

Indian Institute of Technology Madras

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Chemical Engineering Thermodynamics

by

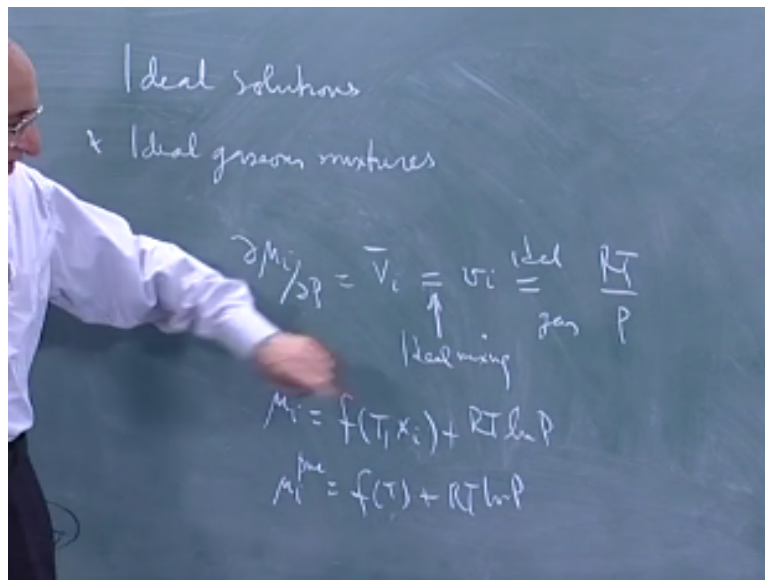
Prof. M.S. Ananth

Department of Chemical Engineering, IIT Madras

Lecture 15

Gaseous and Liquid Mixtures

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Okay we get started in order to discuss ideal solutions actually one topic I left out is equations of state discuss that couple of classes put on the line because we have gone on to liquid mixtures and also discuss ideal gaseous mixtures and do equations of state later okay in the case of ideal gas gaseous mixtures is one redeeming features that helps you solve the Gibbs Duhem equation using experimental data.

The way this happen is you have  $\partial \mu_i / \partial P = \bar{V}_i$  bar is an exact expression in the case of ideal mixtures ideal gaseous mixtures first of all for ideal this is ideal mixing if the process of mixing

is ideal there is no volume change on mixing because ideally molecules do not occupy volume so there is change in volume and ideally molecules do not interact with one other so there is enthalpy change.

So these are the 2 basic definitions of ideal mixing, if that is true this is then if it is an ideal gas that is with the pressure is sufficiently low  $V_i$  is simply  $RT/P$  so you get  $\mu_i =$  if you integrate this is done at constant pressure and composition and temperature in composition and temperature in composition so you will get some function of temperature and composition  $+ RT \ln P$ .

In particular you have  $\mu_i$  pure is some function of temperature  $+ RT \ln P$  to find this function of composition you separate out  $M_i$  and  $RT \ln P$  you have  $F(T)$  and  $X_i$  you know you have to solve the Gibbs Duhem equation now let us put down the I will come back to this in a minute.

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Handwritten notes on a chalkboard:

$$\underline{GD\epsilon}$$

$$\sum_i X_i \frac{\partial \mu_i}{\partial X_j} = 0$$

Simplest solution is  $\mu_i \sim \ln X_i$

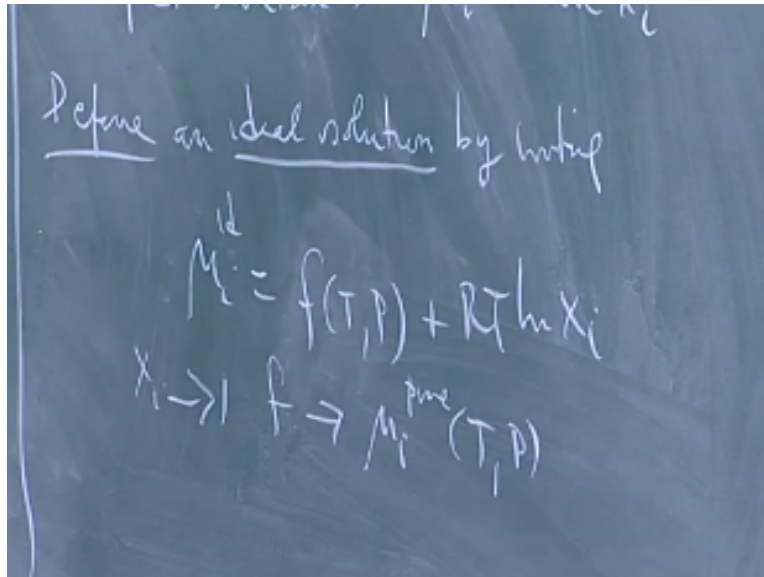
Define an ideal solution by writing

$$\mu_i^{id} = f(T, P) + RT \ln X_i$$

But let me put down the Gibbs Duhem equation we have already seen this is simply  $X_i \partial \mu_i$  with respect to  $X_j$  I said the simplest solution is  $\mu_i$  varying has  $\log X_i$  right varying has because one left side is energy per the mole the right hand side is dimensionless so you have to do some corrections, so I can define this thing an ideal solution I will show you they I can also come from another root from the gaseous behavior and come to the same conclusion.

First of all define an ideal solution by writing  $\mu_i^{\text{ideal}} = \mu_i$  if you okay I will just write or I will write this has function of T in fact tell me get this function of T, P this is the definition of solution now clearly as  $X_i$  goes to 1  $f$  should go  $\mu_i$ .

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This ideal refers only to the mixture I mean only to this mixing process so in the limit as  $X_i$  goes to one you do not have mixing left hand side will become  $\mu_i^{\text{pure}}$  so  $f(T)$  as to be  $= \mu_i^{\text{pure}}$  wherever the value of the chemical potential at that temperature and pressure is.

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$$\mu_i^{\text{id}} = \mu_i^{\text{pm}} + RT \ln X_i$$

$$V_i = \frac{\partial \mu_i^{\text{id}}}{\partial p} = v_i$$

$$-\bar{H}_i = \frac{\partial \mu_i^{\text{id}}}{\partial T} = -\frac{h_i}{T^2}$$

So I have  $\mu_i^{\text{id}}$  this will give all the results that you need to define an ideal solution because  $\mu_i^{\text{id}}$  ideal by P this side is simply  $V_i^{\text{bar}}$  ideal on the right hand side I simply get  $V_i$  then  $H_i^{\text{bar}}$  ideal by  $T^2$  or - of this is  $\partial \mu_i^{\text{id}}$  of T with respect to T this is regress thermodynamics but if I do the right side this comes from the right hand side on the right hand side.

If I divide by T and differentiate with respect to T at constant pressure and composition is simply get  $\partial \mu_i^{\text{pure}}$  with respect to T  $\mu_i^{\text{pure}}$  by T with respect to T which is  $H_i$  sorry -  $h_i/T^2$  so you get  $h_i^{\text{bar}}$  is equal to  $h_i$   $V_i^{\text{bar}} = V_i$  which is a definition of ideal mixing. Ideal mixing says mixing without change in volume and without change in energy enthalpy I can go further because of this if I have an ideal gas Then  $\mu_i = \mu_i^{\text{pure}} + RT \ln P$ .

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$$\mu_i^{id} = \mu_i^{pure} + RT \ln X_i \quad \dots (8)$$

$$\bar{V}_i^{id} = \frac{\partial \mu_i^{id}}{\partial P} = v_i$$

$$-\frac{\bar{H}_i^{id}}{T^2} = \frac{\partial (\mu_i^{id}/T)}{\partial T} = -\frac{h_i}{T^2}$$

For an ideal mixture of ideal gases

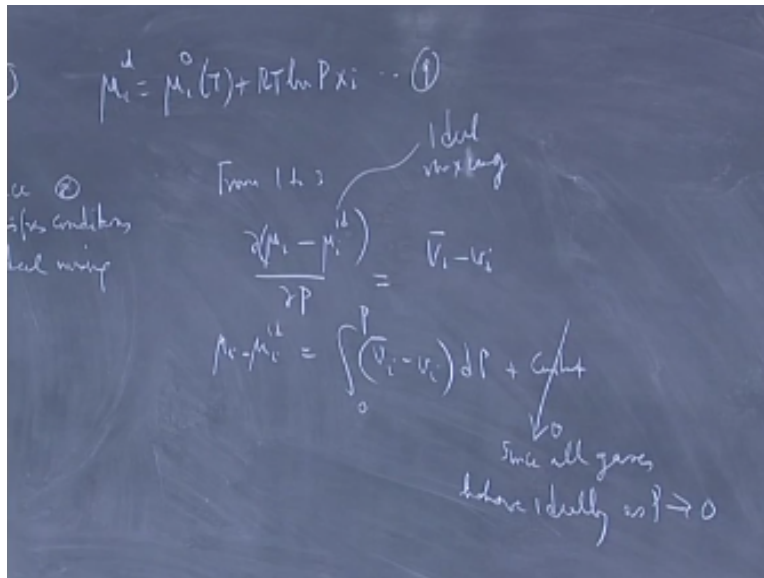
$$\mu_i^{id} = f(T) + RT \ln P X_i$$

I am sorry for a pure substance again this let me put this in here so if I have  $\mu_i$  ideal for an ideal gas mixture for a mixture of ideal gases I have to write two lines there 2 words there for an ideal mixture of ideal gases the ideal mixing process in can be in the liquid phase can be in the gas phase but I am taking this special case of ideal mixture of ideal gases for  $\mu_i$  pure for an ideal gas I got this result.

All of this is for this is for ideal gas right because I have substituted ideal gas here ideal mix of ideal gases so I get  $\mu_i$  ideal = some function of T +  $RT \ln P X_i$  you could have to give an interpretation for  $f(T)$  simply to say what it represents when  $X_i$  goes to 1 and P goes to 1 this term goes to 0 when  $X_i$  goes to 1 you get pure substance pure i so pure i at  $P = 1$  is sufficiently low pressure we usually measure P in bars.

So if you use those units then when P is = 1 bar you get  $f(T)$  is simply  $\mu_i$  the chemical potential of pure i at the temperature of the mixture at pressure = unit T.

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So we write this as  $\mu_i = \mu_i^d$  this is  $\mu_i^d = \mu_i^o(T) + RT \ln P x_i$ . Also from this equation I will be give these equations numbers this is 1 and  $\bar{V}_i = v_i$  is for ideal mixing this is for ideal gas this is 3 so this is will be 4 this will be 5 this of course 1 number every day 6, 7 then 8, this simply proves that this is hence 8 satisfies conditions of ideal mixing are simply  $\partial V_{mix} = 0$   $\partial h_{mix} = 0$  then finally this here 8 let us say this is 9 first of all from equations 1 to 3i have  $\partial \mu_i^{actual} - \mu_i^{ideal} / \partial P$  this when I write ideal here it only means only ideal mixing from  $\mu_i^d$  okay where I have detailed mixing of ideal gas from  $\mu_i$  okay where I have detailed mixing of ideal gases I write it explicitly.

But normally when I write  $\mu_i^{ideal}$  I only mean ideal mixing that is the gases need not be individually ideal but I am assuming the process of mixing is ideal this controller happen at very high pressures between say ethane, methane mixtures similar substance will mix without change in volume without change in so if I take this difference this is  $\bar{V}_i$  first one if I take differentiae this I get  $\bar{V}_i$  now  $\mu_i - \mu_i^{ideal}$  therefore = I am  $\int$  from 0 to P of  $\bar{V}_i - v_i dP +$  some constant.

Now I can determine the constant very simply in the case of gas phase because in the limit has P goes to 0 all gases behave ideally therefore this  $\mu_i$  also becomes a mixing of ideal gases therefore I have  $\mu_i - \mu^{ideal}$  is 0 at  $P = 0$ . So the constant is this is because this is 0 I will write here since all gases behave ideally as P goes to 0.

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The image shows a chalkboard with handwritten mathematical derivations. On the left, a box contains the following text:

$\Delta G_m = S + VM$   
 $\phi_i = \phi_i^*$   
 $\phi_i^{pure} = \phi_i^*$

The main derivation on the right is as follows:

$$\int_0^P (\bar{V}_i - v_i) dP = \int_0^P \left( \bar{V}_i - \frac{RT}{P} \right) dP$$

$$= \int_0^P \bar{V}_i dP - \int_0^P \frac{RT}{P} dP$$

$$= RT \ln \phi_i - RT \ln \phi_i^{pure}$$

I have this  $\int_0^P \bar{V}_i - v_i dP$  I write this as  $\int_0^P$  or  $-$  I am adding and subtracting  $RT/P$  inside the  $\int$  and splitting the integral into two parts this is defined as this is 3 lines is just a definition write this as  $RT \ln(\phi_i) - RT \ln(\phi_i^{pure})$  when  $\bar{V}_i$  is replaced by small  $v_i$  this becomes the specific volume of the pure substance so I recognize this by writing  $\phi_i^{pure}$  indecently smith and van ness is a slightly different notion for  $\phi_i$  here writes  $\phi_i^*$  for this he writes  $\phi_i$  so when he writes  $\phi_i$  he is talking about pure fugacity.

The mixture value fugacity coefficients this is called fugacity coefficient I will come back to it in a minute  $\phi_i$  is called  $\phi_i^*$  is the value on the mixture  $m_2$  used to this notion to change if I change  $\frac{1}{2}$  with  $\partial$  will make a mistake again, I will stick to this notion, so  $\phi_i$  for us is  $\phi_i^*$  our notion smith and van ness our notion is  $\phi_i^{pure}$  and this is  $\phi_i$  similar  $\mu_i^*$  in smith and van ness is chemical potential in the mixture and  $\mu_i$  is the chemical potential of pure.

The reason we give these symbols is because you need default options in chemical engineering you can but  $\phi_i = 1$  as an approximation where as you would not know what the default option for chemical potential was, so we will not really done any significant at this stage we are not made progress in terms of defining the composition dependence is a chemical potential but we have expressed it in terms of quantity for example in a gas phase this is completely measurable  $\bar{V}_i$  is measurable.

So if I know the composition dependence of the chemical potential if I know the composition dependence of this molar  $\partial$  molar volume that is all I am saying so if I know 1 I can get the other I cannot make this connection in the liquid phase because liquids do not have a asymptotic

behavior either at a limit of temperature or a limit of pressure because all gases behave ideally at  $P = 0$  this integration constant vanished for you we would not such a facility in the liquid phase.  
S

On in the liquid phase you have stuck with solving this equation directly here if you experimental data on  $V_i$  bar you have a solution of the Gibbs Duhem equation because the Gibbs Duhem equation simply tells you then that  $\mu_i - \mu_i^{\text{ideal}}$  is this and  $\mu_i^{\text{ideal}}$  is given by this so you got your whole this thing worked out so let me get back and write this equations.

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$$\int_0^P (\bar{V}_i - v_i) dP = \int_0^P \left( \bar{V}_i - \frac{RT}{P} \right) dP$$

$$= \int_0^P (\bar{V}_i - \frac{RT}{P}) dP$$

$$\approx RT \ln \phi_i - RT \ln \phi_i^{\text{pure}}$$

$$\mu_i^{\text{pure}} = \mu_i^{\phi} + RT \ln \phi_i^{\text{pure}}$$

Notation  
 $G_{\text{mix}} = S + Vx$   
 $\mu_i = \phi_i$   
 $\mu_i^{\text{pure}} = \phi_i$

$\mu_i^{\text{pure}}$  in particular okay let me write this out  $\mu_i^{\text{pure}}$  is  $= \mu_i^0$  sorry this is  $\mu_i^0$  I can just simply write  $\mu_i^0$  because  $\mu_i^0$  that I have the notation I have used there is the chemical potential of pure  $i$  at the temperature in question and pressure = 1 + this would have been ideal case times  $V_i^{\text{pure}}$  because if I did the same  $\mu_i$  for the pure case  $\mu_i^{\text{pure}} = 0$ .

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$$\frac{d(\mu_i - \mu_i^0)}{dP} = \bar{V}_i - v_i$$

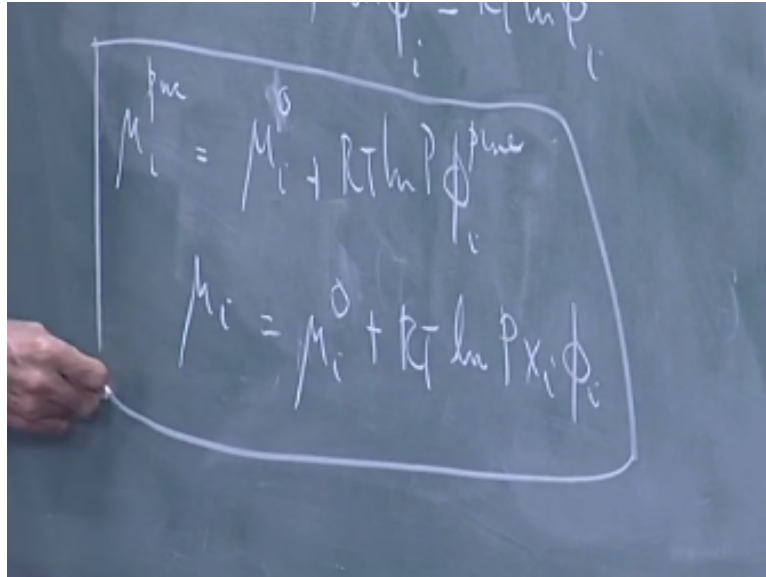
$$\mu_i - \mu_i^0 = \int_0^P (\bar{V}_i - v_i) dP + C_i$$

Since all gases  
behave ideally as  $P \rightarrow 0$

$$\mu_i^{\text{pure}} - \mu_i^0 = \int_0^P \left( \bar{V}_i - \frac{RT}{P} \right) dP$$

$\mu_i$  ideal pure would have been simply  $\bar{V}_i - RT/P$  may be is should write it here same follow us that  $\mu_i$  pure -  $\mu_i$  ideal simply = 0 to P because it is pure  $\bar{V}_i$  bar becomes simply  $\bar{V}_i$  the second case because it is ideal it becomes way - RT dP this is  $\mu_i$  ideal pure in  $\mu_i$  ideal pure we had already that is  $\mu_i^0$  you would have  $\mu_i$  ideal pure would have been  $\mu_i^0 + RT \ln P$  or  $RT \ln$  if I multiply by oh there is no  $x_A P \phi$  pure  $\mu_i^0 + RT \ln P$  would be the value for  $\mu_i$  pure ideal add on a correction for non ideality which is  $\phi_i$  pure.

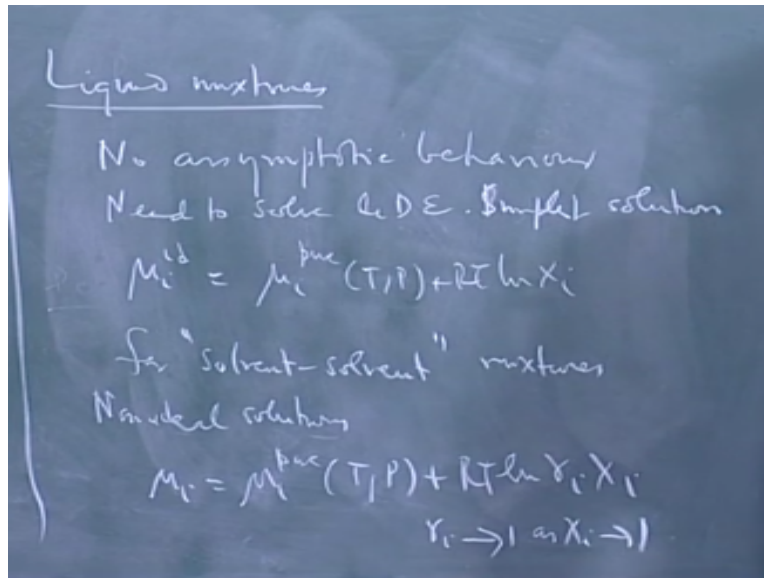
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Then I have  $\mu_i$  in the actual mixture =  $\mu_i$  ideal mixture ideal mixture +  $RT \ln P x_i$  into  $\phi_i$  I have actually  $\mu_i = \mu_i^0 + RT \ln P x_i \phi_i$   $\mu_i$  ideal mixture is already there  $P x_i$  is there so this is  $\mu_i^0$  sorry I have to come back. These are the unique models for gas phase  $\phi_i$  is given in terms of  $V_i^{\text{bar}} - RT/P$   $\phi_i$  pure is simply  $V_i^{\text{bar}} RT/P$  for gases this completes a description remember that the Gibbs Duhem equation is different return for a homogeneous phase.

When you write  $Dg = -SdT + VdP$  now writing for a homogeneous phase so you have to solve this since luckily we have only 3 phases of aggregation you have to solve it for the solid state the gaseous state and the liquid state separately so I have done the gas phase now we will do the liquid phase.

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Liquid mixtures for liquid mixtures first know asymptotic behavior if your able to find an asymptotic behavior you are in for a noble price here but some of these things are not worth searching when the liquid range has been covered people have looked everywhere so you have to solve need to solve Gibbs Duhem equation what we do is to simplest solution is  $\mu_i$  ideal again I still have okay possible simplest solution is however you must notice that in liquids you have all kinds of mixtures I can have gas dissolved in liquid.

So suppose  $i$  refers to carbon dioxide in water and  $I$  referred to carbon dioxide then has if you go to the temperature of the solution if you go to the temperature in pressure of the solution go to the pure state the pure state is a gaseous state it is not realizable in practice that is I cannot take a mixture of composition  $X_i$  where  $X_i$  represents the composition of carbon dioxide mole fraction of carbon dioxide in a mixture and take their mole fraction continuously to one without change of phase and Gibbs Duhem equation is written for a single phase.

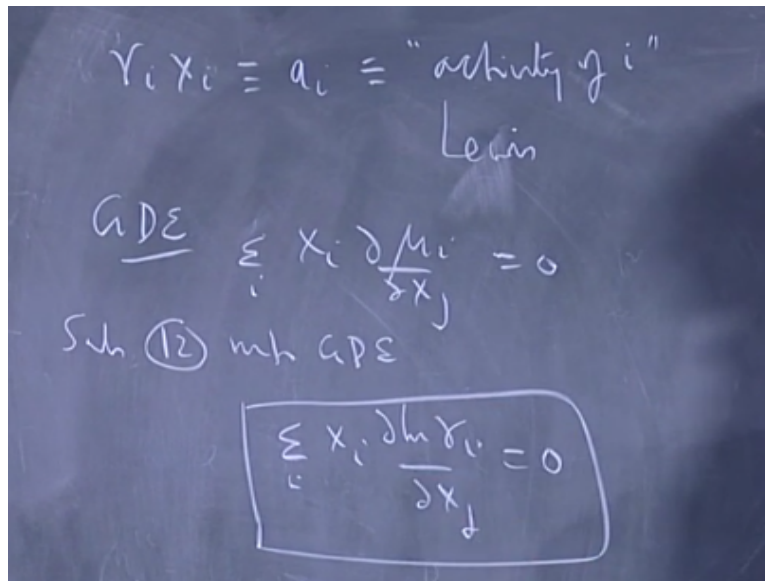
So I have to solve within that phase so what we do is write this has simplest solution for give name for mixtures in we classify mixtures into solvents, solvent mixtures and solvent solute mixtures, so this is valid only for solvent – solvent mixtures in solvent – solvent mixtures you can take the mole fraction of all the components individually to unity without change of phase that is the definition of a solvent – solvent mixture.

For every component I can go to mole fraction unity and still get preserve liquid state so this is a simplest solution this is A solution so what you do in practice is write for non ideal solutions or

real solutions as here you have  $\mu_i - \mu_i^{\text{pure}}$  at TP again for solvent – solvent mixtures only will treat the other case separately.

I write  $\mu_i = \mu_i^{\text{pure}} + RT \ln \gamma_i X_i$  that where I have to tell you what  $\gamma_i$  is in this case when  $X_i$  goes to one left hand side becomes equal to this so  $\gamma_i$  is 1 in the pure state so I you write a along with this you notice that  $\gamma_i$  goes to one as  $X_i$  goes to 1.

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This quantity  $\gamma_i \times X_i$  is called  $a_i$  is been called activity and to the term coined by Lewis long ago incidentally Lewis is credited with being the first expositor of classical thermodynamics because Gibbs need an interpreter what Gibbs said was absolutely right but very tedious Gibbs would say in the foot note obviously and he will say in the foot note if necessary this can be derived and so on then Lewis will do the derivation.

So Lewis is book at all the just like he took notice from the master and filled them up so in ideal solutions the role of non ideal solutions the role of mole fraction is played by  $a_i$  the activity and the activity default option and all chemical engineering design programs as I said  $\gamma_a = 1$  but yiu can go terribly wrong, there are mixtures in which  $\gamma_i$  can be 1500 which means  $X_i$  being the being a representative of  $\mu_i$  is meaningless  $\log X_i$  does nor any longer represent.

But those are exceptions but typical  $\gamma$  values of 2, 1.5, 2, 3 etc very common many mixtures so we will do lots of examples in which these and that will make a lot of differences I told you finally you are going to use  $\Delta G$  is the work done  $\Delta G$  is  $X_i \mu_i - X_i \mu_i^{\text{pure}}$  for the of  $\mu_i^{\text{pure}}$  so that is

the change when you do mixing or unmixing you take air separated into oxygen and nitrogen you can calculate how much work is done in the process.

I will do that calculation when you do difference essentially this  $RT \ln \gamma_i$  represents the work in bring you show that it is equal to the work in bring 1 mole of a  $i$  into a mixture of  $j$  so if you have a binary system and since the work depends on log of  $X_i$  if it is  $\log \gamma_i X_i$  and  $\gamma_i$  turns out to be 2 you will make a mistake of  $R \ln 2$  it can be large differences that is all so this is your activity coefficient.

Now you have not really done anything only change words because if substitute this into the Gibbs Duhem equation now let me go to the Gibbs Duhem equation it will play it you solve it in some way or other now substitute this quantity give these finished 9 I think last we will call this 10 this can be 11, 12 so substitute 12 into the Gibbs Duhem equation do this by inspection  $\mu_i$  pure non function of  $X_j$  at all so if it will cancel  $RT \ln X_i$  satisfies the Gibbs Duhem equation.

So when I do sum over  $i$  of  $X_i \partial$  of  $\log X_i$  with respect to  $X_j$  you know we get 0 right for come  $ij$   $i=j$  you will get 1 for the last component  $r$  you will get  $-1$  so you will get 0 so only thing that remains is  $\log I$  and  $RT$  is constant so this will give you sum over  $i \partial$  of  $\log \gamma_i$  with respect to  $X_j = 0$ , to solve this you go back to the other formalism remember we wanted to look at  $\partial G$  you model  $\partial G$  and you get all your equations for  $\mu_i$  instead of  $\mu$  I now you will get  $\log \gamma_i$  so the equation.

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Binary

$$\frac{g^E}{RT} = x_1 \ln \gamma_1 + x_2 \ln \gamma_2$$

Swa

$$\Delta g - \Delta g^{id}$$

$$= (g_{am} - g_{am}^{id}) - (g_{am}^{id} - g_{am}^{id})$$

$$= \sum x_i (\mu_i - \mu_i^{id})$$

$$= RT \sum x_i \ln \gamma_i$$

So the equations are very simple you get g excess by RT I will write this for a binary X1 now log  $\gamma_1 + X_2 \log \gamma_2$  reason is g excess is  $\Delta g - \Delta g_{\text{ideal}}$   $\Delta g$  is g after mixing – g before mixing – g after mixing ideal – g before mixing g excess / definitions  $\Delta g - \Delta g_{\text{ideal}}$  so this g after mixing – g after mixing ideal g after mixing is sum over  $X_i \mu_i$  these 2 will cancel before mixing these 2 will cancel I am looking at the ideal mixing process now.

$X_i \mu_i - \mu_i^{\text{ideal}}$  so  $\mu_i^{\text{ideal}}$  is  $\mu_i^{\text{pure}} + RT \ln X_i$   $\mu_i^{\text{non ideal}}$  is  $\mu_i^{\text{pure}} + RT \ln \gamma_i X_i$  so the only difference is  $\log \gamma_i$  so this is =  $RT \sum$  SO g excess for binary this in particular will give you  $x_1 \log \gamma_1 + x_2 \log \gamma_2$  okay suppose I should say here since this is okay so then do the same thing that I did before.

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Binary

$$\frac{g^E}{RT} = x_1 \ln \gamma_1 + x_2 \ln \gamma_2$$

$$\frac{\partial (g^E/RT)}{\partial x_1} = \ln \gamma_1 - \ln \gamma_2 + \left( x_1 \frac{\partial \ln \gamma_1}{\partial x_1} + x_2 \frac{\partial \ln \gamma_2}{\partial x_1} \right)$$

$GDE$

$$\ln \gamma_1 = \frac{g^E}{RT} + x_2 \frac{\partial (g^E/RT)}{\partial x_1} \quad (14)$$

Differentiate this  $\partial$  of  $\partial g$  excess by  $RT / \partial X_1$  is  $\log \gamma_1 - \log \gamma_2 +$  and this term is 0 because is the Gibbs Duhem equation I just to solve these two equations algebraically equation 12 this is 13 this 14 and if you recall and we did one last equation we really got an expression for g excess not

for  $\partial g$  the models for solid mixtures are identical we do not often use them in chemical engineering.

We use them in metrological engineering because basically between solids and liquids is not really much difference they both represent the condense phase molecules reasonably closely packed in the density of the solid is not very different from that of the liquid we have very well defined crystal structure in solids in a amphora solids you do not have a crystal structure but you have a well defined reasonably well defined structure in the case of the liquids you have a little more entropy.

So the model for solids mixtures are similar but we do not use them often in chemical engineering to it main solid mixture problems are in metrological thermodynamics so primarily we will deal with liquid mixtures we will of course deal with gaseous mixtures but most of the time your gaseous mixtures will behave themselves when you will have to calculate frigorific coefficient but you will know the equation of state and you can calculate it that way.

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Solvent-solvent mixtures		
Name	Model for $g^E/RT$	$\ln \gamma_1$
Ideal	0	0
Pitzer's	$A x_1 x_2$	$A x_2^2$
Margules 2-suffix	$A x_1 x_2 (1 + B x_1)$	0 Walas Phase Equilibrium

For this you will have to go through for liquid mixtures I will write the model for  $g$  excess in the corresponding  $\log \gamma_1$  you can get this from tables this is you will have to write this separately for 2 classes of mixtures and deal with solvents solute mixtures later for solvent solute mixtures the model for the solvent will remain the same but for the solute I will have to write it differently  $g_x$  as can be 0 is a ideal solution  $\gamma_1 = 1$  or I will just write  $\gamma$  its is okay  $\log \gamma$  or not  $g_x$  is can be  $A x_1$

$x_2 g_X/R_T$  this is called the misses ideal given name for the model this is called porters equations and if you do that solution that I have given in 14 you take this expression differentiate it and so on.

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Handwritten mathematical derivation on a chalkboard:

$$g^E/R_T = x_1 \ln \gamma_1 + x_2 \ln \gamma_2$$

$$\frac{\partial g^E/R_T}{\partial x_1} = \ln \gamma_1 - \ln \gamma_2 + \left( x_1 \frac{\partial \ln \gamma_1}{\partial x_1} + x_2 \frac{\partial \ln \gamma_2}{\partial x_1} \right)$$

$$\ln \gamma_1 = \frac{g^E/R_T}{x_1} + x_2 \frac{\partial (g^E/R_T)}{\partial x_1} \quad (14)$$

The chalkboard also has the word "Brey" written at the top left and "GDE" written at the top right with an arrow pointing to the right.

With Aax22 for log  $\gamma_1$  to like for log  $\gamma_1$  here you get 0  $\gamma_1$  is 1 log  $\gamma_1$  is equal, so you can built models of this kind get  $Ax_1 x_2$  into  $1 + Bx_1$  you can write higher polynomial models this is Margulies it is called two suffix mod.

As I told you, you can write this is A11 and this is B12 so it is called two suffix I think he even writes is AA12 and A2 1 or something does not matter two parameters existing and you have to solve that I do not remember the solution but you can solve those equations simultaneously and get the values what I will do is put a table of this instantly this is very nice personally do not like the book points but it is well written and it is written by an industrial practice so it is written typically with the law out of redundancy.

It is got everything worked for an open book examination it is ideal classify and tell you the model you do not have to start solving those equations and the examiner make mistakes and

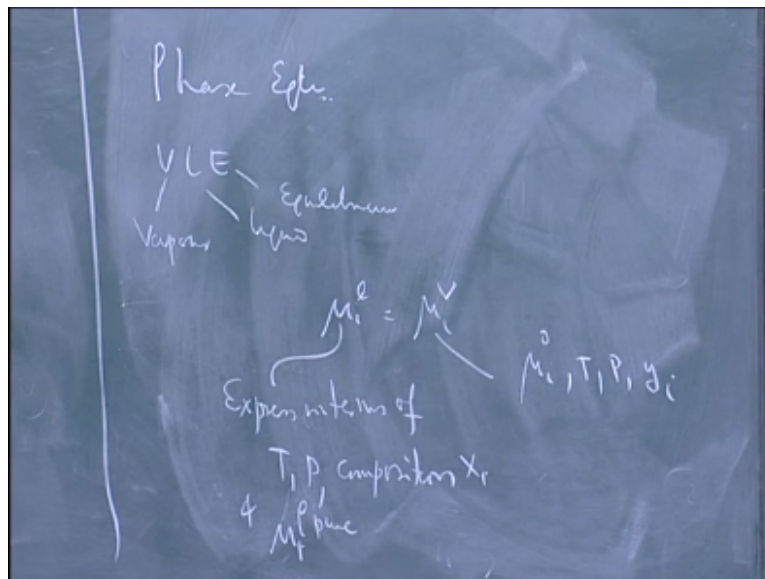


solving simultaneous equations because I will never find out where you know your thermo you will make a silly mistake there so the best thing is for you to keep these things wall as a table I will and put that table on the web itself, he has this model and he has the solution for  $\log \gamma_1 \log \gamma_2$  is got it for ternary mixtures.

For any number of multi component mixtures in case you do not know how to simplify the multi component mixture equation for a binary he will separately give you a binary table, so he is got all that it is called I think phase equilibrium thing just got a phase equilibrium I do not remember exactly, but wall is a practicing engineer for 15 20 years okay so I just said once you have models with the chemical potential the various models in and your also have to go through the model for venlor.

Which is in expression for  $R_T / gx$  is only thing you have to be careful about is at  $gz$  is has to go 0 in the limits otherwise you can propose your own model.

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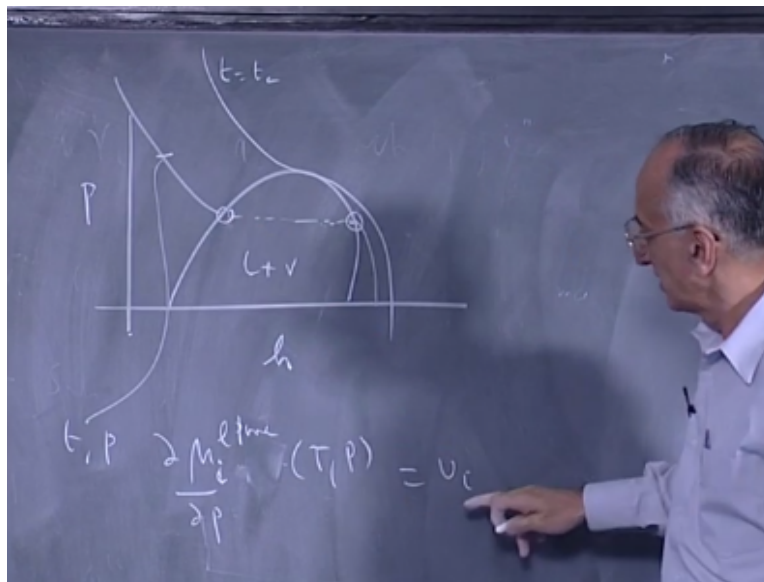


But if you have a phase equilibrium if you have vapor liquid equilibrium and you will get  $\mu_i$  liquid =  $\mu_i$  vapor this is we have already shown now the whole purpose of all this exercise was expressed  $\mu_i$  liquid in terms of measurable quantities and we will shown that everything has measurable except the composition dependents the chemical potential, so this alone you have to do modeling so with the model you get it in terms of composition we get so what you do is when you solve this phase.

Equilibrium problem for  $\mu_i$  liquid I have to go through a little more so here you express in terms of T,P and composition and you will get 1 term here that represents and  $\mu_i$  liquid pure, beaks your model is  $\mu_i = \mu_i \text{ pure} + R_T \ln \gamma_A x_i$  in your rated for the liquid phase this is  $\mu_i$  liquid pure this will come out similarly in terms of  $\mu$  this will come out in terms of  $\mu_i$  is 0 always we will come out in terms of T,P what will do hereafter for composition and the gas phase will use y for liquid phase we use x it seems to be a convention.

Composition xi so the only thing that is that is still gives you a little difficulty is this  $\mu_i$  0 that appears in the gas phase equations in the  $\mu_i$  liquid pure that appears in the liquid phase equations, so you have to related  $\mu_i$  liquid to  $\mu_i$  vapor and I will do that then I will go back and do in solids in solids solvent mixtures if you go back to your pure component phase diagram P was a H.

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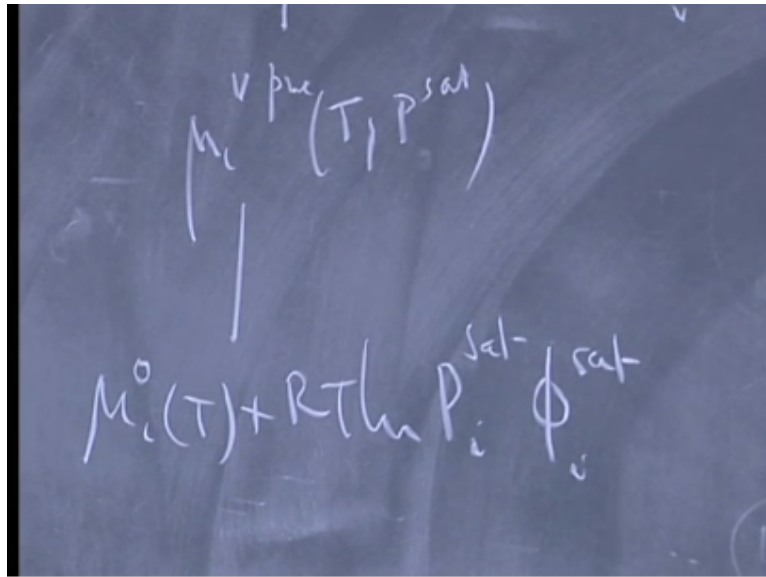


This  $t = t_c$  and you have liquid + vapor diagram here so if you talking of liquid pure you are talking of substance below the critical point it has to be that is the ways of critical point right, so the way you connect the two which to recognize that at this point the chemical potential of the liquid is same as the liquid in chemical potential the vapor, so if I want to cancel this  $\mu_i$  0 I have to express to  $\mu_i$  liquid in terms of  $\mu_i$  gas if I do that the  $\mu_i$  gas will have the  $\mu_i$  0 in it which will cancel on both sides.

So having introduced a  $\mu_i$  0 which is a hypothetical quantity which is the chemical potential of pure I at the temperature and pressure equal to 1 I do not care what it is it will cancel on both

sides, so I will simply write the equations down and cancel so to do this all I do is to take at any temperature  $T$  let say  $T, P$  I write  $\mu_i$  liquid pure this is what I want I know from thermo dynamics that this is equal to  $\mu_i$  so I integrate this between this point where I can make the connection with the gas along that temperature this is the saturation pressure.

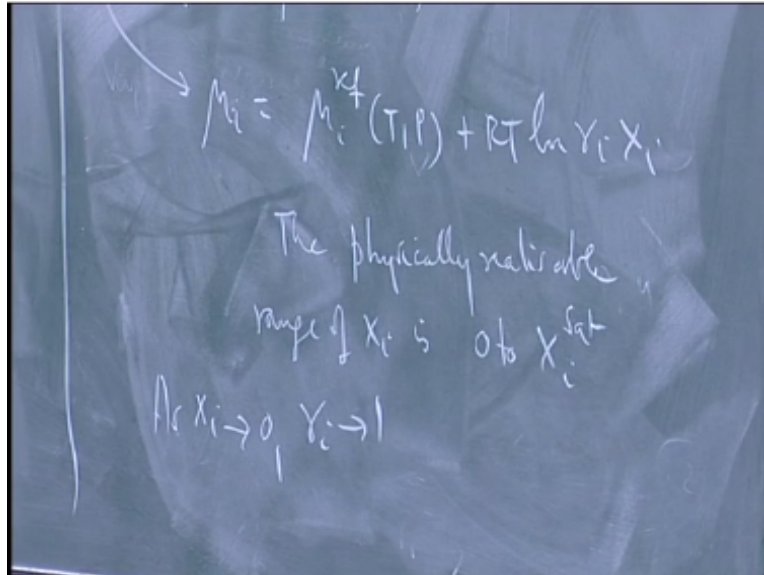
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So integrate this I get  $\mu_i$  liquid pure  $T, P$  –  $\mu_i$  liquid pure at  $T$  and  $P$  saturation =  $\int V_i \text{ liquid } dT$  from  $P$  saturation to  $P$  but this is the same as  $\mu_i$  vapor pure and  $T, P$  saturation because the pure substance in the pure substance at this saturation pressure and temperature the vapor and liquid are at equilibrium, because this is given in term this is simply  $\mu_i^o$  of  $T + RT \ln A_i$  saturation  $V_i$  saturation this is referring back to the way I treated this once I have got  $\mu_i^o$  on both sides of the equation.

I can cancel it if I do not have to worry what it is I do not even care if it goes to – it is the identical quantity on both sides, so to complete the picture let me go ahead and do solvents solute mixtures.

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For the solvent the model is the same  $\mu_i$  is  $\mu_i$  pure liquid T,P this is the complete specification because  $\gamma_i$  will go to 1 as  $x_i$  goes to for solute mixtures I still have  $\log x_i$  s an ideal solution and simply writ  $\mu_i$  reference but for the solute the physical state that I can realize is between  $x_i = 0$  and  $x_i = \text{saturation value}$  if it is carbon dioxide dissolved in water the saturation value at room temperature and normal pressure would be something like  $10^{-2}$  mole fraction whatever the value so  $x_i$  can varying.

$X_i$  is physically realizable or it physically realizable range of  $x_i$   $S_0$  to  $S_i$  saturation whatever the solubility limit is now there are two constants here one is this  $\mu_i$  reference in this  $\gamma_i$  it is a function of composition add only one unknown to began with, so I must be able to specify one number uniquely so I will have to choose an  $x_i$  reference at which I choose  $\gamma_i$  to be 1 in this case of pure substances I would go all the way to mole fraction 1 I could use the pure substances is a reference.

So here I will simply say since  $x_i$  is going to 0 is a realizable we said  $\gamma_i = 1$  you will have this conceptual difficulty that as  $\gamma_i$  goes to 1  $x_i$  goes to 0 you get  $\log$  of 0 so you get  $-\infty$  there but we would not worry about it what I am going to do in thermodynamics we also cheat you on this actually it is very valid it is not just we think what you do is get an animal that diverges but the same animal on both sides of equation then you can cancel it, so that is one way of doing it the other things in simply ways.

Although this quantity this is an expression has a function of  $x_i$  so I can divide this T and

differentiate with respect to  $x_i$  differentiate with respect to T or P if I differentiate if I divide by T and differentiate with respect to  $T_i$  will get a quantity here which represents – enthalpy of I bar partial more or enthalpy by  $T^2$  on this side I will get some reference enthalpy this term will vanish because I am differentiating with respect to temperature, so if I do that in the if I it will vanish in the limit as  $x_i$  goes to 0.

So I got very dilute solutions so what I will do is although I do not know this animal I know it is derivative with respect to T or it is derivative with respect to T and I will show you in treating phase equilibrium I do not need to know  $\mu_i$  reference I only need to know it is derivatives.

This lecture Series Edited by  
Kannan Krishnamurthy

Online Recording &  
Post Production team

Subash  
Soju Francis  
Selvam  
Pradeepa  
Robert  
Karthikeyan  
Ram Kumar  
Ram Ganesh  
Sathiraj

Producers  
K R Ravindranath  
Kannan Krishnamurthy

Recorded at NPTEL Video Studios  
IIT Madras