Momentum Transfer in Fluids
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I welcome you to this lecture on Momentum Transfer in Fluids. I am going to discuss Flow through Porous Media. This is going to be the last lecture in this series though porous media by itself is a subject on its own, but since we are talking about fluid mechanics and momentum transfer with fluids and we have been discussing so far the flow through a pipe or flow through a capillary it could very well be that instead of a capillary it could be a porous material. In particular, you must note that this porous media is pretty much ubiquitous that human body is porous, Mother Earth is porous, and almost every material we encounter they are porous in some way or the other, and flow through this, we rely on that flow every day. So, we at least the basics of this I want to put one session on this flow through porous media at least the how fundamentals of fluid mechanics, how fundamentals of momentum transfer in fluids is moved from flow because porous medium essentially is a bundle of capillaries, there could be inter interconnected network of capillaries. So, how our whatever we learnt in so far on fluid mechanics how those concepts are ported how those concepts are extended to flow through porous media that is my objective in this session.

When it when we have flow through porous medium, and we also discuss a little bit about fluidization because this is also a type of flow which we use particularly in chemical processes. When it when it comes to flow through a porous medium we have something called permeability and porosity. Porosity also goes by the name void fraction you have a porous medium you can think of as if I have a cylinder and this cylinder in case of a pipe it was all open and you are having a flow, but in case of a porous medium this is let us say we have these particles present here and the flow will take place through the nook and cranny of this of these particles through this it will form some interconnected network and flow through this. This type of porous medium you might have encountered in sand packs where you have a sand bed and you pour water and if water has some suspended solid they get accumulated on top and you see that you get cleaner water.

In fact, the household filters they have one preliminary filter before it goes to a standard UV device or standard other say reverse osmosis or other membrane devices. Before it goes you have a pre filter which where you have this type of this type of particles earlier there used to be carbon we have this carbon particles in a packed bed and now we see a lot of monoliths made of carbon that is also used as a filter bed. So, what is void fraction?

It is the volume of this void space divided by total volume. So, void space it is it is referred as ϕ it is it is the void fraction or porosity it is called by both names and then that is void volume that is the volume that is that is void through which the fluid travels divided by total volume. And permeability is as before I go to permeability, let me define what is superficial interstitial velocity.

Superficial velocity is if I if I take this entire cross sectional area as A and suppose I have a flow through this which is Q then superficially if I do not enter into this porous medium if I see these as a black box the net flow going in is Q in so many let us say meter cube per second again this is a very high flow rate and milliliter per hour would be the other extreme very low flow rate. So, Q is so many meter cube per second, these divided by A which is in meter square. So, if I write Q by A, that gives me a velocity, which is you can call it a volumetric flux, that means volumetric flow per unit area, or you can call it some velocity. So, what that velocity is you are treating as if the entire cross sectional area is available for flow which is not true. So, if you are interested in such hypothetical velocity, we call this superficial velocity or super visually talking about a velocity.

Whereas, if you want to if you are truly interested in the velocity local velocity that is that is there in the in the nook and cranny of those particles then what you have to do is you note that this velocity would be much higher because the entire cross sectional area is not available for flow you have a much smaller cross-sectional area and that is what would be that cross sectional area. One representation of this porous medium is probably a bundle of capillaries you can think of as if you have you are running some capillaries through these porous medium and the remaining part is all solid absolutely solid. So, they will not allow any flow. So, you have some areas that are available for flow though you are in a superficial in calculation of superficial velocity we assumed entire cross sectional area, but actual flow area available is these smaller areas. So, you can see what is the void volume in this case if this is the length L and if this cross-sectional area is A, and let us say this cross-sectional area is let us say A_1.

So, A_1 into L that gives me the void volume right this gives me the void volume if A_1 is sum of all these areas and with this gives me the void volume and A into L where A is the overall cross sectional area that gives me the total volume and if I divide the two. So, this void volume by total volume that is equal to porosity and L and L will cancel out. So, A_1 by A is equal to porosity or in other words A_1 is equal to ϕA. So, now, interstitial velocity will be defined as if I write interstitial velocity that would be defined as Q divided by the smaller area that is available. So, Q divided by A_1 that is equal to Q divided by ϕA on the other hand v superficial is defined as Q divided by A.

So, you can see that superficial interstitial velocity they are linked by the fact that

$v_{superficial} = v_{interstitial} \cdot \phi$.

You can see here that which one is higher interstitial velocity because ϕ is fraction the value of ϕ will change from 0 to 1, 0 means all solid 1 means entire no solid the entire channel is available for flow. So, ϕ range is 0 to 1. So, it is less than 1 always. So, superficial velocity would be always be less than the interstitial velocity.

Now, once we have this known there is this Darcy's law which goes by the name goes by cause and effect relationship just like you have flux of heat from higher temperature to lower temperature and that you have you call that you have that equation heat flux is proportional to d/dx the temperature gradient in x direction. Similarly, you have current because of voltage gradient. So, by that token, if you write Darcy's law,

$$V_{superficial} = -\frac{k}{\mu}\frac{dP}{dx}$$

So, essentially if it is linear if I assume that the pressure drop is linear. So, it would be equal to

$$\frac{dP}{dx} = -\frac{\Delta P}{L} = -\frac{P_{upstream} - P_{downstream}}{L}$$

So, if I assume this to be linear the pressure changing with x linearly. So, then this would be p upstream minus Δx I have this to be valid. So, if I assume this to be linear then over this entire length L this is the case p upstream minus p downstream in whatever direction you have flow. So, you have to make sure that it is there this minus sign we when you put this minus sign and this minus sign they will cancel out because flow would take place only when the upstream pressure is higher than the downstream pressure.

So, that is why one has to have a minus sign to take into account this. So, this is this is known as the Darcy's law μ here is the viscosity and k is referred as permeability. So, unit of this permeability is meter square and also there is a common terminology for unit of permeability which is which is known as Darcy. Darcy is also another unit of permeability. So, this is the way typically the flow through porous medium is handled.

So, one can have some flow rate in fact, how to find out permeability one will plot one will have a porous medium and then they will continue to change the flow rate and find out the corresponding delta p that means, p upstream minus p downstream and change the flow rate and they will get different points and then they pass a straight line through this true 0 0 by some least square feet because lines will be scattered and whatever is the slope of this line from there you can calculate the permeability this is commonly practiced. Now, this is may like to note here that whatever we have learnt the pressure drop here we are writing. So, one can write pressure drop versus flow rate using this cause and effect relationship, but how did you find out pressure drop in case of flow through a pipe we have found out what is friction factor and from there we found out the pressure drop right. So, so if we want to if we are to implement that if we want to find out what is the friction factor in case of flow through a porous medium for that we have to find out what is the Reynolds number, how we define Reynolds number for a porous medium and then friction factor chart. So, to that effect how to define a Reynolds number for a porous medium, you must note that Reynolds number for when the cross section is of arbitrary shape, one has to go by something called hydraulic diameter or equivalent diameter, and that is defined as

$$d_h = 4 \times \frac{wetted\ area}{wetted\ perimeter}$$

So, that means, if I have a cross section which is square. So, in this case the weighted area is a^2. So, 4 into a^2 is the weighted area and perimeter is one side is a. So, cross sectional area is a^2 and perimeter is 4 a, a plus a plus a plus a. So, here in this case the hydraulic diameter d h would be equal to a.

Similarly, you can have a rectangular cross section you can find out and you can use this hydraulic diameter to find out what is the corresponding Reynolds number. So, that is in case of a porous medium how you define this weighted area you can think of the as we said that any say let us say I have I have this is the porous medium I have let us say this is the porous medium I have and here I have this is running over the length L. So, I have the channels that are that are running running over the length L or you can think of what is the weighted area weighted area is area that is that is there I mean how do we how do we find out the let us say I have this cross section here. What is the volume in this case if this length is L? The volume would be a^2 into L that is the volume of this conduit with square cross section and what is the perimeter of this. So, this is the volume right and what is the inner surface area of this would be 4 a into L.

So, whatever is the weighted area if I multiply by L I get the corresponding volume and if whatever is the weighted perimeter if I multiply by L we get the corresponding perimeter weighted perimeter. So, the same thing here I will have some you can think of bundle of capillary or what you have these are of length L. So, if I take total void volume here, I have taken the total volume of a square L divided it by L. I get the weighted area, total surface area divided by L, and I get the weighted perimeter. So, here it is then total void volume because flow is taking place to the void space total void volume by L that gives me the weighted area and total surface area of particles divided by L that gives me the weighted perimeter. So, this weighted area in this case what is the total void volume? Total void volume is what is the void volume? Void volume is divided by total volume that is equal to ϕ, so that means void volume is total volume into ϕ.

So, that is or here instead of phi I have written ϵ you can you can use ϕ or you can use ϵ as a void fraction. So, total volume if it is total volume is v, ϵ into v gives me the void volume. So, total void volume is given as ϵ into v and this L and this L cancel out. And total surface area of particles that is written as this one this one can be written as surface area of one particle multiplied by number of particles. You are assuming that these particles are touching each other at I mean there is no common area shared by particles that means, you I have one particle of this nature and another particle of this nature and there they are just locked to each other it is not that way.

It is I have a another one particle I have another particle and there is no common area which is shared. So, this number surface area of one particle and multiplied by number of particles that gives me total surface area of particles. And then it is so, you have total void volume is ϵV I have already written and total surface area of the particle is given by this into surface area of one particle it is this, please correct it surface area of one delivered. So, this has to be multiplied by surface area of one particle. Because, number of particles is defined as total volume of particle by volume of one particle.

So, this gives me the total volume of the particles by the volume of one particle, which gives me the number of particles. So, the number of particles multiplied by the surface area of one particle gives me the total surface area of particles. So, this has to be in the denominator. So, this now you can write this total volume of particle total volume of particle is what? If, ϵV is the void volume what is the solid volume? Solid volume would be 1 minus ϵV because, sum of these two has to be equal to v total volume. Total porous medium is consisting of void volume plus solid volume.

If, void volume is ϵV solid volume has to be $1-\epsilon V$. So, we write the total volume of particles which is essentially solid volume as 1 minus ϵV and then you have surface area of one particle by volume of one particle. And, what is surface area of one particle? If, I have spherical particle it is 4 π r^2 and if it is spherical particle the volume of one particle is $\frac{4}{3}\pi r^3$. So, what would be this ratio surface area of one particle by volume of one particle? You can write this as $\frac{6}{d_p}$ is the diameter of the particle. So, we can write

$$d_h = 4 \times \frac{\epsilon V}{(1-\epsilon)V \times \frac{surface\ area\ of\ one\ particle}{Volume\ of\ one\ particle}}$$

$$\Rightarrow d_h = 4 \times \frac{\epsilon V}{(1-\epsilon)V \times \frac{6}{d_p}}$$

So, now, I have some handle over the hydraulic diameter and then I think the problem is mostly solved becaus and then I get Reynolds number. Now, there is one catch you may not have all these as spherical particles. So, then one has to bring in something called a sphericity. So, for a spherical particle $\frac{6}{d_p}$ is understood I have already established that $\frac{4\pi r^2}{\frac{4}{3}\pi r^3}$. For a non spherical particle you have to find out something called an equivalent diameter of the particle.

This is diameter of sphere having same volume as particle think of a particle which is of arbitrary shape and that particle is forming the bed that particles of that type is forming the bed. So, what is the how do you find the equivalent diameter? First of all you need to find the volume of that particle. So, you dip it in a liquid in which the that does not get dissolved and see how much volume gets displaced. So, from there you find out what is the volume. Now, this volume if it would have been a spherical particle of that volume. For spherical particle,

$$\frac{S_p}{V_p} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3}$$

$$\Longrightarrow \frac{S_p}{V_p} = \frac{6}{d_p}$$

For non-spherical particle,

$$\Longrightarrow \frac{S_p}{V_p} = \frac{6}{\phi_s d_p}$$

Where $\phi_s = sphericity = \frac{surface\ area\ of\ sphere\ with\ diameter\ d_p}{surface\ area\ of\ non\text{-}spherical\ particle}$

$d_p = Equivalent\ diameter$

$= diameter\ of\ sphere\ having\ same\ volume\ as\ particle$

So, what should have been the diameter of that particle? So, that will give you the equivalent diameter diameter of sphere having same volume as particle. So, that diameter you use as dp and then you have to find out the area of the particle of arbitrary shape and then that area surface area of sphere with the diameter dp that you I mentioned just now divided by surface area of non spherical particle. You take that ratio that gives you so called sphericity of the particle I mean how spherical how close to spherical the particle is anyway that does not matter to us you have to somehow measure this sphericity or there are first standard shapes of particles there are sphericity values listed tabulated and this factors surface area per volume for a particle for of non spherical particle you have to multiply this sphericity and dp that you count there is the equivalent diameter. So, that is that is how you handle once you do this now you bring in you must agree that interstitial velocity is something we have to consider because that is the real velocity through the pore space and hydraulic diameter we just now calculated rho by mu. Hydraulic diameter that we have calculated is this $4\left[\frac{\epsilon}{1-\epsilon}\frac{d_p}{6}\right]$.

If I do not consider ϕ_s sphericity otherwise sphericity will come in there as well. So, what you have in that case the Reynolds number for porous medium you will write in terms of superficial velocity if you write then interstitial velocity would be superficial velocity divided by epsa right and that epsa and this epsa will cancel out. So, that is why we do not have any epsa here and we have a two-third term dangling there, but for a dimensionless number typically we do not have those numbers I you might have seen already we talked about Reynolds number capillary number and other numbers there we do not have any numerical value put there in the beginning. So, two-third dv $\frac{\rho}{\mu}$ does not make sense. So, that is why two-third we are ignoring and this is the Reynolds number for porous medium.

$$N_{Re_{porous\,media}} = d_h \cdot v_{interstitial} \cdot \frac{\rho}{\mu}$$

$$\Rightarrow N_{Re_{porous\,media}} = 4\left[\frac{\epsilon}{1-\epsilon}\frac{d_p}{6}\right] \cdot v_{interstitial} \cdot \frac{\rho}{\mu}$$

$$\Rightarrow N_{Re_{porous\,media}} = \left(\frac{2/3}{1-\epsilon}\right) \cdot d_p \cdot v_{superficial} \cdot \frac{\rho}{\mu}$$

$$\Rightarrow N_{Re_{porous\,media}} \approx \frac{1}{1-\epsilon}\ \frac{d_p\, v_{superficial}\, \rho}{\mu}$$

So, now, with this in mind you have something called a one has done first of all friction factor versus Reynolds number you have that chart now if you do if you if you work with that same chart f versus Re, but in this case it is Reynolds number for porous medium what is noted here is first of all when it comes to travel over a length L if it is traveling over traveling inside a porous medium it follows a zigzag path. So, if it is taking a 45 degree angle from the actual point to point distance then it is this would be then there would be 64 by Re even if I go by the Darcy friction factor 64 by Re would be changing to there it will be multiplied further. So, what is done by these researchers they have reported that for laminar part of the region if I go by the Reynolds number for porous medium it would be

$$f = \frac{150}{(Re)_{PM}} \;\; for\ low\ Reynold\ number\ \left(Kozney - Carman\ Equation\right)$$

and here it is friction factor would be for the turbulent region it would be 1.75 and there is a combination available which is $\frac{150}{(Re)_{PM}}$ these are these are these numbers they have analytical basis it is not and also supported by experimental data. And so, this looks this looks very similar to what friction factor chart we have for a pipe only 64 by Re twice and then some other factors have to be considered to be considered and so, it become 150 by Re and then you know that in the turbulent region the friction factor becomes almost constant.

$$f = 1.75 \;\; for\ high\ Reynolds\ number\ (Blake - Plummer\ Equation)$$

$$f = \frac{150}{(Re)_{PM}} + 1.75\ (Ergun\ Equation)$$

$$\left(\frac{\Delta P}{L}\right)\frac{\phi_{s\,d_p}}{\rho \overline{V}_0^2}\frac{\epsilon^3}{(1-\epsilon)} = \frac{150(1-\epsilon)}{\phi_s d_p \overline{V}_0 \frac{\rho}{\mu}} + 1.75$$

So, that is that is how it makes sense. So, if we convert now the friction factor to the pressure drop this is the expression that we get which is referred as Ergun equation. So, all these see here you can see that this $\frac{\Delta P}{L}$ is it is a non-linear function it is earlier we had in Darcy's law $\frac{\Delta P}{L}$ the superficial velocity is proportional to $\frac{\Delta P}{L}$. So, it is a linear velocity and the pressure drop they were linear linearly related.

Here it is not because I can see here one is v o bar square one v o bar. So, there is a quadratic that it is it is it is a different type of relationship altogether and implicit type relationship we have. So, this is a different relationship and it is probably a parallel to Darcy's law. Darcy's law was also relating flow rate with pressure drop this in this is also relating the flow rate with pressure drop, but these are the it is a different equation probably more comprehensive because it has you have both void fraction or porosity ϵ appearing there you have dP appearing there diameter of the particle sphericity is appearing there ϕ_s is there. So, say it is in case of Darcy's law you have only one term taking care of all that is permeability here you have other parameters they are coming in.

So, I would say this is more comprehensive more or worse, but point is in many cases you do not have all these information for a porous medium. For example, if you pick up a monolith which is porous. So, in that case you may not know what inside what are the equivalent diameter of particles which the bed is made of. So, in that case probably you have to rely on permeability a single value that that would work out better for you.

So, once you have this. So, this is this is typically a flow through porous medium. So, once you have this I would like to look into another case where you have this is the porous medium and you are having flow from the bottom. So, you have you have a flow from the bottom going in there and then you are measuring Δp across this across this and over length L. So, but it is following let us say if this is pressure drop versus superficial velocity if I plot then it would be a straight line I mean I am collecting points those are aligned along a straight line because this is a classical Darcy's law we have right. If I follow Darcy's law it will be a straight line like this.

So, we typically I mean if you go by Ergun equation there would be slight deviations, but it is mostly following a similar line similar trend. Now, this is you have this superficial velocity and a pressure drop is changing. At one point you will find that the when you increase the velocity to a very large value you see that this particle that is sitting there the particle has a gravity force acting downward and if it is suspended in a fluid there is a buoyancy force acting upward. So, you have a buoyancy corrected gravity acting downward and the fluid and since you are putting a pressure drop here. So, there is a pressure force acting on this particle at one point you will find that they balance each other.

So, in that case particle is remaining simply suspended it is not it is not holding on the plate rather it gets completely suspended. So, in that case at that time you have pressure drop across the bed would be equal to weight of bed per unit cross sectional area per unit cross sectional area allowing for the buoyant force. So, you have volume of the solid here is $AL(1-\epsilon)$,

$$\Rightarrow \Delta p = \frac{AL(1-\epsilon)\left(\rho_p - \rho\right)g}{A}$$

$$\Rightarrow \frac{\Delta p}{L} = \frac{AL(1-\epsilon)\left(\rho_p - \rho\right)g}{A}$$

$$\Rightarrow \frac{\Delta p}{L} = g(1-\epsilon)(\rho_p - \rho)$$

Therefore, at incipient fluidization

$$\frac{\Delta P}{L} = g(1-\epsilon)\left(\rho_p - \rho\right) = \frac{150\mu\overline{V}_0}{\phi_s^2 d_p^2}\frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75\rho\overline{V}_0^2}{\phi_s d_p}\frac{(1-\epsilon)}{\epsilon^3}$$

So, this is weight divided by the area. So, that gives that is the force divided by the area and that is to be suppo who has to follow this expression. So, in that case now this is one expression and before this I mean when the when the flow rate was less you we had to follow Ergun equation which say $\frac{\Delta P}{L}$ is this. So, $\frac{\Delta P}{L}$ is equal to this which was a which is arriving from Ergun equation we discussed this in the previous slide whereas, $\frac{\Delta P}{L}$ at this time when the particle gets suspended this will follow this. So, when these two becomes equal so, that that that time particle gets suspended on its own.

So, particle gets suspended on its own and we call this condition as incipient fluidization fluidization means the particle gets suspended on its own. So, beyond this point if I increase the superficial velocity what is going to happen is the pressure drop will no longer increase only the bed height will continue to increase beyond this point. That means, I have this porous medium and then I had a pressure drop, but beyond this point you will find that particles get they are not come they are not settling in a compact manner rather they get suspended they tend to be become bubbly. So, the bed starts rising, but the pressure drop remains same. So, that is a classical condition of fluidization and in many cases this fluidization is of immense benefit.

For example, if you use a packed bed of catalyst to perform certain industrial reaction. So, if you want to improve the improve the performance you want to have a better contact packed bed of catalyst means catalyst particles which are primarily you can think of tablets inside you have noble metals the other materials which are performing as catalyst. So, these tablets they are all packed together in a bed. Now, if you have this type of tablets they are suspended on their own. So, you have better contact between the fluid and these solid particles.

So, that is desirable. So, because these reason you will find many many reactions are performed in this type of condition you might have heard of fluidized catalytic cracker FCC units. So, these are in refinery in petrochemicals. So, this is where it that is basically the and that the packed bed is in fluidized condition you want to have better more intimate contact between the catalyst particles and fluid. And, so, or it could be something else you have a packed bed you are using as a filter and lot of suspended solids have accumulated in the in those nook and cranny of that packed bed.

And, then now you want to flush it now you want to clean it. So, you have a back flush and you want those particles to churn and so, that those suspended solids those are which are of much smaller dimension they would be taken out and carried up from there. So, there could be many reason you want to get into this fluidized condition. One thing you have to ensure and at what velocity this fluidization starts that is given by this expression.

$$\frac{\Delta P}{L} = g(1-\epsilon)(\rho_p - \rho) = \frac{150\mu\overline{V}_0}{\phi_s^2 d_p^2}\frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75\rho\overline{V}_0^2}{\phi_s d_p}\frac{(1-\epsilon)}{\epsilon^3}$$

So, this is a quadratic equation. So, you solve this equation to find out what is the v o bar and that v o bar will g

If that is what you do not want that that is required for grain transport and for other reasons. So, probably if your objective is to transport grain from one place to the silo and you have this pneumatic conveying is a better way instead of putting it on a conveyor and traveling there. So, so the in those cases probably you will get there you will then you will work with the terminal velocity and all this, but otherwise you want your superficial velocity to be sufficiently small that terminal velocity than terminal velocity. Because you must you must note here that is entire analysis is based on an average dp, but you never have in a packed bed dp would be there would be a distribution there would be larger than dp smaller than dp and moment you when you reach the incipient fluidization may be smaller particles they are already will start first getting into fluidized mode larger particles yet to be then all these all these dynamics are there. So, you must have sufficient window when it comes to terminal velocity of the smallest particle and the incipient fluidization.

So, that you do not mess with this I mean you do not get into a situation where slightest change in flow rate the superficial velocity because of some something with the do with the control of the apparatus and you go into that more pneumatic conveying mode instead of fluidized mode because you want a steady fluidization to happen. So, these are some of the issues with fluidization which also comes up as a part of the flow through porous medium. This is all I have as far as the present module is concerned. Thank you very much for your attention.