Momentum Transfer in Fluids

Prof. Somenath Ganguly
Department of Chemical Engineering
IIT Kharagpur

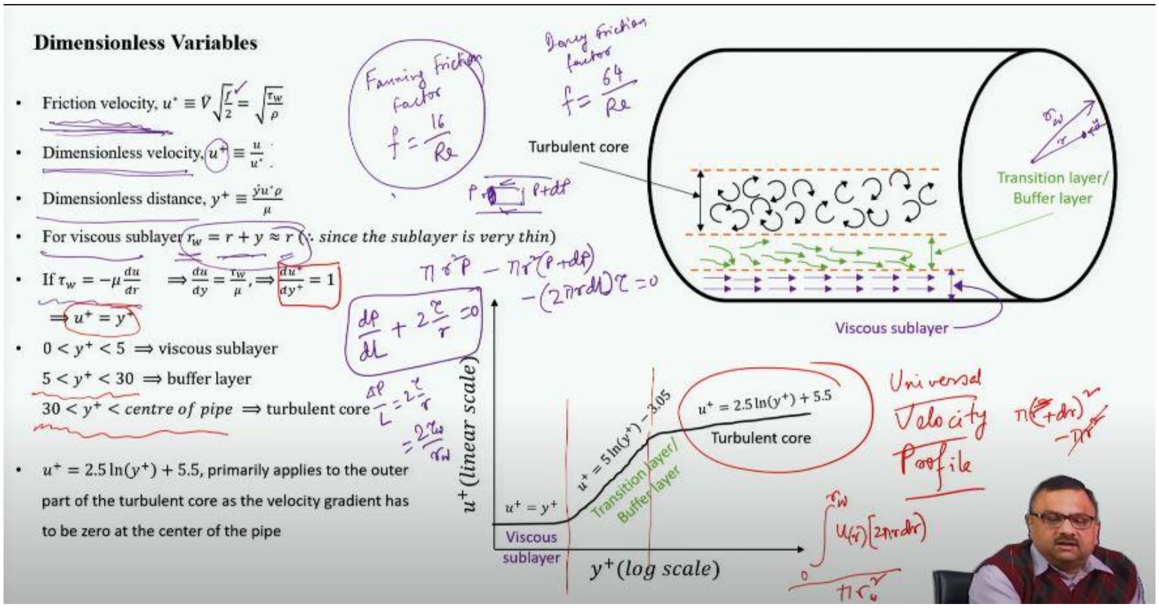
Week-12

Lecture-59

I welcome you to this lecture on Momentum Transfer in Fluid. Today, I am going to talk about something new which is Turbulence. You all have studied already at the very at the initial stage I talked about this classification about laminar flow and turbulent flow. Maybe I would repeat that exercise. This is primarily the contribution of Reynolds. You may recall that there was an experiment performed by Reynolds.

What he found is when he has a flow through a tube. So, flow is taking place through a tube a very low flow rate he is maintaining and he was maintaining and at that time he introduced some dye. What he saw is that he introduced some dye and that dye moves like a straight line. If this is the cross section he will see a straight line the dye is traveling.

There would be some fuzziness we expect at the downstream end because it the tracer is miscible with the let us say it is it is miscible with water and the water is flowing through the pipe. So, he might have seen some diffusion happening there some because of Brownian motion that the straight line will be little thicker and little lighter as it travels the further downstream, but it is maintaining a straight line. But as he as he increases the flow rate he finds that at one point the line becomes wavy and then if he increase the when he increase the flow rate further he found that the lines are rolling like this and at one point there would be totally chaotic movement of color. So, he classified that as the turbulent flow. What he said is that during laminar flow one layer was sliding against other and so, you have classical parabolic velocity profile you may recall you have studied in Poiseuille flow and all.



And then at a higher flow rate what he said is there is random movement of eddy and this is this is this is a different kind of flow and he called that turbulence. So, now, if you want to characterize this turbulence I mean you have already studied turbulence in terms of friction factor in terms of how friction factor changes and from there by referring to the friction factor chart you can you can find out the pressure drop. Now, this part I mean if you if you see the laminar part you have 64/Re etcetera that that that is all well defined obtained from first principle. But beyond the transition region what you have is many different lines depending on the value the depending the extent of roughness of the pipe and that part is somewhat empirical I mean that is what that is what we have studied when we looked that is what we learnt when we studied the friction factor chart. So, we try to see in this in today's lecture is there any way to put some theory behind this because I understand there would be chaotic eddy movement and chaotic movement is not I mean that is definitely that the treatment of chaos the way it is handled in physics that is definitely not that does not come under the purview of this course.

But is there any way we can we can find out what would be the velocity profile just like we have for a parabolic velocity profile you all know if the if for a circular cross section u is equal to u max into 1 minus r by r whole square small r by capital R where small r is the radial location and the capital R is the radius of the capillary this you have studied. So, now, is there any way we can have a similar velocity distribution for turbulence? So, when we looked at it we found that people have very nicely mixed the empirical information with the with theories to come up with something called universal velocity profile by which they can not only obtain, but then they cannot you can not only predict the velocity you can also find you can also give some rationale behind that friction factor versus Reynolds number in the turbulent zone. So, by the use of universal velocity profile. So, what I will do in this in this into in present lecture is discuss about these universal velocity profile.

Another point I must I must point out another another point I must emphasize here is that the turbulence you must keep in mind that turbulence is what is first of all Reynolds number Reynolds number is the it gives you the ratio or the impact of it is viscosity is in the denominator. So, that means, inertia force by viscous force I mean that is that we have all understood. So, it is it is essentially ratio of inertia force by viscous force when inertia force dominates over viscous force then you have turbulence. So, you have viscous force dominating when you have small cross-sectional area when you have one layer sliding against the other you have viscous force having control over the over the flow, but when inertia force dominates I can give you an example where you do not have to increase the velocity a lot just simply you can you can you can fiddle with other parameters of Reynolds number to in to increase the inertia or to put the domination of inertia force over the viscous force. Think of it when you have flow over a flat plate you see that let us say I have a flat plate.

So, I have you all see that upstream side it is all potential flow. So, the so, it is it is all fluid moving in mass no sliding against the other because there is no wall when you have wall then only we have one less sliding against the other. So, you have so, called boundary layer that that is the idea right. So, you have here at the wall you have to satisfy that the velocity to velocity has to be 0 on the other hand little bit away from the wall you have to satisfy the condition that free stream velocity is maintained because free stream velocity is maintained. So, it has to go through a transition velocity is 0 here and velocity is little bit more here little bit more here and it reaches the same velocity that is the free stream velocity.

So, this transition has to be accounted. So, to have this transition to account for this transition one has to consider something called a boundary layer within which this transition that the increase in velocity from 0 to free stream velocity takes place. But as the boundary layer continues to grow at one point you will find that this thickness of a boundary layer it increases to that extent that one cannot have this viscous impact. That means, one layer sliding against the other cannot be maintained cannot be retained and in that case you will see that you have a reversal of flow taking place. Reversal of flow means it is the velocity is here and in this region the velocity is in reverse direction because one layer sliding against the other is possible when you are working with some compact layers.

But when you let it grow over a larger thickness you cannot just by mere application of viscous force and one layer sliding against the other you could not retain all layers like a pack of cards. So, there would be some reverse flow occurring. So, in that case we call that to be turbulence right because in this case our Reynolds number is calculated as x v ρ

by µ where x is the distance from the from the left end. So, as the x increases the Reynolds number increases at and at one point you find that the boundary layer becomes unstable and you call that to be turbulence. So, it is something like it is a turbulence means inertia force dominating our viscous force.

I just wanted to tell you these at the at the very outset. Now, this when it comes to we want to discuss this universal velocity distribution that is the idea. So, first thing that is assumed is that if within a pipe you have turbulent flow then one condition that has to be satisfied is at the wall the velocity is 0. So, that means, the velocity so, there has to be a viscous layer present very near the wall that thickness whatever small it is one has to have the viscous sub layer present. Otherwise, you cannot define the flow I mean it is travelling random it is travelling all the way to the wall and getting reflected and coming back that is not feasible.

So, one has to have a viscous sub layer. So, that is what it is and here one layer is sliding against the other velocity is 0 at the wall and velocity is slightly higher as you move away from the wall. And on the other hand since it is a turbulent flow we expect the random movement of eddies that has to be there and that is in the core part there. So, these transition has to take place. So, somewhere and there has to be some buffer region where the transition from viscous sub layer and the random movement of eddies take place and that takes place and that is referred as the transition layer or buffer layer.

So, here in this case eddy diffusion is insignificant in viscous sub layer instead viscous shear driven sliding of layer is present and velocity gradient is 0 at the central line that is assumed most of the kinetic energy of the eddies lie in the buffer zone. So, this is one thing that is assumed when it comes to the viscous layer when it comes to the when it comes what are the assumptions that we have to make when it comes to defining the universal velocity profile. So, here first some dimensionless vary variables introduced and these variables are here we you can see

Friction velocity, $u^* \equiv \bar{V}\sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w}{\rho}}$

mind it these f is is not the friction factor that was defined earlier in as a part of this course in the friction factor chart etcetera. These there are there are essentially two friction factors if you go to standard textbooks you will find that there are two friction factors one is known as fanning friction factor another is Darcy friction factor. So, you will see that one is 4 times the other.

So, you will find that is as per the fanning friction factor this laminar part gives you f is equal to 16 by Reynolds number whereas, the other friction factor which goes by the Darcy or Moody friction factor that is given by friction factor is equal to 64 by Re. So, one is 4 times the other. So, it is just the choice of I mean some somebody is following a meter as a unit somebody is following kilometer or somebody is doing centimeter as unit. So, it is something like this. So, here this friction factor that is used here this friction factor is essentially fanning friction factor which goes by the 16 by Re concept.

And then you can you can always note that when it comes to the shear stress you always note that you have you can relate τ_w with friction factor you note that if you if you have let us say within these let us say I have if I have a laminar flow if I assume and a cylinder of radius r a cylindrical element of radius r moving and then you have the pressure at this point is P and pressure at this point is P + dP then you will have the force that is acting on this face is pi r square if this is this radius is π r square P and this side the force is π r square P plus dP if I assume and then these difference in forces that would be accounted by these would be accounted by the shear stress that is acting here. So, it is 2 π r dl 2 pi r is the perimeter l is the length of the dl is the length differential length of this element and then that into tau and that is equal to 0. So, if you if you perform such force balance you will with the term dP dl plus 2 tau by r is equal to 0 and as I pointed out since dP dl would be same for any value of r. So, you can write in this case it could be 2 tau by r or 2 tau w by r w tau by r is equal to tau w by r w. So, that is that is the other thing you have.

So, you can write in this case delta P by l that is equal to 2 tau by r and that is equal to 2 tau w by r w. So, if you if you do this and then you write this F as 16 by r e then these v the velocity average velocity in square root of F by 2 that will come to $\sqrt{\frac{\tau_w}{\rho}}$ you yourself can check this out. So, we introduce a friction velocity which is given by this we introduce a dimensionless velocity. Now, friction velocity has a is not dimensionless I believe friction velocity has a unit, but dimensionless velocity in that case is defined as velocity divided by this friction velocity and that we are calling u plus. So, this is a dimensionless velocity and similarly we introduce dimensionless distance which is y plus and that is equal to y u star rho by mu.

Dimensionless velocity, $u^+ \equiv \frac{u}{u^*}$

Dimensionless distance, $y^+ \equiv \frac{yu^*\rho}{\mu}$

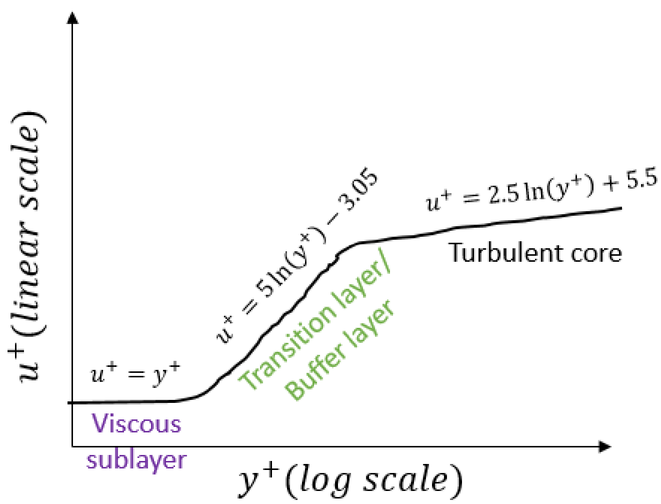
So, we that is what dimensionless distance that is how it is defined and then you assume that this viscous sub layer if it is very thin. So, you can you can write here if this is if this radius is r w this radius is r w radius inner radius and then you can you can pick up any value any r value here. So, this is any point here its radial coordinate would be r and distance from the wall would be y. So, you can write

For viscous sublayer $r_w = r + y \approx r\ (\therefore\ since\ the\ sublayer\ is\ very\ thin)$

So, in that case you can also you know that $\tau_w = -\mu \frac{du}{dr}$ and in that case you can convert that to d u d y and then you have here because you can see that r is in this direction y is in reverse direction. So, that is why you do not have the minus sign anymore and if you work further on this

$$\frac{du}{dy} = \frac{\tau_w}{\mu}, \Longrightarrow \frac{du^+}{dy^+} = 1$$

So, you can see here that we what we are doing here is we are plotting u plus in this in this curve we are plotting u plus with y plus and furthermore y plus is in log scale it is it is known as semi log I mean if you want to do it on a on a graph paper by pen and by pencil and eraser then you have to get what is called semi log graph paper. One axis is log scale the other axis is in linear scale or otherwise you can use any software they will you can change the option of that axis. So, you have option you can choose either linear or logarithmic or if you have some other options they are possible.



So, you have this x axis is here you are plotting y plus and the y axis you are plotting u plus. So, I am defining viscous layer transition layer and turbulent core right. So, u plus as a function of y plus here you can see in the viscous sub layer we just now established $\frac{du^+}{dy^+} = 1$

$\Longrightarrow u^+ = y^+$ Now, if you had it been a linear scale it would have been a 45 degree line, but since it is a semi log scale it is this curve this part this part of the curve that is appearing somewhat different and that is because of this y axis we have the y axis as linear and x axis in log scale.

$0 < y^+ < 5 \implies$ viscous sublayer

$5 < y^+ < 30 \implies$ buffer layer

$30 < y^+ < centre\ of\ pipe \implies$ turbulent core

Then we have the buffer layer and then we have the turbulent core, buffer layer we define for y plus between 5 and 30 and the from 30 to the center of the pipe we call that turbulent core. So, we are putting some value of now we here we are getting u plus as 5 ln y plus I mean this expression we are putting for buffer layer and this expression we are putting for turbulent core. Now, the point is who gave you this expression it is obtained from empirical information, but the fact is this is I am people have demonstrated that you can bank on these expressions. So, this is because and beauty of this is these are all in terms of dimensionless numbers you can see u plus is a dimensionless velocity y plus is the dimensionless distance. So, this for another system for any system you can one thing you can say for sure that velocity dimensionless velocity at a dimensionless distance in this turbulent flow will follow I mean when you are inside the core will follow this expression.

This is referred as universal velocity profile the origin where from you got it is the experiments researchers have performed and they found that the experimental data they are aligned to this expression. So, they said that this is universal and using this expression using these expressions rather for every other parts also, they can now come up with what would be the pressure drop, what would be the friction factor and how you can find out the velocity at a particular location average velocity by integrating this expression because you have to then integrate this. Remember what we did in case of finding out average velocity from a velocity profile. You recall that for when you have a circular cross section we had taken we said that this u we put it u as a function of r right u as a function of r. So, that is the velocity at a radial location and that is applicable for an annular region around that position r and that annular region the area would be 2 π r dr right.

If dr is the width around that dr is the width of that annular region at a position r. So, then it is π r plus dr whole square minus π r square and then you have the dr whole square term you have to you have to ignore you have you have already seen this right π r plus dr whole square minus π r square and then π r square and this π r square we can cancel out dr whole square you ignore assuming this to be a differential element and then you are left with 2 π r dr that is the area of the annular region. So, velocity in meter per second into meter square. So, this gives you meter cube. So, this gives you meter cube per second.

So, so this is the flow rate you have. So, this you integrate if you integrate this over the cross section over this 0 over the r changing from 0 to what you have r w right the inner wall. So, so this gives you the this gives you the total flow rate and when you divide it by pi r w square that gives you the average velocity. So, you can apply all these because your y plus is simply dimensionless distance and that is linked to that is that is linked to r right. So, you can you can you can play with this find out average velocity this is something which we are looking for only thing is how we got it that part is the that you have to rely on the wisdom and experience of previous researchers who have done enough study and they came up with this universal velocity profile.

So, now, with this if you ignore the contribution of viscous sub layer and buffer layer and you focus only on the on the only on the turbulent core and then this was the expression for the turbulent core. So, any intermediate location this is the expression at the center of the pipe this is the expression and then you can you can if you subtract this you get this expression.

Ignore the contribution of viscous sublayer and buffer layer.

At the center of the pipe $u_c^+ = 2.5\ln(y_c^+) + 5.5$

At intermediate location $u^+ = 2.5\ln(y^+) + 5.5$

Upon subtraction $u_c^+ - u^+ = 2.5\ln\left(\frac{y_c^+}{y^+}\right)$

Furthermore if you try to find out what is the average velocity

$$\bar{V} = \frac{1}{\pi r_w^2}\int_0^{r_w} u\left(2\pi r\, dr\right) = \frac{2}{r_w^2}\int_0^{r_w} u(r_w - y)\, dy$$

$$\Rightarrow \bar{V} = \frac{5\left(\frac{\mu}{\rho}\right)^2}{r_w^2\, u^*}\int_0^{y_c^+}\left(0.4u_c^+ + ln\left(\frac{y^+}{y_c^+}\right)\right)(y_c^+ - y^+)dy$$

$$\Rightarrow \frac{\overline{V}}{u^*} = u_c^+ - 3.75$$

$$\Rightarrow u_c^+ = \frac{1}{\sqrt{f/2}} + 3.75$$

Since $u_c^+ = 2.5\ln(y_c^+) + 5.5$

$$\frac{1}{\sqrt{f/2}} = 2.5 \ln\left(Re\sqrt{\frac{f}{8}}\right) + 1.75$$

$$r = r_w - y \implies dr = -dy$$

$$when\, r = 0 \implies y = r_w$$

$$when\, r = r_w \implies y = 0$$

Also,

$$y_c^+ = \frac{r_w \overline{V}}{\left(\frac{\mu}{\rho}\right)}\sqrt{\frac{f}{2}}$$

$$\implies y_c^+ = \frac{r_w \overline{V}}{\left(\frac{\mu}{\rho}\right)}\sqrt{\frac{f}{2}}$$

$$\implies y_c^+ = \frac{D_w \overline{V}}{2\left(\frac{\mu}{\rho}\right)}\sqrt{\frac{f}{2}}$$

$$\implies y_c^+ = \frac{Re}{2}\sqrt{\frac{f}{2}}$$

So, this is an expression now look at this expression what do we have here we have friction factor we have Reynolds number. So, this is a classical expression of friction factor versus Reynolds number. So, think of that chart I mean whether you plot your fanning friction factor or I mean the trend remains same right.

So, so you have f versus f versus R e you have in this case not 64 by R e maybe 16 by R e and then you have multiple lines right. So, you have depending on the value of f sub i d f sub i d the surface roughness. So, now, in this case you have now this part what expression will you use for friction I mean suppose somebody does not want to use this friction factor chart. So, this is the expression provided and this expression mind it this expression what is the genesis where from we got it. It is only assumption we made in this whole show is this is the only assumption we made that in the turbulent core this universal velocity profile is valid.

The fact is this expression when someone fitted for smooth pipes they are I mean the experiment this friction factor at least for the Reynolds a high Reynolds number for the turbulent region it is primarily experimental data. So, that experimental data is matched very well by this expression I mean certain level of confidence it they match very well. So, that shows the universal velocity profile has that shows a merit of universal velocity profile that based on that universal velocity profile you are coming up with a friction factor which you predict a friction factor which matches very well with the experimental values of friction factor versus Reynolds number chart in the turbulent region probably for smooth pipe assumption and for rough pipes you can make modifications to this. So, this is a major observation and you must appreciate the role of universal velocity profile in this. So, if it comes to finding average velocity if you when it comes to finding local velocity in case of turbulent flow you one may with certain level of confidence use universal velocity profile that is the take home message.

So, that is at least one way you can handle turbulence without resorting to other first principle analysis the theories which are not which does not come under the purview of this course. So, that is what is universal velocity profile. Apart from that I have I do not have any other major message for turbulence one thing is that typically the velocity profile if someone looks at the velocity profile tends to be more flat in case of turbulence I mean what is commonly understood is that if this is the velocity profile for a laminar flow in case of turbulent flow the velocity profile is much flatter. So, that is one thing and so, accordingly the value of you might recall kinetic energy correction factor we discussed.

So, those values will change from laminar to turbulent. So, turbulent flow will have more the value of α close to 1 because of this flatter velocity profile. So, this is another observation of turbulent flow since we have been going through this turbulence. So, you may like to make note. So, I believe this is all I have as far as the this topic turbulence is concerned. I want to stop here now and keep an open mind think about it that at least somehow we have come up with something to express something to analyze or at least something to theorize and quantify turbulence and this is what this is how it is done and it has it is consistent in terms of pressure drop information friction factor chart etcetera.

So, this is a consistent treatment. So, that is where the merit of universal velocity profile lies. That is all I have for this session. Thank you very much for your attention.