

**Course: Adsorption Science and Technology: Fundamentals and Applications**

**Instructor: Prof Sourav Mondal**

**Department: Chemical Engineering**

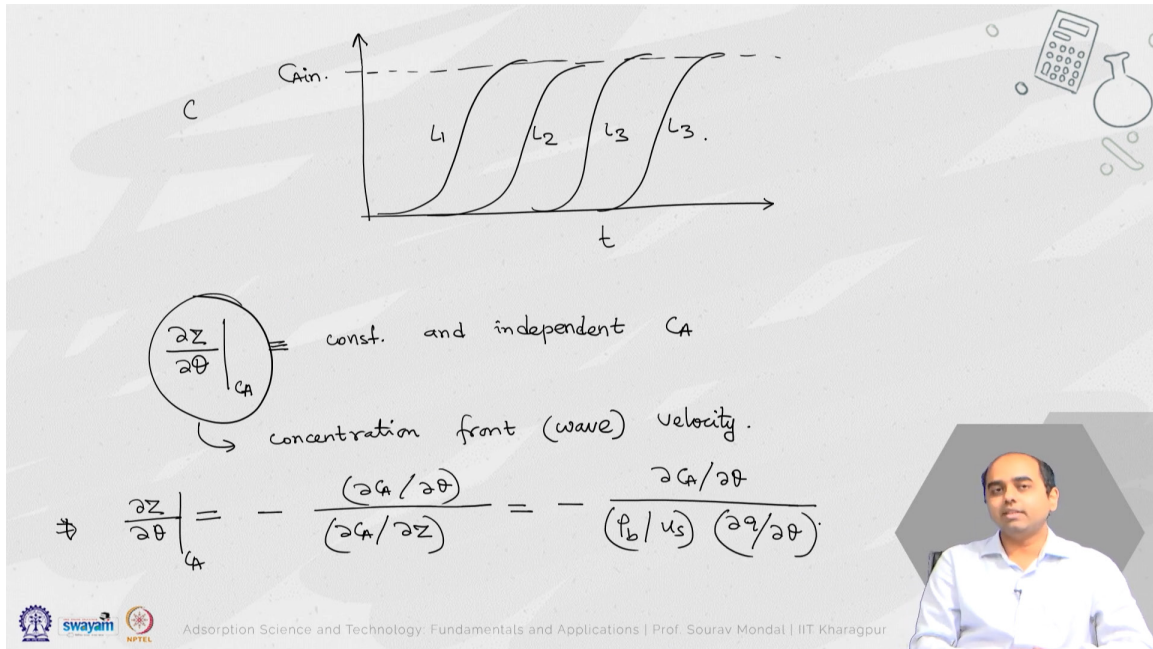
**Institute: Indian Institute of Technology Kharagpur**

**Week 05**

**Lecture 25 | Fixed Bed Design: Constant Pattern Behaviour**

Hello everyone in this last lecture of this week we are going to talk about the constant pattern behaviour of this fixed bed design so specifically the concentration wave front model or this concentration profile or the saturation breakthrough profile in the bed generally follows the same pattern or the same trend even on changing the length of the bed. So specifically what I tried to imply here is that the concentration profile with respect to time all follows the same trend even for different lengths. So, the mass transfer zone it is assumed that the mass transfer zone because of this dispersion or because of this adsorption or because of both of these two phenomena happens you know more or less in the same way for different bed lengths and it is independent and it is independent of the length of the bed. So, this packing in the bed can be viewed is like three parts one could be like the saturated zone one could be like the fresh adsorbent zone and one could be like the transition zone. So, in this constant pattern analysis the transition zone length remains constant irrespective of the total length of the bed. Now of course exploiting this unique feature of this constant patterns analysis the calculations can be simplified for the bed design and this is something we are going to talk about in this class today.

So, mathematically speaking this different you know beds showing or exhibiting the same breakthrough curve or the same nature of the breakthrough curve implies that  $\frac{dz}{d\theta}$  for constant  $a$  or for a particular  $c_a$  is like constant and independent of  $CA$ . So, essentially this quantity  $\frac{dz}{d\theta}$  represents nothing, but the concentration front or this wave velocity. So by definition  $\frac{dz}{d\theta}$  ( $\frac{\partial z}{\partial \theta}$ ) can be represented as  $\frac{\partial CA}{\partial \theta} \frac{\partial \theta}{\partial CA} / \frac{\partial z}{\partial CA}$ . So, this  $\frac{dz}{d\theta}$  represents the zone or the concentration zone or the front the velocity of which within the bed and this is constant or independent of  $CA$  and this represents actually the mass transfer zone how it is propagating inside there.



So, mathematically this can be also written down in this form. Now, taking the expression of the fluid phase you know conservative equation one can also write this. So this  $dz / d\theta$  for this constant  $c$  can also be written down. I mean this right hand side expression denominator can be written down in terms of  $dq / d\theta$  as like this. This is something from the conservation equation and the characteristic you know time that we say for absolutely no dispersion effects writing the characteristic time with respect to the convective time scale.

the  $dC_A$  by  $d\theta$  with respect to  $dQ$  by  $d\theta$  can also be written down. So, this concentration wave front here can be you know this denominator in this expression can be written down in terms of  $Q$  like this or like this form where of course, this is related only for the situations when we ignore the mass transfer effect or when we ignore the particularly the axial dispersion the time can be scaled with respect to the this characteristic convective time scale and  $dc/dz$  can be represented in this form. So, in terms of the dimensionless version in dimensionless version, this can be written down as  $dz_{\text{plus}} / d\theta_{\text{plus}} \text{ at } C_A \text{ plus} = dC_A \text{ plus} / d\theta_{\text{plus}} \text{ by } dQ \text{ plus} / d\theta_{\text{plus}}$ , where  $C_{\text{plus}}$  is  $C_A$  by  $C_{A \text{ in}}$   $Q_{\text{plus}}$  is  $Q$  by  $Q_{\text{ref}}$   $Z_{\text{plus}}$  is  $Z$  by  $L$ .  $\theta_{\text{plus}}$  is of course  $\theta$  into  $U_s$  by  $L$  reference which can be written down as  $T Z_{\text{plus}}$  where is denoted by and this quantity is nothing but  $\rho_b Q_{\text{ref}} (C_A)_{\text{in}}$ . Of course  $L$  reference is a scaling factor for this bed that we can define later on.

So from this scaling argument this above expression is obtained from the previous relation. Now we already know that  $dq / d\theta$  we can apply the chain rule and say that  $dq / d\theta$  can be represented as  $dq / dC_A$  plus product of  $dC_A / d\theta$  plus. Then combining this expression and combining the above dimensionless equation one can get a form of  $dz$  this concentration wave front expression as  $1 / (dq / dC_A)$  plus. Now, this is something which is particularly can be obtained from isotherm. Now for linear isotherms for linear isotherms  $dq / dC$  is actually constant for Langmuir or Fendt which where  $n$  is not less than not equal to 1 this is a function of  $C$ , but for linear isotherm or dilute solutions.

In dimensionless version,

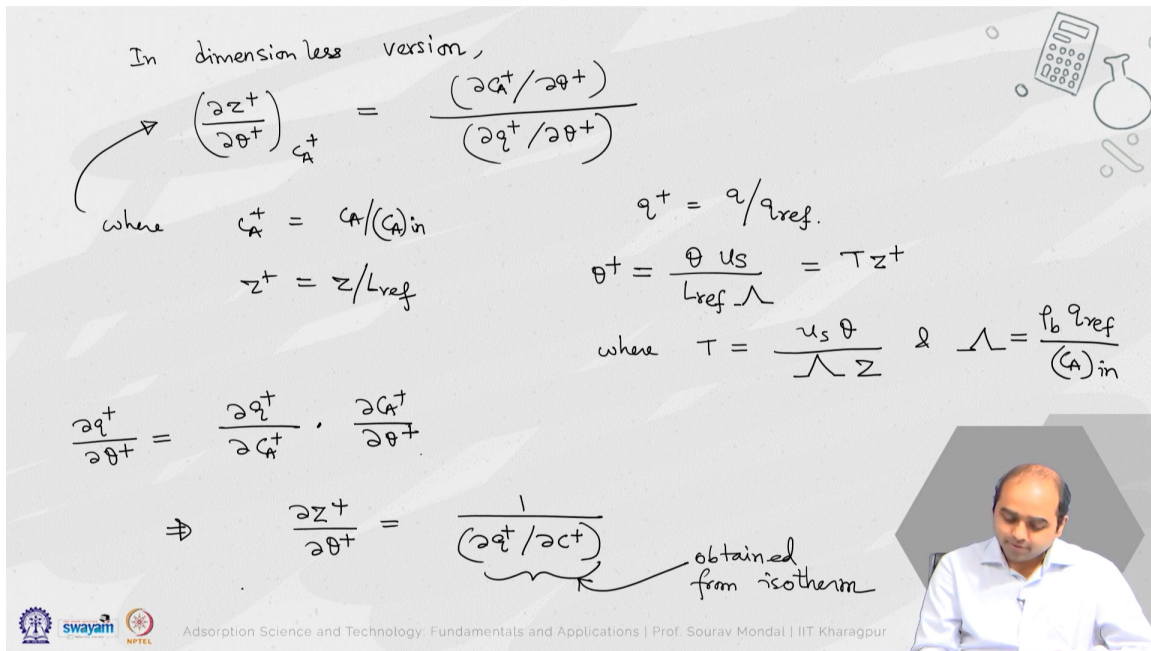
$$\left( \frac{\partial z^+}{\partial \theta^+} \right)_{C_A^+} = \frac{(\partial C_A^+ / \partial \theta^+)}{(\partial q^+ / \partial \theta^+)}$$

where  $C_A^+ = C_A / (C_A)_{in}$   $q^+ = q / q_{ref}$   
 $z^+ = z / L_{ref}$   $\theta^+ = \frac{\theta u_s}{L_{ref} \Lambda} = \tau z^+$   
where  $\tau = \frac{u_s \theta}{\Lambda z}$  &  $\Lambda = \frac{f_b q_{ref}}{(C_A)_{in}}$

$$\frac{\partial q^+}{\partial \theta^+} = \frac{\partial q^+}{\partial C_A^+} \cdot \frac{\partial C_A^+}{\partial \theta^+}$$

$$\Rightarrow \frac{\partial z^+}{\partial \theta^+} = \frac{1}{\left( \partial q^+ / \partial C_A^+ \right)}$$

obtained from isotherm



Adsorption Science and Technology: Fundamentals and Applications | Prof. Sourav Mondal | IIT Kharagpur

$dq / dC$  is constant. So, with this logic  $dz / d\theta$  is constant and this also suggest that  $q$  plus is equal to  $C_A$  plus. Now, it is possible if the constant is treated to unity, then one can say that  $q$  plus can be equal to  $C_A$  plus, which is of course true for linear isotherms anyway. But a reverse you know this idea works that if the constant pattern does exist then it also implies that there is linear equilibrium isotherm which exist in this case. Now leveraging this idea of the constant pattern analysis we can use the design calculations and let us see that how this can work out particularly in determining the adsorption zone height.

So, for fixed bed adsorption, adsorption under constant pattern behaviour, the zone or this mass transfer zone generally moves you know, along the inflow direction or around the flow direction. So, for an observer moving along with this zone the problem becomes something like the adsorbent or the adsorbent phase or the solid phase moves counter

current to the direction of the feed. So, in this scenario for an absorber we are just fixing the frame of reference of the absorber moving along the zone for an absorber sorry observer moving along this mass transfer zone the problem can be reposed in which the packed bed adsorbent or the solid phase moves counter current to the flow of feed. So, just to give an example or just to relate this more clearly what we mean that if this is the bed and we say that the adsorbent front is in at a small location this is the adsorbent front and we say that the fluid flow is happening in this direction. Then the solid flow happens in the counter current direction.

$$\frac{\partial z^+}{\partial \theta^+} \equiv \text{const.}$$

This also suggest,  $q^+ \sim C_A^+$  (linear isotherm)

For fixed-adsorption under const. pattern behaviour, the MTZ moves along the flow direction.

For an observer moving along this MTZ, the problem can be reposed in which the packed bed adsorbent (solid phase) moves counter-current to the flow of feed

CA(in). Fluid flow  $G_s$ .  
 MTZ.  
 Solid phase  $S_s$ .

Adsorption Science and Technology: Fundamentals and Applications | Prof. Sourav Mondal | IIT Kharagpur

So, we are trying to relate this as a counter current mass exchanger where there is a flow of liquid and there is also a flow of this solid phase even though practically it is not the case, but if we consider this with respect to the you know a moving reference frame fixed to the mass transfer zone, and then one can say that the solid phase is moving actually counter currently with respect to the fluid with respect to the fluid flow or the solution phase. So, the solution phase flow rate can be set as like  $G_s$  which remains unchanged the concentration is  $C$  in at the inlet And the solid phase one can consider is this whatever this equivalent flow rate as something like  $S_s$ . So,  $S_s$  is the solid mass velocity and the solid phase adsorbate concentration is  $q$  star. So, with this idea of course, this  $q$  star is in equilibrium with  $C_A$  in. So, with this you know idea or this picture you can write down counter current mass balance as  $G_s$  into  $C_A$  that exist within this mass transfer zone is equal to  $S_s$  with  $q$ .

Now of course, this can be also true. So, what I mean that  $G_s$  by  $S_s$ , which is true at the

inlet and the exit conditions. So, one can relate that as at the inlet this can be related as this is true at any point within the you know column and this is also equal to  $q$  generic  $q$  A by C A. So, this with this I you know logic or with this relation this analysis scenario now becomes very similar to you know this staged Here for example, a counter current absorption column or a counter current humidification tower this becomes very similar in that respect. So, if you consider a small elemental segment.

$\frac{\partial z^+}{\partial t^+} = \text{const.}$   
 This also suggest,  $q^+ \sim C_A^+$  (linear isotherm)  
 For fixed-adsorption under const. pattern behaviour, the MTZ moves along the flow direction.  
 For an observer moving along this MTZ, the problem can be reposed in which the packed bed adsorbent (solid phase) moves counter-current to the flow of feed  
 $G_s C_A = S_s q$   
 $\frac{G_s}{S_s} = \frac{q^*(C_A)_{in}}{(C_A)_{in}} = \frac{q}{C_A}$   
 $G_s \downarrow$  Fluid flow  
 $G_s \downarrow$  Solid phase  
 $S_s \uparrow$

Adsorption Science and Technology: Fundamentals and Applications | Prof. Sourav Mondal | IIT Kharagpur

So, we look into a small elemental segment of you know thickness let us say  $\Delta z$ , and we say that this inlet of the flow rate is  $G_s$  and this is at a particular location of  $z$  and this is  $z$  plus  $\Delta z$ . So, we are considering the location of  $z$  from top to bottom and similarly you have the flow rate of this  $S_s$ . So, this  $C_e$  or whatever  $C_A$  is getting converted to reaching equilibrium and this is reaching equilibrium here. So, for small elemental segment, doing an appropriate mass balance results in this equation that the change in the adsorbate concentration is equal to the this  $\Delta z$   $q$  minus  $q^*$  which is nothing but equal to this overall mass transfer coefficient, it could be the absorption kinetic, it could be the mass transfer due to diffusion or whatever multiplied to the specific surface area  $A$ , very similar to the scenario of absorption. Of this where of course this is the pseudo overall mass transfer coefficient in the particle phase or from the particle side because that is where the adsorption is happening.



For small elemental segment :  
mass balance.

$$-G_s dC_A = dz (q - q^*)$$

$$= dz (\tilde{K}_{of} a) (C_A - C_A^*)$$

pseudo overall mass transf. coeff.

$$a = \frac{3}{r_p} = \frac{6}{r_p} \quad \text{particle radius}$$

$$\int_0^L \frac{\tilde{K}_{of} a}{G_s} dz = - \int_{C_b}^{C_e} \frac{dC_A}{C_A - C_A^*} = \int_{C_b}^{C_e} \frac{dC_A}{C_A - C_A^*}$$

$C_A^* \equiv$  value of  $C_A$  in equilibrium with  $q$

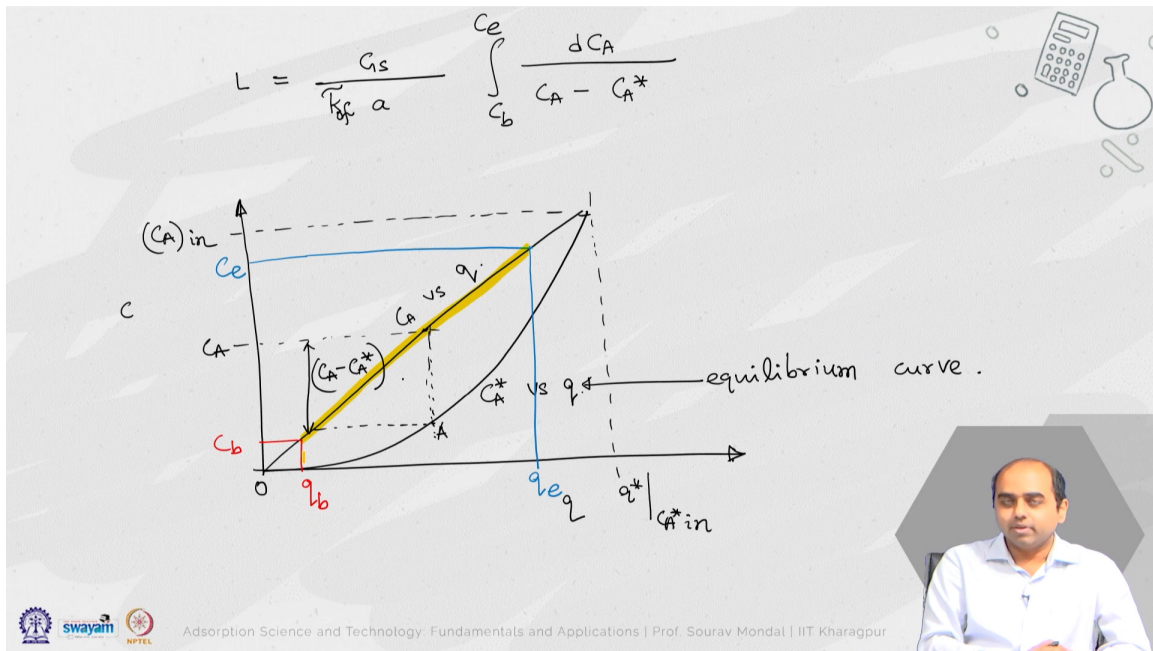
Adorption Science and Technology: Fundamentals and Applications | Prof. Sourav Mondal | IIT Kharagpur

This 'a' is the specific surface area per unit volume of the adsorbent. So whether it is radius or diameter, this could be like 6 by rho p ap or 6 by, 3 by, so this is like the particle radius. You can also write in terms of the particle diameter. I hope all of you realize that what would be your next approach is to use the method of HTU and the NTU. So, if one integrates the above equation on both sides, you get integration of Kof and the right hand side is integration of or this can be also written down as Cb to Ce as d CA by CA minus CA\*.

So, CA\* is the value. So, C a star is the value of C a in equilibrium u, C b and C e are the breakthrough concentration and the exhaustion concentration. So, you can understand that one needs to work out what would be the, what would be the integration and or the numerical integration of this curve to work out the value or the length of this you know column or the length of this fixed bed adsorption column. So, now here I would like to emphasize that this length of the column L can be Gs /. So, one needs to draw operating line and an equilibrium curve for this calculation.

So, this is C, this is q. So, let us say we draw this you know relation from up to the point of C A in to q star at can. So, this is a straight line. So, this is ca versus q and the operating line sorry the equilibrium line would look something like this and here it is important to denote that at any stage the difference in the concentration is this value. So, this is like C A and the corresponding value at any particular point here is CA star.

So, the difference between these two points is needs to be evaluated for this integration for different points within this operating line. So, this is the operating line, the straight line and this is the equilibrium curve. So, the starting point could be here. This could be defined as the breakthrough point and like  $q_b$  and the end point could be defined as the exhaustion limit. So, within this range from this point to from  $C_b$  to  $C_e$ .



from this portion of the operating line one needs to work out this value of the integration of the denominator of this integration in terms of taking the help of these auxiliary lines to calculate what is the equivalent value of the  $C_A$  star for this particular  $C_A$  and this is nothing but referred to as the you know tie lines. So in this case the tie lines would be all vertical that we can use here. So, I hope all of you got a glimpse of how this constant behavior pattern can be approach can be used and they can be related to this staged calculation in this case. And we will see perhaps in one or two lectures in the next class I mean after the next to next lecture we will see an example problem of this constant pattern behavior or this analysis and perhaps it will be more clear at that time. I hope all of you found this lecture to be useful in using or utilizing this constant pattern behavior and how that approach can be used to form like the stage wise calculation or this calculation of this NTU HTU type methods very similar to absorption AB or humidification of tower designs.

Thank you. See you everyone next week.