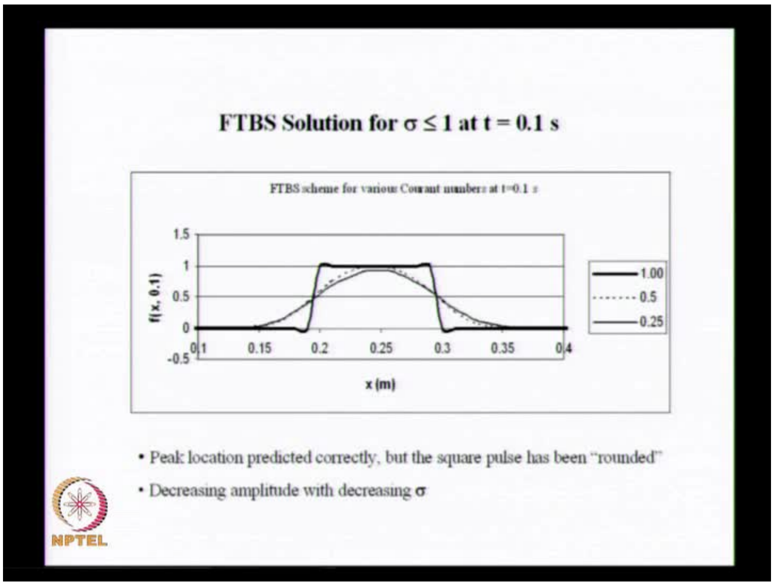

We see that in all the cases ,either we look at this ,or this one, where we are getting a pulse which is centered at point 25 ,which means that the computed pulse is travelling in the positive x direction ,at the correct speed .Now but what about the accuracy part ? We are expecting better accuracy, with point 25 courant number, because the time step is less and worse accuracy, if there is anything at a courant number of 1. Whereas what we are getting here in this is that the solution obtained with a courant number of 1, seems to be more like the square pulse that you started out with.

At point 5 and point 25, it is more smeared .You can see that here. It should be 0 but you have a value here. So this seems to be some, the diffusion of this particular pulse, which is making it from a square shape, into something like a gradually sloping hill type of variation ,more like a normal distribution.

So ,one can see that the predicted ,or the computed results are counterintuitive in the sense, that we expect more accurate results ,with smaller courant number because the delta t is less. So we expect more accuracy, but we are not getting that here. The pulse that we should be getting as a square pulse,is not retained as a square pulse, although we have considered a diffusion less equation in our equation, there is no diffusion The computed solution seems to be showing some diffusion type of behavior ,for certain courant numbers point 5 and point 25, but not for a courant number of 1. Not only that the diffusion is more appears to be more for a smaller courant number, than ,for a slightly higher courant number point 5.

So it is we can see that the computed solution, is showing a different kind of physics ,compared to what is expected . What we are hoping to get in our computed solution, and finally ,because we are smearing it the pulse ,the peak pulse value, is decreasing with increasing sigma ,all at the same time of point 1 seconds

During which it has travelled from point 15 to point 25, so this is what we are getting from the FTBS scheme ,forward in time and backward in space And what about the other 2 schemes ,we have FTFS and FTSC.

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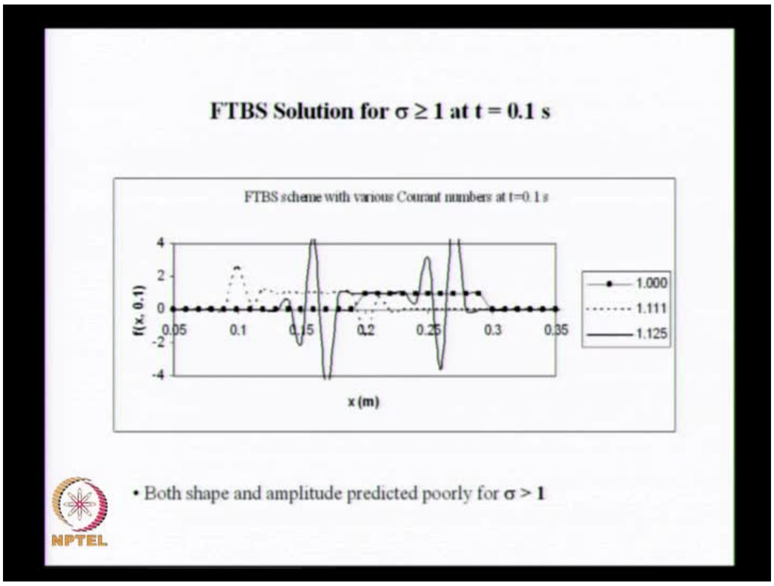

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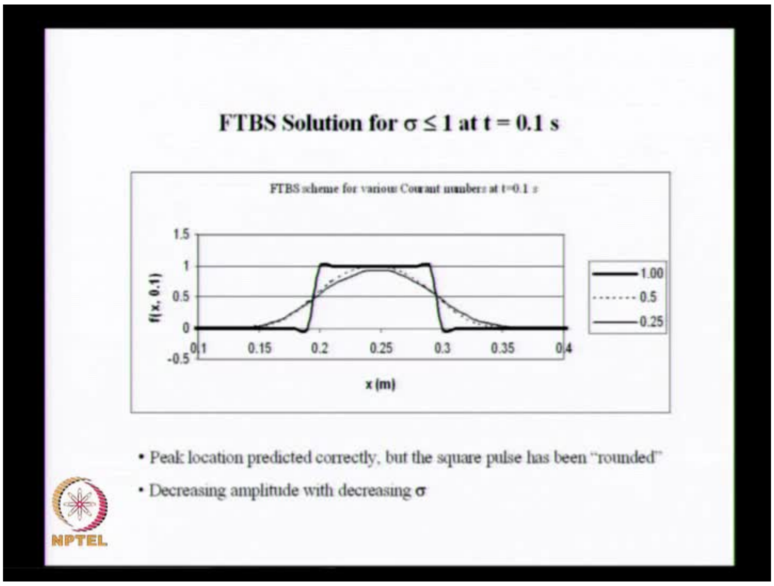

Now if you go to FTBS solution ,for even larger values of courant number , because we seemed to be getting better solution ,with increasing value of courant number .So what can we get ,when we increase the value of sigma ?So this is a solution ,that we have got from FTBS scheme ,for 3 different values of sigma and you seemed to be getting a better solution, as sigma is increasing and paradoxically as delta t is increasing.

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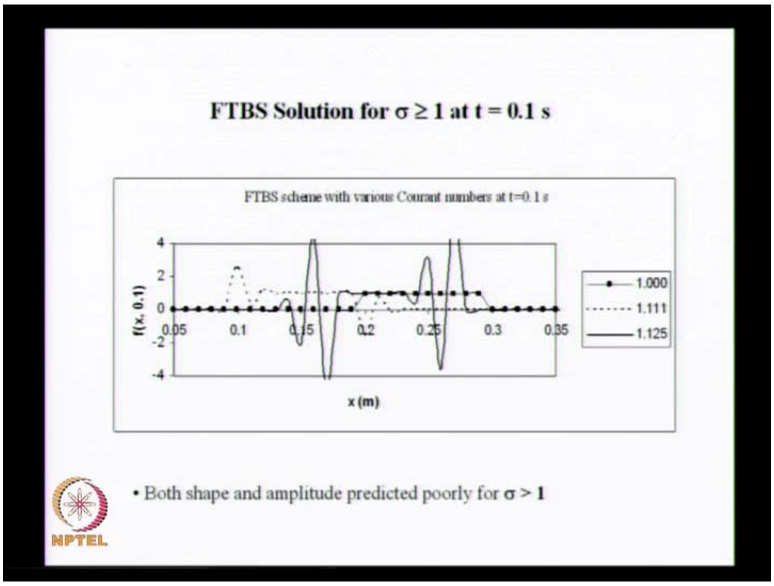

Now, what do we get if sigma is increased more ?So here we have solutions. 3 different solutions, the solution for sigma equal to 1 and sigma increased by about 10 percent here

,and by about 12 percent here. So there is a small increase in delta t ,as compared to this ,and this ,and we are again looking at the solution predicted solution at point 1 seconds.

Now ,what we see is something very disturbing ,and very strange here. We see that the function at the same time, is now ,becoming negative. It should not be negative and it looks like something erratic behavior is there.

At with point 1, we still have a solution like this .We can see that here it is 1, and the next point it is 0 ,so it is actually a sharp change, which is not good .So that looks like the correct solution ,with sigma equal to 1. But when we increase the delta t by about 11 percent, then we get a solution, which is shown by the dash line ,and it is showing a nonzero value at point 1. Where as it should be 0 here and it is increasing ,and then coming down like this, and here it is becoming negative ,and then it is going like this. And when we increase the delta t by not by 11 percent but by, 12 point 5 percent, we are getting even stranger result. The value is negative here ,shooting up to greater than plus 4 ,and immediately falling down to less, than minus 4 .Then it is showing this kind of erratic behavior and we can see that the shape of the pulse ,which should be a square and the amplitude of the pulse ,are predicted very poorly for values of sigma which are slightly greater than 1 .

And how do we get these values of sigma, increased value of sigma ?Just by changing the value of delta t by about 10 percent or 12 percent ,than where we are getting the best solution .So the value the computed solution seems to be so sensitive to the value of delta t ,that we have chosen for this particular scheme.

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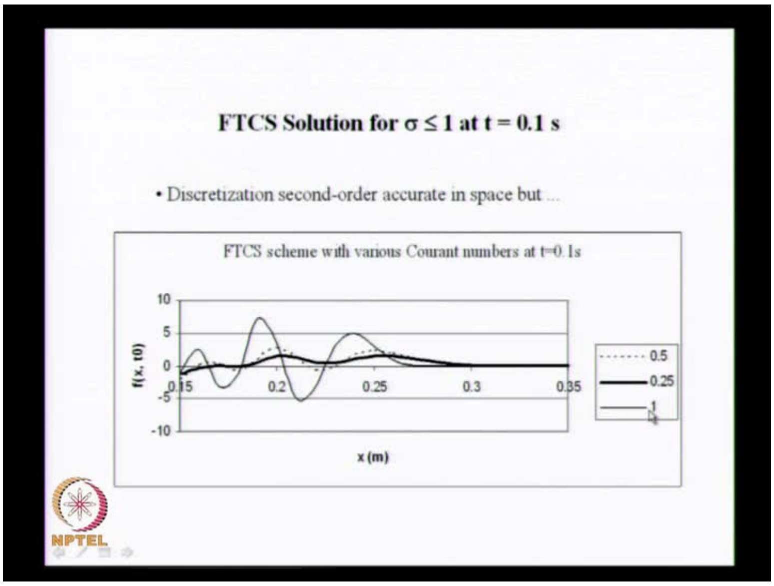

What about the other schemes? If you look at FTCS solution ,CS is central in space .So we are expecting better accuracy ,as compared to the FTBS, which is only first order accurate . This is second order accurate in space and we are looking at spatial variation So we hope that ,we will get much better solution .But our hopes, are actually delayed ,because for the same 3 courant numbers ,that we originally looked at for sigma equal to 1 ,and point 5 and point 25.

We are getting results, which are not at all resembling, that kind of expected square pulse ,we are centered around point 25, that we are hoping to see.

We are hoping to see a pulse here, located between point 2 and point 3, over this region ,with an amplitude of 1. What we are getting here, is an amplitude which goes up to something like 7 here ,and minus 5 here ,and is showing nonzero values which are well beyond the expected zone of nonzero variation.

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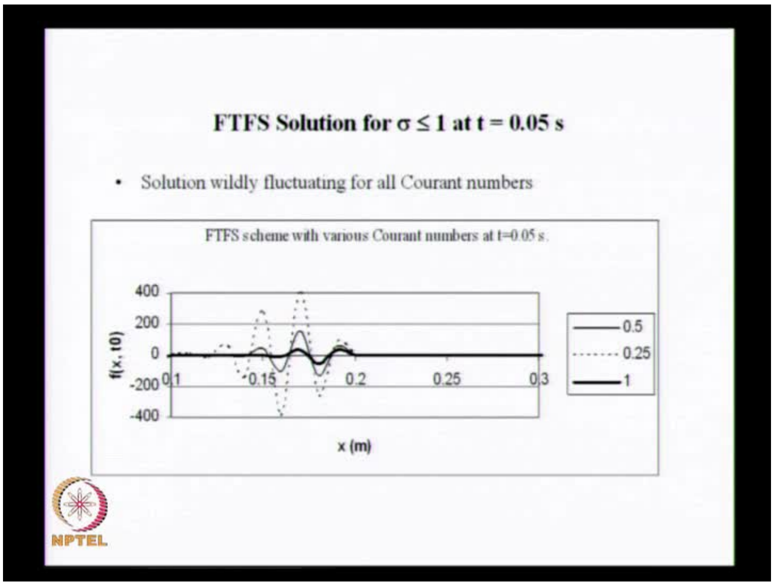

So, this is clearly not correct and the solution is also showing increased erraticness, increased values of these things .When lambda is increased, when the courant number sigma is increased, for sigma less than 1 ,and what about the other method FTFS forward in time, and forward in space? And here again we are looking at sigma less than 1 ,and now we are looking at elapse time of only point 05 seconds. The reason being that, even for this short times, half the time as what we are looking e,arlier the computed value, has an amplitude of going up to 400.

Whereas the expected amplitude is only 1 for phi .So we can see that here again for different values of courant number point 25 point 5 and 1 .We are getting very erratic solution. We should be seeing something centered between point 2 and point 3 ,with an amplitude of 1. We are getting highly unacceptable values at these, and here we see that as delta t is reduced further from 1 to point 5 and point 25. The amplitude of this pulsation is increasing much more, so here we have solutions ,which are which are behaving very erratically, compared to what we are expecting.

So ,our first foray ,to compute a solution of a very simple equation ,has proved to be close to a disaster.

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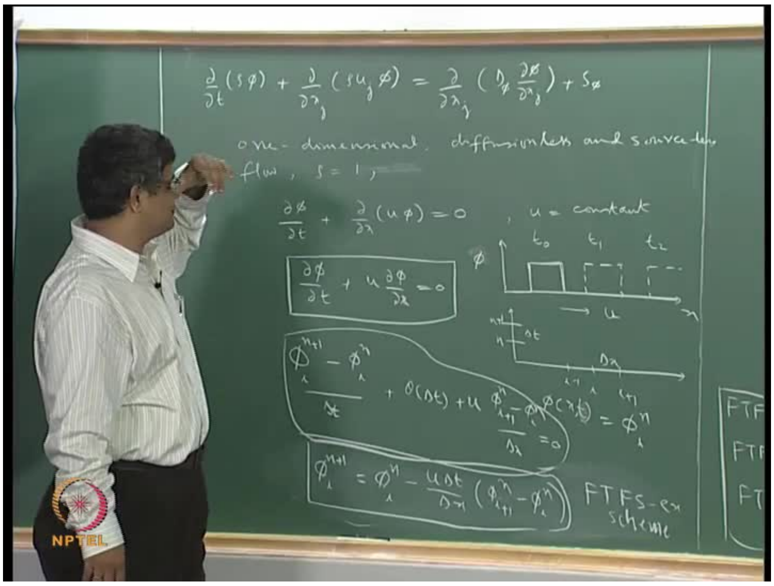

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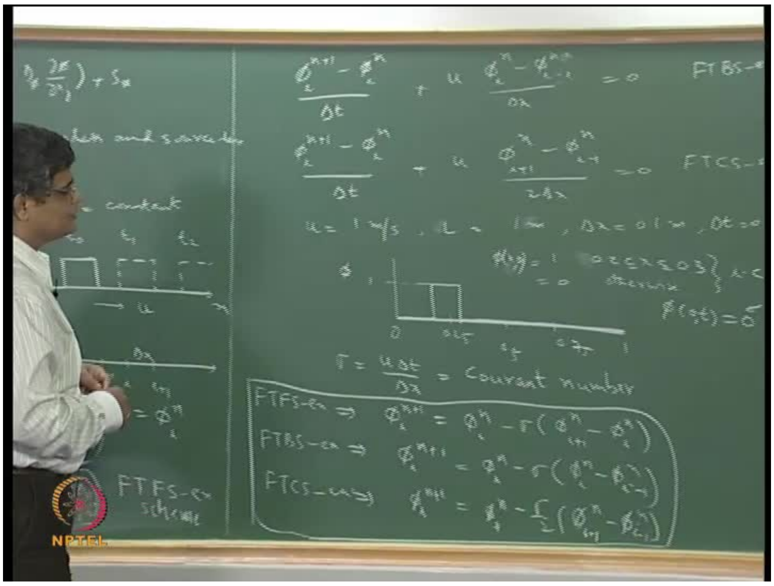

We have considered the simple 1 dimensional wave equation ,which is not as complicated as the whole equation, that we are trying to solve, and we have simplified it such that u is constant. We have written 3 eminently possible finite difference approximations ,containing certain distinctive features ,that we are expecting .That is with 1 has more accuracy, than the other, and therefore the solution should be more accurate . We have chosen reasonable values for phi and all these things, we have chosen a domain ,in which we are expecting the solution to go from here to here.

So that there is no bound effect of the end conditions, coming into picture. So in spite of this, carefully chosen simplified case ,the solution that we are getting from the all the 3 ,that we have considered, is very poor except .For a specific value of the courant number, which is equal to 1 only, when the courant number is equal to 1, we got a solution ,which is matching with the expected solution ,that it is travelling at constant speed of u in the x direction ,with an unchanging amplitude and shape.

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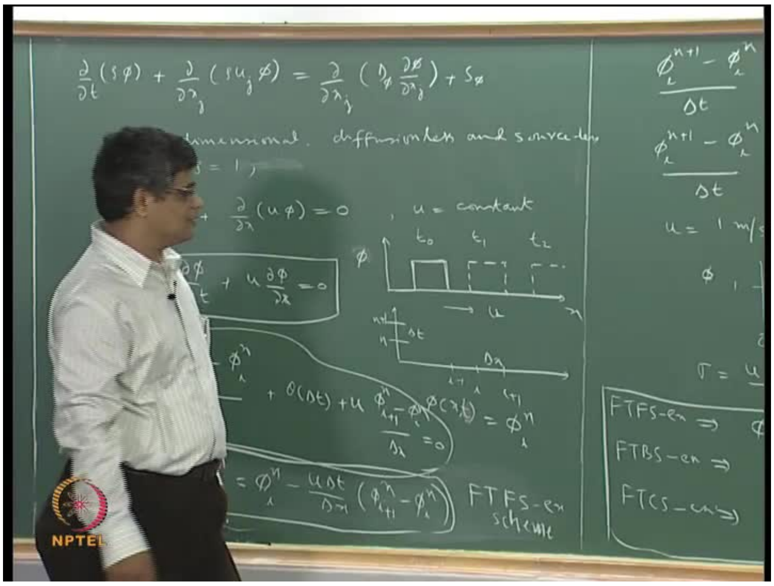

We found that when the courant number is reduced, thereby employing a smaller delta t which should give us more accurate solution .We found that when sigma is reduced with the FTBS scheme, we still got a reasonable solution. Reasonable in the sense, that we got a solution, which is propagating in the positive x direction at the speed of u .But instead of getting a square pulse ,we are getting a smeared pulse, more like a Gaussian distribution, which means that is more like a diffusion which is entering in our computed solution.

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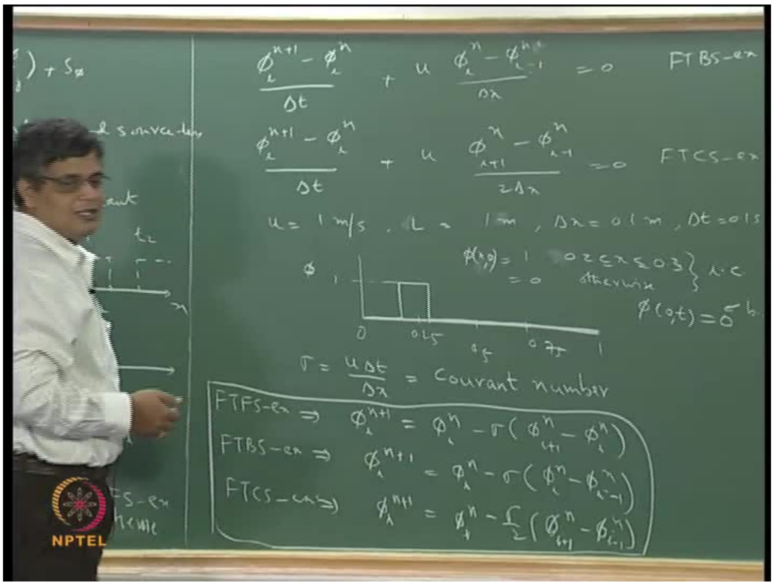

Whereas the true equation does not have any diffusion term, and not only that as we decrease the sigma further by decreasing the delta t further. That is by making the approximation here, more accurate, we got more smearing ,which is counterintuitive part with FTFS scheme ,which should behave roughly in the same way as FTBS scheme .As for as the accuracy is concerned ,we are getting a totally different solution .We got a solution, which is nowhere close to what we expect . We did not even get the sense of propagation of the lorry ,or the wave correctly. Not only that the value of phi ,which should be one, we are getting as plus 400 and minus 400 ,so that was totally unacceptable for the 3 different values of sigma .It did not seem to be very sensitive, to the value of sigma .In the sense that no matter what value of sigma, we used we are getting a clearly unsatisfactory solution.

Now ,if we when we look at FTCS method ,which is second order accurate in space .Therefore it should give us a better solution .Especially for the spatial variation, we again had a shock there .For any value of sigma ,we are getting a solution ,which is not as good as what we are getting with the first order accurate solution. So the solution, that we are getting to the simple equation using straight forward uncomplicated discretization method, finite difference approximations is giving results ,which are not correct even with a very few number of steps. It is not that there is some sort of round of error ,which is actually spoiling the results , because the solution of plus or minus 400 or 5 ,that we got for FTFS scheme was obtained ,at as shorter time as point 05 seconds.

So, that is within 5 time steps, the solution exploded ,from the initial value of 5, to 400 .It is clearly not because of any kind of round of errors .Round of errors have a tendency, to accumulate over a large number of repeated calculations. So what we find damaging in this,what we find unknowing in this ,is the fact ,that the solution ,the approximation, which is mathematically equivalent to .There are 2 solution in the limiting case of delta t and delta x going to 0, is exhibiting a behavior, which is sensitive to the very parameters, that we are choosing, that we need to choose when we attempt a computer solution.

The parameters that we need to choose, are delta x and delta t .Because that is all that is there, and the other parameters, that we choose is what kind of approximations that we make .So if we go for more accurate solution ,by going for a central in space ,which seems to be a reasonable approximation ,we are not getting any better accuracy .If you are decreasing the value of delta t, we are not getting the any better accuracy .

So in that sense for the choices,that we have the choices, that we make are profoundly influencing the solution that we are getting in this very simple case. With or without any kind of nonlinearity, that one can expect to complicate the matters and that to without going through a large number of repetitive calculations so the solution is blowing up just like that.

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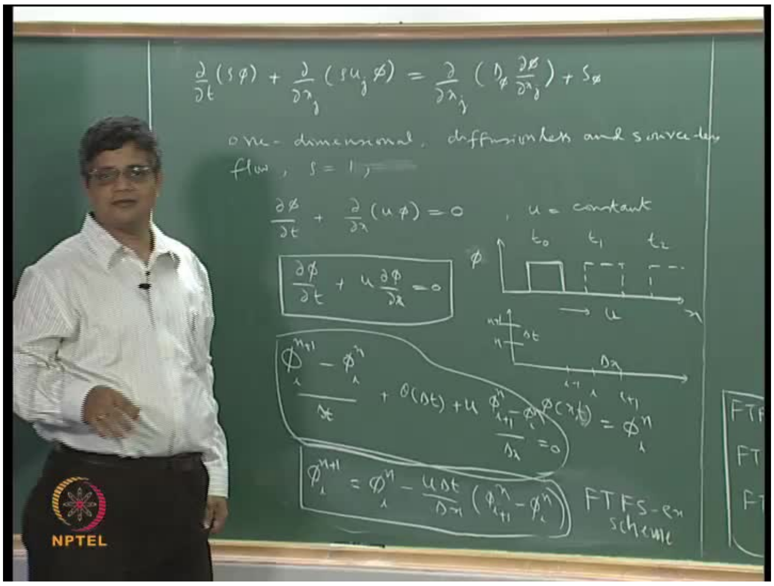

So, we need to have a more systematic way in which we can identify what kind of parameters that we need to have .So that we can get a satisfactory solution .So our initial bravado, that we are now able to solve, write ,finite difference approximations, for these things ,and then go for a solution is now totally misplaced .Because even for a subset of this equation, we are not able to get a method of finite difference, or combination of finite difference approximations ,which is giving us a reasonably satisfactory solution .So we need to understand more about the nature of the discretized equation ,because there is nothing wrong with the actual equation .

This is a simple equation which has a solution like this .So the numerical ,the partial differential equation here ,is very clear and the boundary conditions and initial conditions are nothing at fault here .It is just the approximations, that we are making here ,are really causing all the problems .So this is where ,we have need to be careful, about what kind of approximations, that we make in order to get a computed solution, and then what kind of values, that we choose for delta x and delta t.

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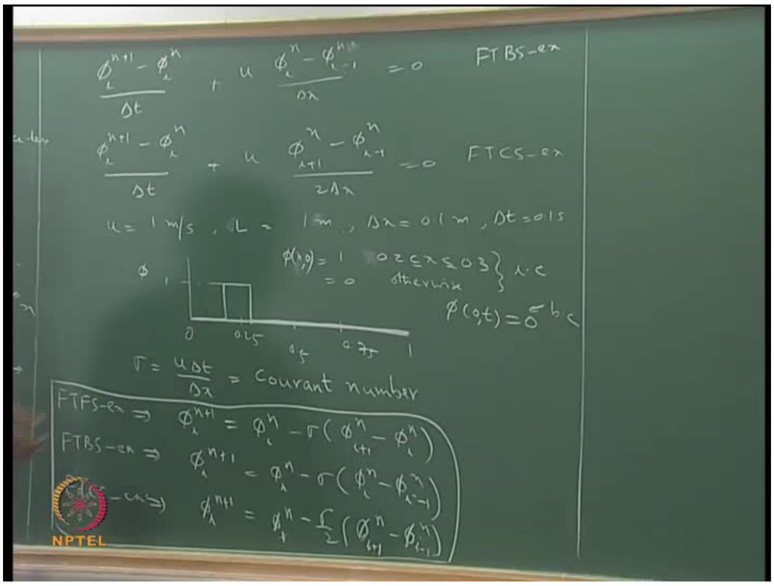

In this particular case, we have fixed the value of delta x .We have changed the value of delta t, you can fix the value of delta t ,and change the value of delta x . See what will happen ,and you can change both of them, and see whether you can get a satisfactory solution. But unless we have a systematic way of analyzing this , we cannot go on trying different values of delta x and delta t. Because this is the case, with which is one dimensional .

In the general case we may have a 3 dimensional thing .We cannot arbitrarily keep on trying all possible values of delta x and delta y and delta z and delta t. So that kind of thing ,is not really at any kind of trial and error approach ,to finding the right combination of parameters, or the right combination of the difference approximations, is not appropriate .So we have to have a systematic approach, to identify the kind of discretization method for the overall equation .

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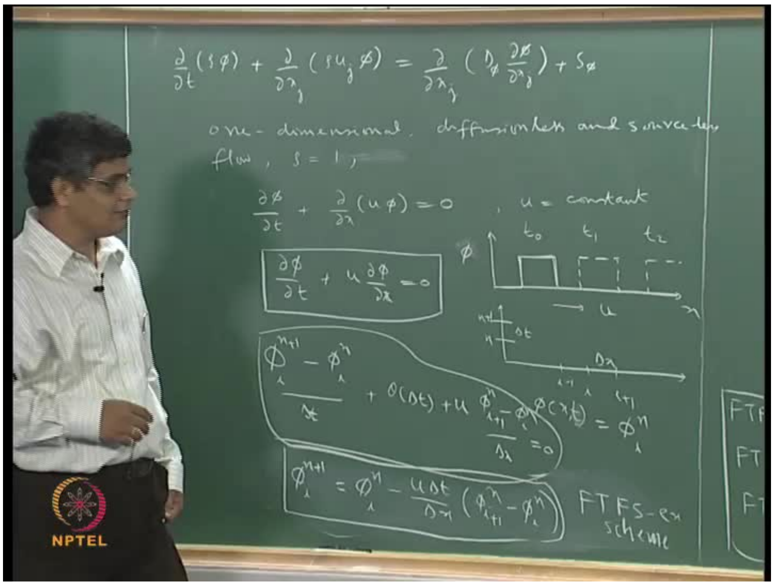

What we need to understand here ? what we need to emphasize here, is that taken in themselves ,a finite difference approximation of the first derivative, in the form of this ,or the spatial derivative here, in the form of forward differencing, and backward differencing ,or central differencing .They are it is putting them together into 1 equation ,and then looking at the evaluation of five subject to both the terms is causing problems.

So what kind of combination of discretization methods and approaches, is going to give us to proper solution or a satisfactory solution, for different cases is something that has to be systematically analyzed . Only when we do that, we can hope to have the assurance of getting a solution .Even as we set out ,to write the solution scheme for a phi equal to b, and so on that sort of analysis, of the discretized equations is necessary . That is an important step ,before we go from the partial differential equation, to spatial discretization, to finite difference approximation and finally to the solution of a phi equal to b .

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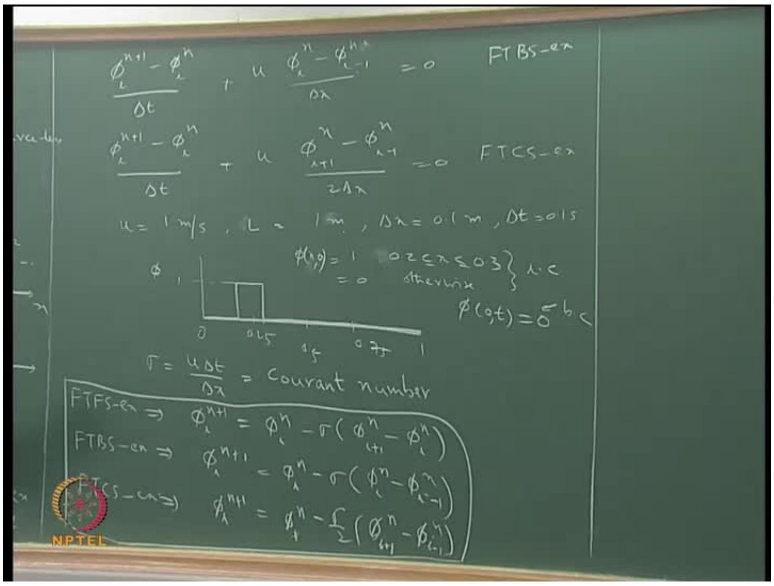

So the solution of a phi equal to b ,must be preceded by an analysis ,to make sure that we are solving the right combination ,of finite definite approximations, for the governing equation.

So ,this is what we need by means of analysis of the discretized equations and the analysis is very well laid out, and cleanly defined and understood for a linear equation.

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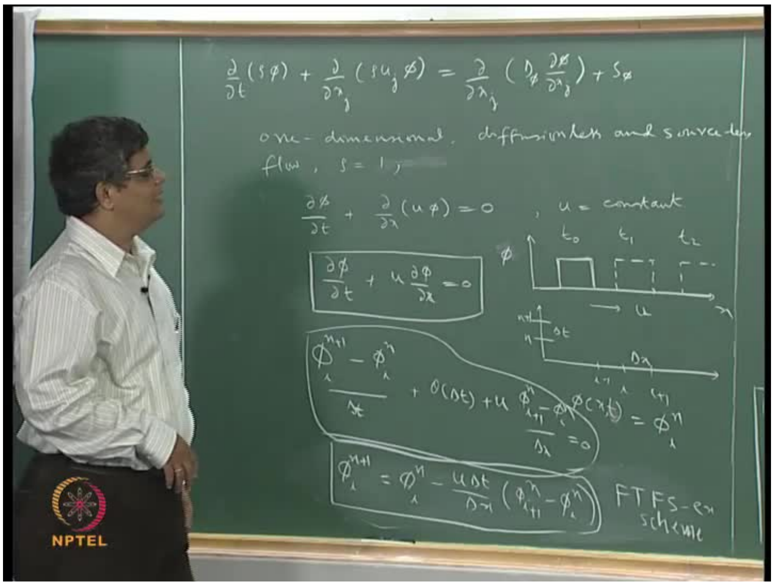

When we are dealing with the linear partial differential equation ,then analysis is really understood .But for non-linear kind of equation ,which is what we normally have in the momentum equation .When you put u i here ,then this term becomes non-linear .When you have a diffusion here ,which may dependent on some other things ,then it may become non-linear .

So there is a possible source of nonlinearity .There is nonlinearity in the advection term, when we consider the momentum equation. There is a possible source of nonlinearity here, and there is a possible source of nonlinearity .For a non-linear governing equation, we do not yet have a full proof method of analysis . But for a linear equation, we have a full proof method of analysis ,which tells us a priori ,whether or not, we are going to get a proper solution .

We will see in the next lecture ,what we mean by a proper solution, and how we can assess a given discretization scheme ,for this properness of the expected solution expected computer solution.